note: unpublished figures had to be removed from the slides

Polarons in ultracold atomic gases

- flowing spectral functions -

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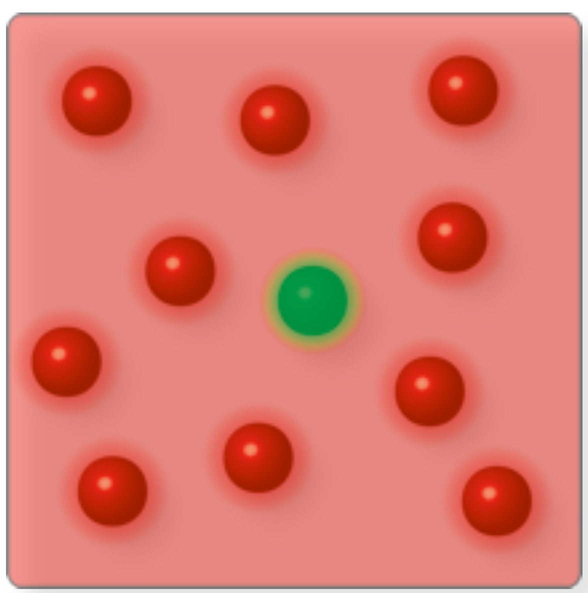
ERG 2014, Lefkada, 26 September 2014

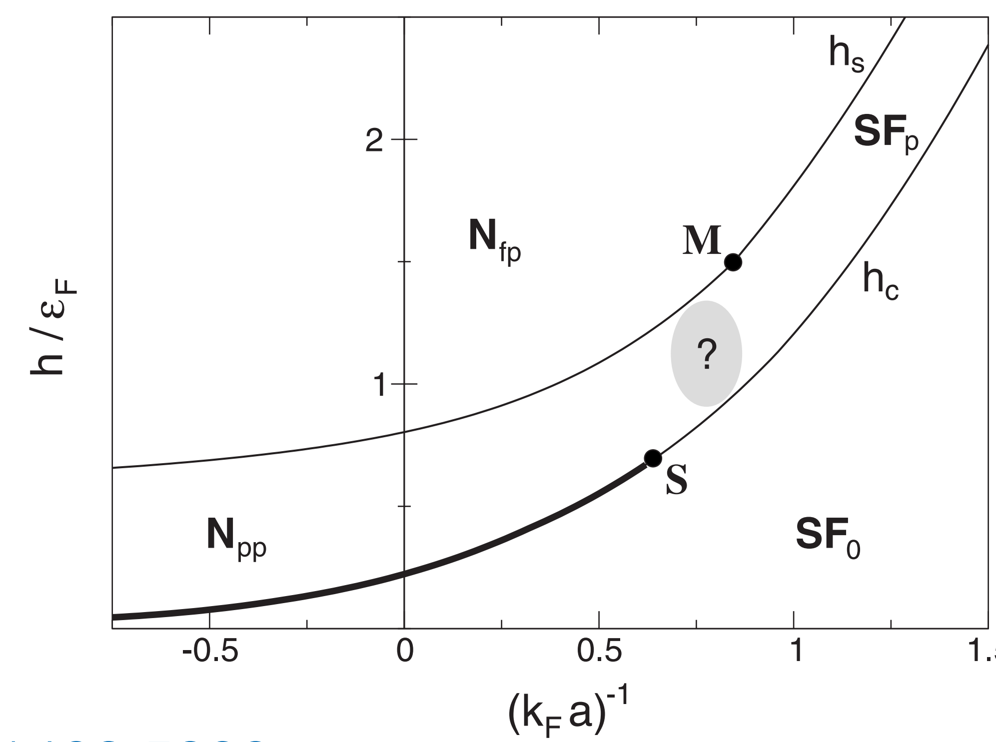
The Fermi polaron

- impurity coupled to environment, fundamental condensed matter problem
  Feynman 1955; Anderson 1967
- here: single mobile ↓ fermion in ↑ Fermi sea:



ferromagnet: Nagaoka 1966

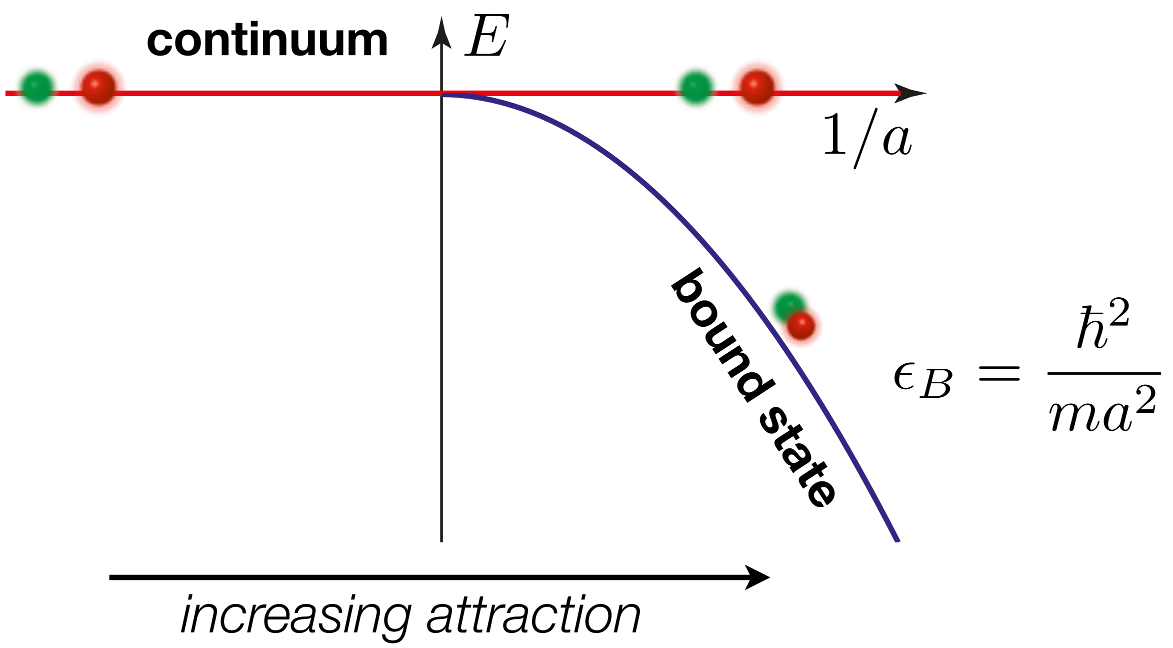
ultracold atoms:
Chevy 2006 (variational);
Combescot et al. 2007 (T-matrix);
Prokof'ev & Svistunov 2008 (diagMC);
Punk, Dumitrescu & Zwerger 2009 (var)



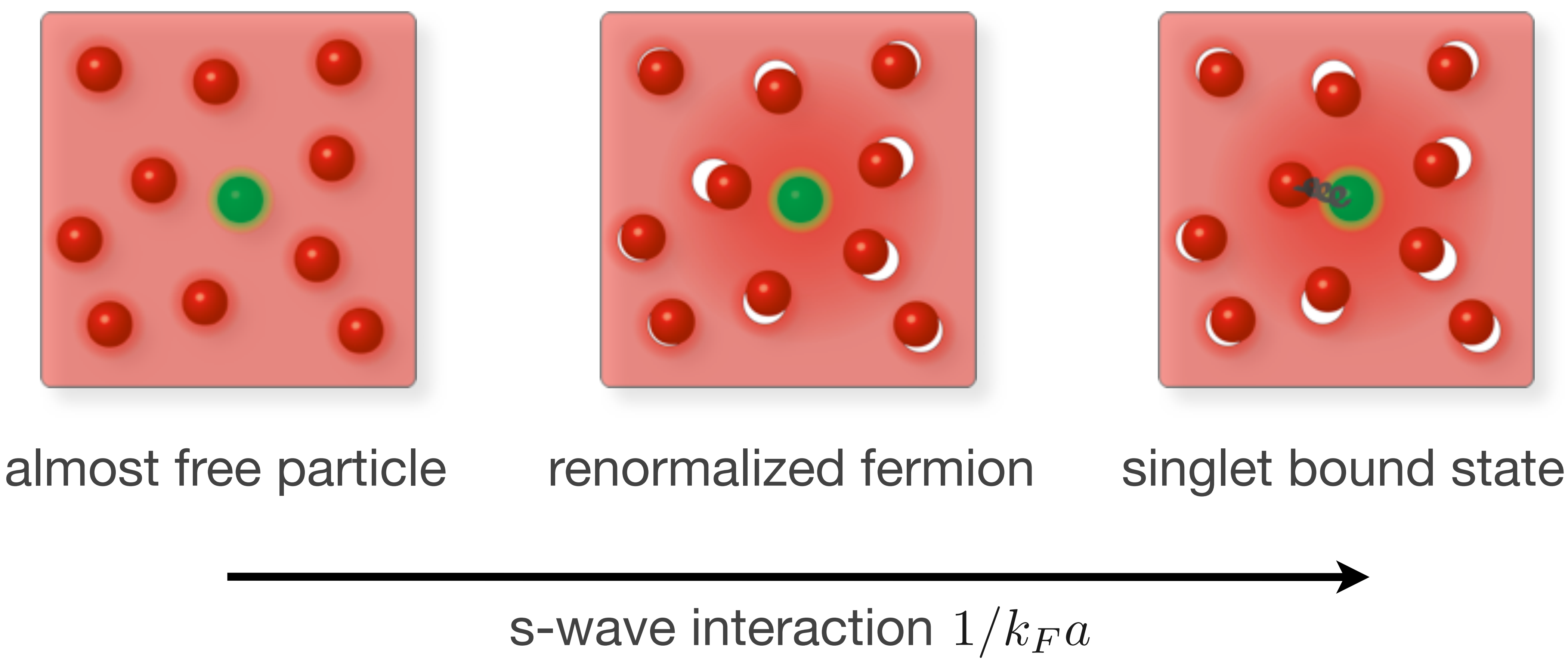
- strong polarization limit of:
  BEC-BCS crossover (attractive)
  Stoner ferromagnetism (repulsive)
- cf. Sarma phase Strack+ 1311.4885, Boettcher+ 1409.5232

Ultracold atoms

- contact interactions, s-wave scattering length a
- two-body problem:



Polaron to molecule transition

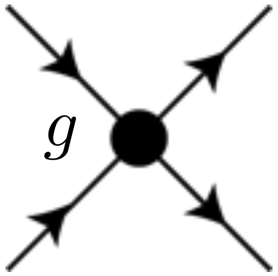


- ground state properties well understood (variational, Monte Carlo, experiment)
  Chevy 2006; Prokof'ev & Svistunov 2008; Schirotzek et al. 2009
- here: dynamical properties, decay rates, linear response (more involved)

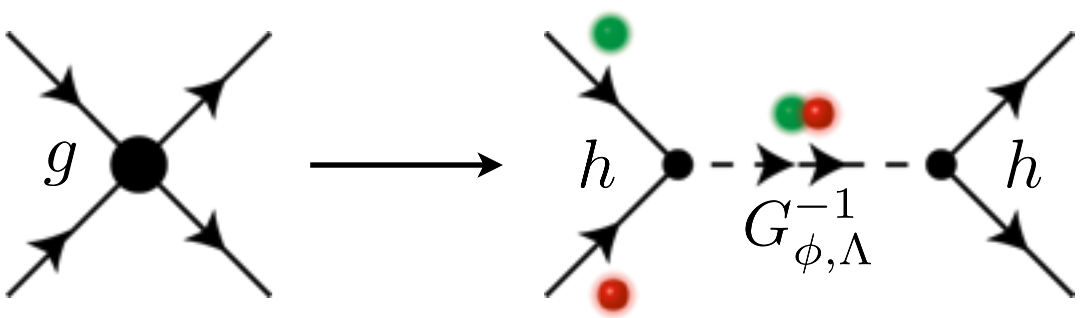
The model

- two-component Fermi gas with contact interaction, **microscopic action**

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^*[-i\omega + p^2 - \mu_\sigma]\psi_\sigma + g\int_X \psi_\uparrow^*\psi_\downarrow^*\psi_\downarrow\psi_\uparrow$$



- Hubbard-Stratonovich transformation in **Cooper channel:** exchange of virtual molecules (T-matrix)



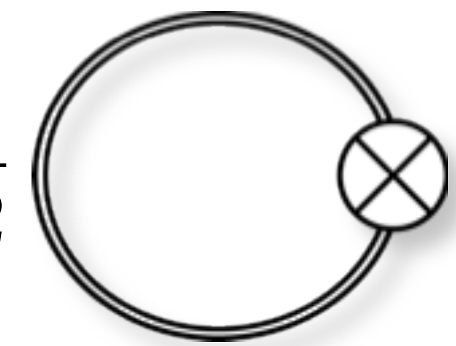
$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^*[-i\omega + p^2 - \mu_\sigma]\psi_\sigma + \phi^* G_{\phi,\Lambda}^{-1}\phi + h\int_X (\psi_\uparrow^*\psi_\downarrow^*\phi + h.c.)$$

Functional Renormalization Group

- include quantum and thermal fluctuations successively:

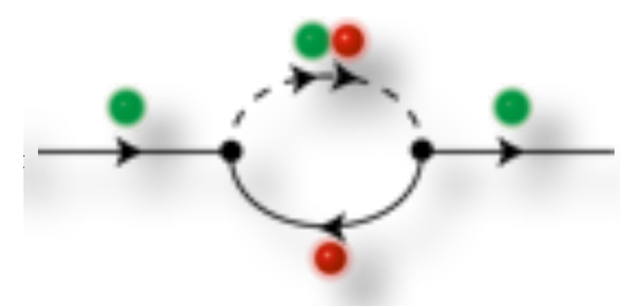
functional renormalization group equation Wetterich 1993

$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_k R_k = \frac{1}{2}$$

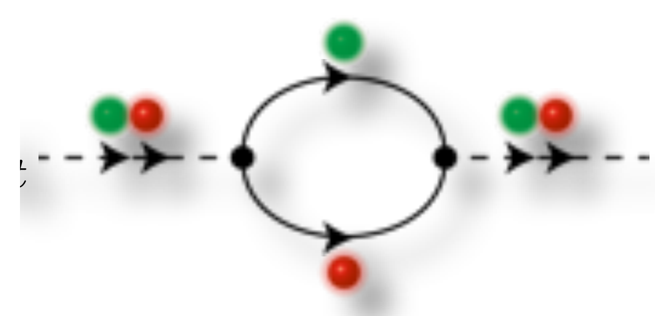


- compute two processes:

