Correlation functions of homogeneous and isotropic turbulence

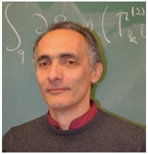
Metropolitan Museum of Art, NY

Léonie Canet

FRG, Heidelberg

7/03/2017

In collaboration with ...



Bertrand Delamotte

LPTMC

Univ. Paris 6

Nicolás Wschebor

Univ. República

Montevideo

Vincent Rossetto

LPMMC

Univ. Grenoble Alpes

Guillaume Balarac

LEGI

Grenoble INP

LC, B. Delamotte, N. Wschebor, Phys. Rev. E **91** (2015)

LC, B. Delamotte, N. Wschebor, Phys. Rev. E **93** (2016)

LC, V. Rossetto, N. Wschebor, G. Balarac, Phys. Rev. E **95** (2017)

M. Tarpin, LC, N. Wschebor, in preparation (2017)

Malo Tarpin, LPMMC

Presentation outline

1. Navier-Stokes turbulence
   - Fully developed turbulence
   - Universality and power laws
   - Kolmogorov theory and intermittency
   - RG approaches to turbulence
2. Non-Perturbative Renormalization Group for turbulence
   - Navier-Stokes equation and field theory
   - Exact flow equations for two-point correlation functions
   - Solution in the inertial range
   - Behavior in the dissipative range
   - Bi-dimensional turbulence
3. Perspectives

Fully developed turbulence

very old . . .

studied since (at least) Da Vinci . . .

Fully developed turbulence

very old . . . and very challenging

studied since (at least) Da Vinci . . .

. . .and yet Feynman's words still hold :

"turbulence is the most important unsolved problem of classical physics"

Fully developed turbulence

very old . . . and very challenging

- non-equilibrium driven-dissipative state
- characterized by rare and extreme events (rogue waves, tornados, ...) : intermittency

- technological implications : design of boats, aircrafts, wind power plants, tidal power plants, weather forcast, etc.
- fundamental physics : understanding and computing the statistical properties of turbulent flows

Fully developed turbulence

very old . . . and very challenging

- non-equilibrium driven-dissipative state
- characterized by rare and extreme events

(rogue waves, tornados, ...) : intermittency

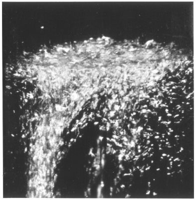
- technological implications : design of boats, aircrafts, wind power plants, tidal power plants, weather forcast, etc.
- fundamental physics : understanding and computing the statistical properties of turbulent flows

Universality and power-laws

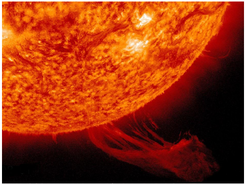
tidal channel, pipe, wake, grid . . .

wind tunnel and atmosphere

liquid helium

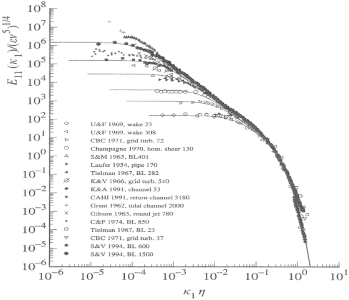


solar wind plasma

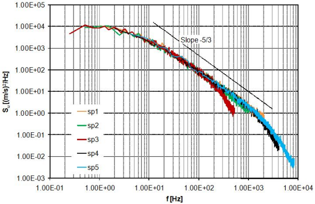


Universality and power laws : kinetic energy spectrum

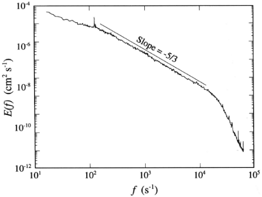
tidal channel, pipe, wake, grid . . .



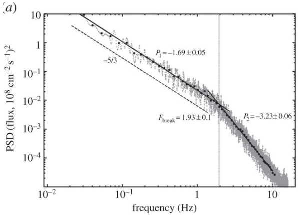
wind tunnel and atmosphere



liquid helium



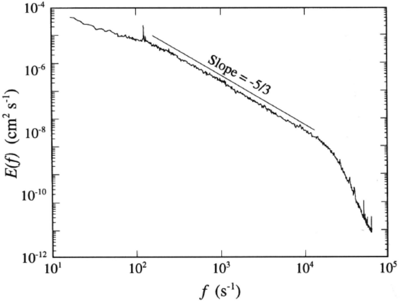
solar wind plasma



Kinetic energy spectrum

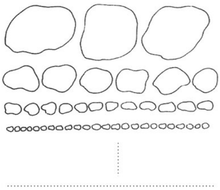
Universal features, energy cascade

liquid helium Maurer,Tabeling, Zocchi EPL **26** (1994)



L integral scale

η Kolmogorov scale



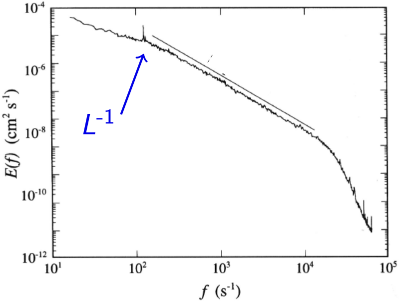
Frisch, Turbulence, Camb. Univ. Press (1995)

$$E(k) = 4\pi k^2 \, \mathrm{TF} \, (\langle \vec{v}(t,\vec{x}) \cdot \vec{v}(t,0) \rangle) = C_K \epsilon^{2/3} k^{-5/3}$$

Kinetic energy spectrum

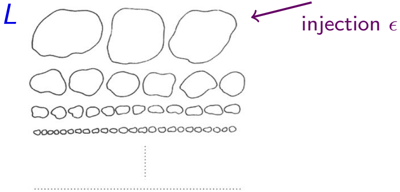
Universal features, energy cascade

liquid helium Maurer,Tabeling, Zocchi EPL **26** (1994)



