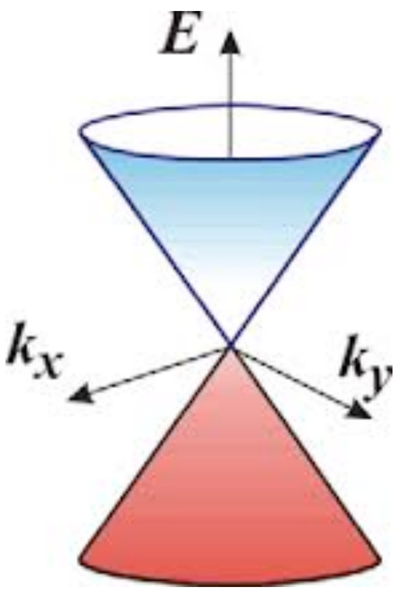
Two dimensional metals from disordered QED$_3$

Srinivas Raghu (Stanford)

Pallab Goswami, Hart Goldman and SR, arXiv:1701.07828



Motivation: Presence of quantum diffusion in two dimensions?

Perfect metals, metals, and insulators

In this talk, we will use the following classification (N. Mott):

dc conductivity: $$\sigma_{dc} = \lim_{T \to 0} \lim_{\omega \to 0} \sigma(\omega, T)$$

If σ_{dc} is:

(i) infinite: Perfect metal, superconductor.

(ii) finite: metal.

(iii) zero: insulator.

From perfect to diffusive metals

Let us start with a perfect metal:

$$S = \int d^d x d\tau \psi^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \psi + \cdots$$

There is no lattice and there are no impurities: $\sigma_{dc} = \infty$

For the moment we ignore interactions. To this system, add disorder:

$$S_{dirt} = \int d^d x d\tau V(x) \psi^\dagger(x,\tau) \psi(x,\tau)$$

From perfect to diffusive metals

The disorder is specified by moments of a disorder distribution:

e.g. $$\overline{V(x)} = 0$$

$$\overline{V(x)V(x')} = \Delta\delta^{(d)}(x - x')$$

Naive expectation: since V is a chemical potential, it has dimension 1.

$$[V] = 1 \qquad \Rightarrow \qquad [\Delta] = 2 - d$$

So you might have guessed that the perfect metal is stable when $d > 2$.

However, this is false! Where did we go wrong?

From perfect to diffusive metals

The previous argument missed the finite DOS at the Fermi energy.

Fermi's Golden Rule:

$$\frac{1}{\tau} \sim V^2 \rho \sim \Delta \frac{k_F^{d-1}}{v_F} \qquad [1/\tau] = 1$$

The finite DOS introduces a new scale below which the perfect metal is almost always destroyed.

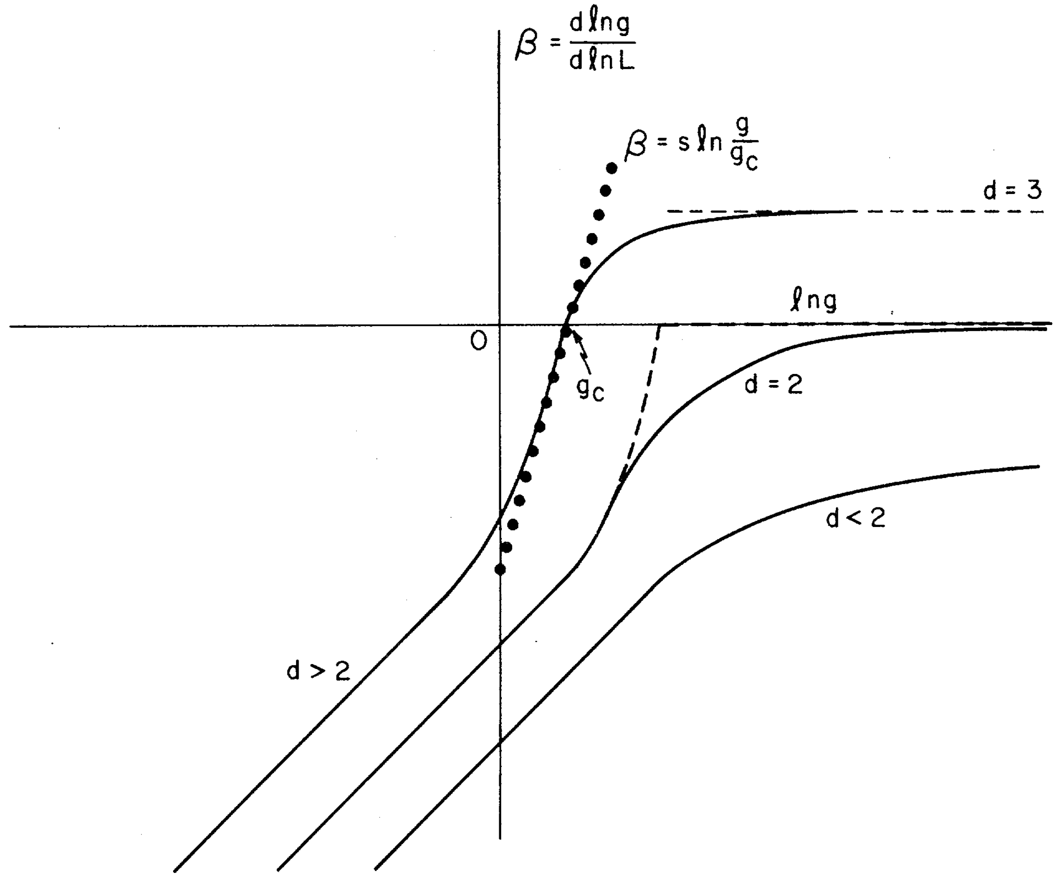
Instead of ballistic motion, we have quantum diffusion.