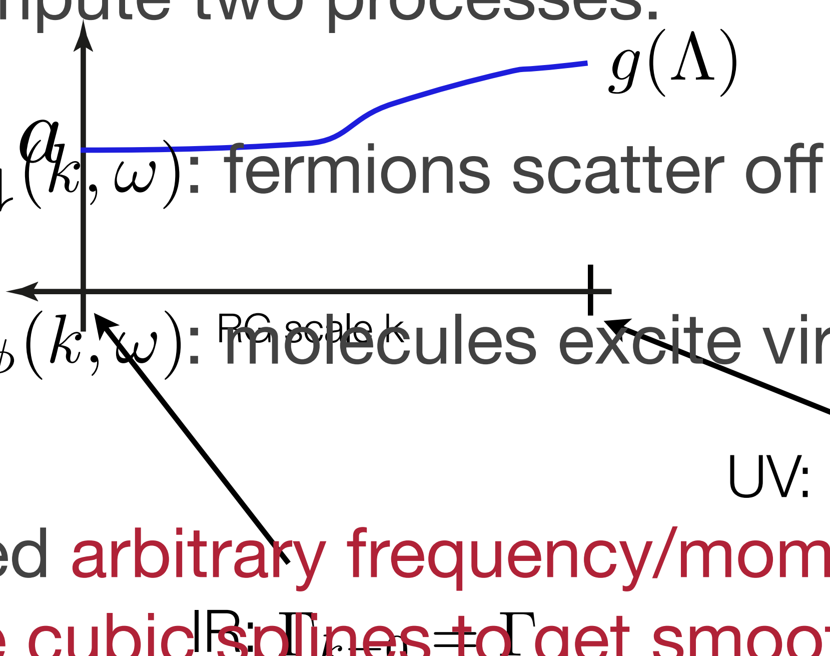
(a) $\Sigma_\downarrow(k,\omega)$: fermions scatter off virtual molecules



(b) $\Sigma_\phi(k,\omega)$: molecules excite virtual fermion pairs



- need arbitrary frequency/momentum dependence; use cubic splines to get smooth right-hand side



Need for full frequency dependence

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* [-i\omega + p^2 - \mu_\sigma]\psi_\sigma + \phi^* G_{\phi,\Lambda}^{-1}\phi + h\int_X (\psi_\uparrow^*\psi_\downarrow^*\phi + h.c.)$$

derivative expansion: $\Gamma_{k,\phi,\mathrm{kin}} = \int_{\mathbf{p},\omega} \phi^*[-iA_k\omega + B_k\mathbf{p}^2 + C_k]\phi$

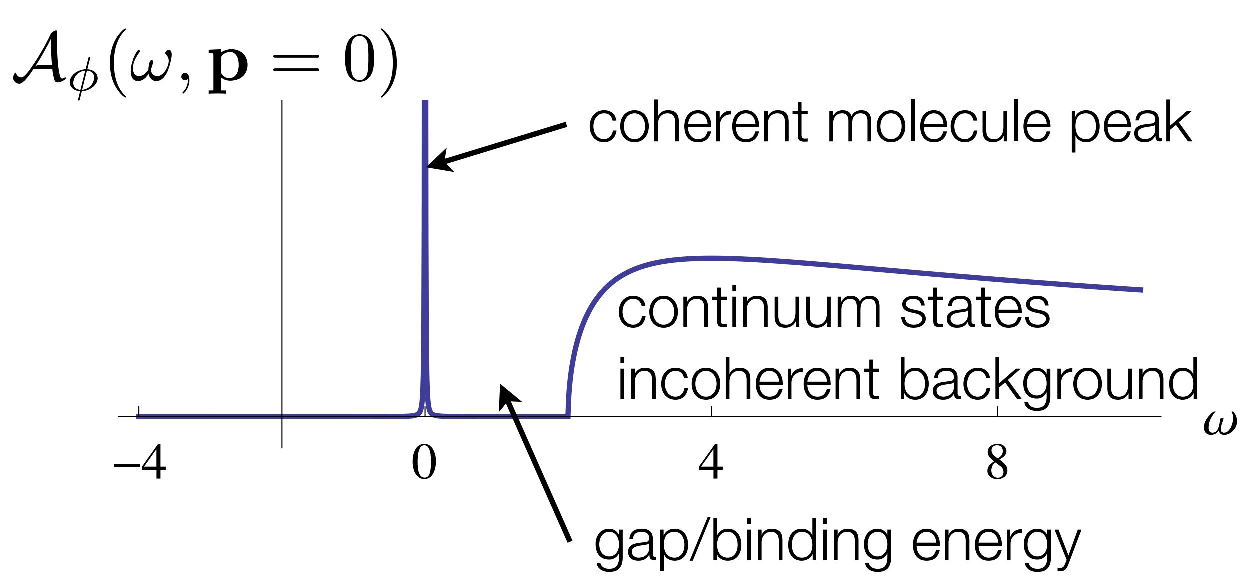
- single coherent quasi-particle excitation
- no anomalous dimension

Ellwanger+ 1994/98, Pawlowski+ 2002, Kato 2004, Fischer+ 2004, Blaizot+ 2006, Diehl+ 2008, Bartosch+ 2009, Benitez, Blaizot+ 2009/10

analytical solution for zero density:

$$G_\phi(\omega,\mathbf{p}) \sim \frac{1}{-a^{-1} + \sqrt{-\omega/2 + \mathbf{p}^2/4 - \mu - i0^+}}$$

Diehl, Krahl, Scherer 2008, Moroz, Flörchinger, Schmidt, Wetterich 2009, Schmidt, Moroz 2010



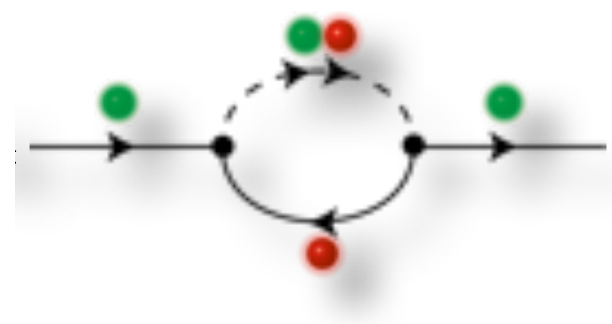
not captured in expansion:

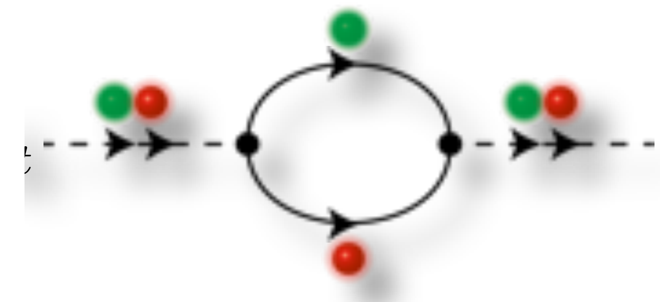
- most weight in continuum
- anomalous dimension $\eta = 1$

Flowing spectral functions

Schmidt & Enss 2011

$$\Gamma_k = \int_{\mathbf{p},\omega} \left\{ \psi_\uparrow^* [-i\omega + \mathbf{p}^2 - \mu_\uparrow]\psi_\uparrow + \psi_\downarrow^* G_{\downarrow,k}^{-1}(\omega,\mathbf{p})\psi_\downarrow + \phi^* G_{\phi,k}^{-1}(\omega,\mathbf{p})\phi \right\}$$
$$+ \int_{\vec{x},\tau} h(\psi_\uparrow^*\psi_\downarrow^*\phi + h.c.)$$





(1) sharp momentum cutoff:

$$G_{\downarrow,k}^c(\omega,\mathbf{p}) = \frac{\theta(|\mathbf{p}|-k)}{P_{\downarrow,k}(\omega,\mathbf{p})}, \quad G_{\phi,k}^c(\omega,\mathbf{p}) = \frac{\theta(|\mathbf{p}|-k)}{P_{\phi,k}(\omega,\mathbf{p})}, \quad G_{\uparrow,k}^c(\omega,\mathbf{p}) = \frac{\theta(|\mathbf{p}^2-\mu_\uparrow|-k^2)}{P_{\uparrow,k}(\omega,\mathbf{p})}.$$

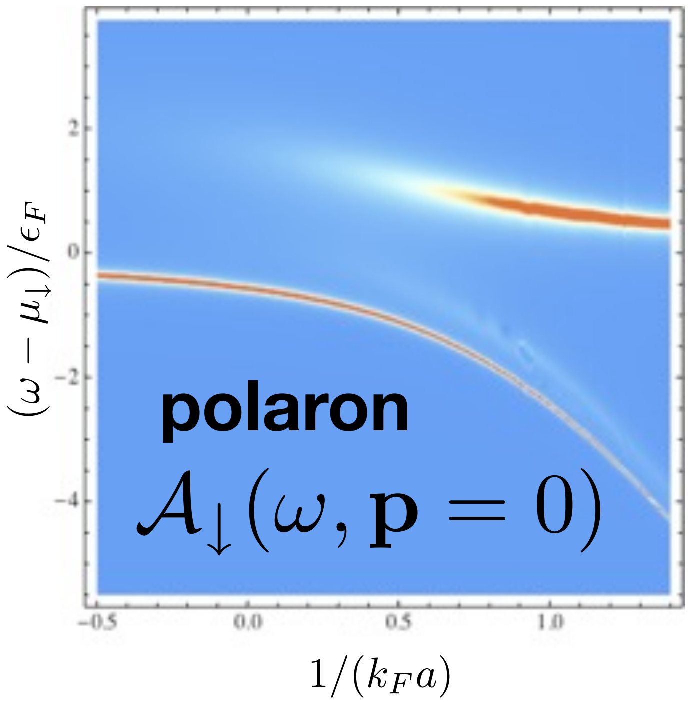
(2) reconstruct $P_k(i\omega,\mathbf{p})$ from bicubic spline interpolation of $P_{kij} = P_k(i\omega_i,\mathbf{p}_j)$

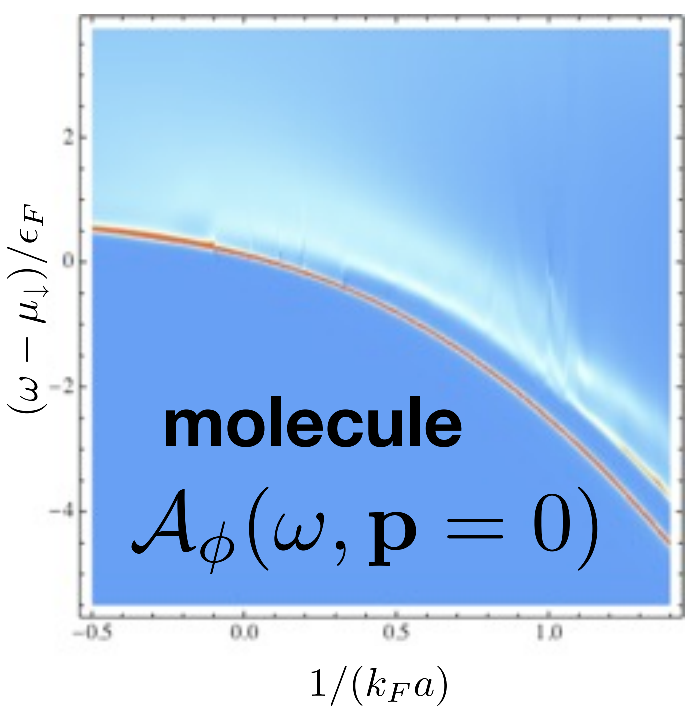
(3) analytical continuation $P_k(i\omega,\mathbf{p}) \to P_k(\omega+i0,\mathbf{p})$ for spectral function at scale k

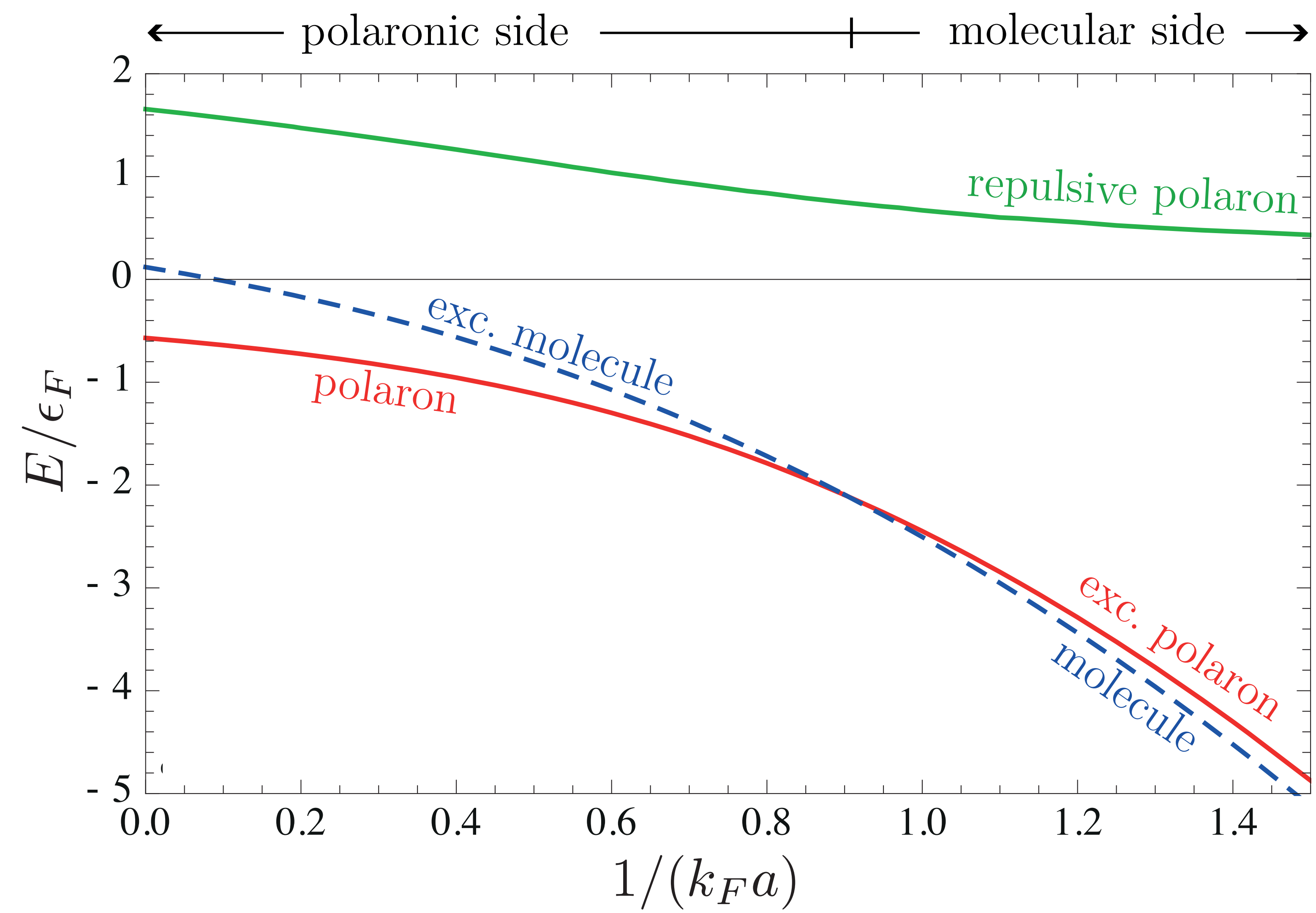
(4) compute smooth RHS of flow equation, $\tilde{\partial}_k P_{kij}$ and integrate flow down to IR

