QCD correlation functions from the FRG

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based on

- AKC, Mitter, Pawlowski, Strodthoff, $N_f = 2$ Vacuum QCD, in preparation
- AKC, Mitter, Pawlowski, Strodthoff, $T > 0$ Yang-Mills, in preparation

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QCD phase diagram with functional methods

fQCD-collaboration:

This talk:
- Vacuum Yang-Mills
- YM at Finite Temperature
- Vacuum QCD

Aim:
- Qualitative understanding
- Quantitative precision

Motivation:
- No sign problem
- Understanding of confinement

QCD from the functional renormalization group

- Only perturbative QCD input
  - $\alpha_S(\mu = \mathcal{O}(10) \text{ GeV})$
  - $m_q(\mu = \mathcal{O}(10) \text{ GeV})$

- Wetterich equation with initial condition $S[\Phi] = \Gamma_\Lambda[\Phi]$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} - \text{[diagram]} - \text{[diagram]}$$

- Effective action $\Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi]$

- Exact equation

- $\partial_t$: integration of momentum shells controlled by regulator

- Full field-dependent equation with $(\Gamma^{(2)}[\Phi])^{-1}$ on rhs
**Vertex expansion**

- Approximation necessary – vertex expansion:

  \[
  \Gamma[\Phi] = \sum_n \int_{p_1, \ldots, p_{n-1}} \Gamma_n^{(n)}(p_1, \ldots, p_{n-1}) \Phi_1(p_1) \cdots \Phi_n(-p_1 - \cdots - p_{n-1})
  \]

- Wanted: “apparent convergence” of \( \Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi] \)

- Current state-of-the-start truncation:

  - Classical tensor

  - Functional derivatives of \( \Gamma_k[\Phi] \) with respect to fields yield equations
Truncation – closed set of equations

\[ \partial_t \rightarrow^{-1} = \text{diagram 1} \]

\[ \partial_t \rightarrow^{-1} = \text{diagram 2} \]

\[ \partial_t = \text{diagram 3} + \text{perm.} \]

\[ \partial_t = \text{diagram 4} + \text{perm.} \]

\[ \partial_t = \text{diagram 5} + \text{perm.} \]

momentum dependent coupled tensor equations

tracing necessary
FormTracer – Mathematica tracing package using FORM

- Mathematica: very powerful, flexible and convenient
- FORM: very fast and efficient

**FormTracer** uses FORM while it keeps the usability of Mathematica:
- Lorentz/Dirac traces in arbitrary dimensions
- Arbitrary number of group product spaces
- Intuitive, easy-to-use and highly customizable Mathematica frontend
- Support for finite temperature/density applications
- Support for FORM’s optimization algorithm
- Convenient installation and update procedure within Mathematica:

**Preprint:** AKC, Mitter, Strodthoff; arXiv:1610.09331 [hep-ph]

**Open source:** https://github.com/FormTracer/FormTracer
FormTracer – installation and usage

Installing

```mathematica
Import["https://raw.githubusercontent.com/FormTracer/FormTracer/master/src/FormTracerInstaller.m"]
```

Tracing

Space-Time

Define syntax for space-time

```mathematica
DefineLorentzTensors[\delta[\mu, \nu](*Kronecker delta*), \text{vec}[p, \mu](*vector*), p.q(*inner product*)];
```

Take traces:

```mathematica
FormTrace[\text{vec}[p + 2 r, \mu] \delta[\mu, \nu] \text{vec}[s, \nu]]
FormTrace[\delta[\alpha, \nu] (\delta[\nu, \rho] + \delta[\nu, \rho] \delta[\sigma, \sigma]) \delta[\rho, \alpha]]
FormTrace[\delta[1, \nu] \text{vec}[s, \nu]]
```

s. (p + 2 r)

20

vec[s, 1]
Truncation – closed set of equations

\[ \partial_t \stackrel{-1}{= \ldots} = \ldots + \ldots \]

\[ \partial_t \stackrel{-1}{= \ldots} = \ldots - 2 \ldots - \frac{1}{2} \ldots \]

\[ \partial_t = - \ldots - \ldots + \text{perm.} \]

\[ \partial_t = - \ldots + 2 \ldots + \text{perm.} \]

\[ \partial_t = + \ldots + \ldots - 2 \ldots - \ldots + \text{perm.} \]

momentum dependent
coupled tensor equations

tracing necessary
Consequences of the Regulator

- Regulator breaks BRST symmetry
- \( \implies \text{modified STIs} \)
- mSTIs reduce to STIs at \( k = 0 \)
- mSTIs determine initial action at \( k = \Lambda \)
- More practical solution: choose \( \Gamma_{\Lambda} \approx S \) such that STIs are fulfilled \( k = 0 \)

We mainly show scaling results, for which the gluon mass parameter is uniquely fixed.