In a diffusive regime: we can have finite conductivity. What happens at T=0?

Absence of quantum diffusion in 2d

F. J. Wegner, *Z. Phys. B* **25**, 327 (1976).

E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, PRL **42**, 673 (1979).



G(L)=dc conductance

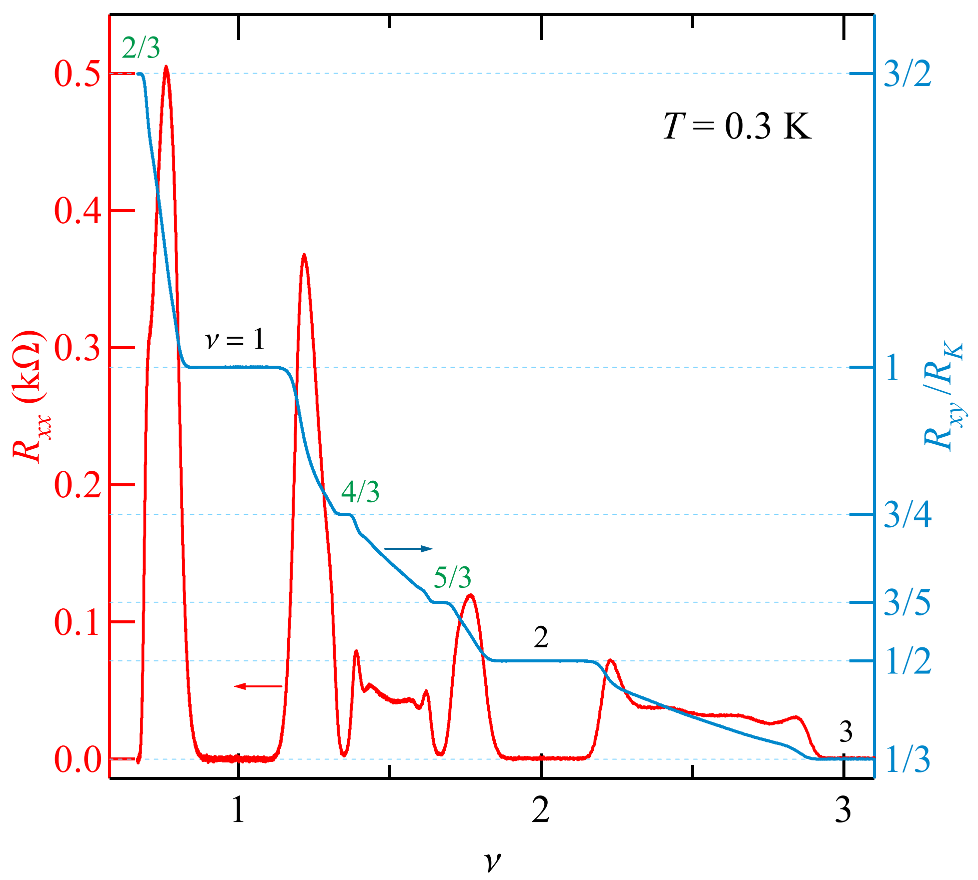
$$g(L) = G(L)\frac{h}{e^2}$$

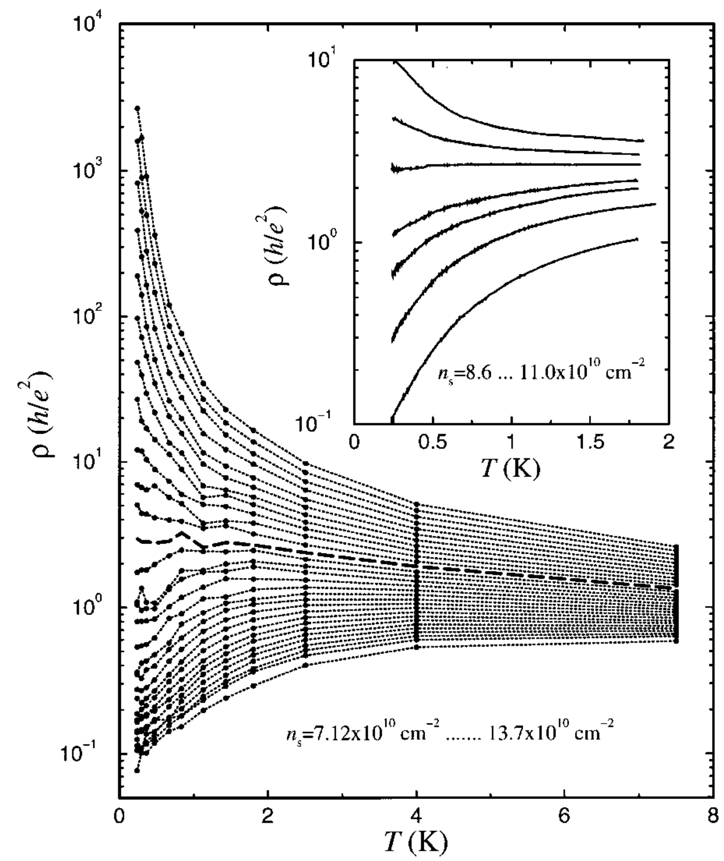
Systems without spin-orbit coupling are never metals in 2d at T=0.

Absence of quantum diffusion in 2d?

Can strong interactions alter this conclusion?

Some experimental evidence for 2d metals:





But this question can only be settled by theory!