Excitation spectrum

Schmidt & Enss 2011

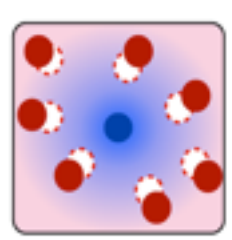




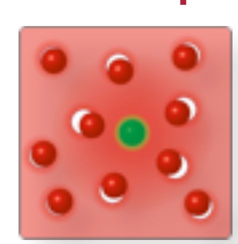


polaron has **three characters:**

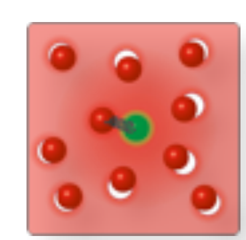
repulsive polaron



attractive polaron

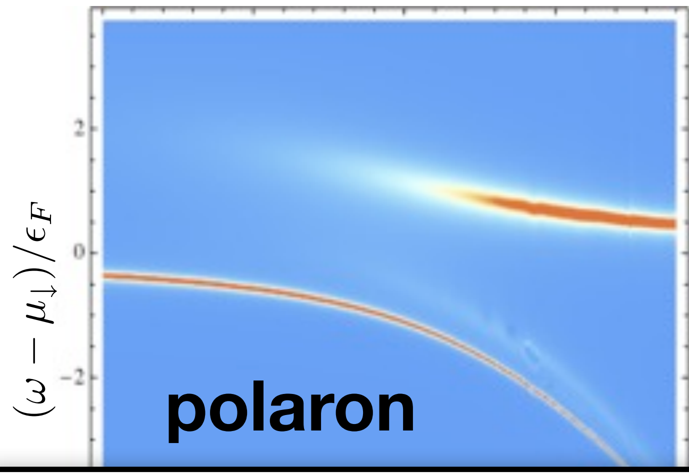


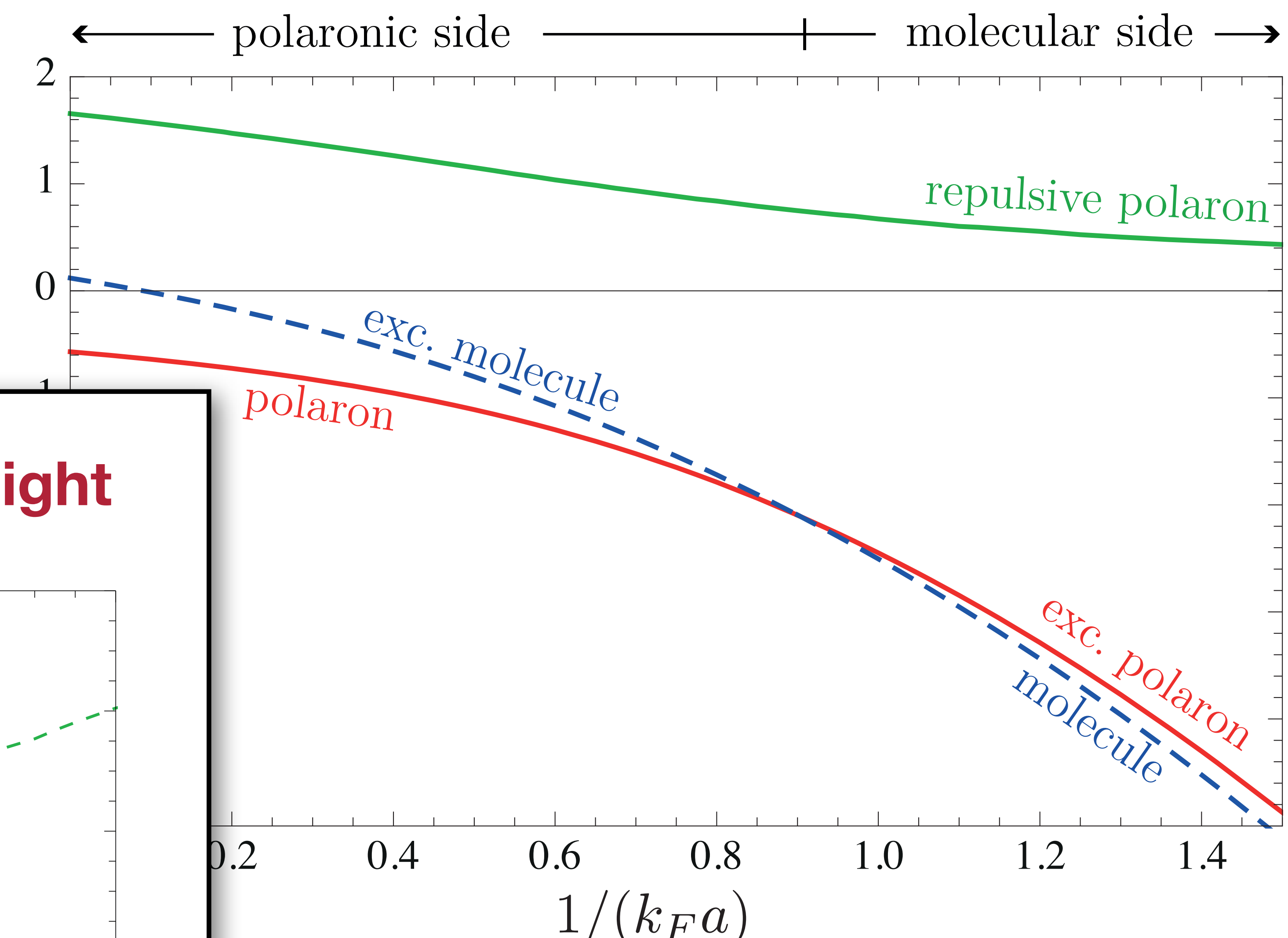
bound molecule

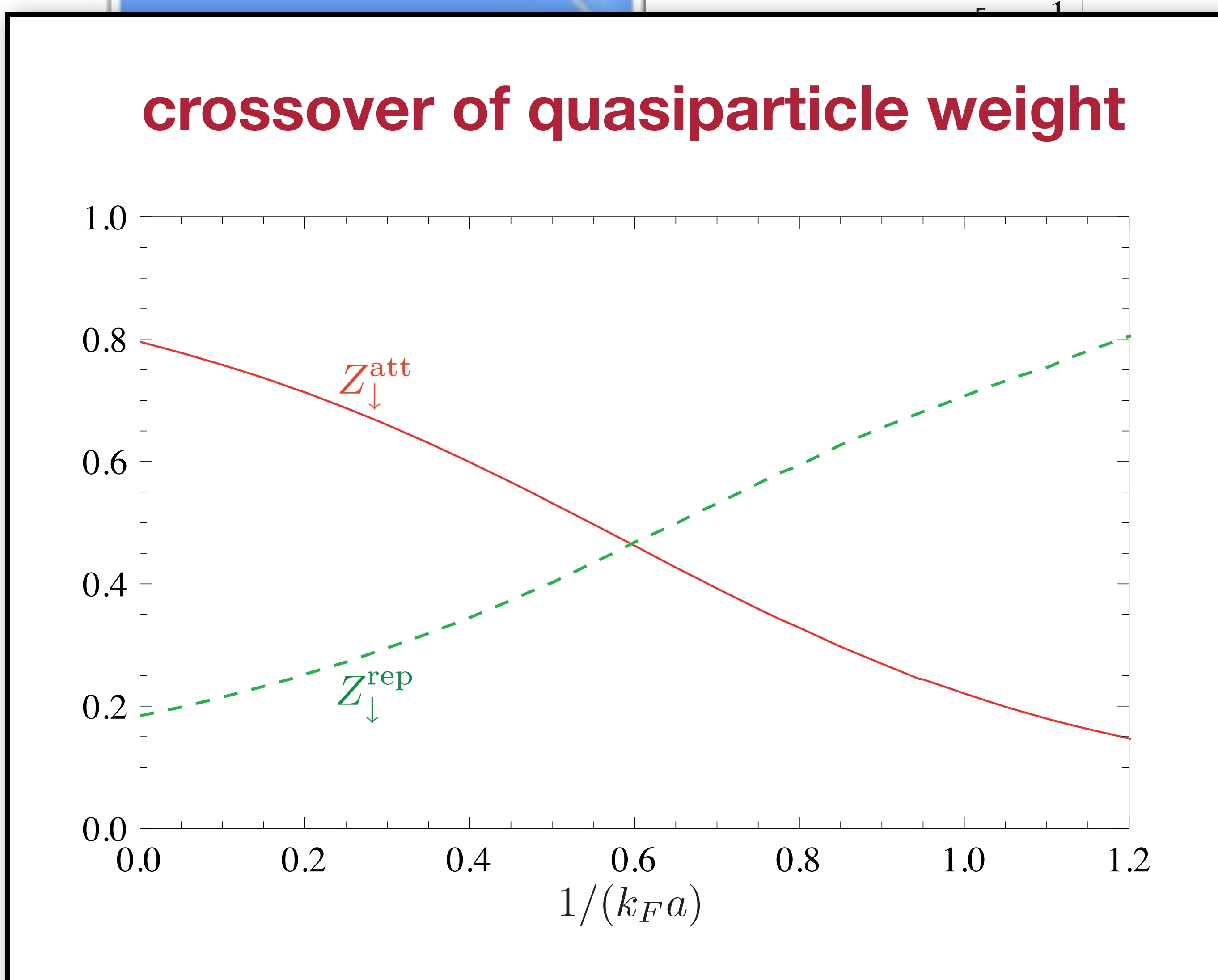


Excitation spectrum

Schmidt & Enss 2011

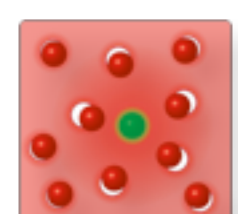






characters:

attractive polaron



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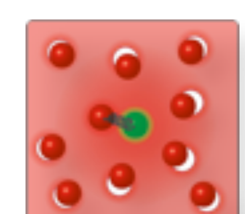
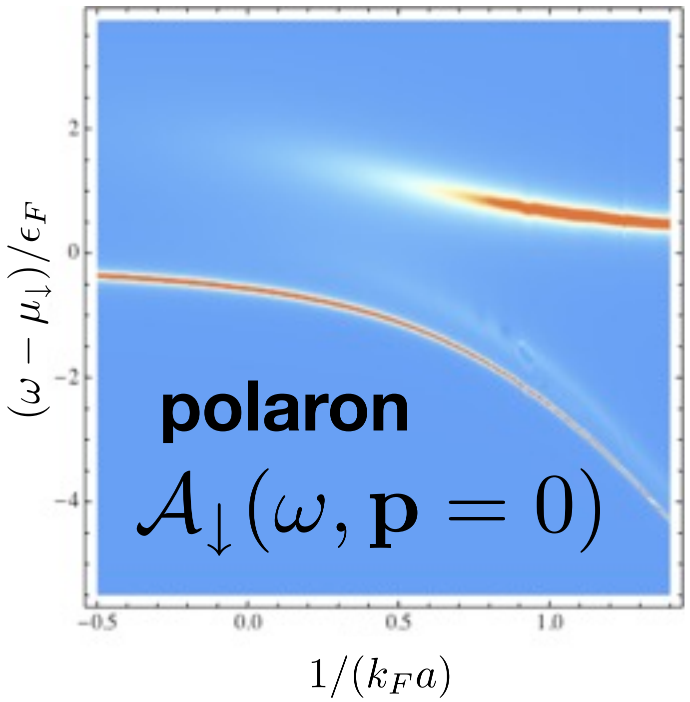
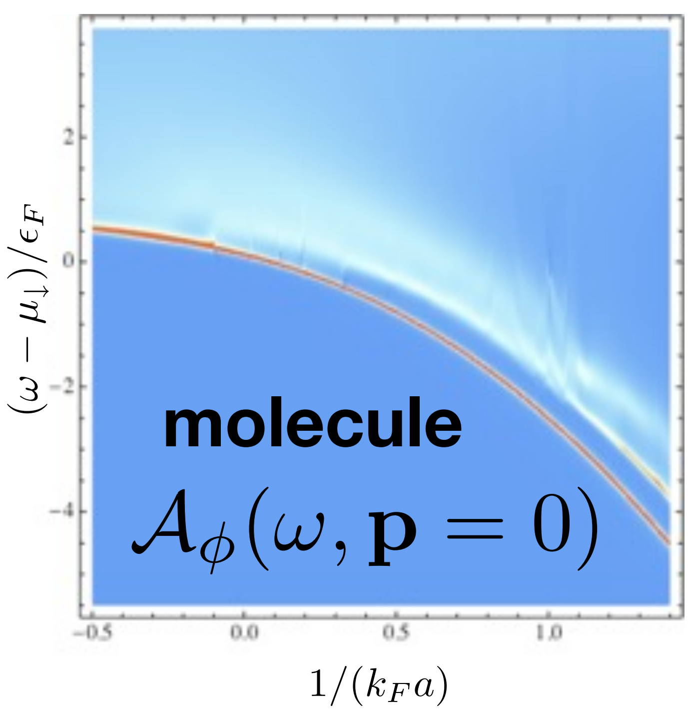
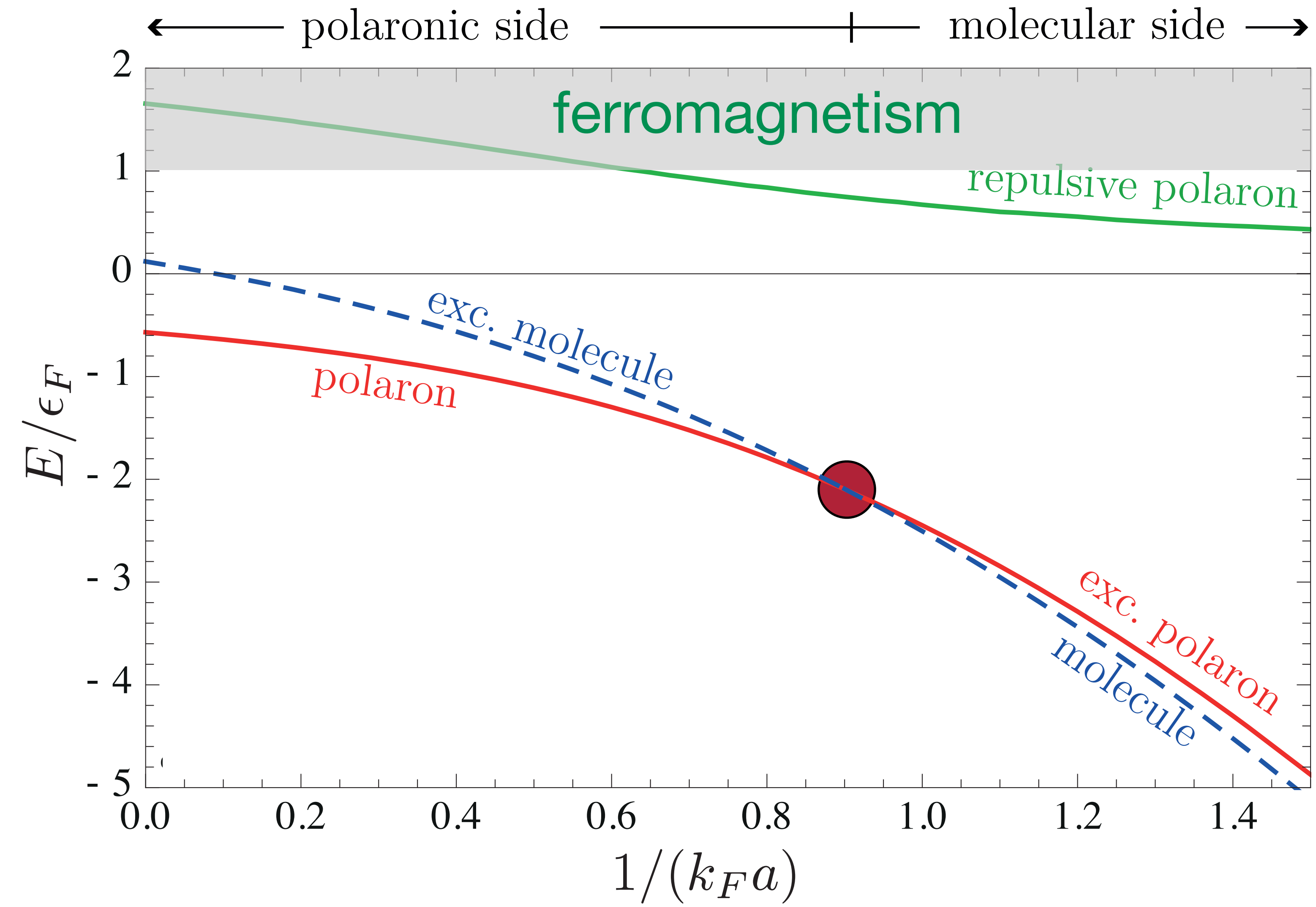