L integral scale

η Kolmogorov scale



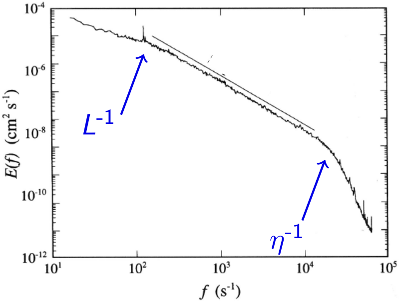
Frisch, Turbulence, Camb. Univ. Press (1995)

$$E(k) = 4\pi k^2 \, \mathrm{TF} \left(\langle \vec{v}(t,\vec{x}) \cdot \vec{v}(t,0) \rangle \right) = C_K \epsilon^{2/3} k^{-5/3}$$

Kinetic energy spectrum

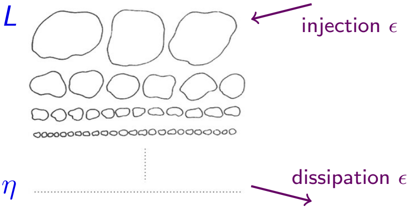
Universal features, energy cascade

liquid helium Maurer,Tabeling, Zocchi EPL **26** (1994)



L integral scale

η Kolmogorov scale



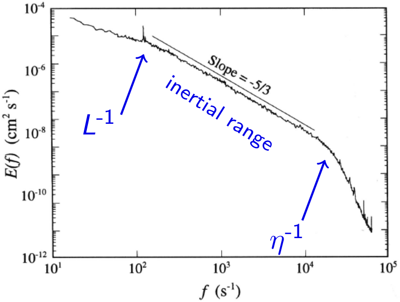
Frisch, Turbulence, Camb. Univ. Press (1995)

$$E(k) = 4\pi k^2 \,\mathrm{TF}\,(\langle \vec{v}(t,\vec{x}) \cdot \vec{v}(t,0) \rangle) = C_K \epsilon^{2/3} k^{-5/3}$$

Kinetic energy spectrum

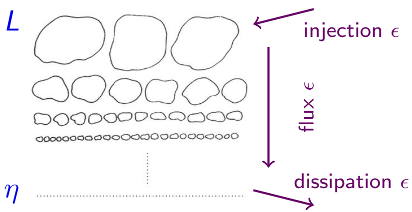
Universal features, energy cascade

liquid helium Maurer,Tabeling, Zocchi EPL **26** (1994)



L integral scale

η Kolmogorov scale



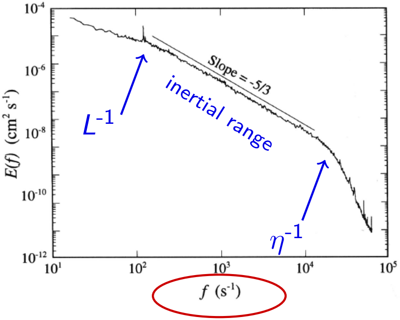
Frisch, Turbulence, Camb. Univ. Press (1995)

$$E(k) = 4\pi k^2\, \mathrm{TF}\, \left(\langle \vec{v}(t,\vec{x}) \cdot \vec{v}(t,0)\rangle\right) = C_K \epsilon^{2/3} k^{-5/3}$$

Kinetic energy spectrum

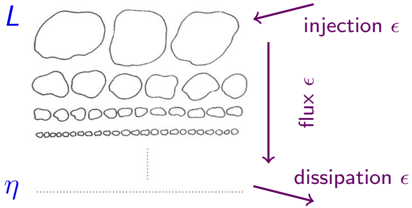
Universal features, energy cascade

liquid helium Maurer,Tabeling, Zocchi EPL **26** (1994)



L integral scale

η Kolmogorov scale



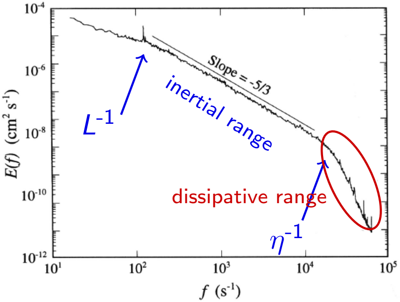
Frisch, Turbulence, Camb. Univ. Press (1995)

$$E(k) = 4\pi k^2 \, \mathrm{TF} \left(\langle \vec{v}(t,\vec{x}) \cdot \vec{v}(t,0) \rangle \right) = C_K \epsilon^{2/3} k^{-5/3}$$

Kinetic energy spectrum

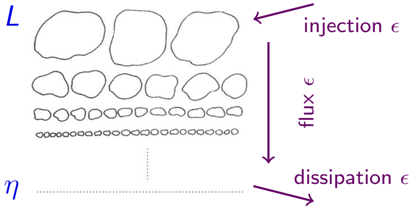
Universal features, energy cascade

liquid helium Maurer,Tabeling, Zocchi EPL **26** (1994)



L integral scale

η Kolmogorov scale



Frisch, Turbulence, Camb. Univ. Press (1995)

$$E(k) = 4\pi k^2 \, \text{TF} \left(\langle \vec{v}(t,\vec{x}) \cdot \vec{v}(t,0) \rangle\right) = C_K \epsilon^{2/3} k^{-5/3}$$

Scale invariance and Kolmogorov theory

power law behaviors

- velocity increments

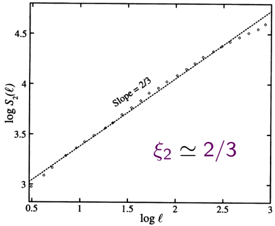
$$\delta v_{\ell\parallel} = [\vec{u}(\vec{x}+\vec{\ell}) - \vec{u}(\vec{x})]\cdot\vec{\ell}$$

- structure function

$$S_p(\ell) \equiv \langle (\delta v_{\ell\parallel})^p \rangle \sim \ell^{\xi_p}$$