Metallic phases in systems with vanishing density of states

We consider metallic phases in systems with vanishing DOS.

Such systems can still have a finite DC conductivity: hence ‘metals’.

Helpful example: Graphene + 1/r Coulomb interactions.

$$\frac{1}{\tau} \sim \alpha^2 T \qquad DOS \sim T \qquad \sigma_{dc} \sim \frac{1}{\alpha} \qquad \alpha = e^2/v_F$$

This system is unstable to disorder -> disorder leads to a finite DOS and a vanishing conductivity.

In this talk, I show that Dirac fermions + strong gauge interactions can host metallic phases.

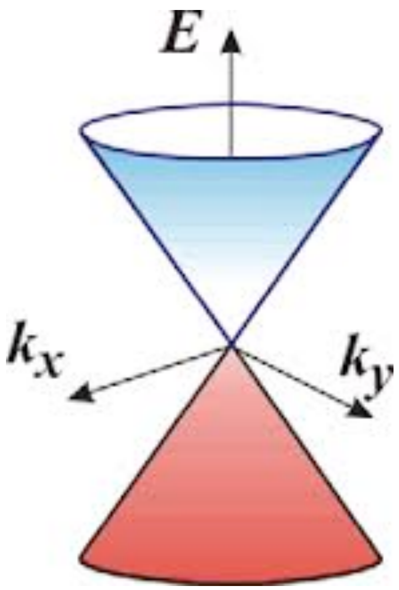
We will study QED_3 + disorder (solvable in large N limit).