Excitation spectrum

Schmidt & Enss 2011





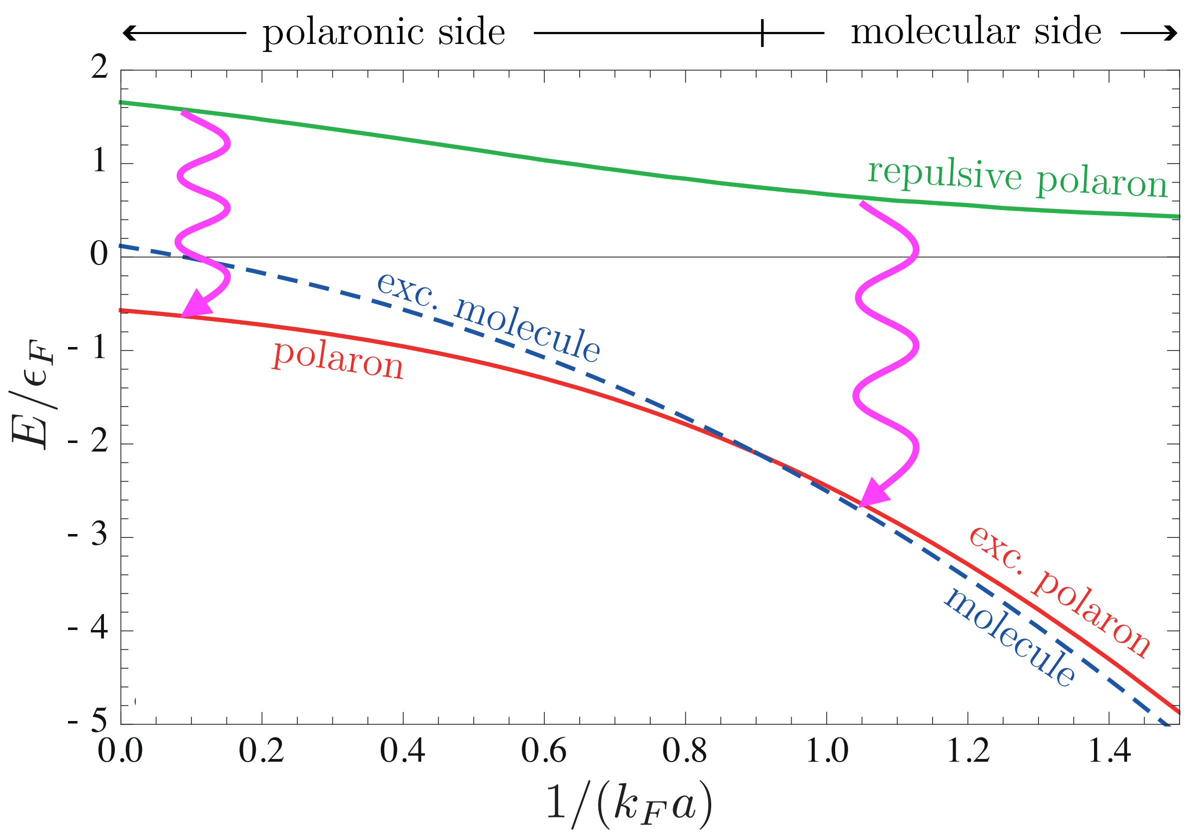


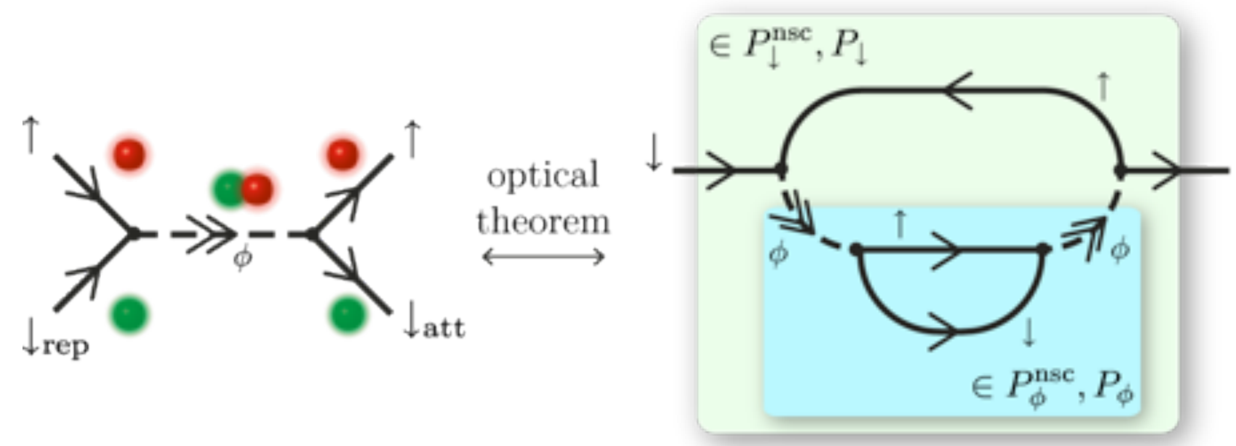
- polaron to molecule transition at $(k_F a_c)^{-1} = 0.904(5)$
  cf. bold diagMC $(k_F a_c)^{-1} = 0.90(2)$

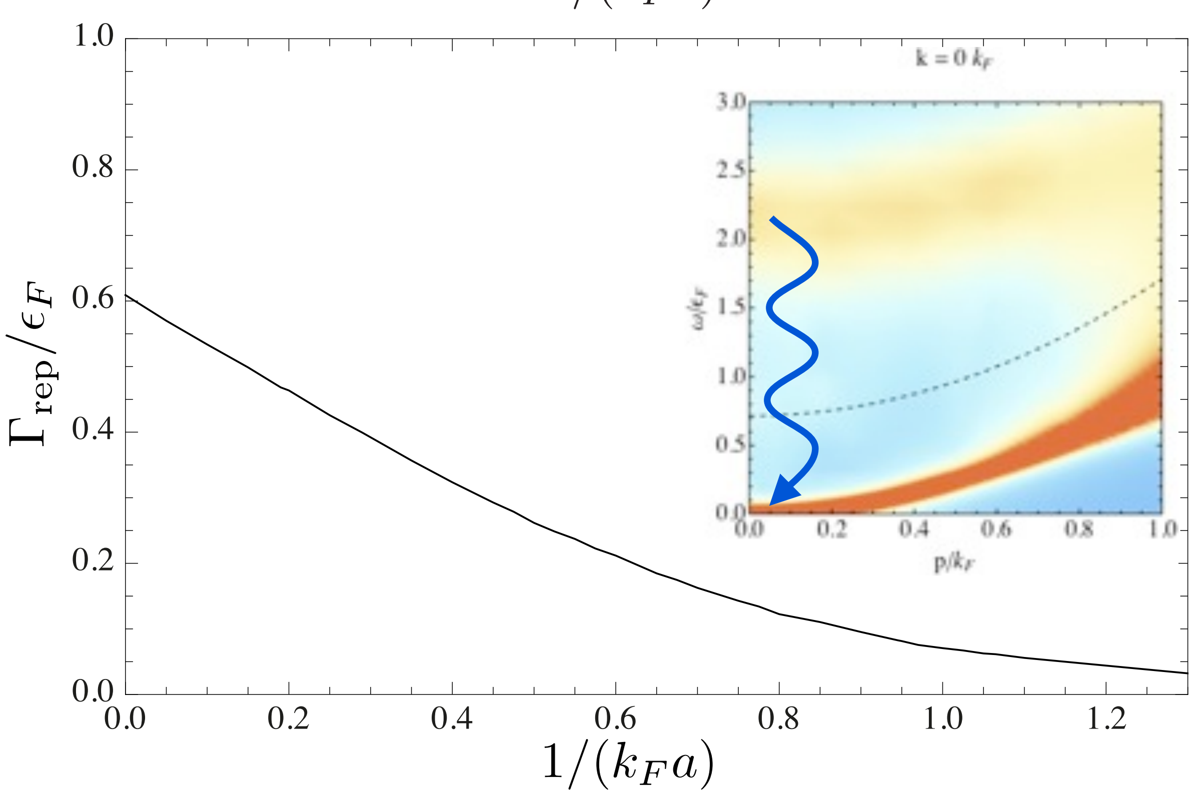
- $E_{\mathrm{rep}} > E_F$: ferromagnetism favored