ONERA wind tunnel

Anselmet et al., J. Fluid Mech. **140** (1984)



Kolmogorov K41 theory for homogeneous isotropic 3D turbulence

A.N. Kolmogorov, Dokl. Akad. Nauk. SSSR **30, 31, 32** (1941)

assumptions : local isotropy and homogeneity, finite ϵ in the limit $\nu \to 0$

- exact result : $$S_3(\ell) = -\frac{4}{5}\,\epsilon\,\ell$$
- universality and self-similarity :

$$\begin{cases} E(k) &= C_K\,\epsilon^{2/3}\,k^{-5/3} \\ S_p(\ell) &= C_p\,\epsilon^{p/3}\,\ell^{p/3} \end{cases}$$

Intermittency, multi-scaling

deviations from K41

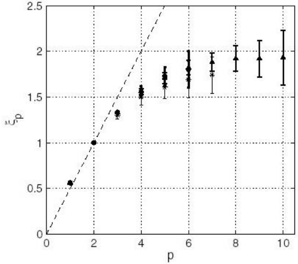
in experiments and numerical simulations :

$$S_p(\ell) \equiv \langle (\delta v_{\ell_{||}})^p \rangle \sim \ell^{\xi_p}$$

$$\xi_p \neq p/3$$

- violation of simple scale-invariance
  $\Longrightarrow$ multi-scaling
- rare extreme events
  $\Longrightarrow$ intermittency

illustration :
von Kárman swirling flow



●	exp.
▲ , *	num.
- - -	K41

Mordant, Lévêque, Pinton,

New J. Phys. **6** (2004)

RG approaches to turbulence

theoretical challenge : understand intermittency from first principles
universality and power laws $\Longrightarrow$ RG approach

perturbative RG approaches

formal expansion parameter through the forcing profile $N_{\alpha\beta}(\vec{p}) \propto p^{4-d-2\epsilon}$

- *early works* de Dominicis, Martin, PRA **19** (1979) , Fournier, Frisch, PRA **28** (1983) Yakhot, Orszag, PRL **57** (1986)
- *reviews* Zhou, Phys. Rep. **488** (2010)

  Adzhemyan *et al.*, *The Field Theoretic RG in Fully Developed Turbulence*, Gordon Breach, 1999

Functional RG approaches

Tomassini, Phys. Lett. B **411** (1997), Mejía-Monasterio, Muratore-Ginnaneschi, PRE **86** (2012)

Fedorenko, Le Doussal, Wiese, J. Stat. Mech. (2013).

RG approaches to turbulence

theoretical challenge : understand intermittency from first principles
universality and power laws $\Longrightarrow$ RG approach

Non-Perturbative and Functional RG : one (big) step further

exact closure based on symmetries
in the limit of large wave-numbers

LC, Delamotte, Wschebor, PRE **93** (2016), LC, Rossetto, Wschebor, Balarac, PRE **95** (2017)

Presentation outline

Microscopic theory

Navier Stokes equation with forcing for incompressible flows

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\frac{1}{\rho} \vec{\nabla} p + \nu \vec{\nabla}^2 \vec{v} + \vec{f}$$
$$\vec{\nabla} \cdot \vec{v}(t, \vec{x}) = 0$$

- $\vec{v}(\vec{x}, t)$ velocity field and $p(\vec{x}, t)$ pressure field
- ρ density and ν kinematic viscosity
- $\vec{f}(\vec{x}, t)$ gaussian stochastic stirring force with variance

$$\left\langle f_\alpha(t, \vec{x}) f_\beta(t', \vec{x}\,') \right\rangle = 2\delta_{\alpha\beta}\delta(t - t') N_L(|\vec{x} - \vec{x}\,'|).$$

with N_L peaked at the integral scale (energy injection)

Non-Perturbative Renormalisation Group for NS

MSR Janssen de Dominicis formalism : NS field theory

Martin, Siggia, Rose, PRA **8** (1973), Janssen, Z. Phys. B **23** (1976), de Dominicis, J. Phys. Paris **37** (1976)

$$\mathcal{S}_0 = \int_{t,\vec{x}} \bar{v}_\alpha \left[\partial_t v_\alpha + v_\beta \partial_\beta v_\alpha + \frac{1}{\rho} \partial_\alpha p - \nu \nabla^2 v_\alpha \right] + \bar{p} \left[\partial_\alpha v_\alpha \right]$$
$$- \int_{t,\vec{x},\vec{x}'} \bar{v}_\alpha \left[N_L(|\vec{x} - \vec{x}'|) \right] \bar{v}_\alpha$$

Non-Perturbative Renormalization Group approach

- Wetterich's equation C. Wetterich, Phys. Lett. B **301** (1993)
- aim : compute correlation function and response function

$$\left\langle v_\alpha(t,\vec{x}) v_\beta(0,0) \right\rangle \quad \text{and} \quad \left\langle v_\alpha(t,\vec{x}) f_\beta(0,0) \right\rangle$$

in the stationary non-equilibrium turbulent state

Non-Perturbative Renormalisation Group for NS

Wetterich's equation for the 2-point functions

$$\partial_\kappa \Gamma^{(2)}_{\kappa,ij}(\mathbf{p}) = \mathrm{Tr}\int_{\mathbf{q}} \partial_\kappa \mathcal{R}_\kappa(\mathbf{q}) \cdot G_\kappa(\mathbf{q}) \cdot \Big(-\frac{1}{2}\,\Gamma^{(4)}_{\kappa,ij}(\mathbf{p},-\mathbf{p},\mathbf{q})$$
$$+\Gamma^{(3)}_{\kappa,i}(\mathbf{p},\mathbf{q}) \cdot G_\kappa(\mathbf{p}+\mathbf{q}) \cdot \Gamma^{(3)}_{\kappa,j}(-\mathbf{p},\mathbf{p}+\mathbf{q})\Big) \cdot G_\kappa(\mathbf{q})$$

infinite hierarchy of flow equations

- approximation scheme : truncation of higher-order vertices

based on BMW scheme and inspired by similar approximation for KPZ

LC, Chaté, Delamotte, Wschebor, PRL **104** (2010)