Metallic phases of disordered QED_3

Main message of my talk:

1) QED_3 + potential disorder: clean metallic phase with irrelevant disorder and finite interaction strength.

2) QED_3 + mass disorder: dirty metallic phase with finite disorder and finite interaction strengths.



3) If time permits: I will construct non-perturbative examples of stable metals at small N, with finite DOS and without using the replica trick!

Some organizing principles

When is a non-interacting system with vanishing DOS stable to disorder?

Let us consider a slightly generalized disorder problem:

$$\overline{V(x)} = 0 \qquad \overline{V(x)V(x')} = \frac{\Delta}{|x - x'|^{\chi_0}}$$

Note: Gaussian white noise is realized when $\chi_0 = d.$

The clean system is stable to disorder when $\chi_0 > 2.$

Some organizing principles

Next consider the stability of an interacting system with vanishing DOS.

$$\overline{V(x)} = 0 \qquad \overline{V(x)V(x')} = \frac{\Delta}{|x - x'|^{\chi_{int}}}$$

The interacting system can renormalize (or screen) disorder correlations. Let us define

$$\chi_{int} = \chi_0 - 2\eta$$

The clean system is stable to disorder for any d when $\chi_{int} > 2.$

Some organizing principles

$$\chi_{int} = \chi_0 - 2\eta$$

The “anomalous dimension” of disorder correlations has two sources:

(i) Screening of disorder by strong interactions.

(ii) Anomalous dimension effects - provided disorder couples to a <u>non-conserved</u> operator (*e.g.* mass).

Conserved quantities like charge density are protected from anomalous dimension effects but not from screening effects.

Some organizing principles

$$\chi_{int} = \chi_0 - 2\eta$$

Two interesting possibilities are logically possible:

$$(i)\ \chi_{int} > 2 > \chi_0 :$$

In this case, the interacting system is stable to disorder while the non-interacting counter part is unstable.

QED_3 + potential disorder at large N.

$$(ii)\ \chi_{int} < 2 < \chi_0 :$$

Now the non-interacting system is stable but the interacting counterpart is unstable.

QED_3 + mass disorder at large N.

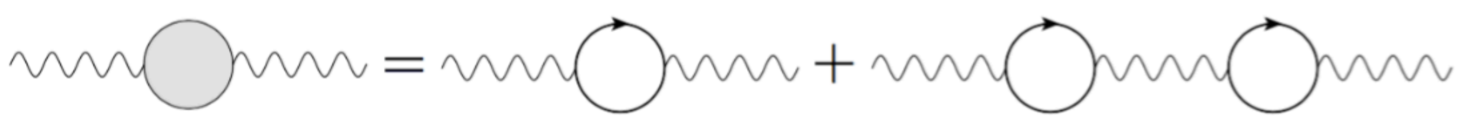
QED_3 + potential disorder: a clean metallic phase

QED$_3$ at large N

$$S_0 = \int d^2x d\tau \left[\bar{\Psi}_j \gamma_\mu D_\mu \Psi_j + \frac{1}{4} f_{\mu\nu}^2 \right]$$

$$j = 1 \cdots N \qquad D_\mu = \partial_\mu + iga_\mu \qquad f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$$

The large N limit: $N \to \infty, \alpha = g^2 N \to \text{constant}$



(RPA is exact).

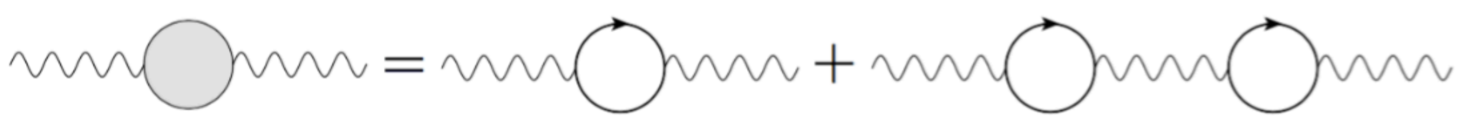
Dynamically screened photon: $$D_{\mu\nu} = \frac{\delta_{\mu\nu} - k_\mu k_\nu / k^2}{k^2 + \alpha k/8} \sim \frac{1}{k} \qquad (k \ll \alpha)$$