Polaron decay

Schmidt & Enss 2011



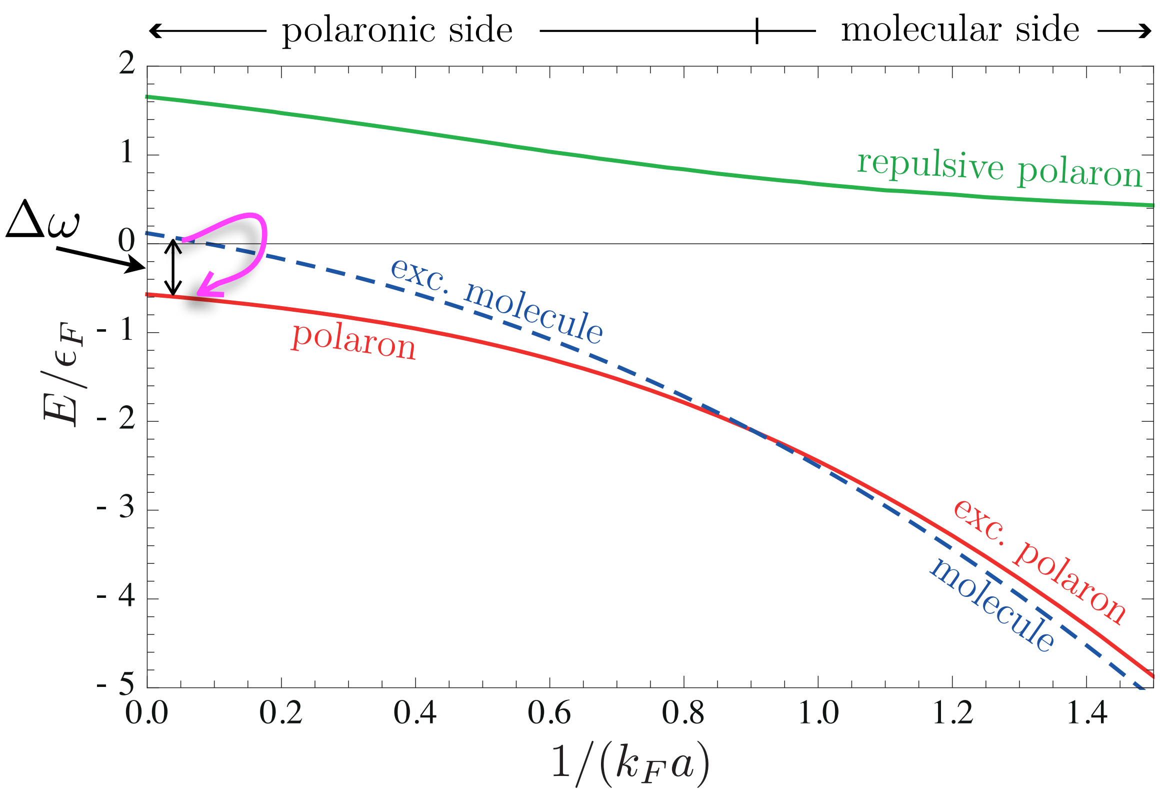


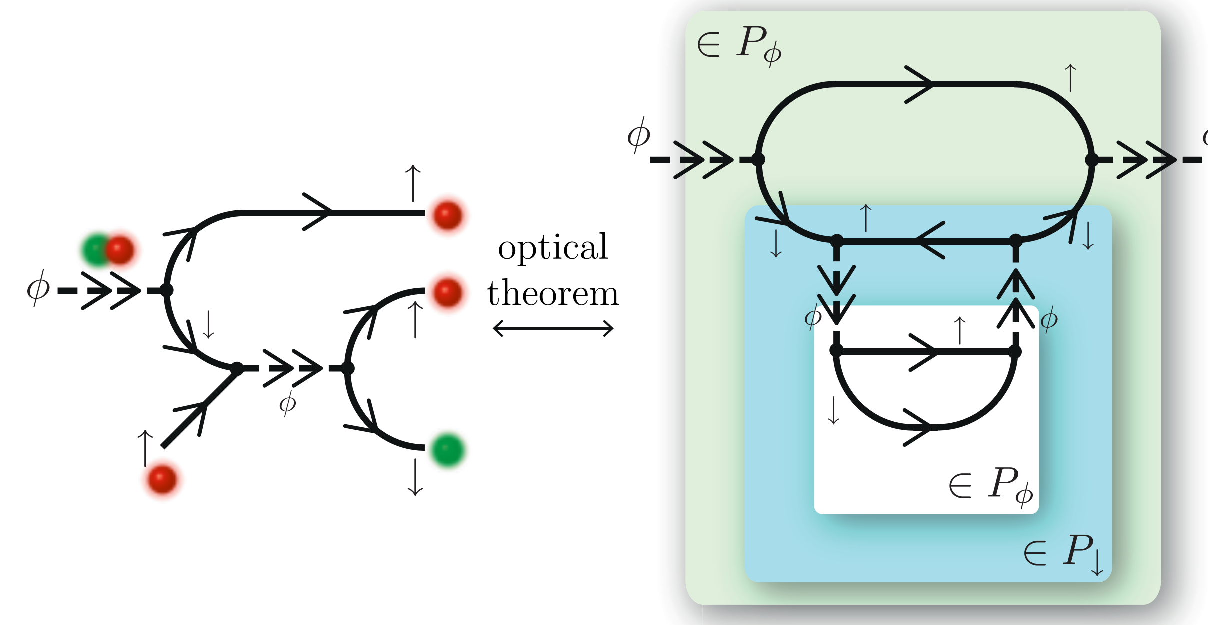


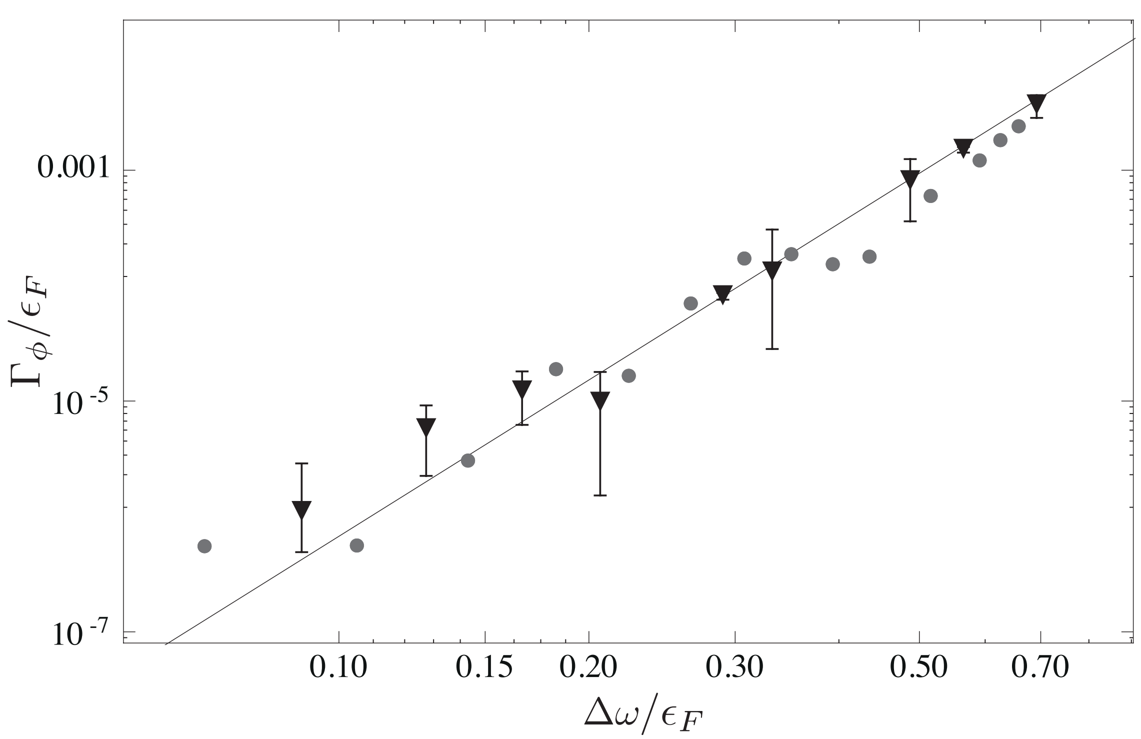
- strong binding: stable repulsive branch
- intermediate binding $(k_F a)^{-1} < 0.6$:
  $E_{\rm rep} > E_F$: onset of ferromagnetism
  $\Gamma_{\rm rep} > 0.2\, E_F$: molecule formation
- competition of dynamical phenomena
  Jo et al. 2009; Pekker et al. 2011; Cui & Zhai 2010

Molecule decay

Schmidt & Enss 2011







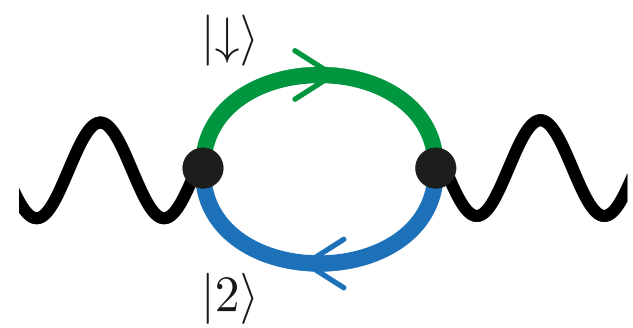
- leading **3-body** process (incl. in fRG)
- molecule stable: $\Gamma \propto \Delta\omega^{9/2}$
  Bruun & Massignan 2010
- 1st order transition

Radio-frequency response

Schmidt & Enss 2011

rf protocol

linear response



bubble + vertex corrections,
cf. transport Enss, Haussmann, Zwerger 2011
but for polaron they vanish

rf current: decay rate of rf photons by coupling to atoms,
imaginary part of photon self-energy:

$$I_{\rm rf}(\omega) = \frac{\pi\Omega_{\rm rf}^2}{2}\int\frac{d\mathbf{p}}{(2\pi)^2}\mathcal{A}_\downarrow(\mathbf{p},\omega+\varepsilon_\mathbf{p}-\mu_\downarrow)\, n_F(\varepsilon_\mathbf{p}-\mu_2)$$

atom inserted in polaron state

atom removed from state 2

Radio-frequency response

Schmidt & Enss 2011

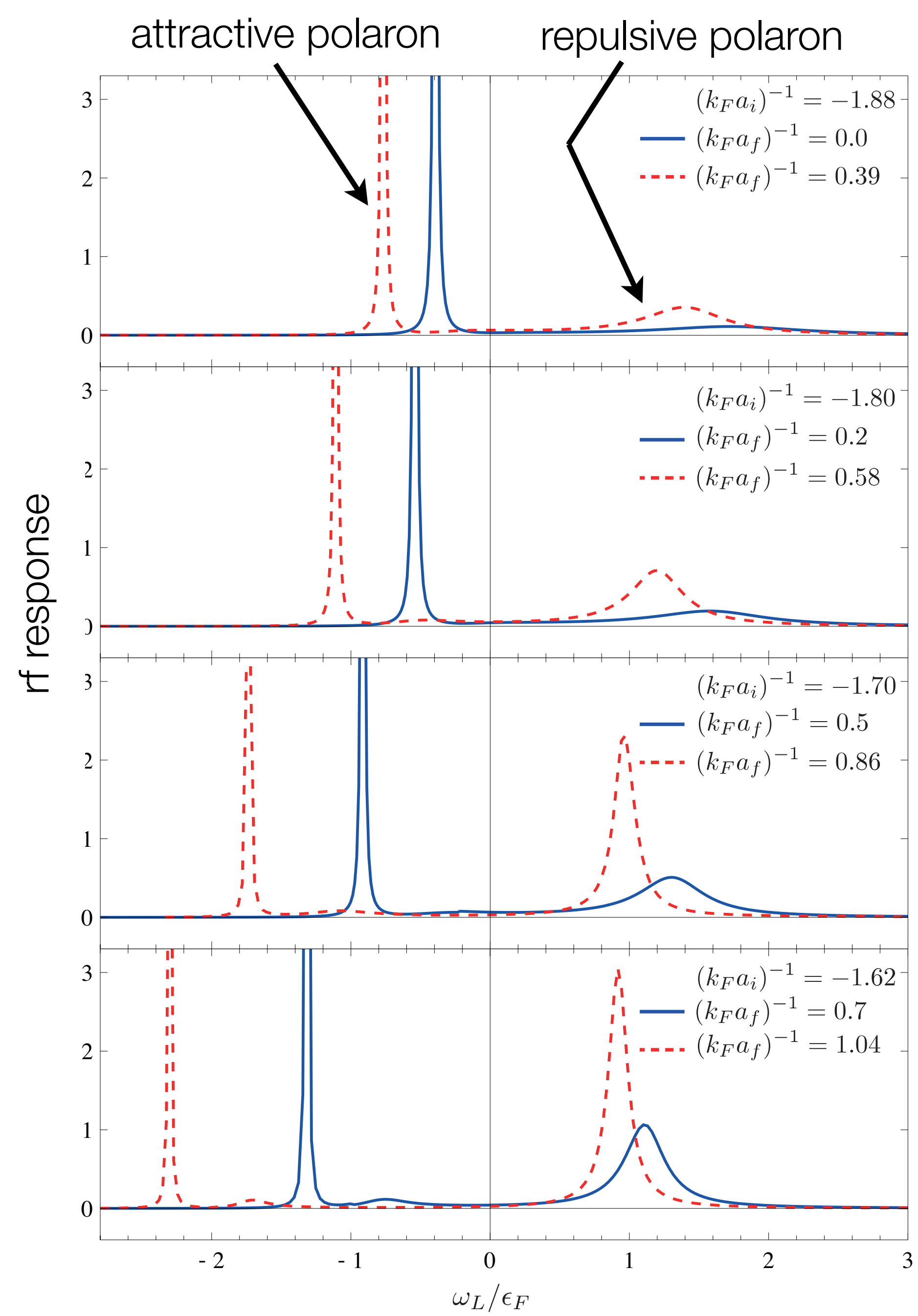
rf protocol

rf current: decay rate of rf phot

imaginary part of phot

$$I_{\mathrm{rf}}(\omega) = \frac{\pi\Omega_{\mathrm{rf}}^2}{2}\int \frac{d\mathbf{p}}{(2\pi)^2}$$

rf spectrum for $^6\mathrm{Li}$



Experimental confirmation: Innsbruck group


LETTER

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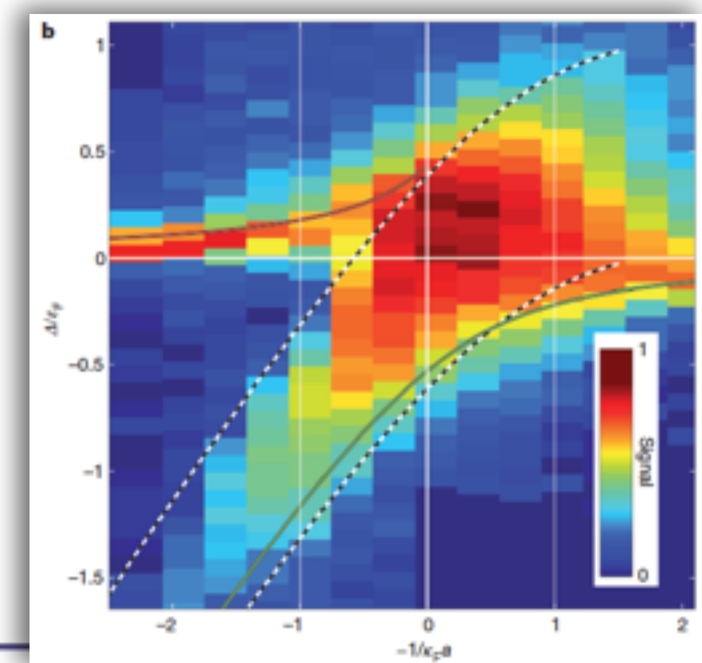
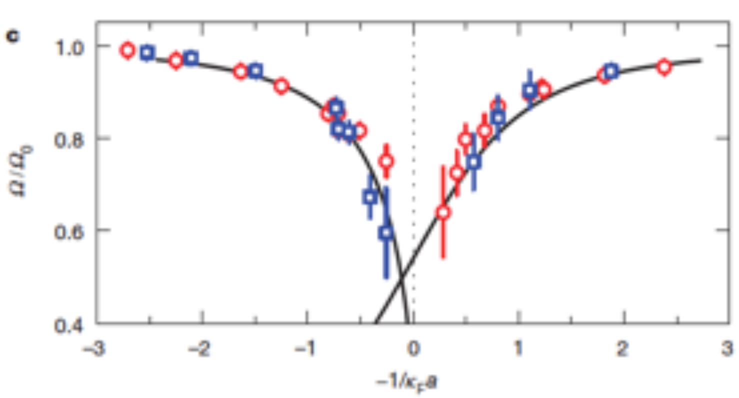
doi:10.1038/nature11065

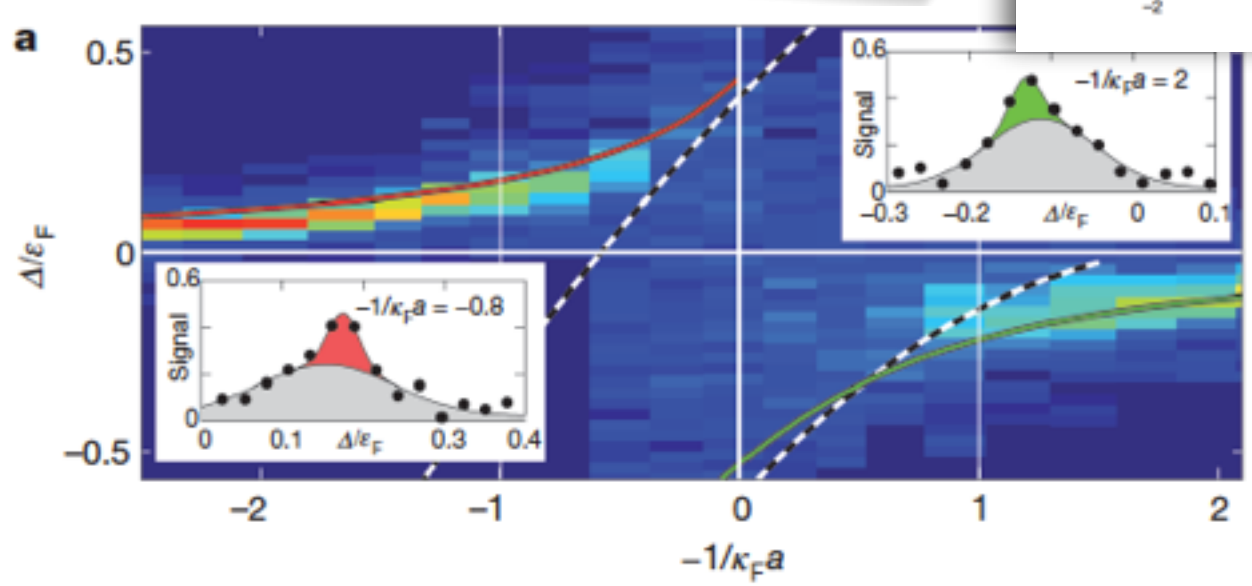
Metastability and coherence of repulsive polarons in a strongly interacting Fermi mixture

C. Kohstall[1,2], M. Zaccanti[1], M. Jag[1,2], A. Trenkwalder[1], P. Massignan[3], G. M. Bruun[4], F. Schreck[1] & R. Grimm[1,2]





quasiparticle weight



energy spectrum
agrees with theory
for narrow resonance
(Richard Schmidt, unpubl.)