- Tomassini, Phys. Lett. B **411** (1997)
- Mejía-Monasterio, Muratore-Ginnaneschi, PRE **86** (2012)
- LC, Delamotte, Wschebor, PRE **93** (2016)

$\Longrightarrow$ RG fixed point

- exact closure in the limit of large wave-numbers

Non-Perturbative Renormalisation Group for NS

Wetterich's equation for the 2-point functions

$$\partial_\kappa \Gamma^{(2)}_{\kappa,ij}(\mathbf{p}) \;=\; \mathrm{Tr}\int_{\mathbf{q}} \partial_\kappa \mathcal{R}_\kappa(\mathbf{q}) \cdot G_\kappa(\mathbf{q}) \cdot \Big(-\frac{1}{2}\,\Gamma^{(4)}_{\kappa,ij}(\mathbf{p},-\mathbf{p},\mathbf{q})$$
$$+\Gamma^{(3)}_{\kappa,i}(\mathbf{p},\mathbf{q}) \cdot G_\kappa(\mathbf{p}+\mathbf{q}) \cdot \Gamma^{(3)}_{\kappa,j}(-\mathbf{p},\mathbf{p}+\mathbf{q}) \Big) \cdot G_\kappa(\mathbf{q})$$

infinite hierarchy of flow equations

- approximation scheme : truncation of higher-order vertices

based on BMW scheme and inspired by similar approximation for KPZ

LC, Chaté, Delamotte, Wschebor, PRL **104** (2010)

- Tomassini, Phys. Lett. B **411** (1997)
- Mejía-Monasterio, Muratore-Ginnaneschi, PRE **86** (2012)
- LC, Delamotte, Wschebor, PRE **93** (2016)

$\Longrightarrow$ RG fixed point

- exact closure in the limit of large wave-numbers

Ingredient 1 : Symmetries of the NS field theory

- infinitesimal time-gauged galilean transformations

$$\mathcal{G}(\vec{\epsilon}(t)) = \begin{cases} \vec{x} \to \vec{x} + \vec{\epsilon}(t) \\ \vec{v} \to \vec{v} - \dot{\vec{\epsilon}}(t) \end{cases}$$

- infinitesimal time-gauged response field shift *not identified yet !*

$$\mathcal{R}(\vec{\bar{\epsilon}}(t)) = \begin{cases} \delta \bar{v}_\alpha(t,\vec{x}) & = \bar{\epsilon}_\alpha(t) \\ \delta \bar{p}(t,\vec{x}) & = v_\beta(t,\vec{x})\bar{\epsilon}_\beta(t) \end{cases}$$

LC, Delamotte, Wschebor, Phys. Rev. E **91** (2015)

infinite set of *local in time* exact Ward identities
for all vertices with one zero momentum

▶ constraints in the velocity field sector

$$\Gamma^{(2,1)}_{\alpha\beta\gamma}(\omega, \vec{q}=\vec{0}; \nu, \vec{p}) = -\frac{p^\alpha}{\omega}\Big(\Gamma^{(1,1)}_{\beta\gamma}(\omega+\nu,\vec{p}) - \Gamma^{(1,1)}_{\beta\gamma}(\nu,\vec{p})\Big)$$

$$\Gamma^{(2,2)}_{\alpha\beta\gamma\delta}(\omega,\vec{0},-\omega,\vec{0},\nu,\vec{p}) = \frac{p^\alpha p^\beta}{\omega^2}\Big[\Gamma^{(0,2)}_{\gamma\delta}(\nu+\omega,\vec{p}) - 2\Gamma^{(0,2)}_{\gamma\delta}(\nu,\vec{p}) + \Gamma^{(0,2)}_{\gamma\delta}(\nu-\omega,\vec{p})\Big]$$

Ingredient 1 : Symmetries of the NS field theory

- infinitesimal time-gauged galilean transformations

$$\mathcal{G}(\vec{\epsilon}(t)) = \begin{cases} \vec{x} \to \vec{x} + \vec{\epsilon}(t) \\ \vec{v} \to \vec{v} - \dot{\vec{\epsilon}}(t) \end{cases}$$

- infinitesimal time-gauged response field shift *not identified yet !*

$$\mathcal{R}(\vec{\bar{\epsilon}}(t)) = \begin{cases} \delta \bar{v}_\alpha(t,\vec{x}) & = \bar{\epsilon}_\alpha(t) \\ \delta \bar{p}(t,\vec{x}) & = v_\beta(t,\vec{x})\bar{\epsilon}_\beta(t) \end{cases}$$

LC, Delamotte, Wschebor, Phys. Rev. E **91** (2015)

infinite set of *local in time* exact Ward identities
for all vertices with one zero momentum

▶ constraints in the response velocity field sector

$$\Gamma^{(2,1)}_{\alpha\beta\gamma}(\nu,\vec{p};-\nu-\omega,-\vec{p};\omega,\vec{q}=\vec{0}) = ip_\alpha\delta_{\beta\gamma} - ip^\beta\delta\alpha\gamma$$

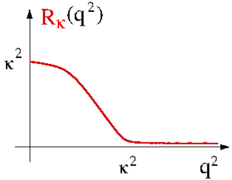
$$\Gamma^{(2,2)}_{\alpha\beta\gamma\delta}(\nu,\vec{p},-\nu,-\vec{p},\omega,\vec{0},-\omega,\vec{0}) = 0$$