Details:
AKC, Fister, Mitter, Pawlowski, Strodthoff
Truncation dependence of the gluon propagator

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016; Lattice: Sternbeck et al. 2006
Gluon propagator dressing

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016; Lattice: Sternbeck et al. 2006
Gluon propagator

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016; **Lattice**: Sternbeck et al. 2006
Ghost propagator dressing

![Graph showing ghost propagator dressing]

- Red solid line: FRG, scaling
- Red dashed line: FRG, decoupling
- Black dots: Sternbeck et al.

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Running couplings (scaling solution)

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
\[ \beta = \frac{\mu^2}{4\pi} \frac{d\alpha}{d\mu^2} \] Functions – Scaling
\[ \beta = \frac{\mu^2}{4\pi} \frac{d\alpha}{d\mu^2} \]

Functions – Decoupling

\[ \begin{align*}
\beta^{\text{Acc}} & \quad \beta^{A^3} \\
\beta^{A^4} & \quad \beta^{1\text{-loop}} \\
\beta^{2\text{-loop}} &
\end{align*} \]
Three-gluon vertex dressing (symmetric point)

Zero crossing between 0.1 GeV to 0.33 GeV

DSE: Blum, Huber, Mitter, Smekal, 2014; Lattice: Cucchieri, Maas, Mendes, 2008
Introducing finite temperature:
\[
\int \frac{d^4p}{(2\pi)^4} \rightarrow T \sum \omega_n \int \frac{d^3p}{(2\pi)^3}
\]

Splitting of magnetic and electric components necessary!

\[
P^M_{\mu\nu}(p) = (1 - \delta_{0\mu})(1 - \delta_{0\nu}) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)
\]

\[
P^E_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} - P^M_{\mu\nu}(p).
\]

Other zeroth mode classical tensor structures are degenerate.
Gluon propagator at finite temperature

Solid Lines:
Magnetic propagator

Dashed Lines:
Electric propagator
Magnetic gluon propagator dressing

$Lattice: Maas, Pawlowski, Smekal, Spielmann, 2011$
Finite Temperature Results

Electric gluon propagator dressing

Lattice: Maas, Pawlowski, Smekal, Spielmann, 2011
Debye mass compared to perturbation theory

\[ m_D^0 = \sqrt{\frac{N}{3}} g T; \quad m_D = m_D^0 + \left( c_D + \frac{N}{4\pi} \ln \left( \frac{m_D^0}{g^2 T} \right) \right) g^2 T + O(g^3 T) \]
Finite Temperature Results

Propagators

Ghost propagator dressing

\[ T = 0 \]
\[ T = 0.075 \text{ GeV} \]
\[ T = 0.125 \text{ GeV} \]
\[ T = 0.200 \text{ GeV} \]
\[ T = 0.300 \text{ GeV} \]
\[ T = 0.400 \text{ GeV} \]
\[ T = 0.800 \text{ GeV} \]
Ghost-gluon vertex dressing

Magnetic Vertex

Zeroth mode electric vertex is degenerate.
Three-gluon vertex dressing

Solid Lines: Magnetic vertex
Dashed Lines: Electric vertex
Four-gluon vertex dressing

Solid Lines:
Magnetic vertex

Dashed Lines:
Electric vertex

Finite Temperature Results

Vertices

$T = 0$
$T = 0.075 \text{ GeV}$
$T = 0.125 \text{ GeV}$
$T = 0.200 \text{ GeV}$
$T = 0.300 \text{ GeV}$
$T = 0.400 \text{ GeV}$
$T = 0.800 \text{ GeV}$

$p_\parallel$ [GeV]

four-gluon vertex dressing

$T = 0$
$T = 0.075$ GeV
$T = 0.125$ GeV
$T = 0.200$ GeV
$T = 0.300$ GeV
$T = 0.400$ GeV
$T = 0.800$ GeV