Fermi polarons in two dimensions

Schmidt, Enss, Pietilä & Demler, PRA **85**, 021602(R) (2012)

2D scattering

experimental setup: quasi-2D “pancakes”

longitudinal motion frozen if

$k_B T, E_F \ll \hbar\omega_0$

exact 2D scattering amplitude:

2D: always bound state

3D: $f(k) = \dfrac{1}{-1/a_{3D} - ik}$ → 2D: $f(k) = \dfrac{1}{\ln(1/k^2 a_{2D}^2) + i\pi}$

$\varepsilon_B = \dfrac{\hbar^2}{m a_{2D}^2}$

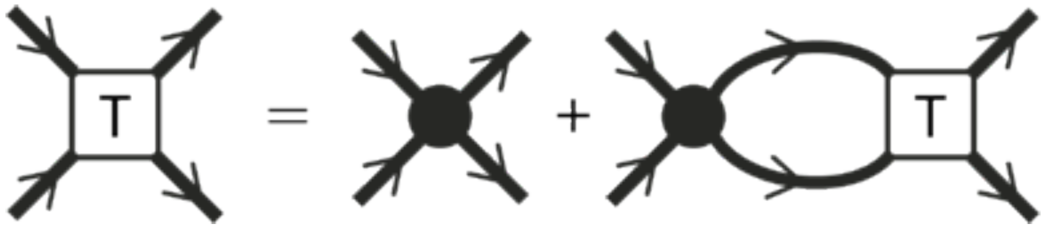
Adhikari 1986

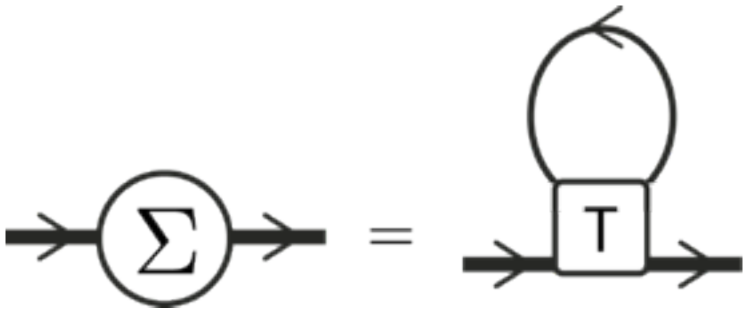
quasi-2D scattering:

$\varepsilon_B = 0.905\,(\hbar\omega_0/\pi)\exp(-\sqrt{2\pi}\ell_0/|a_{3D}|)$ Petrov & Shlyapnikov 2001

determines a_{2D} from a_{3D}

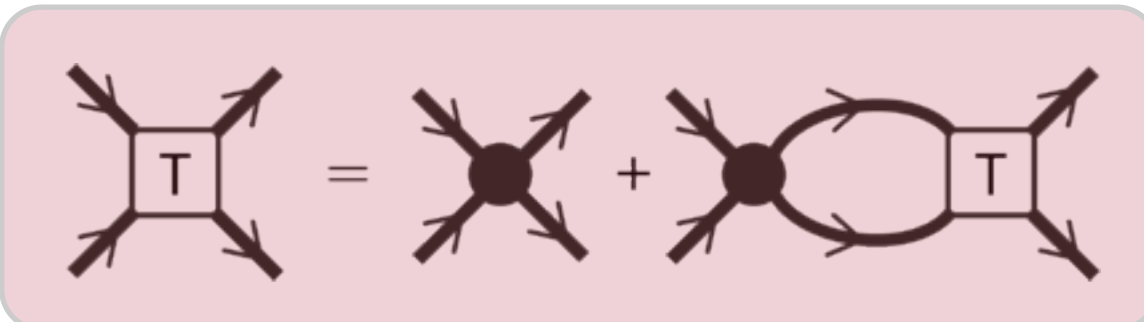
Many-body T-matrix

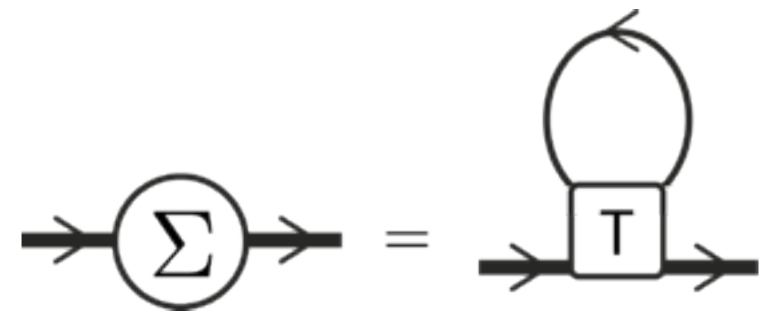




Nozières & Schmitt-Rink 1985;
2D: Engelbrecht & Randeria 1990

Many-body T-matrix





Nozières & Schmitt-Rink 1985;
2D: Engelbrecht & Randeria 1990

step 1: compute many-body T-matrix

two-body T-matrix: $$T_0(E) = \frac{4\pi/m}{\ln(\varepsilon_B/E) + i\pi}$$

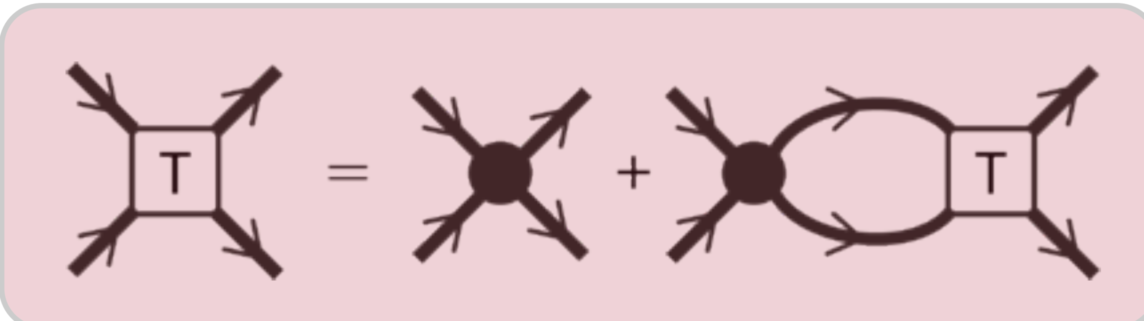
many-body: finite density medium scattering Schmidt, Enss, Pietilä & Demler 2012

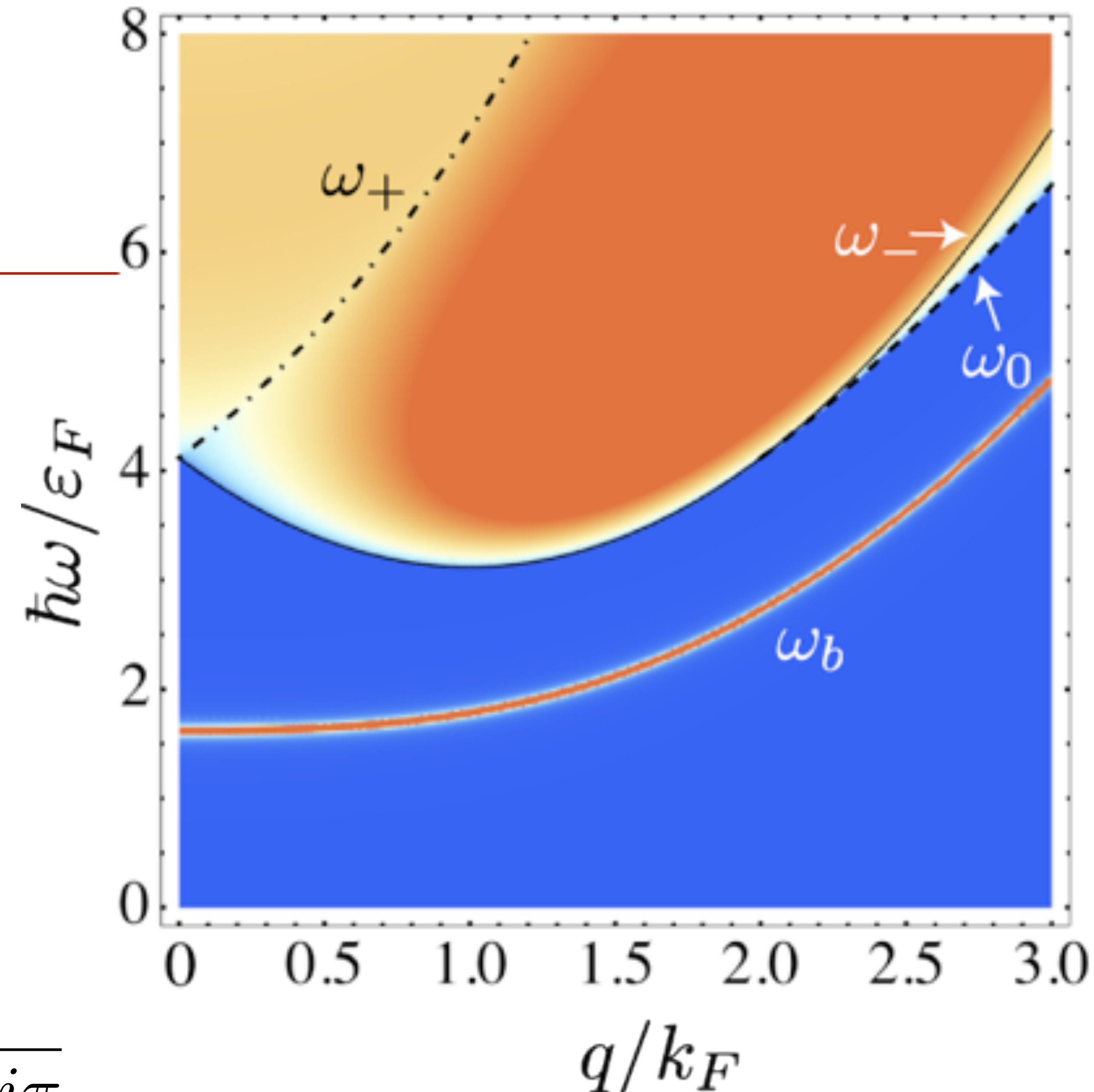
$$T^{-1}(\mathbf{q},\omega) = T_0^{-1}(\omega + i0 + \mu_\uparrow + \mu_\downarrow - \varepsilon_{\mathbf{q}}/2) + \int \frac{d^2k}{(2\pi)^2} \frac{n_F(\varepsilon_{\mathbf{k}} - \mu_\uparrow) + n_F(\varepsilon_{\mathbf{k+q}} - \mu_\downarrow)}{\omega + i0 + \mu_\uparrow + \mu_\downarrow - \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k+q}}}$$

we find compact solution

$$T(\mathbf{q},\omega) = T_0\left(\frac{1}{2}z \pm \frac{1}{2}\sqrt{(z-\varepsilon_{\mathbf{q}})^2 - 4\varepsilon_F\varepsilon_{\mathbf{q}}}\right) \qquad z = \omega + i0 - \varepsilon_F + \mu_\downarrow$$

Many-body T-matrix





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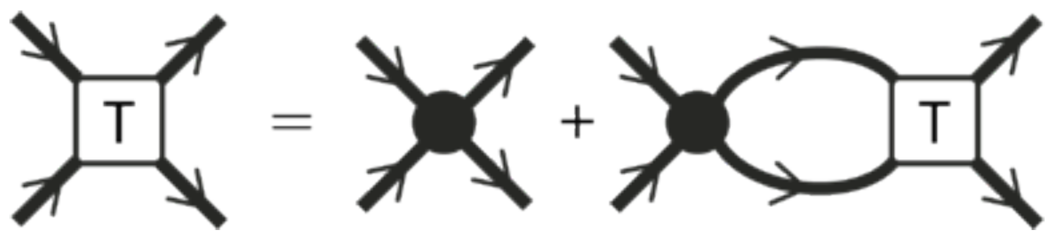
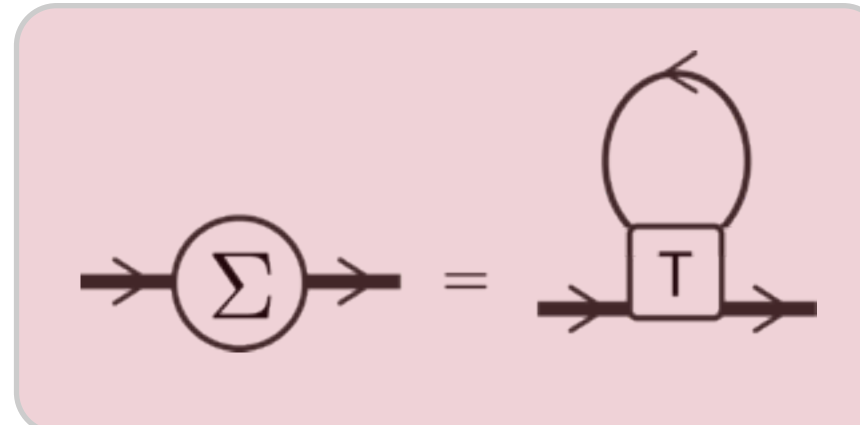
we find compact solution

$$T(\mathbf{q},\omega) = T_0\left(\frac{1}{2}z \pm \frac{1}{2}\sqrt{(z-\varepsilon_\mathbf{q})^2 - 4\varepsilon_F\varepsilon_\mathbf{q}}\right) \qquad z = \omega + i0 - \varepsilon_F + \mu_\downarrow$$

Polaron self-energy



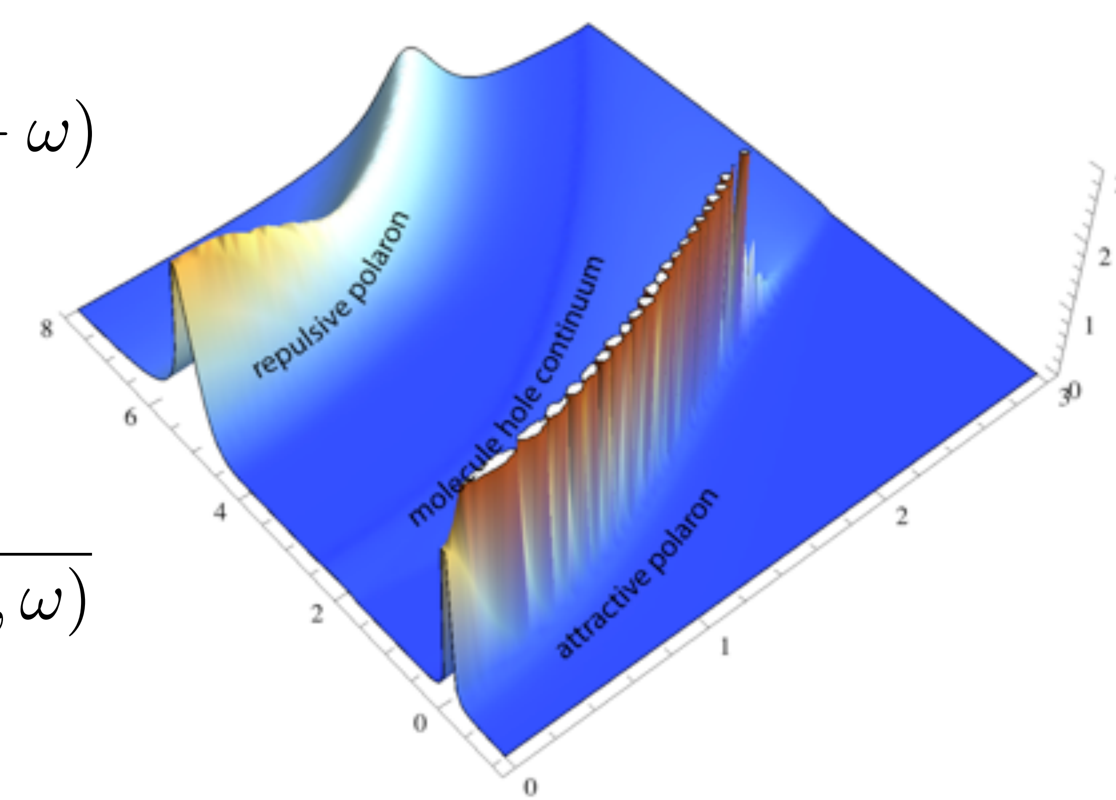


step 2: polaron self-energy

$$\Sigma_{\downarrow}(\mathbf{p},\omega) = \int_{k<k_F} \frac{d^2k}{(2\pi)^2}\, T(\mathbf{k}+\mathbf{p}, \varepsilon_{\mathbf{k}} - \mu_{\uparrow} + \omega)$$

step 3: polaron spectral function

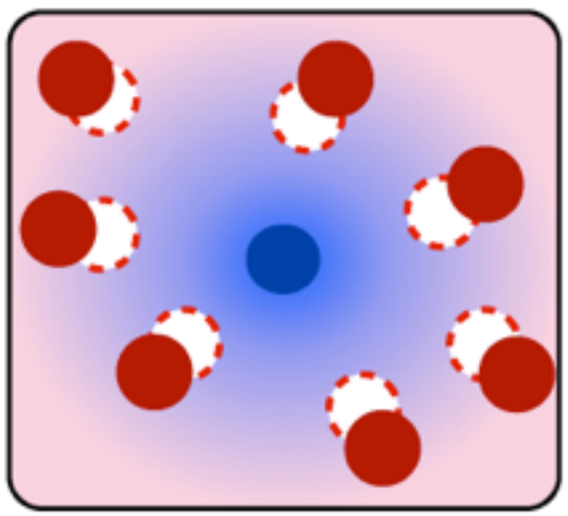
$$\mathcal{A}_{\downarrow}(\mathbf{p},\omega) = -2\,\mathrm{Im}\,\frac{1}{\omega + i0 + \mu_{\downarrow} - \varepsilon_{\mathbf{p}} - \Sigma_{\downarrow}(\mathbf{p},\omega)}$$



