Nonperturbative dynamics of scalar fields in de Sitter space

Julien Serreau

APC - University Paris Diderot

- Renaud Parentani (Orsay)
- Florian Gautier (Munich)
- Maxime Guilleux (APC)

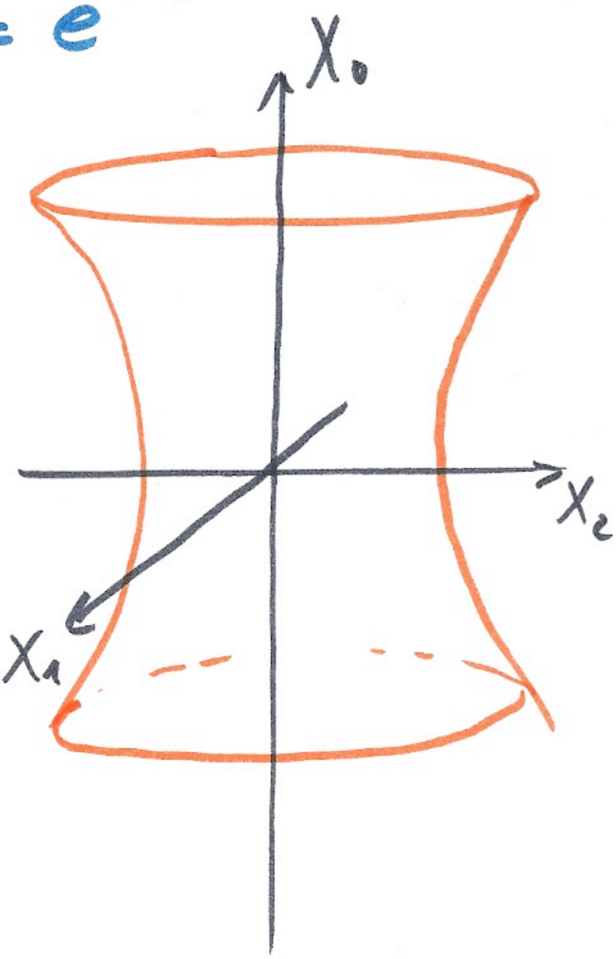
Motivations

- Radiative corrections to inflationary dynamics
- (Analog) black hole radiation
- Curvature-induced phase transitions
- Foundations of QFT in curved space-times
- ⋮

Scalar fields in de Sitter space (I)

$$ds^2 = -dt^2 + \bar{a}^2(t)\, d\vec{X}^2$$

$$\bar{a}(t) = e^{Ht}$$



$$\boxed{d\eta = dt / \bar{a}(t)}$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\vec{X}^2\right)$$

spatially homogeneous
but nonstationary

$$\boxed{\vec{x} = a(t)\,\vec{X}}$$

$$ds^2 = -(1-x^2)\,dt^2 - 2\,\vec{x}\cdot d\vec{x}\,dt + d\vec{x}^2$$

stationary but inhomogeneous

Scalar fields in dS space (II)

$$S = \int d^D x \sqrt{-g(x)} \left(\frac{1}{2} \phi \Box \phi - \frac{m^2}{2} \phi^2 \right)$$

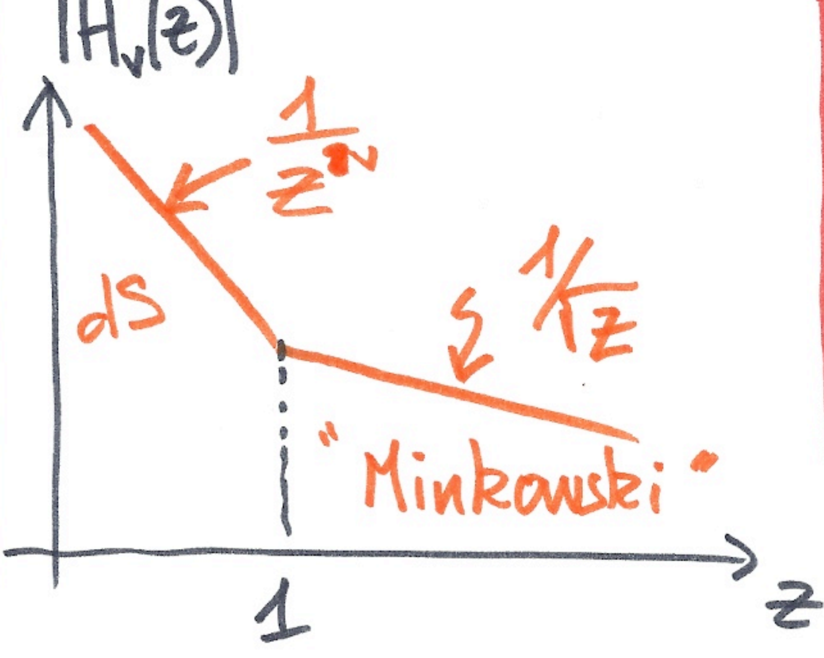
$$D = d+1 \; ; \quad \Box = \frac{1}{a^2(\eta)} \left(-\partial_\eta^2 + \frac{d-1}{\eta} \partial_\eta + \vec{\nabla}_x^2 \right)$$

$$\boxed{(-\Box + m^2)\phi = 0}$$

$$\phi(\eta, \vec{x}) \sim \int \frac{d^d k}{(2\pi)^d} \left(e^{i\vec{k}\cdot\vec{x}} H_\nu\left(\frac{k}{a(\eta)}\right) a_k + h.c. \right)$$

$$\nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}$$

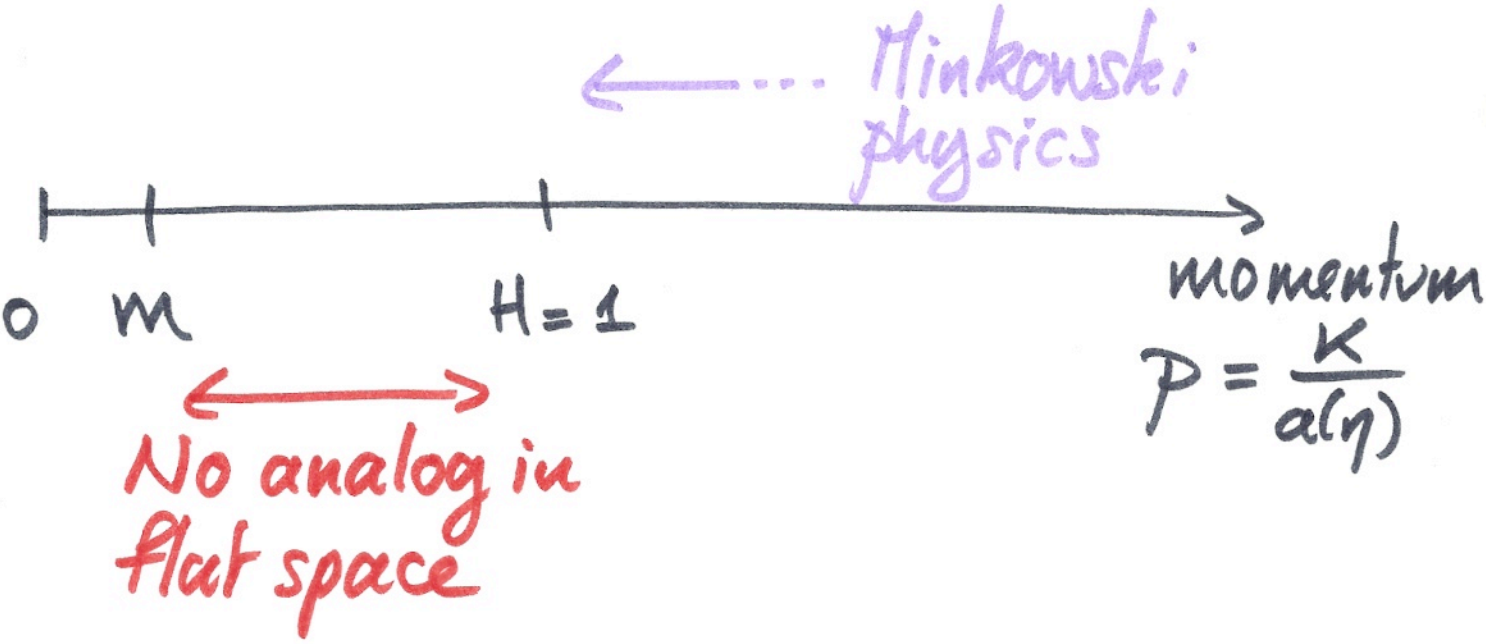
Redshift.