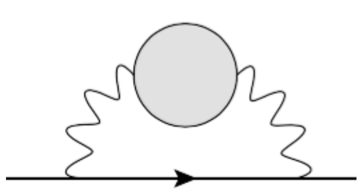
QED$_3$ at large N

$$S_0 = \int d^2x d\tau \left[\bar{\Psi}_j \gamma_\mu D_\mu \Psi_j + \frac{1}{4} f_{\mu\nu}^2 \right]$$

$$j = 1 \cdots N \qquad D_\mu = \partial_\mu + iga_\mu \qquad f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$$

The large N limit: $N \to \infty, \alpha = g^2 N \to \text{constant}$





Fermion anomalous dimension: $\eta_\psi \sim \mathcal{O}(1/N)$

QED_3 + disorder at large N

We add potential disorder to S_0

$$S_{dirt} = \int d^2x d\tau V(x) \psi_i^\dagger(x,\tau)\psi_i(x,\tau)$$

Gaussian white noise disorder:

$$\overline{V(x)} = 0$$

$$\overline{V(x)V(x')} = \Delta\delta^{(2)}(x-x')$$

Disorder averaging is done using the replica trick:

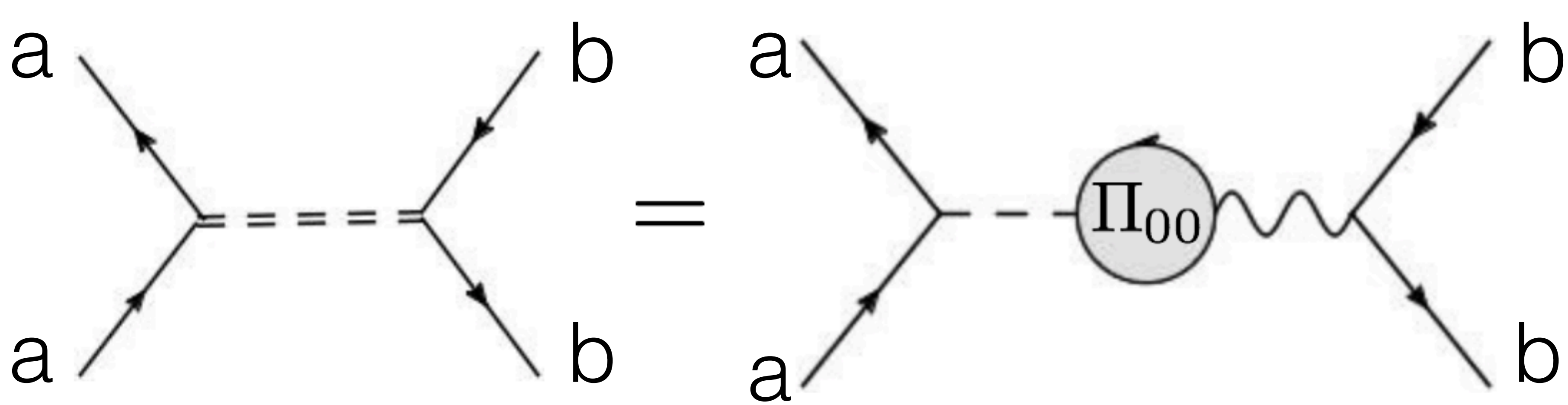
$$S_{dirt} = -\frac{\Delta}{2}\int d^2x d\tau d\tau' \bar{\psi}_{ia}\psi_{ia}(x,\tau)\psi^\dagger_{jb}\psi_{jb}(x,\tau')$$