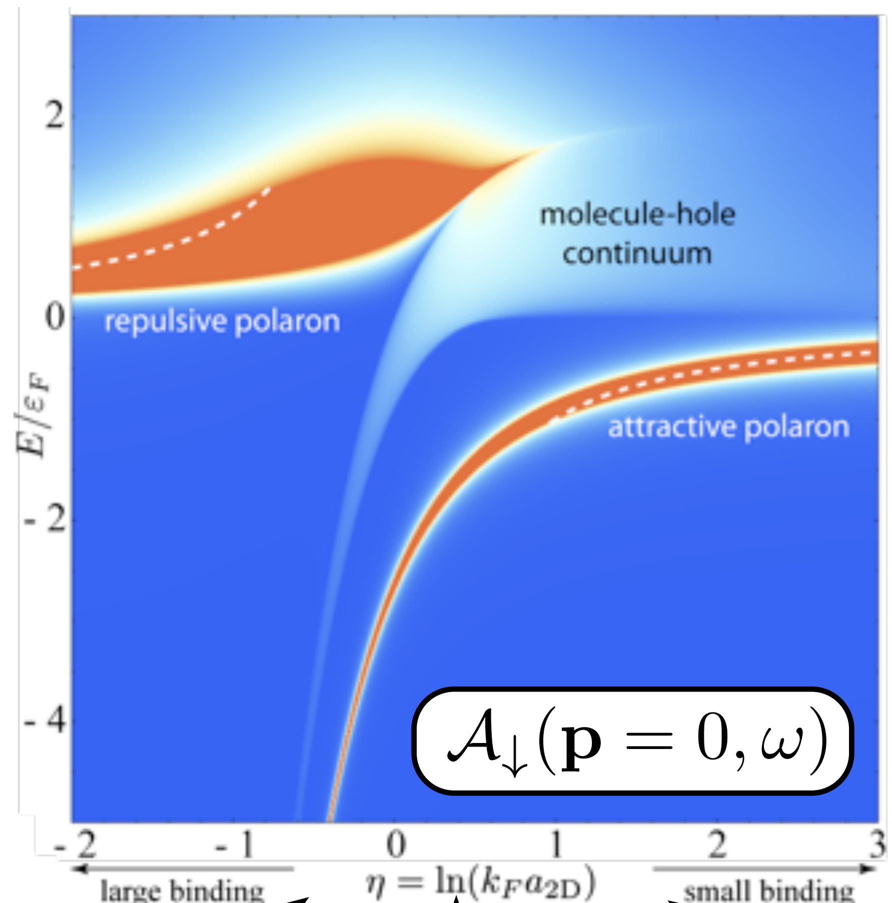
contains full information about
energy spectrum, quasiparticle weights, decay rates...

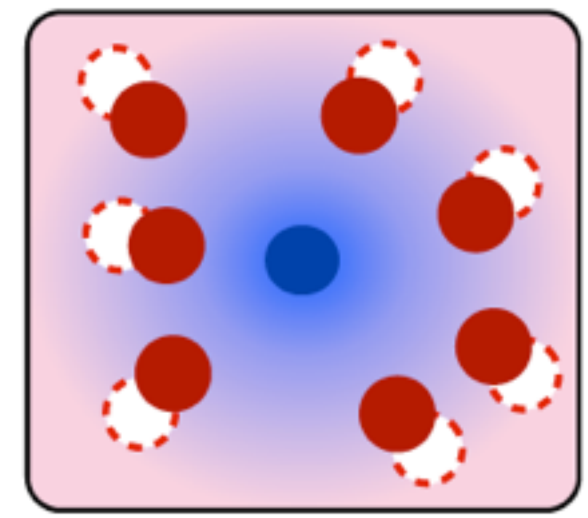
Polaron spectral function

Schmidt, Enss, Pietilä & Demler 2012

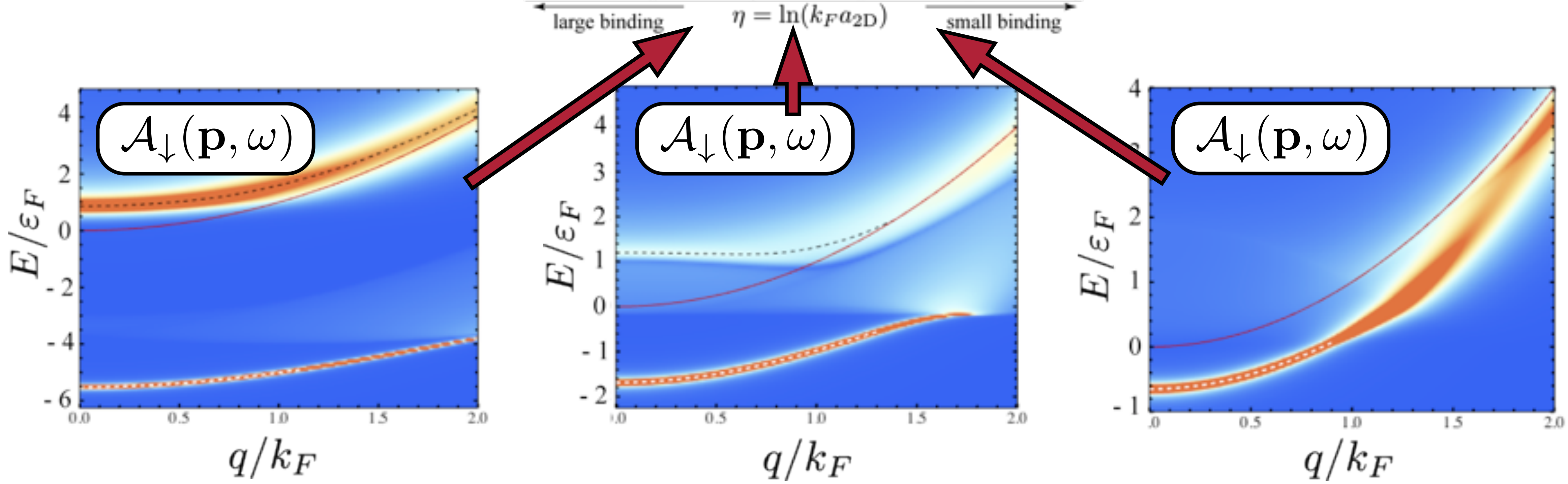


repulsive polaron



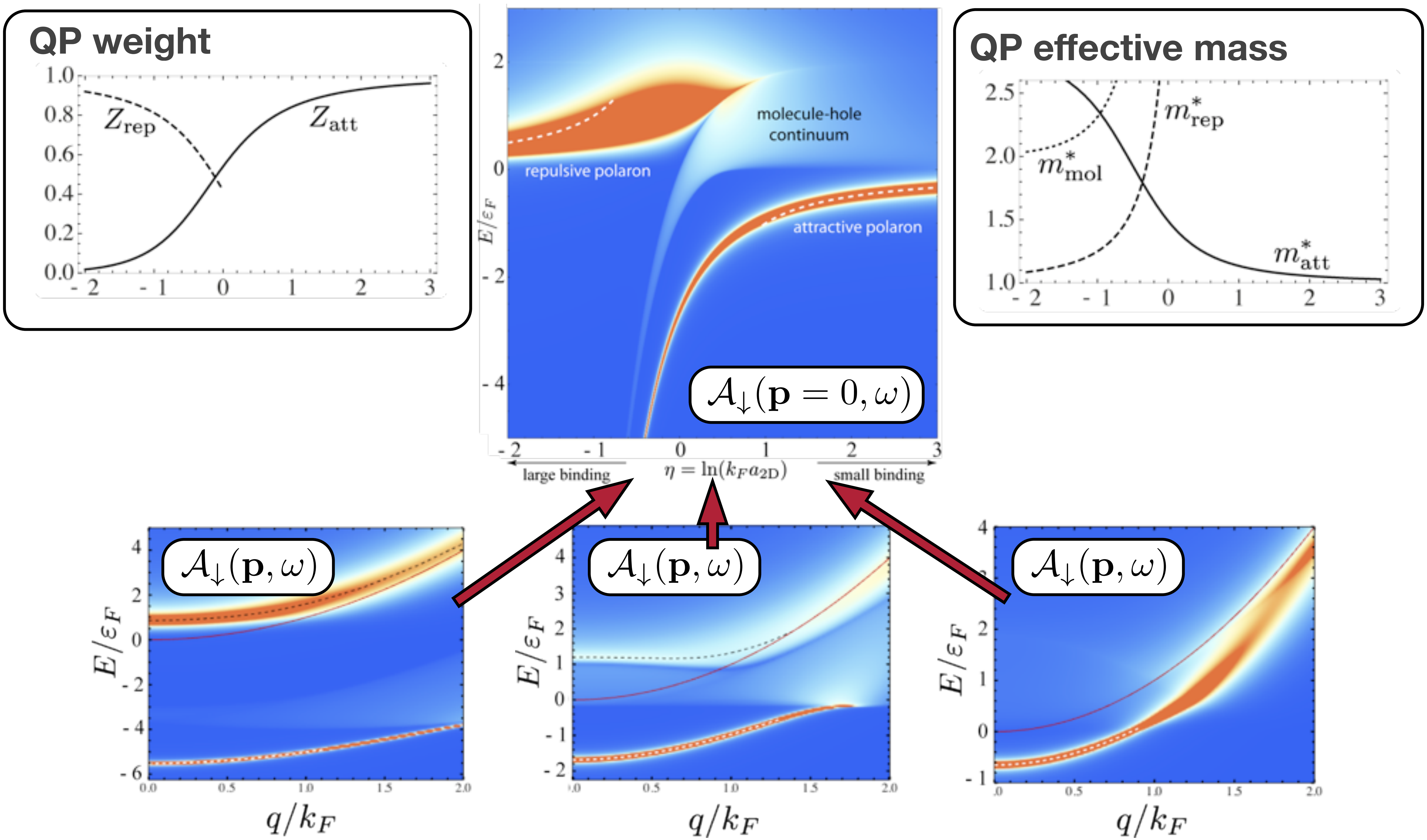


attractive polaron



Polaron spectral function

Schmidt, Enss, Pietilä & Demler 2012



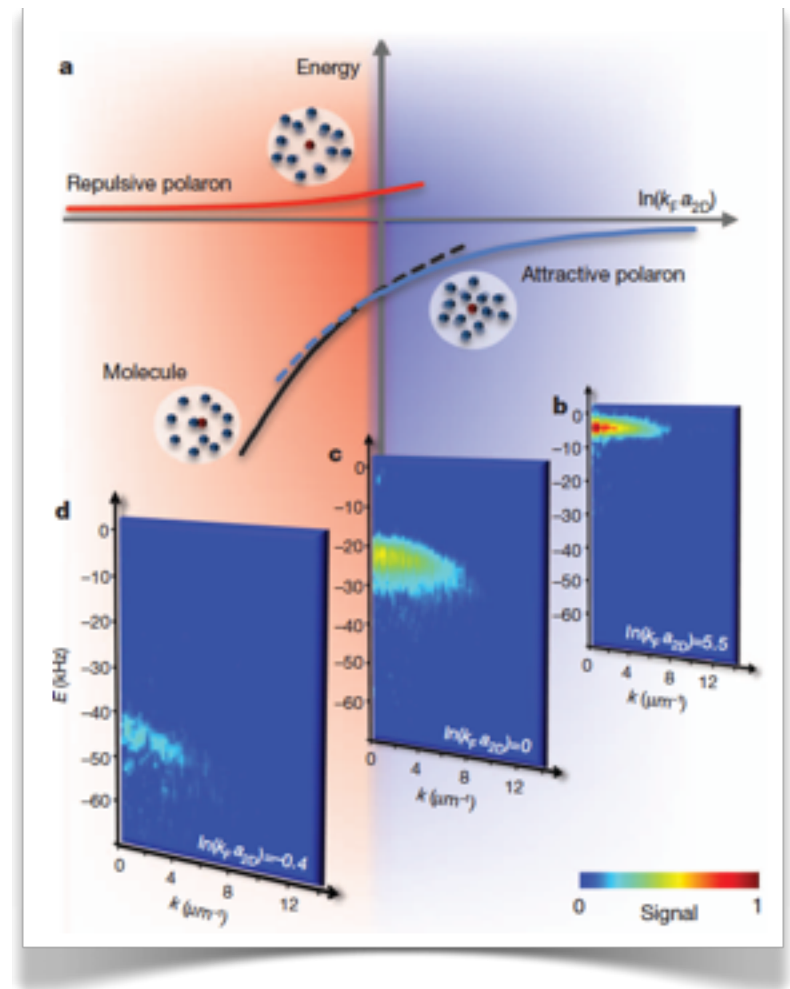
Cambridge experiment

LETTER

31 MAY 2012 | VOL 485 | NATURE | 619
doi:10.1038/nature11151

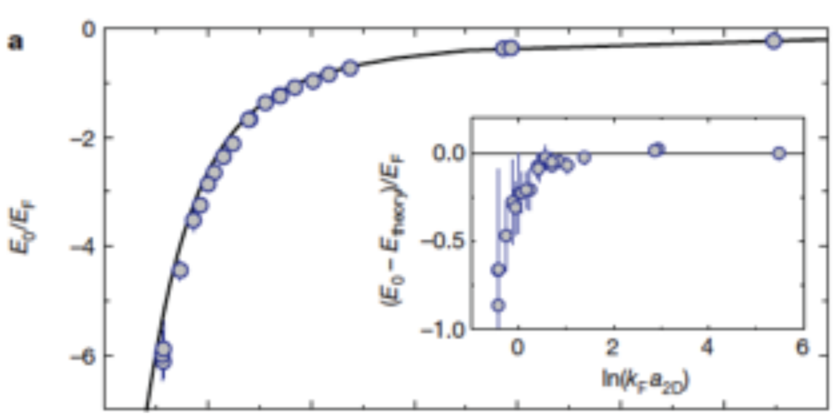
Attractive and repulsive Fermi polarons in two dimensions