Stationary gravitational redshift leads to strong infrared (IR) fluctuations

Scalar fields in dS (III)

The case of light fields $m \ll H = 1$



Loop corrections :

 $\sim \frac{H^4}{m^2}$: IR divergencies

 $\sim H^2 \ln(p/H)$: large logs (secular divergencies)

$\Rightarrow$ $\boxed{\text{NEED FOR RESUMMATION}}$

Resummation / nonperturbative methods in de Sitter.

difficulty : nonequilibrium system

- Stochastic approach
  [Starobinsky, Yokoyama ('94)]
- Euclidean de Sitter
  [Rajaraman ('10); Beneke, Moch ('13)]
- Dynamical R.G.
  [Burgess et al. ('10)]
- Wigner-Weisskopf method
  [Boyanovsky ('12)]

- Large-N
  [Riotto, Sloth ('08); Mazzitelli, Paz ('89); J.S. ('11)]
- Dyson-Schwinger equations
  [Garbrecht, Rigopoulos ('11); Akhmedov, Burda ('12)
  Parentani, J.S. ('12) ; Gautier, J.S. ('13)]
- Nonperturbative / Functional R.G.
  [Kaya ('13) ; Serreau ('14); Guilleux ; JS ('15)]

Large N

[Mazzitelli, Paz (89), Riotto, Sloth (08), Serreau (11)]

Local IR terms :

 $\sim N$

Effective mass resumation

 $$\sim \lambda \int \frac{d^d p}{(2\pi)^d} |H_\nu(p)|^2 \sim \frac{\lambda H^4}{M^2}$$

GAP EQUATION

$$M^2(\phi) = \underbrace{m^2 + \lambda \phi^2}_{m_{cl}^2(\phi)} + c\,\frac{\lambda H^4}{M^2(\phi)}$$

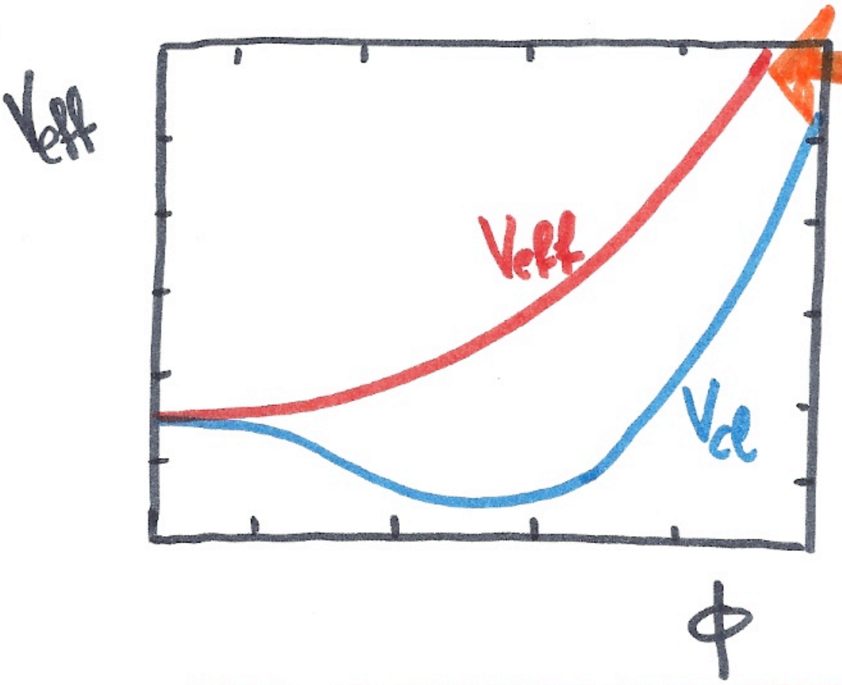
$$\boxed{M^2(\phi) = \frac{m_{cl}^2(\phi)}{2} + \sqrt{\frac{m_{cl}^4(\phi)}{4} + c\lambda H^4}}$$

Large N : Effective potential

[Serreau, PRL (11)]

$$V_{eff}(\phi) = \int_0^{\phi^2} dx \, \frac{M^2(x)}{2}$$

$$= \frac{3}{2\lambda}\left(M^4(\phi) - M^4(0)\right) + \frac{3H^4}{16\pi^2} \ln \frac{M^2(\phi)}{M^2(0)}$$



$$\boxed{M^2(\phi) > 0}$$

$\boxed{\text{Radiative symmetry restoration } \forall d, \forall H}$

[See also: Ratra ('85); Mazzitelli et al. (89), Lazzari et al. (13)]

Phase structure of O(N) theories in dS space

Strong IR fluctuations restore spontaneously broken symmetries in any spacetime dim.

✓ O(2) [Ratra ('85)]

✓ O(∞) [Serreau ('11)]

What about $N=1$; $2<N<\infty$?

- perturbation theory
- Hartree approx.
- Wigner-Weisskopf

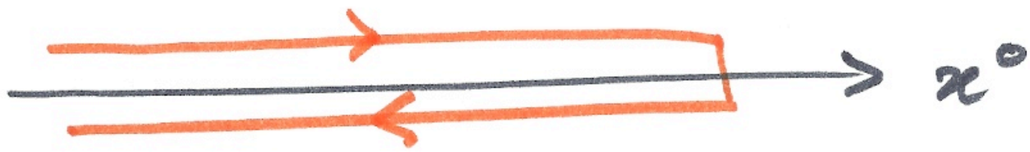
$\Rightarrow$ first order phase transition with H + massive "Goldstone modes"

Non perturbative R.G. in dS space (I)

[Kaya ('13), Serreau ('14)]

Flow equation on the closed time-contour:

[see also: Gasenzer, Pawlowski ('08)
Berges, Mesterhazy ('12)]



$$S \longrightarrow S + \Delta S_\kappa$$

$$\frac{1}{2}\int_{x,y} \phi(x) R_\kappa(x,y) \phi(y)$$

$$\dot{\Gamma}_\kappa[\phi] = \frac{1}{2} \mathrm{Tr}\left\{ \dot{R}_\kappa \cdot G_\kappa[\phi] \right\}$$

$$G_\kappa[\phi] = i \left(\Gamma_\kappa^{(2)}[\phi] + R_\kappa \right)^{-1}$$

NPRG : Local Potential Approx. (LPA)

$$\Gamma_\kappa[\phi] = -\int_x \left\{ \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + \boxed{V_\kappa(\phi)} \right\}$$

Full field dependence

+ choose $\quad R_\kappa(t,t';p) = \delta(t-t')(\kappa^2-p^2)\theta(\kappa^2-p^2)$

$$\boxed{\kappa \partial_\kappa V_\kappa = \frac{C_d\, \kappa^{d+2}}{\kappa^2 + V_\kappa''} \underline{B_d}(\nu_\kappa, \kappa)}$$

$$C_d = \frac{\pi}{16 d} \frac{\Omega_d}{(2\pi)^d} \quad , \quad \nu_\kappa = \sqrt{\frac{d^2}{4} - V_\kappa''}$$

$$\underline{B_d}(\nu,\kappa) = e^{-\pi \mathrm{Im}(\nu)} \Big\{ (d^2 - 2\nu^2 + 2\kappa^2) |H_\nu(\kappa)|^2 + 2\kappa^2 |H_\nu'(\kappa)|^2 - 2d\kappa \,\mathrm{Re}\left[H_\nu^*(\kappa) H_\nu'(\kappa)\right] \Big\}$$