$$a, b = 1\cdots n, \quad n \to 0$$

Screening of potential disorder

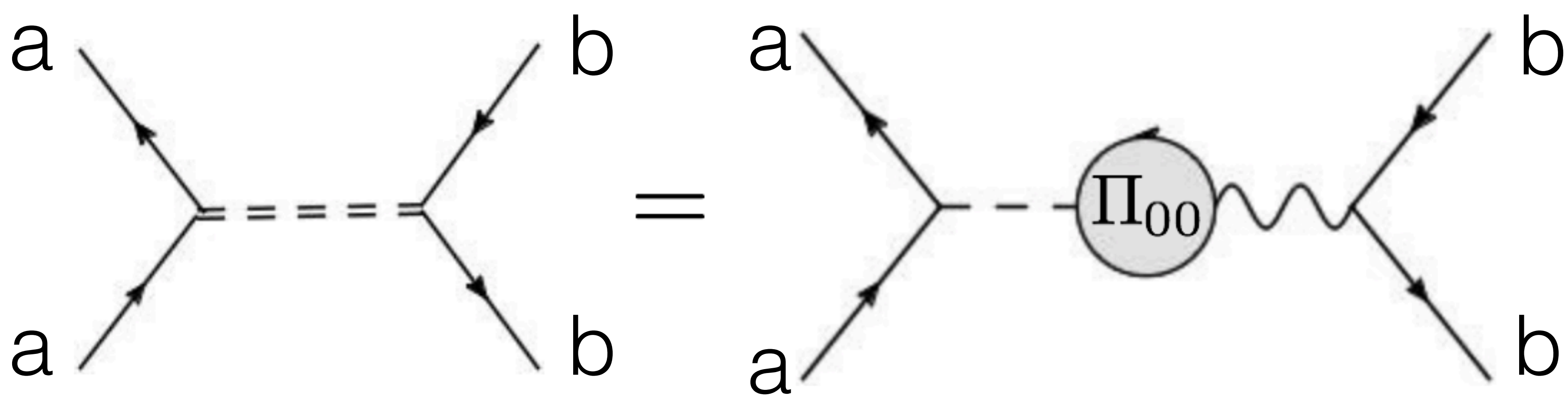
Potential disorder gets screened by interactions. This is similar to dynamical screening of the photon:

At leading order in large N, only one diagram survives the replica limit:



This reflects the renormalization of the disorder variance due to the a_0 fluctuations (which also couple to density).

Screening of potential disorder



As a result, the disorder variance at long distances becomes

$$\overline{V(\boldsymbol{k})V(-\boldsymbol{k})} = \frac{\Delta}{1 + 2\frac{\Pi_{00}(\boldsymbol{k},0)}{\boldsymbol{k}^2}}$$

$$\Rightarrow \quad \overline{V(x)V(x')} = \frac{\Delta}{|x - x'|^3}$$

Screening of potential disorder

Let us summarize. The non-interacting Dirac problem had

$$\overline{V(x)V(x')} = \Delta\delta^{(2)}(x - x') \quad \to \chi_0 = 2$$

By contrast, large N QED screened the disorder with

$$\overline{V(x)V(x')} = \frac{\Delta}{|x - x'|^3} \quad \to \chi_{int} = 3$$

Potential disorder is irrelevant at the large N QED3 fixed point:

$$[\Delta] = -1$$

This is our first example of a stable metallic phase.

Graphene vs QED$_3$

We may naively suppose that the QED$_3$ result is the same as graphene + 1/r interactions.

However, this is not true: transverse gauge fluctuations in QED$_3$ are crucial. Here are the differences.

graphene + 1/r interactions

$$\sigma \sim \frac{1}{\alpha}$$

$$\alpha \to 0$$

unstable to potential disorder

fixed lines in $\alpha - \Delta$ plane

QED$_3$

$$\sigma \sim \frac{1}{\alpha}$$

$$\alpha \to \mathcal{O}(1)$$

stable to potential disorder

fixed point in $\alpha - \Delta$ plane

QED_3 + mass disorder: a dirty metallic phase

QED$_3$ + mass disorder

We previously gave an example of a clean 2d metal.

We next show an example of a dirty metal with a finite disorder, finite interaction fixed point. This will occur with mass disorder:

Mass disorder in graphene: random staggered chemical potential.

$$\mu(x) = M(x)$$

$$\mu(x) = -M(x)$$

QED_3 + mass disorder

So, to the QED_3 Lagrangian, we add mass disorder:

$$S = S_0 + S_{dirt}$$

$$S_0 = \int d^2x d\tau \left[\bar{\Psi}_j \gamma_\mu D_\mu \Psi_j + \frac{1}{4} f_{\mu\nu}^2 \right]$$

$$S_{dirt} = \int d^2x d\tau M(x) \bar{\psi}_i(x,\tau) \psi_i(x,\tau)$$

$$S_{dirt} = \int d^2x d\tau M(x) \bar{\psi}_i(x,\tau) \psi_i(x,\tau)$$

For free 2d Diracs, mass disorder is <u>marginally irrelevant</u>.