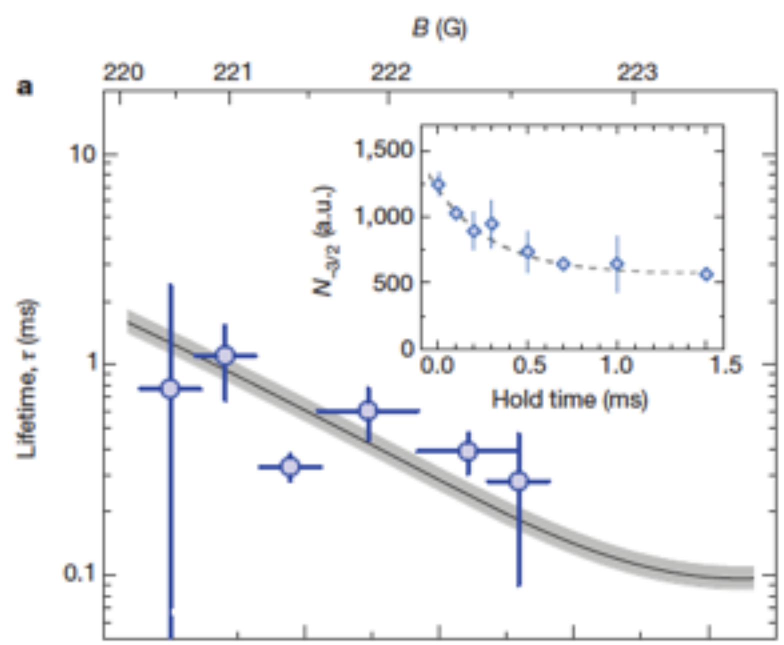
Marco Koschorreck[1]*, Daniel Pertot[1]*, Enrico Vogt[1], Bernd Fröhlich[1], Michael Feld[1] & Michael Köhl[1]



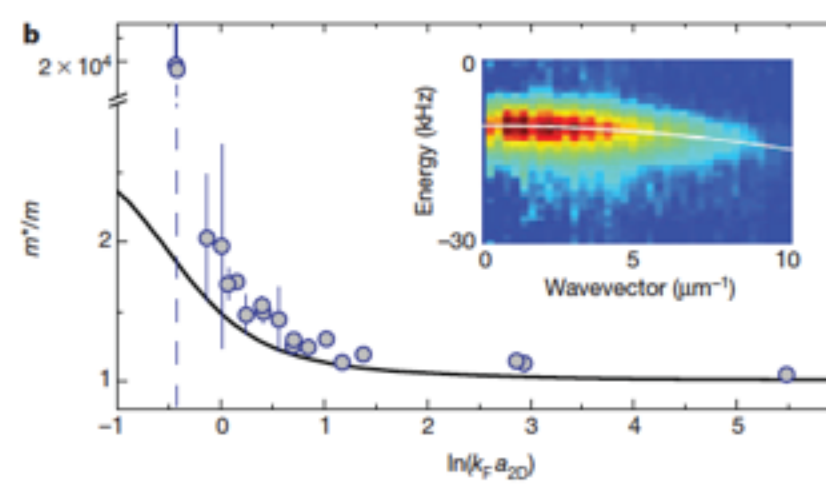
energy spectrum

lifetime of rep. polaron





effective mass



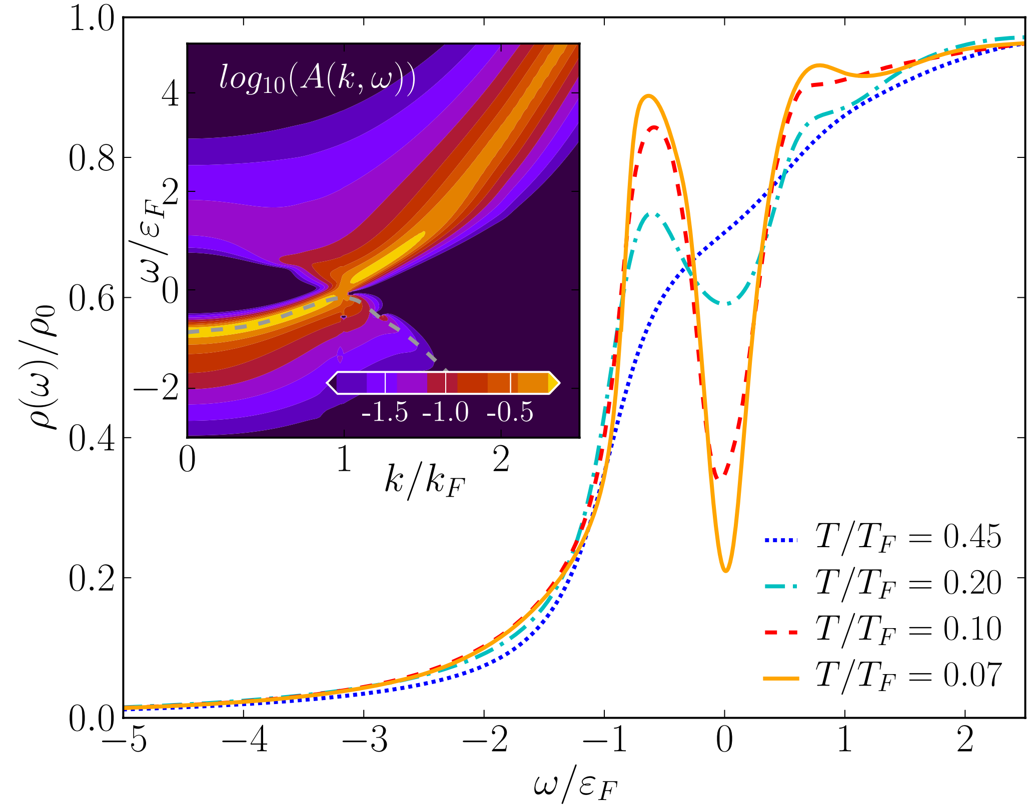
confirms our prediction

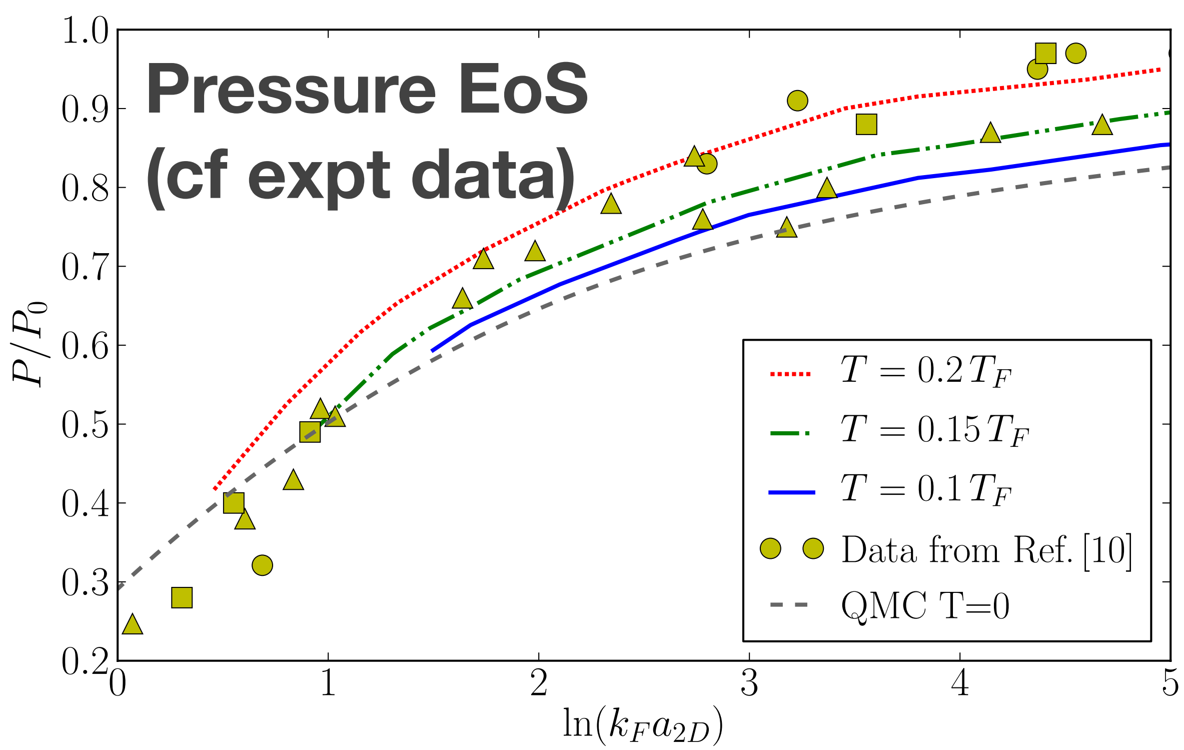
cf. Ngampruetikorn et al. 2012

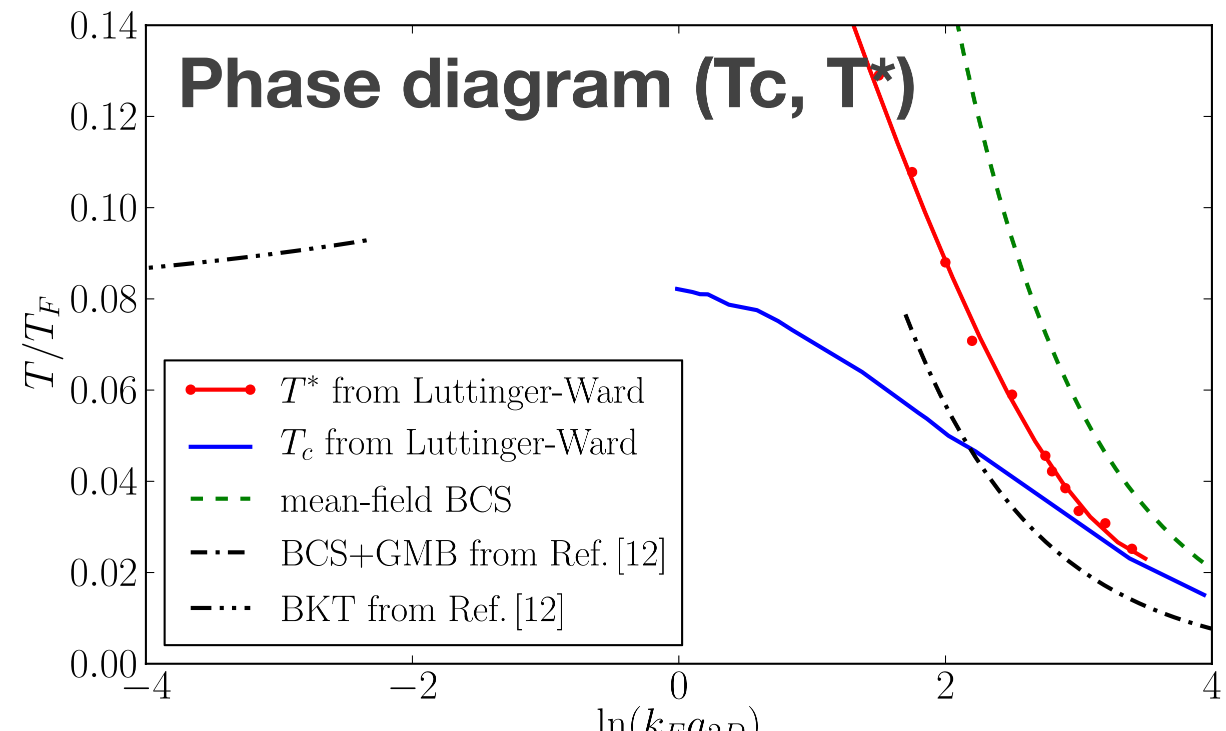
BKT-BCS crossover in 2D Fermi gas

Spectral fct & pseudogap in balanced 2D gas

Bauer, Parish & Enss, PRL **112**, 135302 (2014)

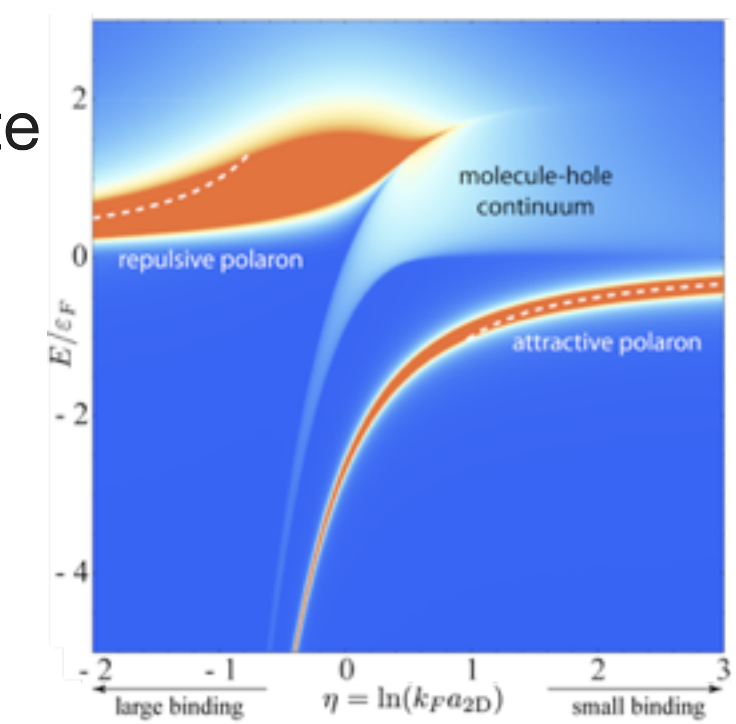






Conclusion & outlook

- **full frequency/momentum dependence of self-energy and Cooper vertex**
  - full self-energy feedback yields transition as accurately as diagMC
  - beyond quasiparticle picture, large anomalous dimension
  - resolve higher excited states, decay rates, power laws
- **RG flow of spectral functions**
  - see how many-body correlations emerge in spectrum
- **predicted repulsive polaron, confirmed in experiment**
  - inverse RF protocol to detect short-lived repulsive state
- **outlook**
  - fRG for 2D polaron to include self-energy feedback
  - interaction between impurities, finite impurity density
  - dynamical and transport processes



Schmidt & Enss, PRA **83**, 063620 (2011)
Schmidt, Enss, Pietilä & Demler, PRA **85**, 021602(R) (2012)
Bauer, Parish & Enss, PRL **112**, 135302 (2014)