Quantum gravity and Standard-Model-like fermions

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Motivation & Introduction

- search for observational tests of quantum gravity
- direct observation of, e.g., graviton scattering is difficult
- indirect tests of quantum gravity theories are more feasible
- compatibility of quantum gravity and (existing) matter

- light fermions $\leftrightarrow$ chiral symmetry
- chiral symmetry forbids mass term $m_\psi \bar{\psi}\psi$
- masses are generated by Yukawa interactions and chiral symmetry breaking in QCD
- masses are generated far below the Planck scale

$\Rightarrow$ Is chiral symmetry an **observational** constraint for **quantum gravity**?

$\Rightarrow$ Is chiral symmetry implied by **quantum gravity**?

Non-minimally coupled Fermions

- in the asymptotic safety scenario we need a (UV) fixed point for the action ($[G_{\mathrm{N}}] = -2$)
- the beta functions of the gravitational sector depend on the matter content and the respective symmetries

$\Rightarrow$ putting the "wrong" matter can destroy the fixed point and thereby the asymptotic safety scenario

- see what happens if we break chiral symmetry explicitly

- choose truncation according to canonical dimension (polynomial and derivative expansion)

$$\Gamma_k = \Gamma_{\mathrm{grav}} + Z_\psi \int_x \bar{\psi}^i \not\nabla \psi^i + \bar{m}_\psi \int_x \bar{\psi}^i \psi^i + \bar{\xi} \int_x R \bar{\psi}^i \psi^i + \bar{\zeta} \int_x \bar{\psi}^i \nabla^2 \psi^i$$

- kinetic term $\bar{\psi}^i \not\nabla \psi^i = \bar{\psi}^i_{\mathrm{L}} \not\nabla \psi^i_{\mathrm{L}} + \bar{\psi}^i_{\mathrm{R}} \not\nabla \psi^i_{\mathrm{R}}$
features $U(N_f)_{\mathrm{L}} \times U(N_f)_{\mathrm{R}}$

- mass term $\bar{\psi}^i \psi^i = \bar{\psi}^i_{\mathrm{L}} \psi^i_{\mathrm{R}} + \bar{\psi}^i_{\mathrm{R}} \psi^i_{\mathrm{L}}$
breaks $U(N_f)_{\mathrm{L}} \times U(N_f)_{\mathrm{R}} \to U(N_f)$

some technicalities I:

- choose Litim-type regulator
- background field approximation and Einstein-Hilbert truncation

$$\Gamma_{\mathrm{grav}} = \frac{-1}{16\pi G} \int_x (R - 2\bar{\lambda}) + S_{\mathrm{gf}} + S_{\mathrm{gh}}$$

- metric split: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{16\pi G}\, h_{\mu\nu} + \frac{\tau}{2}\, 16\pi G\, h_{\mu\rho} h^{\rho}{}_{\nu}$ $(\eta_h = 0)$
- gauge fixing:

$$S_{\mathrm{gf}} = \frac{1}{32\pi\alpha} \int_x \bar{g}_{\mu\nu} F^{\mu} F^{\nu}, \qquad F^{\mu} = \left(\bar{g}^{\mu\lambda} \bar{D}^{\kappa} - \frac{1+\beta}{4} \bar{g}^{\lambda\kappa} \bar{D}^{\mu} \right) h_{\lambda\kappa}$$

- Landau gauge (hard gauge fixing): $\alpha \to 0$
- neglect ghost interactions with the other fields $(\eta_c = 0)$

some technicalities II:

- for the fermion covariant derivative $\nabla_\mu \psi^i = \partial_\mu \psi^i + \Gamma_\mu \psi^i$ we use the spinbase formalism

$$\partial_\mu \gamma^\nu + \left\{ {\nu \atop \mu\rho} \right\} \gamma^\rho + [\Gamma_\mu, \gamma^\nu] = 0, \qquad \mathrm{tr}\, \Gamma_\mu = 0$$

- need to calculate (in a curved space)

$$G_{\mathrm{grav}} = (\Gamma^{(2)}_{\mathrm{grav}} + \mathcal{R}_k)^{-1}$$

$\Rightarrow$ can be done within a curvature expansion:

$$G_{\mathrm{grav}} \simeq {}^{(n)}\tilde{G}_{\mathrm{grav}}, \qquad {}^{(n)}\tilde{G}_{\mathrm{grav}}\,(\Gamma^{(2)}_{\mathrm{grav}} + \mathcal{R}_k) = \mathbb{1} + \mathcal{O}(R^{n+1})$$

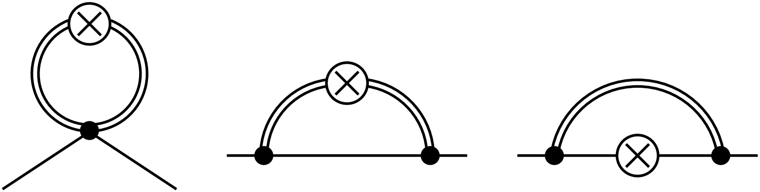
- employ the flow equation

$$\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[(\Gamma_k^{(2)} + \mathcal{R}_k)^{-1}\partial_k \mathcal{R}_k\right]$$