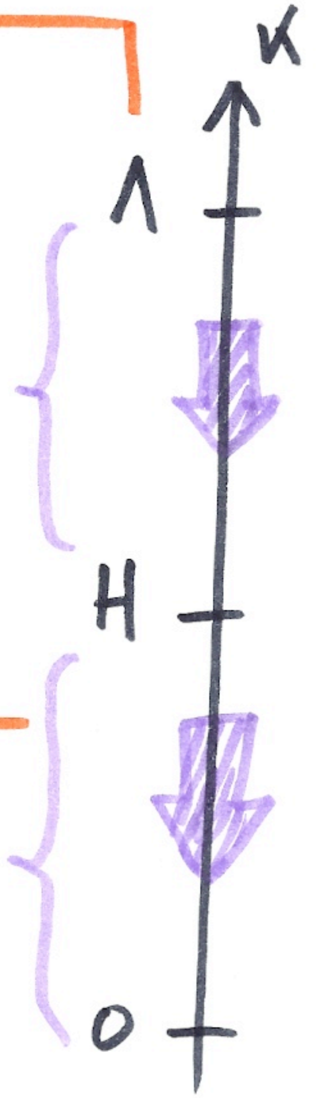
[Kaya ('13) Guilleux, Serreau ('15)]

From UV to IR : onset of gravitational effects

➡ $\kappa \gtrsim H$

$$\kappa\,\partial_\kappa V_\kappa \approx \frac{8C_d}{\pi}\,\frac{\kappa^{d+2}}{\sqrt{\kappa^2+V_\kappa''}}$$

Minkowski Regime



➡ $\kappa \lesssim H$ and $|V_\kappa''| \ll H^2$

$$\kappa\,\partial_\kappa V_\kappa \approx \frac{1}{\Omega_{D+1}}\,\frac{\kappa^2}{\kappa^2+V_\kappa''}$$

[Compare to $\dfrac{\kappa^{D+2}}{\kappa^2+V_\kappa''}$ in flat Euclidean space]

➡ $\boxed{\text{Effective dimensional reduction } D_{eff}=0}$

Zero-dimensional field theory

$$\boxed{e^{-\Omega_{D+1} W_\kappa(J)} = \int d\varphi\, e^{-\Omega_{D+1}\left(V_{\rm eff}(\varphi) + J\varphi + \frac{\kappa^2}{2}\varphi^2\right)}}$$

$$V_\kappa(\phi) = W_\kappa(J) - J\phi - \frac{\kappa^2}{2}\phi^2$$

with $W_\kappa'(J) \equiv \phi$

$$\Rightarrow \boxed{\kappa \partial_\kappa V_\kappa(\phi) = \frac{1}{\Omega_{D+1}} \frac{\kappa^2}{\kappa^2 + V_\kappa''(\phi)}}$$

Adjust $V_{\rm eff}$ by matching at the scale $\kappa \approx H$

$$\boxed{V_{\rm eff}(\varphi) \approx V_H(\varphi)}$$

Relation to the stochastic approach

[Starobinsky, Yokoyama ('94)]

Effective Langevin eqn. for IR modes

$$\dot{\varphi} + \frac{1}{d} V'_{soft}(\varphi) = \xi \quad ; \quad \langle \xi(t)\xi(t') \rangle = \frac{2\,\delta(t-t')}{d\,\Omega_{D+1}}$$

Focker-Planck equation

$$\partial_t P(\varphi,t) = \frac{1}{d}\frac{\partial}{\partial\varphi}\left\{ V'_{soft} P + \frac{1}{\Omega_{D+1}} P' \right\}$$

$$\boxed{P(\varphi, t\to\infty) \propto \exp\left\{-\Omega_{D+1} V_{soft}\right\}}$$

$$\langle \varphi^2 \rangle = \int d\varphi\, P(\varphi)\,\varphi^2 = \frac{1}{\Omega_{D+1}} \underline{\underline{W''_{K=0}(J=0)}}$$

provided $V_{soft} = V_{eff} \approx V_H$

Relation to Euclidean de Sitter

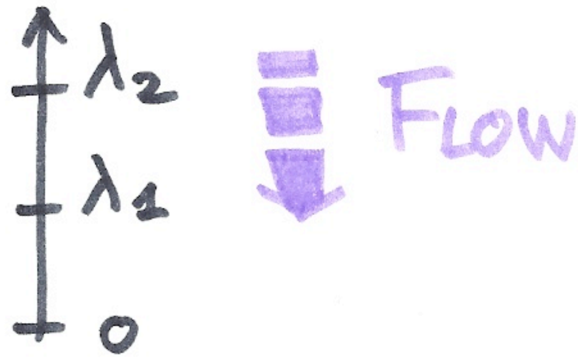
[Rajaraman ('10); Beneke, Moch ('13); Benedetti ('15)]

NPRG on S_D : $S \longrightarrow S + \frac{1}{2}\int_{xy} \varphi R_\kappa \varphi$

$$\boxed{\text{Compact space} \Rightarrow \text{discrete spectrum}}$$

$\Box Y_{\vec{L}} = -\lambda_L Y_{\vec{L}}$

$\lambda_L = L(L+D-1)H^2$



For $\boxed{\kappa^2 < \lambda_1}$, heavy modes decouple

$\Rightarrow$ zero mode only $\boxed{\bar{\varphi}}$

$$e^{-\Omega_{D+1}\bar{W}_\kappa(J)} = \int d\bar{\varphi}\, e^{-\Omega_{D+1}\left(\bar{V}_{eff}(\bar{\varphi}) + J\bar{\varphi} + \frac{\kappa^2}{2}\bar{\varphi}^2\right)}$$

with $e^{-\Omega_{D+1}\bar{V}_{eff}(\bar{\varphi})} = \int \mathcal{D}\hat{\varphi}\, e^{-\bar{S}[\bar{\varphi},\hat{\varphi}]}$

From UV to IR: onset of gravitational effects

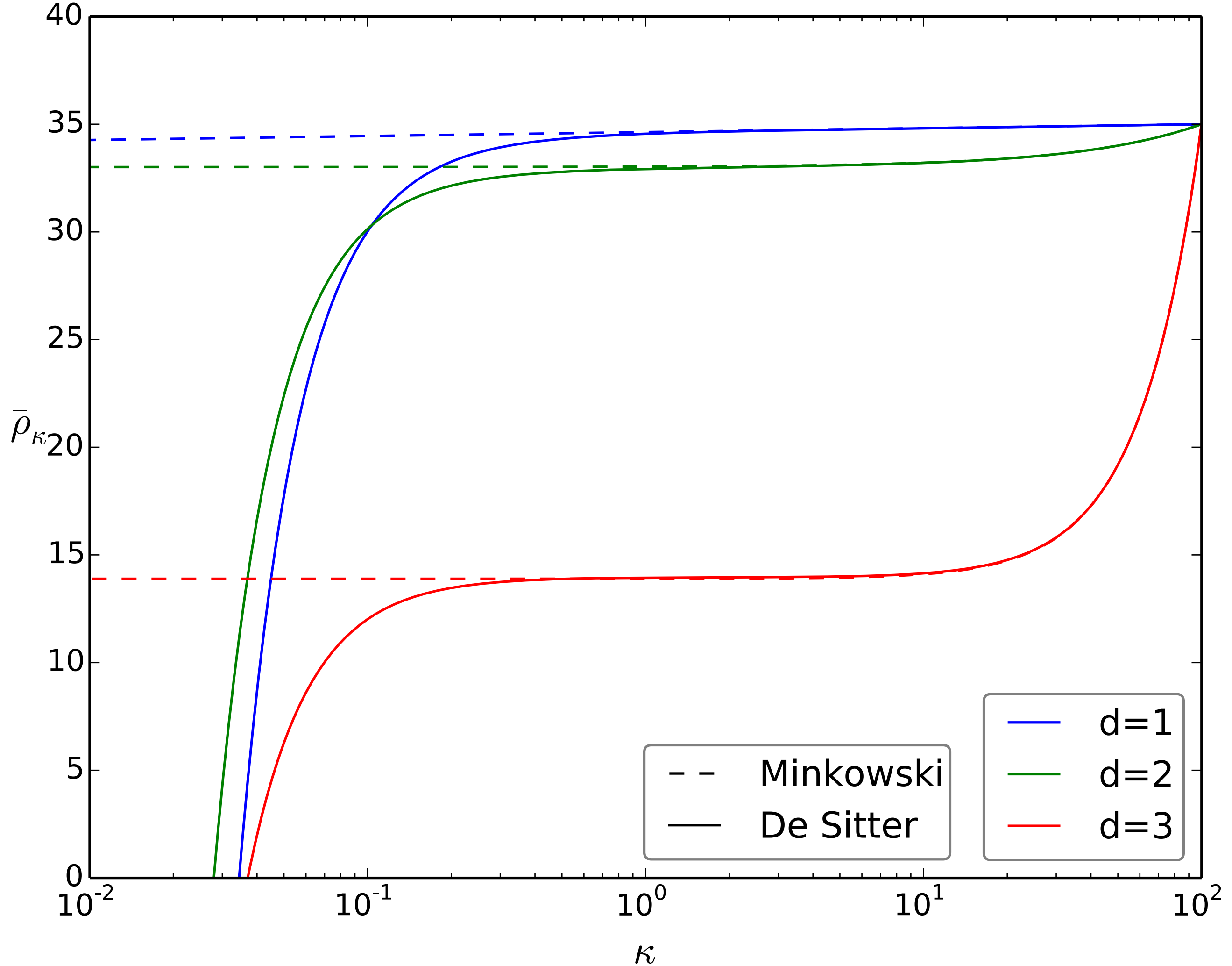
- Massive theory in UV ($\kappa \gtrsim H$)
(i.e. Minkowski symmetric phase)