Ingredient 2 : limit of large wave-numbers

Wetterich's equation for the 2-point functions

$$\partial_\kappa \Gamma^{(2)}_{\kappa,ij}(\nu,\vec{p}) \quad = \quad \mathrm{Tr}\int_{\omega,\vec{q}} \partial_\kappa \mathcal{R}_\kappa(\vec{q}) \cdot G_\kappa(\omega,\vec{q}) \cdot \Big(-\frac{1}{2}\, \Gamma^{(4)}_{\kappa,ij}(\nu,\vec{p};-\nu,-\vec{p};\omega,\vec{q})$$

$$+\Gamma^{(3)}_{\kappa,i}(\nu,\vec{p};\omega,\vec{q}) \cdot G_\kappa(\nu+\omega,\vec{p}+\vec{q}) \cdot \Gamma^{(3)}_{\kappa,j}(-\omega,-\vec{q};\nu+\omega,\vec{p}+\vec{q}) \Big) \cdot G_\kappa(\omega,\vec{q})$$



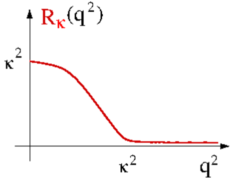
regime of large wave-vector $|\vec{p}| \gg \kappa$

$\Longrightarrow$ $|\vec{q}| \ll |\vec{p}|$ negligible

Ingredient 2 : limit of large wave-numbers

Wetterich's equation for the 2-point functions

$$\partial_\kappa \Gamma^{(2)}_{\kappa,ij}(\nu,\vec{p}) \;=\; \mathrm{Tr}\int_{\omega,\vec{q}} \partial_\kappa \mathcal{R}_\kappa(\vec{q}) \cdot G_\kappa(\omega,\vec{q}) \cdot \Big(-\frac{1}{2}\,\Gamma^{(4)}_{\kappa,ij}(\nu,\vec{p};-\nu,-\vec{p};\omega,\vec{0})$$
$$+\Gamma^{(3)}_{\kappa,i}(\nu,\vec{p};\omega,\vec{0}) \cdot G_\kappa(\nu+\omega,\vec{p}+\vec{q}) \cdot \Gamma^{(3)}_{\kappa,j}(-\omega,-\vec{0};\nu+\omega,\vec{p}+\vec{0})\Big) \cdot G_\kappa(\omega,\vec{q})$$



regime of large wave-vector $|\vec{p}| \gg \kappa$

$\Longrightarrow$ $|\vec{q}| \ll |\vec{p}|$ negligible

- set $\vec{q} = 0$ in all vertices
- close with Ward identities

LC, Delamotte, Wschebor, PRE **93** (2016)

Exact flow equations in the large wave-number limit

exact equation for $C_\kappa(\nu, \vec{p})$ when $|\vec{p}| \gg \kappa$

$$\kappa\partial_\kappa C_\kappa(\nu,\vec{p}) = -2k^2 \int_\omega \frac{C_\kappa(\nu,\vec{p}) - C_\kappa(\nu+\omega,\vec{p})}{\omega^2}\, J_\kappa(\omega)$$

$$J_\kappa(\omega) = -\frac{1}{3}\int_{\vec{q}} \Big\{ 2\partial_s N_s(\vec{q})\,|G_\kappa(\omega,\vec{q})|^2 - 2\partial_s R_s(\vec{q})\, C_\kappa(\omega,\vec{q})\Re G_\kappa(\omega,\vec{q})\Big\}$$

- structure fonction

$$\kappa\partial_\kappa S_\kappa(\vec{p}) = \int_\nu \kappa\partial_\kappa C_\kappa(\nu,\vec{p}) = 0$$

intermittency effects are sub-leading in $\vec{p}$

$\Longrightarrow$ Kolomogorov scaling $\zeta_2 = 2/3$

- time dependence

$$\lim_{|\vec{p}|\to\infty} \frac{\kappa\partial_\kappa C_\kappa(\nu,\vec{p})}{C_\kappa(\nu,\vec{p})} \neq 0$$

violation of scale invariance
$\neq$ critical phenomena

$\Longrightarrow$ intermittency effects are dominant !

Exact flow equations in the large wave-number limit

exact equation for $C_\kappa(\nu, \vec{p})$ when $|\vec{p}| \gg \kappa$

$$\kappa\partial_\kappa C_\kappa(\nu, \vec{p}) = -2k^2 \int_\omega \frac{C_\kappa(\nu, \vec{p}) - C_\kappa(\nu + \omega, \vec{p})}{\omega^2} \, J_\kappa(\omega)$$

$$J_\kappa(\omega) = -\frac{1}{3} \int_{\vec{q}} \Big\{ 2\partial_s N_s(\vec{q}) \, |G_\kappa(\omega, \vec{q})|^2 - 2\partial_s R_s(\vec{q}) \, C_\kappa(\omega, \vec{q}) \Re G_\kappa(\omega, \vec{q}) \Big\}$$

- structure fonction

$$\kappa\partial_\kappa S_\kappa(\vec{p}) = \int_\nu \kappa\partial_\kappa C_\kappa(\nu, \vec{p}) = 0$$

intermittency effects are sub-leading in $\vec{p}$

$\Longrightarrow$ Kolomogorov scaling $\zeta_2 = 2/3$

- time dependence

$$\lim_{|\vec{p}| \to \infty} \frac{\kappa\partial_\kappa C_\kappa(\nu, \vec{p})}{C_\kappa(\nu, \vec{p})} \neq 0$$

violation of scale invariance

$\neq$ critical phenomena

$\Longrightarrow$ intermittency effects are dominant !