- the flow of the dimensionless fermionic couplings

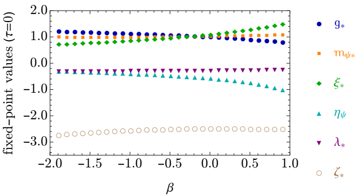
$$\bar{m}_\psi = Z_\psi k\, m_\psi, \qquad \bar{\xi} = \frac{Z_\psi}{k}\,\xi, \qquad \bar{\zeta} = \frac{Z_\psi}{k}\,\zeta$$

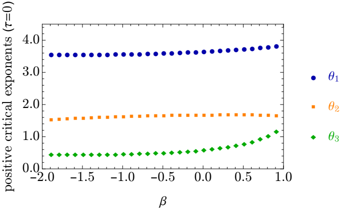
is driven by three diagrams



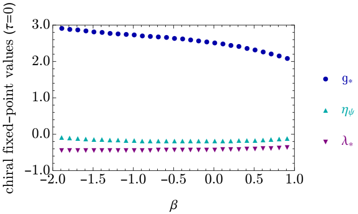
AS with heavy and light Fermions

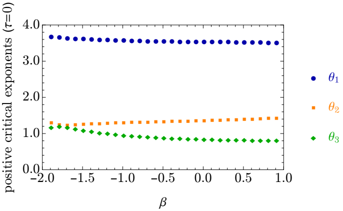
Non-Chiral Fixed Point ($N_f = 1$)





Chiral Fixed Point ($N_f = 1$)





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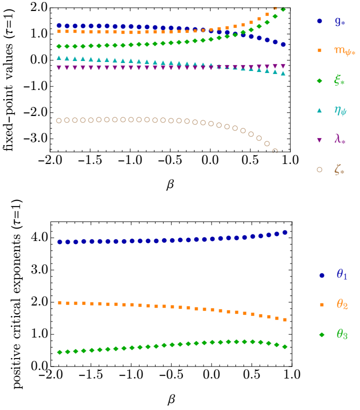
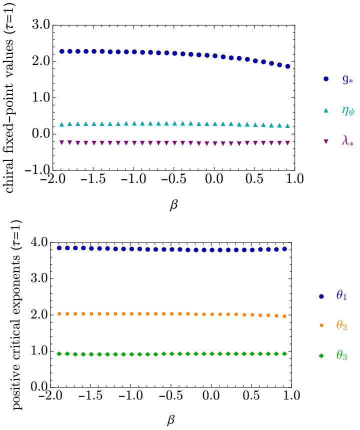
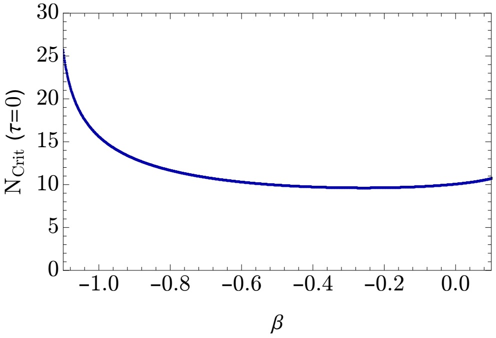


Chiral Fixed Point ($N_f = 1$)





Note: gauge and parametrization dependence increases for larger N_f

$\Rightarrow$ maybe chiral higher terms become important, e.g, $R\bar{\psi}^i \not{\nabla} \psi^i$

$\Rightarrow$ maybe the background approximation breaks down

- observe two fixed points (chiral and non-chiral)
- could lead to a UV completion of a fermionic dark matter model
- N_f chiral fermions and N_d non-chiral (dark) fermions
- example: $N_f = N_d = 1$ $(\beta = 0,\ \tau = 0)$

$$
\begin{aligned}
&g_* = 1.36, && \lambda_* = -0.44, && && \\
&m_{d*} = 0.59, && \xi_{d*} = 0.97, && \zeta_{d*} = 0.97, && \eta_d = -0.38, \\
&m_{f*} = 0, && \xi_{f*} = 0, && \zeta_{f*} = 0, && \eta_f = -0.09
\end{aligned}
$$

- four critical exponents corresponding to the four (canonically) relevant operators

$$
\sqrt{g}, \qquad \sqrt{g}R, \qquad \sqrt{g}\bar{\psi}_f\psi_f, \qquad \sqrt{g}\bar{\psi}_d\psi_d
$$

- for generic quantum gravity models (outside) AS we can use the effective field theory picture
- analogous to:
  microscopic model $\rightarrow$ effective low energy degrees of freedom

$\Rightarrow$ Is chiral symmetry attractive?

- treat g and λ as external parameters
- investigate the stability matrix $-\frac{\partial \beta_{g_i}}{\partial g_j}$ at the chiral fixed point
- the stability matrix has always an IR-repulsive direction for $g \in (0, 30)$, $\lambda \in (-2, 0.4)$

$\Rightarrow$ Is chiral symmetry attractive? **No!**

- chiral symmetry (or an analogue) has to be enforced on the microscopic level

Note: chiral symmetry is a delicate business on the lattice

Summary

- analysis of gravity-matter systems can rule out quantum gravity models with contemporary measurements
- asymptotic safety could imply chiral symmetry in the UV
- analysis of gravity-matter systems is technically **very** challenging
- at this stage dependence on details seems rather strong

Thank you for your attention!