Gauge coupling unification without leptoquarks

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Outline

- Motivation
- Gauge coupling unification without leptoquarks
- Scale and conformal invariance
- Conclusions

Motivation

Standard Model

Standard Model: all interactions are based on different gauge groups.
But this looks rather arbitrary:

- Gauge group $SU(3) \times SU(2) \times U(1)$. Why?
- Quantum numbers and the choice of representations of matter fields appear to be random.
- Electric charge is quantised. Why so - the $U(1)$ group is Abelian?

GUTs

Proposal, going back to 70ties: Strong, weak and electromagnetic interactions are part of the same gauge force and are unified at high energies:

$$SU(3) \times SU(2) \times U(1) \in G$$

- 1973 - Pati, Salam: $G = SU(4) \times SU(2) \times SU(2)$. Lepton number as 4th colour, left-right symmetry
- 1974 - Georgi, Glashow $G = SU(5)$
- 1975 - Fritzsch, Minkowski $G = SO(10)$. All fermions of one generation are in one representation $\underline{16}$!

GUTs

Generic features of GUTs:

- charge quantisation is automatic
- quantum numbers of SM fermions can be understood
- $\sin^2\theta_W$ can be predicted: gauge coupling unification.
- some relations between quark and lepton masses (e.g. bottom quark and τ lepton) can appear
- common prediction: instability of matter, proton decay

Looks great!

Main trouble: hierarchy problem

Extra particles beyond the SM – leptoquarks (vector and scalar) must be very heavy, $M_X > 10^{15}$ GeV

- this is required by the gauge coupling unification
- this is needed for stability of matter, proton lifetime $\tau_p > 10^{34}$ years

$$\text{Hierarchy: } \left(\frac{M_X}{M_W}\right)^2 \simeq 10^{28}$$

Two faces of hierarchy, SU(5)

Gauge bosons are in $\underline{24}$, 15 SM fermions of each generation are in $\underline{5}$ and $\underline{10}$, scalars are in $\underline{24}$, Σ and $\underline{5}$, H

Chain of spontaneous symmetry breaking

$$SU(5) \underset{\mathbf{24}}{\longrightarrow} SU(3) \times SU(2) \times U(1) \underset{\mathbf{5}}{\rightarrow} SU(3) \times U(1) \,.$$

$$\langle \Sigma \rangle = \frac{v_{GUT}}{\sqrt{15}} \text{diag}(1, 1, 1, -3/2, -3/2) \,,$$

$v_{GUT} \sim 10^{15}$ GeV, gives mass to leptoquarks

$$\langle H \rangle = \frac{v_{EW}}{\sqrt{2}} (0, 0, 0, 0, 1)^T \,,$$

$v_{EW} \sim 10^2$ GeV, gives masses to the SM particles.

Tree level tunings

Scalar potential:

$$V = -\frac{1}{2}m_\Sigma^2 \mathrm{Tr}(\Sigma^2) - \frac{1}{2}m_H^2 H^\dagger H + \frac{1}{4}\lambda_{\Sigma\Sigma}\left(\mathrm{Tr}(\Sigma^2)\right)^2 + \frac{15}{14}\lambda'_{\Sigma\Sigma}\mathrm{Tr}(\Sigma^4)$$
$$+ \frac{1}{4}\lambda_{HH}\left(H^\dagger H\right)^2 + \frac{1}{2}\lambda_{\Sigma H}\mathrm{Tr}(\Sigma^2)H^\dagger H + \frac{5}{3}\lambda'_{\Sigma H}H^\dagger \Sigma^2 H\,.$$

Minimum of the potential corresponds to

$$v_{GUT}^2 = \frac{2(\lambda_{HH}m_\Sigma^2 - (\lambda_{\Sigma H} + \lambda'_{\Sigma H})m_H^2)}{\lambda_{HH}(\lambda_{\Sigma\Sigma} + \lambda'_{\Sigma\Sigma}) - (\lambda_{\Sigma H} + \lambda'_{\Sigma H})^2}\,,$$

$$v_{EW}^2 = \frac{2((\lambda_{\Sigma\Sigma} + \lambda'_{\Sigma\Sigma})m_H^2 - (\lambda_{\Sigma H} + \lambda'_{\Sigma H})m_\Sigma^2)}{\lambda_{HH}(\lambda_{\Sigma\Sigma} + \lambda'_{\Sigma\Sigma}) - (\lambda_{\Sigma H} + \lambda'_{\Sigma H})^2}\,.$$