$\Rightarrow$ $V''_H \gtrsim H^2$: Minkowski flow

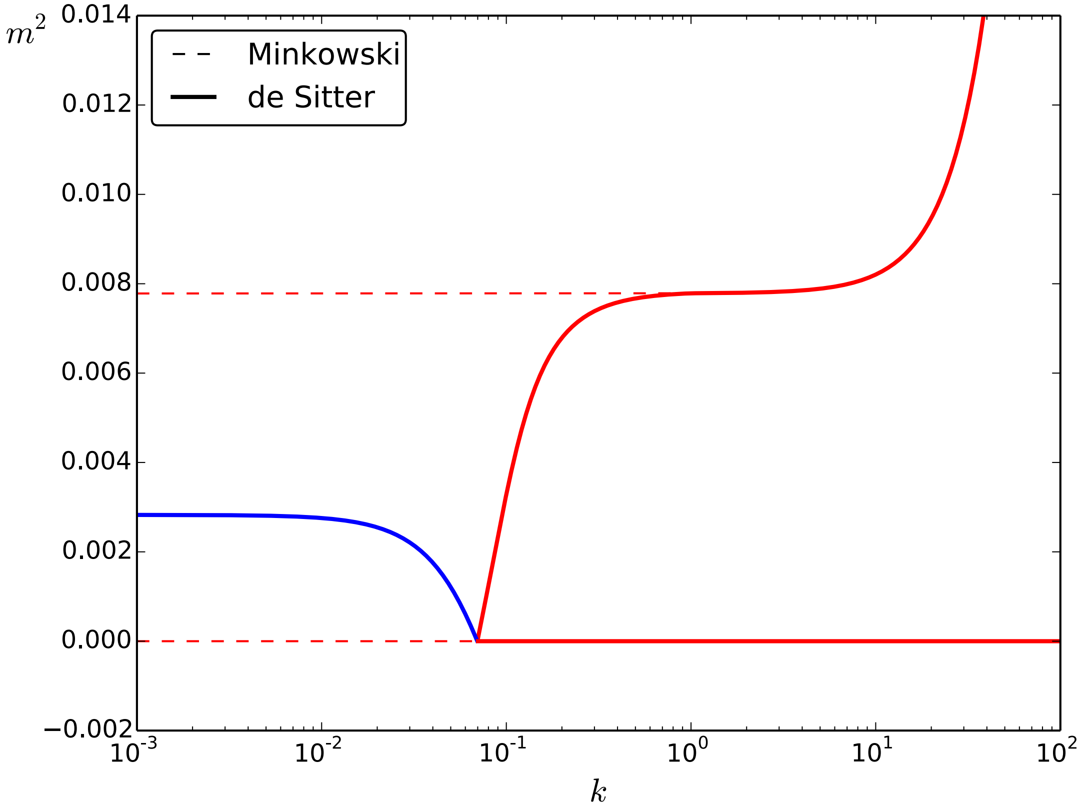
- Massless modes / flat potential
(e.g. Minkowski critical theory or broken phase)

$\hookrightarrow$ Regime of dimensional reduction

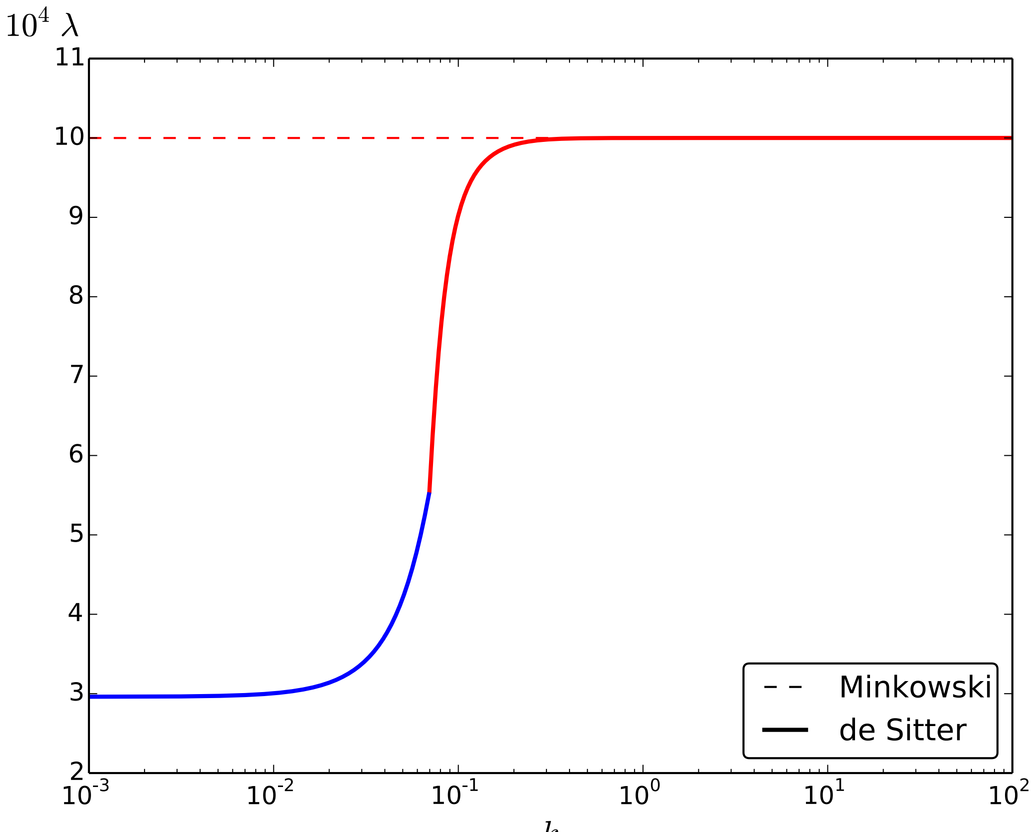
$\Rightarrow$ $\boxed{\text{Symmetry restoration; Mass (re)generation}}$