But the mass is not a conserved object: it can have an anomalous dimension. In large N QED3, the anomalous dimension is known:

$$\eta_M \sim \alpha/N > 0$$

As a consequence, the interacting system is unstable to disorder whereas the non-interacting counterpart is stable.

This is analogous to the story of the Wilson-Fisher fixed point.

QED_3 + mass disorder: RG flows

After some exploration, we found that the simplest treatment of the mass disorder problem involves epsilon and 1/N expansions.

With $$\overline{M(x)} = 0, \quad \overline{M(x)M(x')} = \frac{\Delta_M}{|x - x'|^2}$$

This disorder is marginal for free fermions in <u>any</u> d. But due to anomalous dimension effects, it is now <u>slightly relevant</u>. We expand about

$$d = 3 - \epsilon \qquad\qquad \epsilon, 1/N \ll 1$$

And study the RG flow of the replicated action. We will set d=2 at the end.

QED_3 + mass disorder: RG flows

Since disorder badly breaks Lorentz invariance, there are several running couplings:

$$(i)\ z \qquad \text{(dynamical exponent)}$$

$$(ii)\ v/c$$

$$(iii)\ \bar{\alpha} = \frac{\alpha}{4\pi^2 v}\Lambda^{-\epsilon}$$

$$(iv)\ \bar{\Delta} = \frac{\Delta}{2\pi^2 v^2}$$

The RG flows are obtained with a dimensional regulator, setting c=1 and tracking the running of remaining couplings.

QED$_3$ + mass disorder: RG flows

At leading order the 1-loop RG flows are:

$$z = 1 + \frac{1}{3}\bar{\alpha}\left(1 - v^2\right)$$

$$\frac{dv}{d\ell} = v\left[-\frac{2}{3}\bar{\Delta} - \frac{\bar{\alpha}}{N}g_1(v) + \frac{1}{3}\bar{\alpha}\left(1 - v^2\right)\right]$$

$$\frac{d\bar{\alpha}}{d\ell} = \bar{\alpha}\left[\epsilon + \frac{2}{3}\bar{\Delta} - \frac{2}{3}\bar{\alpha} + \frac{\bar{\alpha}}{N}g_1(v)\right]$$

$$\frac{d\bar{\Delta}}{d\ell} = \frac{2\bar{\alpha}}{N}g_2(v)\Delta - \frac{8}{3}\bar{\Delta}^2$$

g_1, g_2: functions of v.

Infinite N: fixed point has $z_* = 1, v_* = 1, \bar{\alpha}_* = 3\epsilon/2, \bar{\Delta}_* = 0.$

This is the clean QED3 fixed point $(\epsilon \to 1)$.

QED_3 + mass disorder: RG flows

At large but finite N, the clean QED3 fixed point gives way to

$$z_* = 1 + \frac{9\epsilon}{8N}$$

$$v_* = 1 - \frac{9}{8N}$$

$$\bar{\alpha}_* = \frac{3\epsilon}{2}$$

$$\bar{\Delta}_* = \frac{27\epsilon}{16N}$$

The fixed point has both finite interaction and finite disorder strengths. It describes a dirty metal with a vanishing DOS.

Properties of the finite mass disorder fixed point

The density of states vanishes as a universal power law:

$$\rho(E) \sim E^{d/z_* - 1} \sim_{\epsilon \to 1} E^{1 - \frac{9}{4N}}$$

Consequence: universal thermodynamics - e.g. $C \sim T^{2 - \frac{9}{4N}}$

The finite disorder strength leads to a finite Drude conductivity due to elastic impurity scattering:

$$\frac{1}{\tau} \sim \Delta_* T^{2/z_* - 1} \qquad \sigma \sim \alpha_* \tau \rho \sim \frac{\alpha_*}{\Delta_*}$$

This is the second example of a stable 2d metallic phase.

Conclusion and outlook

In this talk I provided two examples of stable interacting 2d metals with vanishing DOS.

These descriptions may describe certain spin liquids with power law correlations - algebraic spin liquids. Our prediction is that such systems are stable to disorder.

Metal-insulator transitions of these systems are not perturbatively accessible - require the analysis of the nonlinear Sigma model. Open problem: nature of the NLSM for these problems.

Open problem: stable 2d metals with finite DOS?