Exact flow equations in the large wave-number limit

exact equation for $C_\kappa(\nu, \vec{p})$ when $|\vec{p}| \gg \kappa$ and large $\nu \gg \kappa$

$$\kappa\partial_\kappa C_\kappa(\nu, \vec{p}) = -I_\kappa \; p^2 \, \partial_\nu^2 \, C_\kappa(\nu, \vec{p})$$

$$\kappa\partial_\kappa C_\kappa(t, \vec{p}) = I_\kappa \; p^2 \, t^2 \, C_\kappa(t, \vec{p})$$

with $\quad I_\kappa = \int_\omega J_\kappa(\omega) \to I_* \quad$ a pure number at the fixed point

LC, Delamotte, Wschebor, PRE **93** (2016)

exact analytical solution of the fixed-point equation

two regimes :

- small time differences t : behavior in the inertial range
- limit $t \to 0$: behavior in the dissipative range

LC, Rossetto, Wschebor, Balarac, PRE **95** (2017)

Solution in the inertial range

- analytical solution in the inertial range

$$C(t,k) = c_C \frac{\epsilon^{2/3}}{k^{11/3}} \exp\big(-\tilde{\alpha} k^2 t^2\big) \qquad \tilde{\alpha} = \frac{3}{2} l_* \gamma \epsilon^{2/3} \eta^{2/3} \sqrt{\mathrm{Re}}$$

- kinetic energy spectrum (in wave-vector)

$$E(k) = 4\pi k^2 C(t=0,k) \propto k^{-5/3}$$ K41 scaling, no intermittency

- kinetic energy spectrum (in frequency)

$$E(\omega) = 4\pi \int_0^\infty k^2 C(\omega,k) dk \propto \omega^{-5/3}$$ intermittency !

$\Longrightarrow$ sweeping effect ! (random Taylor hypothesis Tennekes, J. Fluid Mech. **67** (1975))

standard scaling theory with $z = 2/3 \Longrightarrow E(\omega) \propto \omega^{-2}$

observed for Lagrangian velocities, but not Eulerian ones

Chevillard, Roux, Lévêque, Mordant, Pinton, Arnéodo, PRL **95** (2005)

Solution in the inertial range

- analytical solution in the inertial range

$$C(t,k) = c_C \frac{\epsilon^{2/3}}{k^{11/3}} \exp\left(-\tilde{\alpha} k^2 t^2\right) \qquad \tilde{\alpha} = \frac{3}{2} l_* \gamma \epsilon^{2/3} \eta^{2/3} \sqrt{\mathrm{Re}}$$

- kinetic energy spectrum (in wave-vector)

$$E(k) = 4\pi k^2 C(t=0,k) \propto k^{-5/3}$$ K41 scaling, no intermittency

- kinetic energy spectrum (in frequency)

$$E(\omega) = 4\pi \int_0^\infty k^2 C(\omega,k) dk \propto \omega^{-5/3}$$ intermittency !

$\Longrightarrow$ sweeping effect ! (random Taylor hypothesis Tennekes, J. Fluid Mech. **67** (1975))

standard scaling theory with $z = 2/3 \Longrightarrow E(\omega) \propto \omega^{-2}$

observed for Lagrangian velocities, but not Eulerian ones

Chevillard, Roux, Lévêque, Mordant, Pinton, Arnéodo, PRL **95** (2005)

Solution in the inertial range

- analytical solution in the inertial range

$$C(t,k) = c_C \frac{\epsilon^{2/3}}{k^{11/3}} \exp\left(-\tilde{\alpha} k^2 t^2\right) \qquad \tilde{\alpha} = \frac{3}{2} l_* \gamma \epsilon^{2/3} \eta^{2/3} \sqrt{\mathrm{Re}}$$

- kinetic energy spectrum (in wave-vector)

$$E(k) = 4\pi k^2 C(t=0,k) \propto k^{-5/3}$$ K41 scaling, no intermittency

- kinetic energy spectrum (in frequency)

$$E(\omega) = 4\pi \int_0^\infty k^2 C(\omega,k) dk \propto \omega^{-5/3}$$ intermittency !

$\Longrightarrow$ sweeping effect ! (random Taylor hypothesis Tennekes, J. Fluid Mech. **67** (1975))

standard scaling theory with $z = 2/3 \Longrightarrow E(\omega) \propto \omega^{-2}$

observed for Lagrangian velocities, but not Eulerian ones

Chevillard, Roux, Lévêque, Mordant, Pinton, Arnéodo, PRL **95** (2005)

Solution in the inertial range

- analytical solution in the inertial range

$$C(t,k) = c_C \frac{\epsilon^{2/3}}{k^{11/3}} \exp\left(-\tilde{\alpha} k^2 t^2\right) \qquad \tilde{\alpha} = \frac{3}{2} l_* \gamma \epsilon^{2/3} \eta^{2/3} \sqrt{\mathrm{Re}}$$

- kinetic energy spectrum (in wave-vector)

$$E(k) = 4\pi k^2 C(t=0,k) \propto k^{-5/3}$$ K41 scaling, no intermittency

- kinetic energy spectrum (in frequency)

$$E(\omega) = 4\pi \int_0^\infty k^2 C(\omega,k) dk \propto \omega^{-5/3}$$ intermittency !

$\Longrightarrow$ sweeping effect ! (random Taylor hypothesis Tennekes, J. Fluid Mech. **67** (1975))

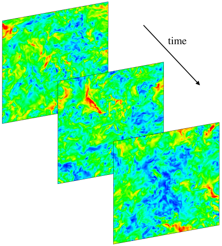
standard scaling theory with $z = 2/3 \Longrightarrow E(\omega) \propto \omega^{-2}$

observed for Lagrangian velocities, but not Eulerian ones

Chevillard, Roux, Lévêque, Mordant, Pinton, Arnéodo, PRL **95** (2005)

Solution in the inertial range : Time dependence

numerical data



- our simulations

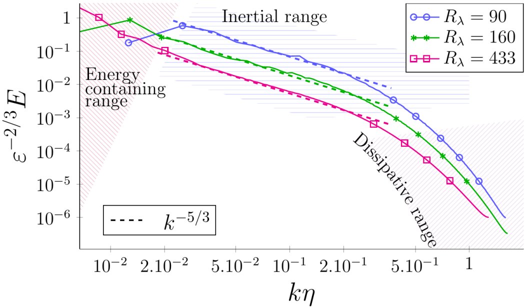
based on pseudo-spectral code

Lagaert, Balarac, Cottet,
J. Comp. Phys. **260** (2014)