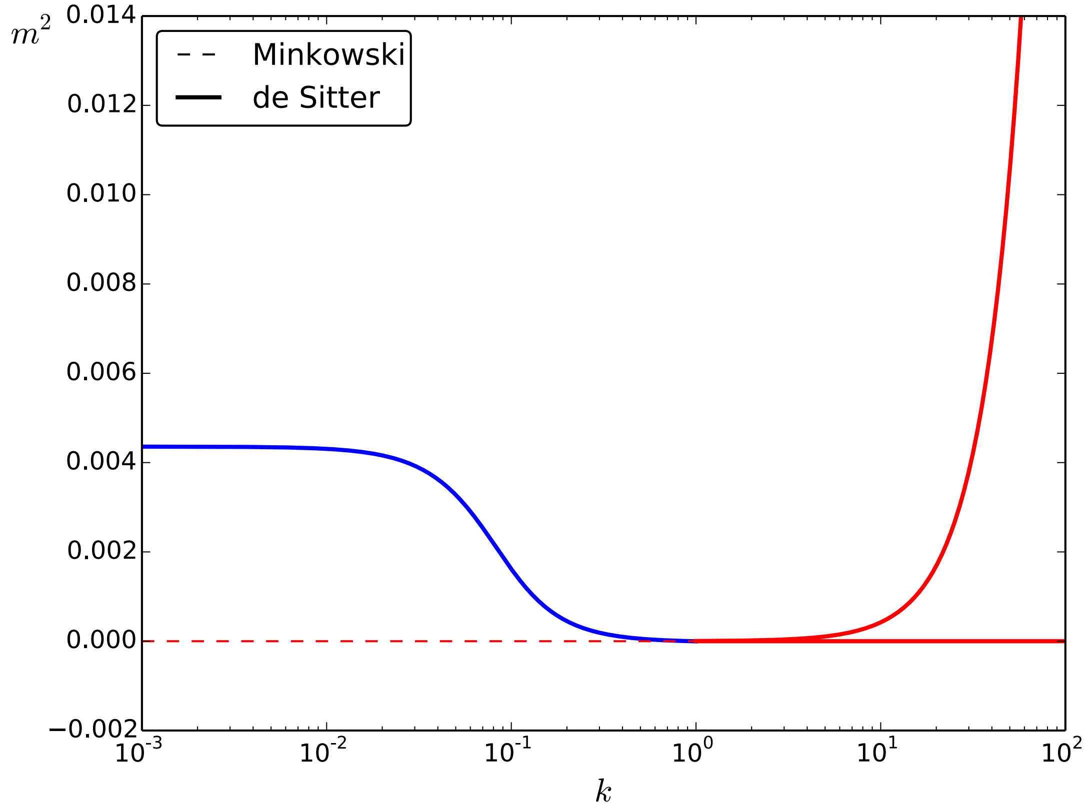


$\bar{\rho}_\kappa$
κ
40
35
30
25
20
15
10
5
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Minkowski
De Sitter
d=1
d=2
d=3

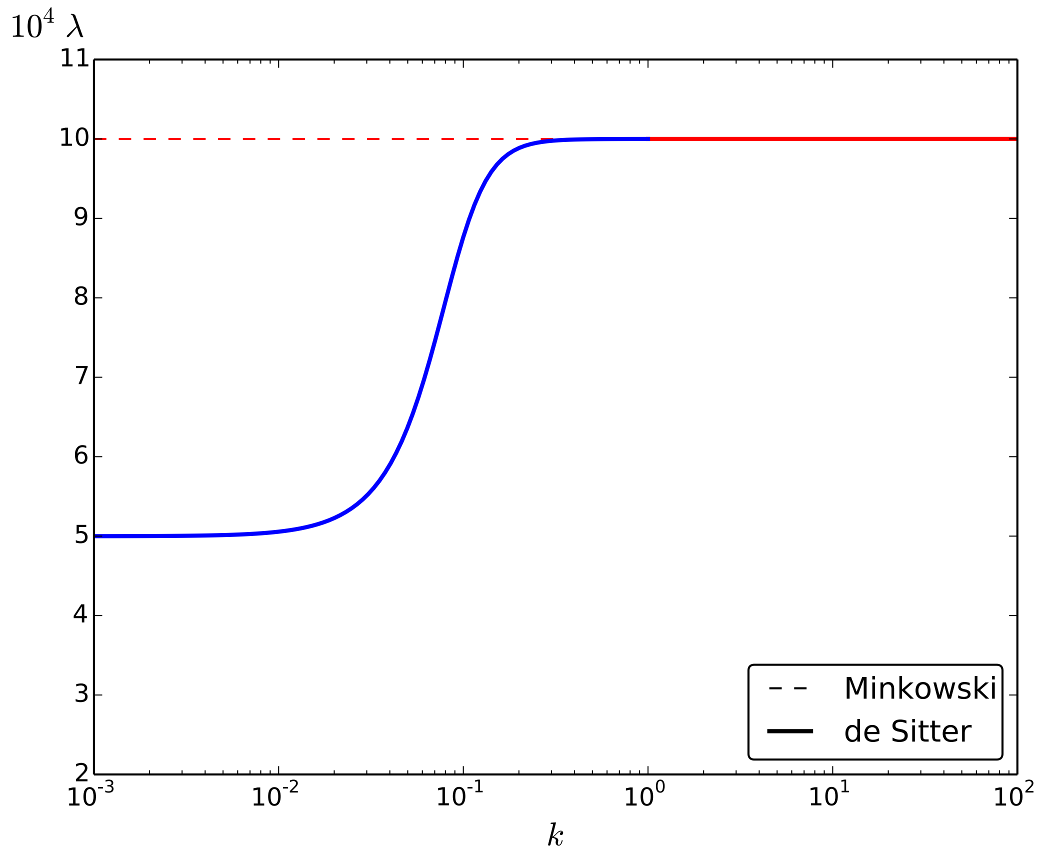


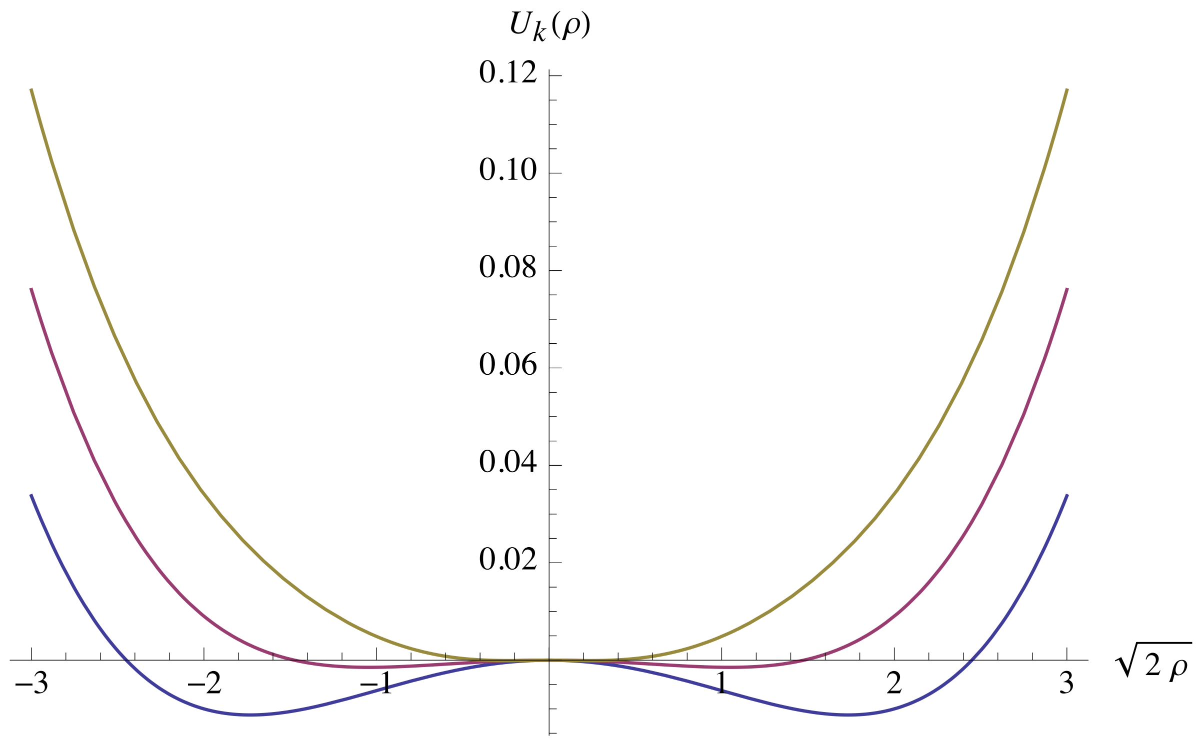
$N \to \infty$, "broken symmetry" case





$N \to \infty$, "critical" case





$$N = 1$$

Beyond LPA : derivative expansion

[Guilleux, Serreau ('16)]