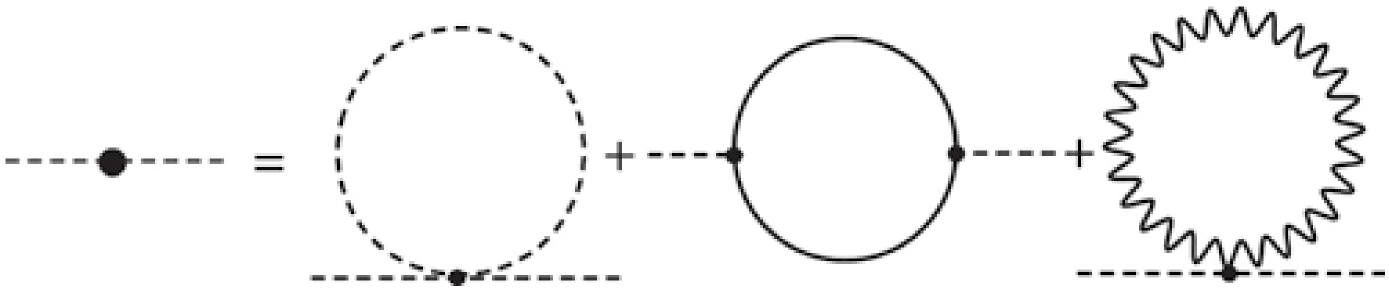
The correct hierarchy between the vacuum expectation values of the fields requires that

$$(\lambda_{\Sigma\Sigma} + \lambda'_{\Sigma\Sigma}) m_H^2 - (\lambda_{\Sigma H} + \lambda'_{\Sigma H}) m_\Sigma^2 \approx 0 \; ,$$

a relation that has to hold with an accuracy of 26 orders of magnitude!

Loop level tunings: stability of EW scale

Stability of the Higgs mass against radiative corrections Gildener, ’76



$$\delta m_H^2 \simeq \alpha_{GUT}^n M_X^2$$

Tuning is needed up to 14th order of perturbation theory!

Proposed solutions

Stability of EW scale: requirement of “naturalness”:

- Low energy SUSY: compensation of bosonic loops by fermionic loops
- Composite Higgs boson - new strong interactions
- Large extra dimensions

All require new physics right above the Fermi scale, which was expected to show up at the LHC

However, the LHC has discovered something quite unexpected : the Higgs boson and nothing else, confirming the Standard Model.

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For $\mathbf{125}$ GeV Higgs mass the Standard Model is a self-consistent weakly coupled effective field theory for all energies up to the quantum gravity scale $M_P \sim 10^{19}$ GeV

Should we abandon Grand Unification?

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Should we accept fine tunings in many orders of perturbation theory?

Main problem of the stability of the Higgs mass against radiative corrections: existence of superheavy particles, $\delta m_H^2 \propto M_X^2$.

Do we need lepto-quarks for GUTs?

Yes, if the Nature we know at EW scale repeats itself at the gauge coupling unification scale!

Physics at EW scale $\equiv$ dynamical Higgs mechanism $\equiv$ true Higgs boson

Perhaps, the physical meaning of the GUT scale is different from that of EW scale?

Gauge coupling unification without leptoquarks

Idea: Take some GUT and remove all heavy degrees of freedom by imposing gauge-invariant constraints.
How does it work? SU(5) example.

- Scalar leptoquarks in 24

Consider eigenvalues σ_i of Σ^2. They are gauge invariant - any condition on them does not break gauge symmetry

$$\sigma_1 = \sigma_2 = \sigma_3 = v_{GUT}^2 \,, \quad \sigma_4 = \sigma_5 = \frac{9}{4} v_{GUT}^2 \,,$$

From the geometrical point of view, this operation confines the theory on a specific manifold in the field-space. When this is done, a generic $\boldsymbol{\Sigma}$ field can be expressed as

$$\boldsymbol{\Sigma^2 = U \begin{pmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5 \end{pmatrix} U^\dagger \,,}$$

with $\boldsymbol{U} \in \boldsymbol{G}$. The above spans the twelve-dimensional space of Goldstones.

- Scalar leptoquarks in 5

$$H^\dagger \Sigma^2 H - \frac{3}{10} \mathrm{Tr}(\Sigma^2) H^\dagger H = 0 \,.$$

This requirement eliminates the color triplet contained in H, but leaves intact the remaining two components which are identified with the SM Higgs field

- Vector leptoquarks in 24

$$\text{Tr}\left([\Sigma, D_\mu \Sigma]^2\right) = 0 \;,$$

All the heavy vector leptoquarks are set to zero, together with corresponding Goldstones. The twelve SM gauge fields are not affected.