- JHTBD

Johns Hopkins TurBulence Database

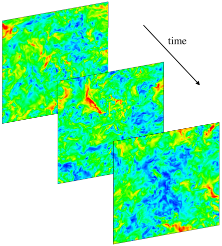
http ://turbulence.pha.jhu.edu/

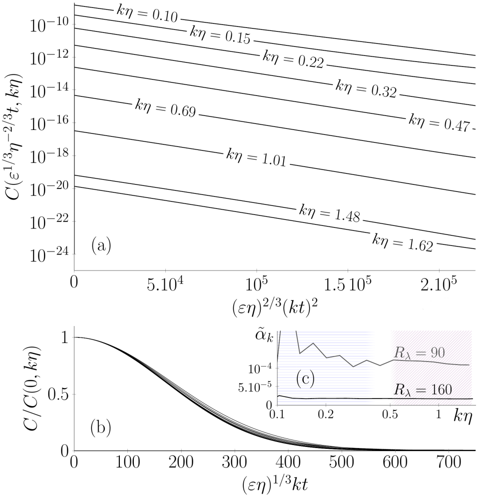


LC, Rossetto, Wschebor, Balarac, PRE **95** (2017)

Solution in the inertial range : Time dependence

numerical data





analytical prediction

$$C(t,k) \propto \frac{\exp(-\tilde{\alpha}k^2t^2)}{k^{11/3}}$$

Behavior in the dissipative range

behavior of the solution in the dissipative range

regime of $p \gg \kappa$, $t \to 0$, but $tp^{2/3} \to \epsilon^{1/3}\tau L^{-2/3} = \eta^{2/3}L^{-2/3}$

$$C(t \to 0, k) = c_C \frac{\epsilon^{2/3}}{k^{11/3}} \exp\left[-\hat{\alpha}\,\eta^{4/3}L^{-2/3}k^{2/3}\right] = c_C \frac{\epsilon^{2/3}}{k^{11/3}} \exp\left[-\hat{\alpha}\lambda^{2/3}k^{2/3}\right]$$

- kinetic energy spectrum

$$E(k) \propto \frac{\epsilon^{2/3}}{k^{5/3}} \exp\left[-\mu(\lambda k)^{2/3}\right] \qquad \lambda \text{ Taylor scale}$$

several empirical propositions $\exp[-ck^{\gamma}]$ with $\gamma = 3/2,\ 4/3,\ 2,\ldots$

Monin and Yaglom, *Statistical Fluid Mechanics : Mechanics of Turbulence* (1973)

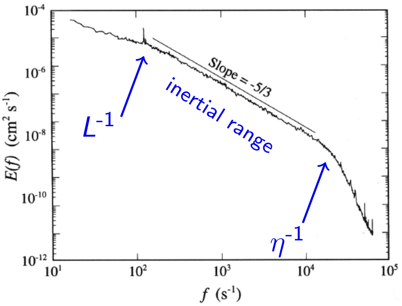
common wisdom : approximately exponential decay

Behavior in the dissipative range

behavior of the solution in the dissipative range

- kinetic energy spectrum

$$E(k) \propto \frac{\epsilon^{2/3}}{k^{5/3}} \exp\left[-\mu(\lambda k)^{2/3}\right]$$

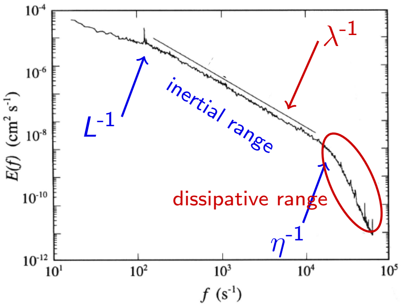


Behavior in the dissipative range

behavior of the solution in the dissipative range

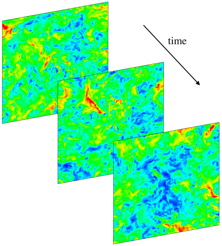
- kinetic energy spectrum

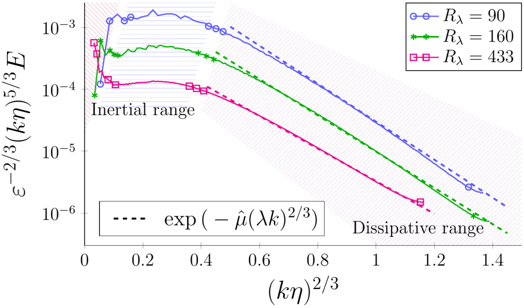
$$E(k) \propto \frac{\epsilon^{2/3}}{k^{5/3}} \exp\left[-\mu(\lambda k)^{2/3}\right]$$



Behavior in the dissipative range

numerical data





analytical prediction

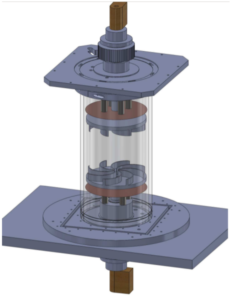
$$E(k) \propto \frac{\exp(-\mu(\lambda k)^{2/3})}{k^{5/3}}$$

LC, Rossetto, Wschebor, Balarac, PRE **95** (2017)

Behavior in the dissipative range

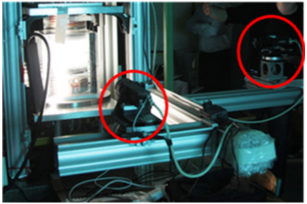
experimental data : SPHYNX team, Iramis/SPEC (CEA/CNRS)

von Kármán swirling flow



PhD Brice Saint-Michel (2013)