Dynamics of long wavelength (superhorizon) modes :

$\boxed{\text{expansion in powers of } \nabla_\mu \phi \text{ ?}}$

Standard in flat (Euclidean) space
[Wetterich ('93) ; Morris ('94) ; Canet et al. ('03)]

$$\Gamma_\kappa[\phi] = \int d^D x \left\{ V_\kappa(\phi) + \frac{Z_\kappa(\phi)}{2} (\nabla\phi)^2 + \cdots \right\}$$

$$\boxed{\eta_\kappa = -\frac{\dot{Z}_\kappa}{Z_\kappa} = -\frac{1}{Z_\kappa} \left. \frac{\partial \dot{\Gamma}_\kappa^{(2)}(k)}{\partial k^2} \right|_{\substack{k^2=0 \\ \text{min}}}}$$

Issues in curved spacetimes

- $\nabla\phi \sim \nabla \overset{\text{metric}}{g} \rightsquigarrow k \sim H$

[e.g. Shapiro, de Morais Texeira, Wipf ('15)]

Not valid for superhorizon modes

➡ $\boxed{\nabla\phi \sim \nabla \overset{\text{curvature}}{R}}$

OK, but many terms at each order

ex: $O(\nabla^2)$

$$\boxed{\begin{aligned} & A(\phi, R, R_{\mu\nu}R^{\mu\nu}, \dots)\, \nabla_\mu\phi\nabla^\mu\phi \\ + \; & B(\phi, R, R_{\mu\nu}R^{\mu\nu}, \dots)\, \nabla_\mu\phi\nabla^\mu R \\ + \; & C(\phi, R, R_{\mu\nu}R^{\mu\nu}, \dots)\, R_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi \\ + \; & \dots \end{aligned}}$$

Issues in curved spacetimes

➕ the expansion is ambiguous

$$\boxed{[\nabla_\mu, \nabla_\nu] V^\lambda = V^\sigma R^\lambda{}_{\sigma\mu\nu}}$$

<u>For example</u> :

$$\nabla_\mu \nabla_\nu \nabla^\mu \nabla^\nu \phi = \Box^2 \phi + \frac{1}{2} \nabla_\mu R \nabla^\mu \phi$$

$$\underbrace{+ R_{\mu\nu} \nabla^\mu \nabla^\nu \phi}_{O(\nabla^2)}$$

$O(\nabla^4)$

Maximally symmetric spaces (dS)

- $\nabla_\mu R_{\alpha\beta\gamma\delta} = 0$
- $R = d(d+1)H^2$; $R_{\mu\nu}R^{\mu\nu} = d^2(d+1)H^4 \dots$

$\hookrightarrow \nabla_\mu \nabla_\nu \nabla^\mu \nabla^\nu \phi = \Box^2\phi + dH^2 \Box\phi$

$\boxed{\text{We can define an expansion in powers of } \nabla_\mu\phi \text{ for fixed } H}$

$$\boxed{\Gamma_k = -\int d^D x \sqrt{-g} \left\{ V_k(\phi) + \frac{Z_k(\phi)}{2} \nabla_\mu\phi \nabla^\mu\phi + O(\nabla^4) \right\}}$$

$$\boxed{\text{Access to } Z_k, \ldots}$$

Flat spacetime:

$$\Box e^{-ik.x} = -k^2 e^{-ik.x}$$

$$\Gamma_k^{(2)}(k) = \int d^D x \, e^{ik(x-x')} \Gamma_k^{(2)}(x,x')$$

$$= -V_k''(\phi) + Z_k(\phi) k^2 + \ldots$$

$$\boxed{Z_k(\phi) = \partial_{k^2} \Gamma_k^{(2)}(k)\big|_{k^2=0}}$$

de Sitter:

$$-\Box_p = +\partial_t^2 + dH\partial_t + \vec{p}^{\,2}$$

$$\boxed{\vec{p} = \vec{k}/a(t)}$$

∂_{p^2} ? No because of grav. redshift

$$\Box_p^2 = \Box_{p=0}^2 - 2p^2 \Box_{p=0} - (2d-4)H^2 p^2 + p^4$$

$$\boxed{\text{All } \Box_p^{n\geq 1} \text{ contribute to } \partial_{p^2} !}$$

Access to z_k ...

A way out : $\|\vec{p}\| \longrightarrow 0$

long wavelength / late times

($k \to 0$) ($a(t) \to \infty$)

$$\Box_{p=0} = -\partial_t^2 - dH\partial_t$$

$$\Box_{p=0}\, e^{-i\omega t} = \underline{\alpha_\omega}\, e^{-i\omega t}$$

with $\boxed{\alpha_\omega = \omega^2 + i d H \omega}$

$$\Gamma_k^{(2)}(\omega) = \int_{\mathcal{E}} dt'\, a^d(t')\, e^{i\omega(t-t')}\, \Gamma_k^{(2)}(t,t',k=0)$$

+ expand in powers of α_ω

$\boxed{\text{Access to } Z_k \ldots}$

$$\Gamma_k^{(2)}(\omega) = \int_{\mathcal{E}} dt'\, a^d(t')\, e^{i\omega(t-t')}\, \Gamma_k^{(2)}(t,t',k=0)$$

1. $\Gamma_k^{(2)}(\omega)$ is time-independent

   (dS isometries)

2. $\Gamma_k^{(2)}(\omega) = -i\, \overline{\Gamma}^{(2)}_{R,k}\left(\omega + i\frac{dH}{2}, k=0\right)$

   (<u>retarded</u> vertex)

   $\Rightarrow$ standard for $H \to 0$

3. $\Gamma_k^{(2)}(\omega) = -V_k'' + Z_k \alpha_\omega + O(\alpha_\omega^2)$

$$\boxed{\begin{aligned} \dot{V}_k'' &= -\dot{\Gamma}_k^{(2)}(\omega=0) \\ \dot{Z}_k &= \left.\frac{\partial \dot{\Gamma}_k^{(2)}(\omega)}{\partial \alpha_\omega}\right|_{\omega=0} \\ &\text{etc.} \end{aligned}}$$

Remark : Lorentz violation

$$R_k(t,t',p) = \underline{\delta(t-t')}\, R_k\left(p = \frac{k}{a(t)}\right)$$

The time direction is not regulated

$\Rightarrow$ breaks (local) Lorentz symmetry

One should allow $Z_k^t, Z_k^s \ldots$

$$Z_k^t\, \phi\left(\partial_t^2 + dH\partial_t\right)\phi + Z_k^s\, \phi\, \frac{\vec{\nabla}^2}{a^2(t)}\, \phi$$

$$Z_k^t = \partial_{\alpha\omega}\, \Gamma_k^{(2)}(\omega)\Big|_{\omega=0} \checkmark$$

$$Z_k^s = ??$$

For now, we assume $Z_k^s \approx Z_k^t$

(ok in flat space)

A practical test : LPA'

$$\Gamma_\kappa^{LPA'}[\phi] = -\int d^Dx \sqrt{-g} \left\{ V_\kappa(\phi) + \frac{Z_\kappa}{2} (\nabla\phi)^2 \right\}$$

Infrared and massless regime
($\kappa \to 0$ and $V_\kappa'' \ll H^2$)

$$\boxed{M_\kappa^2 = \kappa^2 + \frac{V_\kappa''}{Z_\kappa}}$$

$$\dot{\Gamma}_\kappa^{(2)}(\omega) = \frac{\kappa^2}{\Omega_{D+1} M_\kappa^4} \left(1 - \frac{\eta_\kappa}{2}\right) \times \left(\frac{V_\kappa^{(4)}}{Z_\kappa} - \frac{2 V_\kappa^{(3)2}}{Z_\kappa^2} \frac{4M_\kappa^2 - id\omega}{(2M_\kappa^2 - id\omega)^2} \right)$$

$$\dot{V}_\kappa = \left(1 - \frac{\eta_\kappa}{2}\right) \frac{\kappa^2}{\Omega_{D+1} M_\kappa^2}$$

$$\eta_\kappa = \left(1 - \frac{\eta_\kappa}{2}\right) \frac{3 V_\kappa^{(3)2}}{2 Z_\kappa^3} \left.\frac{\kappa^2}{\Omega_{D+1} M_\kappa^8}\right|_{min}$$