Resulting theory: Renormalisable Standard Model which inherits from SU(5)

- fermion quantum numbers
- relations between the gauge couplings
- relations between the Yukawa couplings

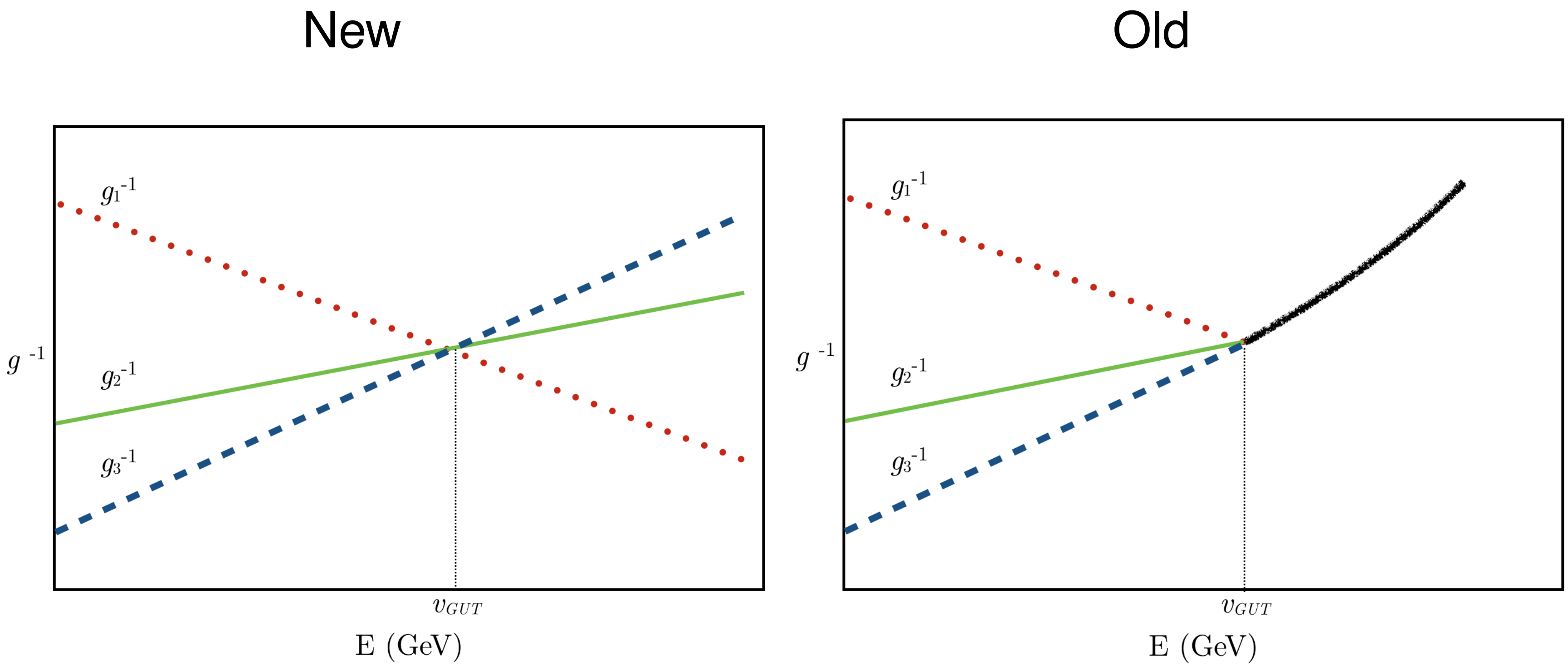
Small Higgs mass requirement:

$$m_H^2 - \frac{1}{2}(\lambda_{HH} v_{EW}^2 + (\lambda_{\Sigma H} + \lambda'_{\Sigma H}) v_{GUT}^2) \sim \mathcal{O}(10^4)\ \text{GeV}^4\ .$$

This relation constitutes a fine-tuning that is not explained. It is, however a technically natural condition due to absence of superheavy particles.

No proton decay!

Gauge coupling unification



As in the Minimal SU(5):

- $v_{GUT} \simeq 10^{14}$ GeV, but no problem with the proton decay
- $\sin^2\theta_W \simeq 0.2$ – too small

How to correct $\sin^2\theta_W$? Proposal goes back to Hill; Shafi and Wetterich: add higher-dimensional operators suppressed by the Planck scale,

$$\mathcal{O}_{4+n} = \mathrm{Tr}\left[F_{\mu\nu}\Sigma^k F^{\mu\nu}\Sigma^{n-k}\right] \ , \quad 0 \leq k < n \ , \quad n > 0 \ ,$$

With our constraint on Σ, these terms modify the relation $g_1 = g_2 = g_3$ at the GUT scale, change the prediction of $\sin^2\theta_W$, and modifying v_{GUT}. The theory is still renormalisable and no new degrees of freedom are introduced!