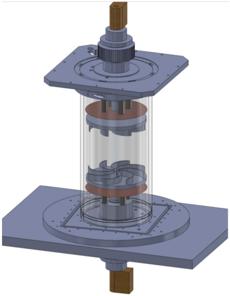
PIV : particle image velocimetry



Behavior in the dissipative range

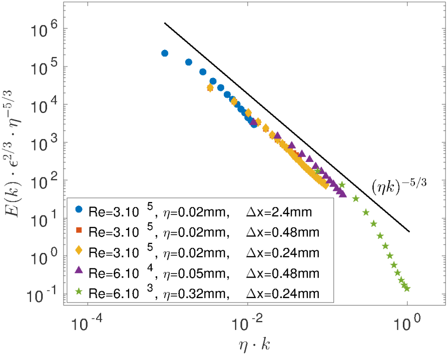
experimental data : SPHYNX team, Iramis/SPEC (CEA/CNRS)

von Kármán swirling flow



PhD Brice Saint-Michel (2013)

kinetic energy spectrum

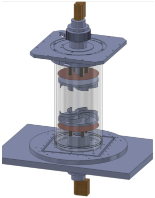


PhD Paul Dubue (in preparation)

Behavior in the dissipative range

experimental data : SPHYNX team, Iramis/SPEC (CEA/CNRS)

von Kármán swirling flow

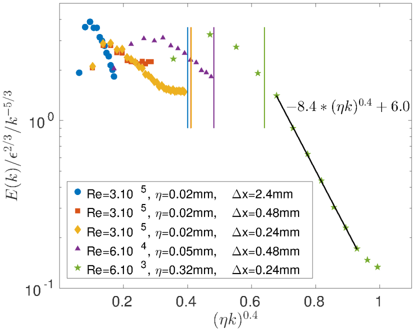


PhD Brice Saint-Michel (2013)

analytical prediction

$$E(k) \propto \frac{\exp(-\mu(\lambda k)^{2/3})}{k^{5/3}}$$

kinetic energy spectrum



Dubue, Kuzzay, Saw, Daviaud, Dubrulle, LC, Rossetto (2017)

Generalisation to n-point correlation functions

exact flow equation for all $C_\kappa^{(n)}(\nu_i, \vec{k}_i)$ when $|\vec{k}_i| \gg \kappa$

see poster by
Malo Tarpin

EXACT FLOW EQUATIONS FOR HIGH ORDER CORRELATION FUNCTIONS IN FULLY DEVELOPED TURBULENCE

Malo Tarpin[1], Léonie Canet[1], Nicolás Wschebor[2]

[1] Université Grenoble Alpes and CNRS, LPMMC, UMR 5493, 38042 Grenoble, France
[2] Instituto de Física, Facultad de Ingeniería, Universidad de la República, J.H.y Reissig 565, 11000 Montevideo, Uruguay

$$\kappa\partial_\kappa C_\kappa^{(n)}(\nu_i, \vec{k}_i) = -\sum_{\ell=1}^{n-1} \int_\omega \frac{2}{\omega^2} \Big[C_\kappa^{(n)}(\nu_i, \vec{k}_i; \nu_\ell + \omega, \vec{k}_\ell) - C_\kappa^{(n)}(\nu_i, \vec{k}_i) \Big] J_\kappa(\omega)$$

in progress ...

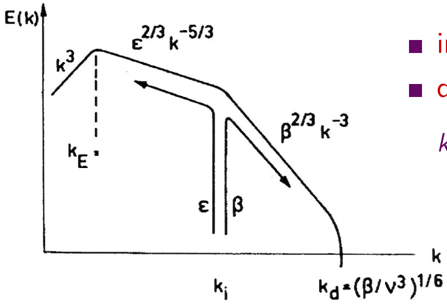
- form of fixed point solutions at large ν_i
- n^{th}-order structure functions $S_n(\ell)$

Tarpin, LC, Wschebor, in preparation (2017)

Bi-dimensional turbulence

two conserved quantities : energy and enstrophy

$\epsilon = \langle \vec{f} \cdot \vec{v} \rangle$: energy injection rate
$\beta = \langle (\vec{\nabla} \times \vec{f}) \cdot (\vec{\nabla} \times \vec{v}) \rangle$: enstrophy injection rate



- inverse energy cascade
- direct enstrophy cascade

  k^{-3} : Kraichnan - Batchelor theory

Lesieur, Turbulence in Fluids, Springer

Bi-dimensional turbulence

two-point correlation function

limit of large wave-number

$\Longrightarrow$ direct cascade

$$C(t,k) = c_C\, \beta^{2/3}\, k^{-4}\, \exp(-\tilde{\alpha} t^2 k^2)$$

$$\tilde{\alpha} = \gamma' \hat{I}_* \beta^{2/3}$$

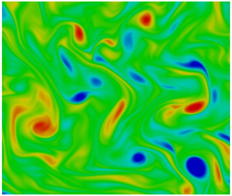
- kinetic energy spectrum

$$E(k) = 2\pi k C(0,k) \propto \beta^{2/3} k^{-3}$$

- dissipative range

$$E(k) \propto \beta^{2/3} k^{-3} \exp(-\mu k^2)$$

numerical data



in progress...

LC, Rossetto, Balarac,

in preparation (2017)

Conclusion and perspectives

conclusion

- symmetry-based closure exact at large wave-numbers
- predictions beyond K41 confirmed by numerical data

numerical solution for $C(\omega, \vec{k})$

in three dimensions :

- intermittency exponent
- nonuniversal constants

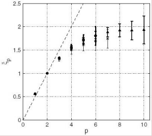
numerical solution for $C(\omega, \vec{k})$

in two dimensions :

- inverse cascade
- finite size effects

structure functions

- derive flow equations for $S_p(\ell)$ at sub-leading order
- intermittency exponents ξ_p




Thank you for attention !