Consequences of dimensional reduction.

$$\dot{V}_\kappa \equiv \beta_V(V_\kappa'', \kappa^2)$$

In the IR (dim. red.) regime:

$$\boxed{\beta_V(\alpha V_\kappa'', \alpha\kappa^2) = \beta_V(V_\kappa'', \kappa^2) \quad \forall \alpha}$$

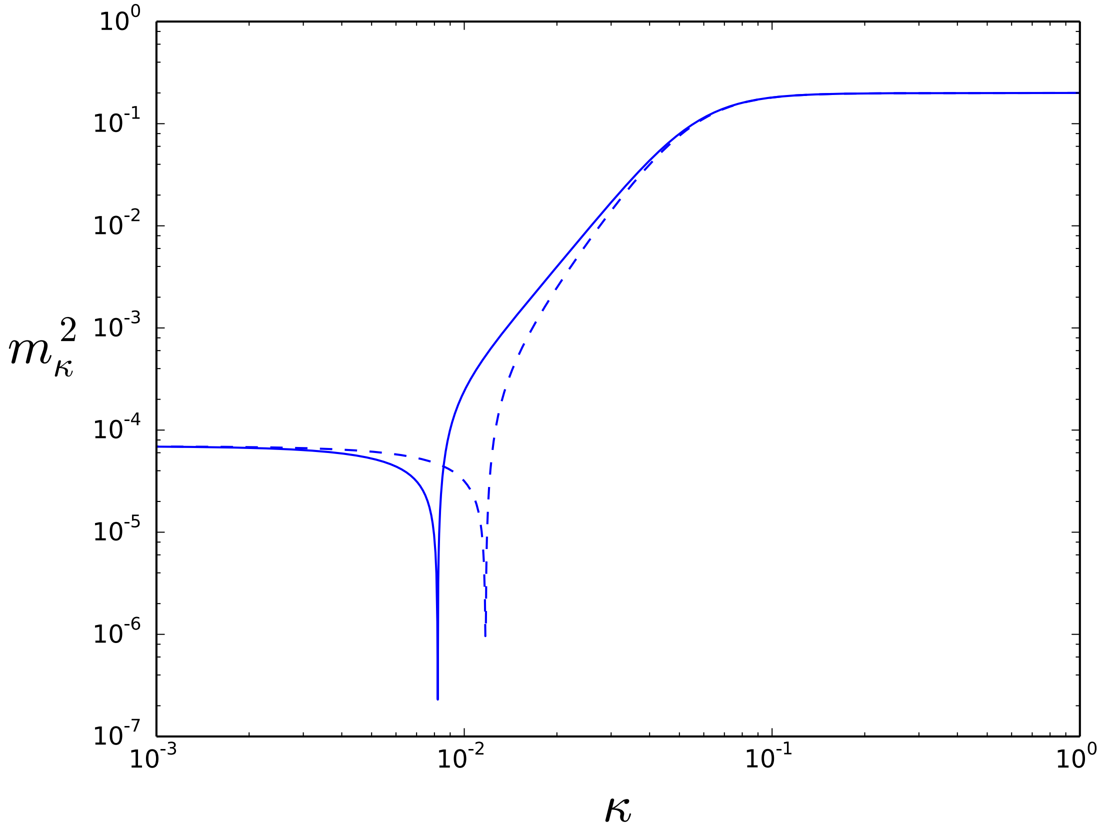
$\Rightarrow$ define $\tilde{\kappa} = \sqrt{Z_\kappa}\,\kappa$

$$\kappa\partial_\kappa = (1 - \eta_\kappa/2)\,\tilde{\kappa}\partial_{\tilde{\kappa}}$$

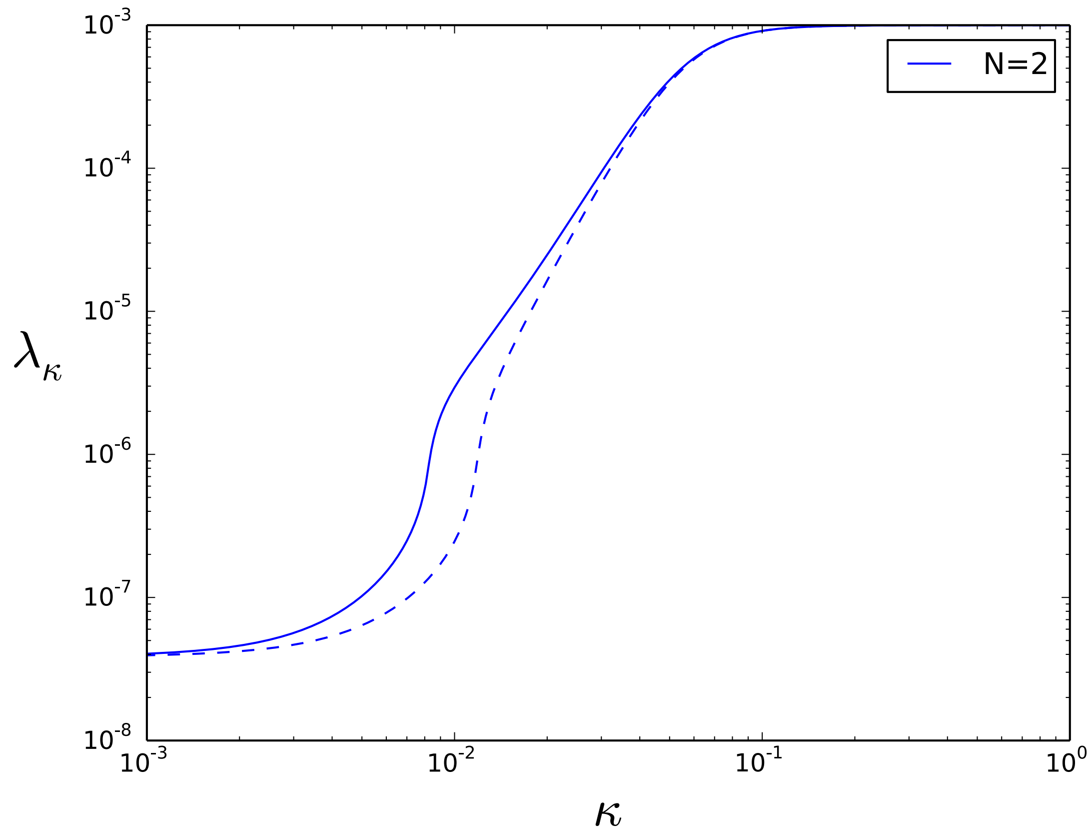
$$\boxed{\tilde{\kappa}\partial_{\tilde{\kappa}} V_\kappa = \frac{1}{\Omega_{D+1}} \frac{\tilde{\kappa}^2}{\tilde{\kappa}^2 + V_\kappa''}}$$

$\Rightarrow$
$$\boxed{V^{LPA'}_{\kappa=0}(\phi) = V^{LPA}_{\kappa=0}(\phi)}$$

(for appropriate initial cond's)



m_κ^2
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-3}
10^{-2}
10^{-1}
10^0
κ



λ_κ
N=2
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-3}
10^{-2}
10^{-1}
10^0
κ