$\rho$ [GeV]
Unquenched $N_f = 2$ QCD

$\Gamma^{(2)}_{AA}(p)$

$\Gamma^{(2)}_{cc}(p)$

$\Gamma^{(3)}_{A\bar{c}c}(\bar{p})$

$\Gamma^{(4)}_{A\bar{c}c}(\bar{p})$

classical tensor

$\Gamma^{(4)}_{\bar{q}\bar{q}}(p)$

$\Gamma^{(3)}_{A\bar{q}q}(\bar{p})$

$\Gamma^{(5)}_{A\bar{q}q}(\bar{p})$

$q\bar{q}D^n q$ complete, $n \leq 3$

mom.–ind. tensors

$\Gamma^{(4)}_{\bar{q}\bar{q}\bar{q}}(p, p, -p)$

$q\bar{q}\bar{q}\bar{q}$ complete, $n \leq 3$

mom.–ind. tensors

$\Gamma^{(2)}_{\phi\phi}(p)$

$\Gamma^{(3)}_{\bar{q}\bar{q}\phi}(p, -p)$

$\Gamma^{(4)}_{\bar{q}\phi\phi}(\bar{p})$

$\Gamma^{(5)}_{\bar{q}\phi\phi}(\bar{p})$

$\phi \in \{\sigma, \bar{\pi}\}$

“classical” tensor

$n \in \{3, \ldots, 12\}$

$\Gamma^{(n)}_{\phi^n}(0)$

“classical” tensor

AKC, Mitter, Pawlowski, Strodthoff, in preparation
Unquenched gluon propagator

\[ \text{Unquenched } N_f = 2 \text{ QCD} \]

\[ \text{Unquenched gluon propagator} \]

[AKC, Mitter, Pawlowski, Strodthoff, in preparation]

Unquenched quark propagator

- Unquenched $N_f = 2$ QCD

- Error estimate
  - $m_\pi = 140$ MeV
  - $m_\pi = 60$ MeV
  - $m_\pi = 285$ MeV

- Lattice data: Orlando Oliveira, Kızılersu, Silva, Skullerud, Sternbeck, Williams, arXiv:1605.09632 [hep-lat]
Unquenched quark-gluon vertex

[AKC, Mitter, Pawlowski, Strodthoff, in preparation]

Angular dependence

\[ \lambda_{\text{qqA}}^{(1)} \]

\[ \lambda_{\text{qqA}}^{(7)} \]

\[ \lambda_{\text{qqA}}^{(4)} \]

\[ m_\pi = 140 \text{ MeV} \]

\[ \bar{p} \text{ [GeV]} \]

\[ p_\perp \text{ [GeV]} \]

\[ \text{quark-gluon vertex dressings} \]

\[ \text{Angular dependence} \]

AKC (U Heidelberg)

QCD from the FRG

May 24, 2017
Conclusion

- FRG first principal approach to QCD
- Big numerical effort $\rightarrow$ tools like FormTracer necessary
- Much progress, but still not enough!
- Promising results for $T > 0$ and the unquenched system.

Outlook

- Towards $\mu > 0$
- Bound states (Bethe-Salpeter eq.), decay widths, . . .
- Nonzero Matsubara modes, gluon spectral function, . . .

Thank you for your attention!
\[ \beta = \frac{\mu^2}{4\pi} \frac{d\alpha}{d\mu^2} \]

Functions at Small Couplings
Regulator breaks BRST symmetry

- Breaking BRST symmetry $\rightarrow$ modified STIs
- mSTIs reduce to STIs at $k = 0$
- $\Rightarrow$ solve mSTIs to get initial action at $k = \Lambda$
- More practical solution: choose $\Gamma_\Lambda \approx S$ such that STIs are fulfilled $k = 0$

\[
\alpha_{A\bar{c}c}(p) = \frac{\alpha(\mu)}{4\pi} \frac{Z_{A\bar{c}c}^2(p)}{Z_A(p) Z_C^2(p)}
\]
\[
\alpha_{A^3}(p) = \frac{\alpha(\mu)}{4\pi} \frac{Z_{A^3}^2(p)}{Z_A^3(p)}
\]
\[
\alpha_{A^4}(p) = \frac{\alpha(\mu)}{4\pi} \frac{Z_{A^4}(p)}{Z_A^2(p)}
\]

Select
\[
Z_{A\bar{c}c}^k=\Lambda(p) = \text{const.}
\]
\[
Z_{A^3}^k=\Lambda(p) = \text{const.}
\]
\[
Z_{A^4}^k=\Lambda(p) = \text{const.}
\]
such that
\[
\alpha_{A\bar{c}c}(\mu) = \alpha_{A^3}(\mu) = \alpha_{A^4}(\mu)
\]
Gluon mass gap

Scaling solution

$$\lim_{p \to 0} Z_c(p^2) \propto (p^2)^\kappa$$

$$\lim_{p \to 0} Z_A(p^2) \propto (p^2)^{-2\kappa}$$

Decoupling solution

$$\lim_{p \to 0} Z_c(p^2) \propto 1$$

$$\lim_{p \to 0} Z_A(p^2) \propto (p^2)^{-1}$$

- **Landau Gauge** gluon STI requires longitudinally mass term to vanish:

  $$p_\mu \left( [\Gamma_{AA,L}^{(2)}]^{ab}_{\mu\nu}(p) - [S_{AA,L}^{(2)}]^{ab}_{\mu\nu}(p) \right) = 0$$

- Splitting between longitudinal and transverse mass term necessary
- Splitting occurs “naturally” for scaling solution
- Decoupling solution requires irregular vertices, e.g. a pole in the longitudinal sector
- Unphysical gluon mass parameter present at $k = \Lambda$, $\Lambda$ can be uniquely determined
Dynamical mass generation

\[
m^2 - m^2_{\Lambda, scaling} \quad [\text{GeV}^2]
\]

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Gluon propagator maximum over UV mass parameter

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Running of the gluon mass parameter
Gluon propagator

![Graph showing gluon propagator](image)

- **Red line**: confined branch
- **Blue line**: Higgs branch

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Ghost propagator dressing

![Graph showing ghost propagator dressing as a function of p [GeV].]

- Red dashed line: confined branch
- Blue solid line: Higgs branch

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Running couplings in comparison with DSE results

\[\alpha_{\text{Acc}} \quad [47, 92]\]
\[\alpha_{\text{Acc}} \quad [91, 93]\]
\[\alpha_A^3 \quad [91, 93]\]
\[\alpha_A^3 \quad [50]\]
\[\alpha_A^4 \quad [42]\]
Momentum dependence of the ghost-gluon vertex

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Ghost-gluon vertex at the symmetric point

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Ghost-gluon vertex with vanishing gluon momentum

![Graph showing ghost-gluon vertex dressing with different methods and their comparison with experimental data. The graph includes lines for FRG scaling, FRG decoupling, DSE scaling, and DSE decoupling, each with corresponding data points and error bars.]
Ghost-gluon vertex with orthogonal momenta

**Graph Description:**
- **Axes:**
  - Y-axis: ghost-gluon vertex dressing
  - X-axis: p [GeV]

**Curves:**
- Solid red line: FRG, scaling
- Dashed red line: FRG, decoupling
- Solid black line: DSE, scaling
- Dashed black line: DSE, decoupling

- **Data Points:**
  - Error bars representing experimental data

- **Legend:**
  - AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016

- **Note:**
  - The graph shows the momentum dependence of the ghost-gluon vertex with orthogonal momenta, comparing FRG and DSE approaches under scaling and decoupling limits.
Momentum dependence of the three-gluon vertex
Three-gluon vertex at the symmetric point

\[ p \text{ [GeV]} \]

FRG, scaling
FRG, decoupling
DSE, scaling
DSE, decoupling

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Three-gluon vertex with vanishing gluon momentum

- FRG, scaling
- FRG, decoupling
- DSE, scaling
- DSE, decoupling

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Three-gluon vertex with orthogonal momenta

FRG, scaling
FRG, decoupling
DSE, scaling
DSE, decoupling

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Momentum dependence of the four-gluon vertex
Four-gluon vertex at the symmetric point

\[ p \text{ [GeV]} \]

- FRG, scaling
- FRG, decoupling
- DSE, scaling
- DSE, decoupling

AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016
Regulator dressing

![Graph showing different gluon dressings versus k [GeV]. The graph has a y-axis labeled "different gluon dressings" and an x-axis labeled "k [GeV]". The graph includes multiple curves representing different gluon dressings. AKC, Fister, Mitter, Pawlowski, Strodthoff, 2016.]