Slowing down the flow

- Generalization to O(N) theory

$$\rho = \frac{Z_\kappa}{2N}\phi^2$$

- Polynomial ansatz

$$V_\kappa(\phi) = \frac{\lambda_\kappa}{2}(\rho - \bar{\rho}_\kappa) + \dots$$

$$\dot{\bar{\rho}}_\kappa = -\eta_\kappa \bar{\rho}_\kappa + \frac{(2-\eta_\kappa)\kappa^2}{2N\Omega_{D+1}}\left[\frac{3}{(\kappa^2+m_\kappa^2)^2} + \frac{N-1}{\kappa^4}\right]$$

$$\dot{\lambda}_\kappa = 2\eta_\kappa \lambda_\kappa + \frac{(2-\eta_\kappa)\kappa^2\lambda_\kappa^2}{N\Omega_{D+1}}\left[\frac{9}{(\kappa^2+m_\kappa^2)^3} + \frac{N-1}{\kappa^6}\right]$$

$$\eta_\kappa = \frac{(2-\eta_\kappa)\kappa^2}{2N\Omega_{D+1}\bar{\rho}_\kappa}\left[\frac{1}{\kappa^4} + \frac{1}{(\kappa^2+m_\kappa^2)^2} - \frac{8}{(2\kappa^2+m_\kappa^2)^2}\right]$$

$$\boxed{m_\kappa^2 = 2\lambda_\kappa \bar{\rho}_\kappa}$$

$\boxed{\text{Slowing down the flow}}$

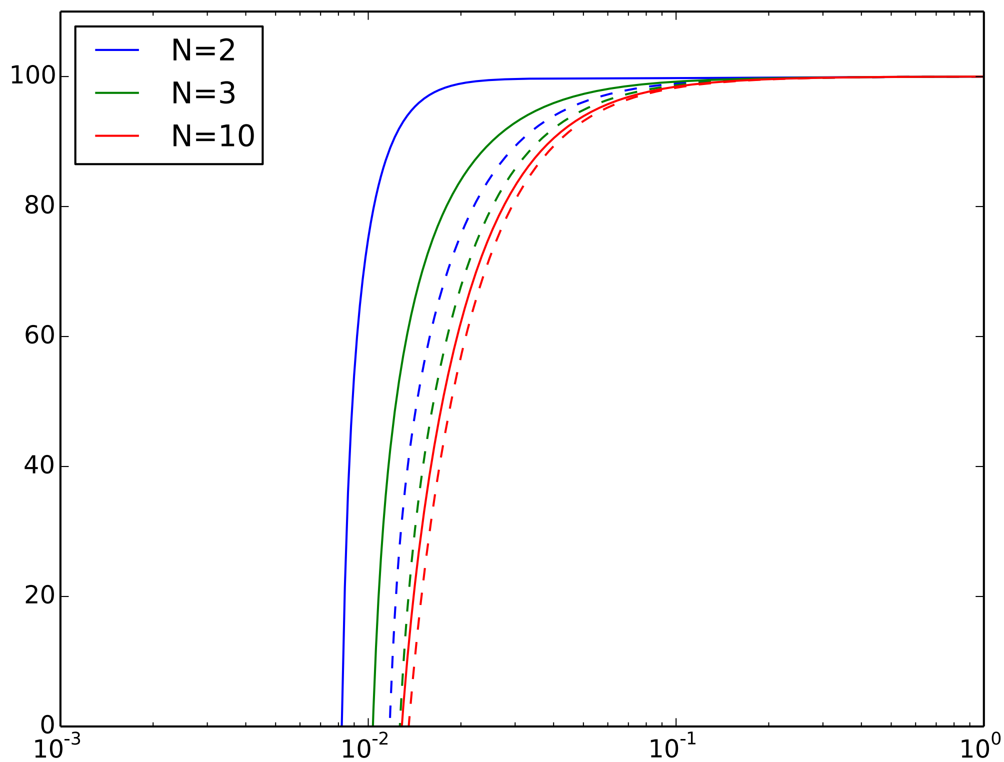
Regime $m_\kappa^2 \gg \kappa^2$
(before symmetry restoration)

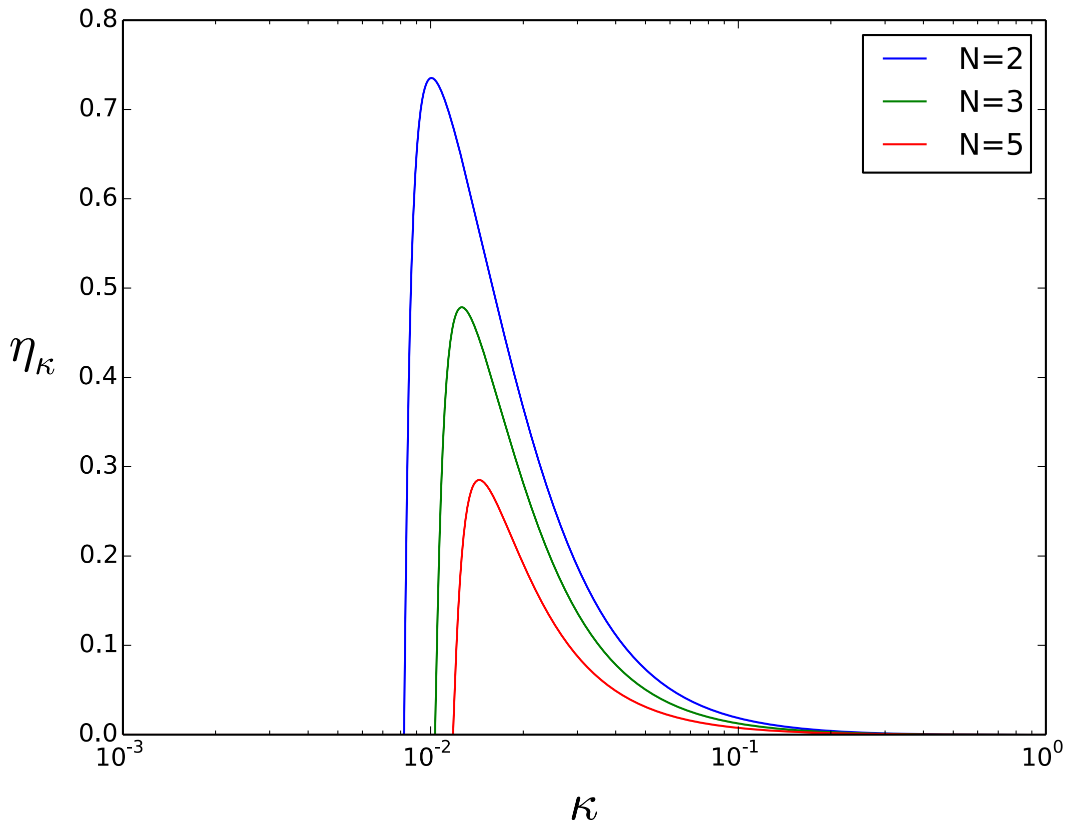
$$\eta_\kappa \approx \frac{2-\eta_\kappa}{2N\Omega_{D+1}\bar{\rho}_\kappa \kappa^2}$$

$$\dot{\bar{\rho}}_\kappa \approx \frac{(2-\eta_\kappa)}{2N\Omega_{D+1}\kappa^2}\left[N-2+O\left(\frac{\kappa^2}{m_\kappa^2}\right)\right]$$

$$\boxed{\tilde{\kappa}\partial_{\tilde{\kappa}}\bar{\rho}_\kappa \approx \frac{N-2}{N\Omega_{D+1}\kappa^2}}$$

LPA-like with $N-1 \Rightarrow N-2$
(cf. KT-transition in flat $d=2$)


N=2
N=3
N=10
100
80
60
40
20
0
10^{-3}
10^{-2}
10^{-1}
10^{0}


N=2
N=3
N=5
η_κ
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10^{-3}
10^{-2}
10^{-1}
10^{0}
κ

CONCLUSIONS

NPRG (on closed time contour)

- Useful to tackle nonperturbativ[e] infrared dynamics in dS space
- Successful implementation at LPA, LPA'
- Nontrivial phenomena (dym. red / sym. rest.) $\underline{\underline{\forall d}}$, $\underline{\underline{\forall N}}$

GO BEYOND !

PERSPECTIVES

- Derivative expansion at complete $O(\partial^2)$: $Z_k(\phi)$
- Blaizot-Mendes Galain-Wschebor (BMW) scheme in dS
- dS-invariant regulator
- Ward identities for Lorentz symmetry
- ⋮