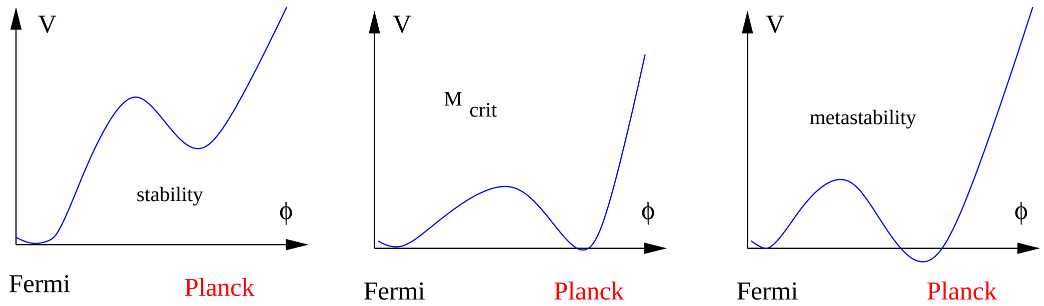
A viable possibility: $v_{GUT} \simeq M_P$ – unity of all forces at the Planck scale?

Other problems of the SM

In our approach we have no new particles up to the gravitational Planck scale. How to deal with the SM problems:

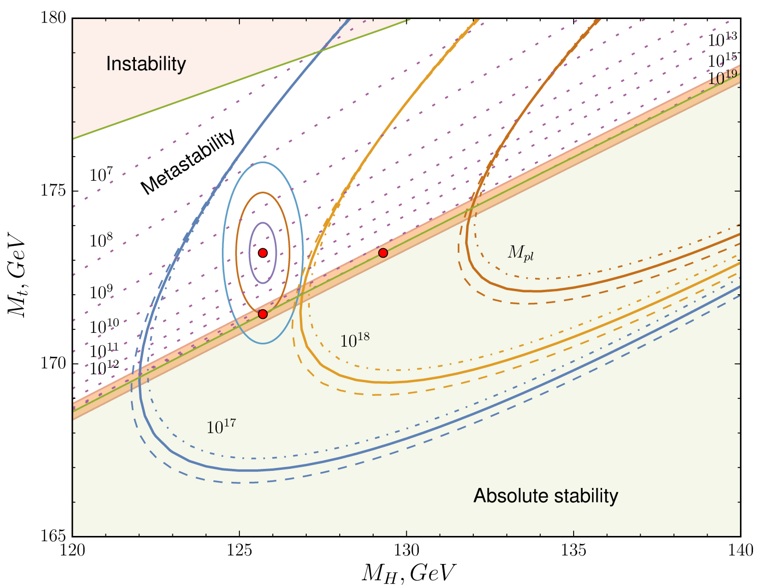
- Observations of neutrino oscillations (in the SM neutrinos are massless and do not oscillate)
- Evidence for Dark Matter (SM does not have particle physics candidate for DM).
- No antimatter in the Universe in amounts comparable with matter (baryon asymmetry of the Universe is too small in the SM)
- Cosmological inflation is absent in canonical variant of the SM
- Accelerated expansion of the Universe (?) - though can be "explained" by a cosmological constant.

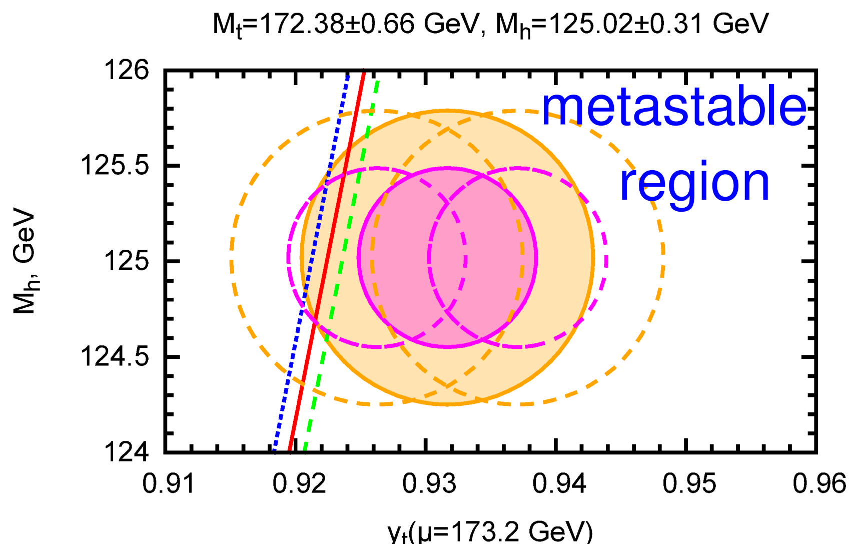
- Marginal evidence (less than 2σ) for the SM vacuum metastability given uncertainties in relation between Monte-Carlo top mass and the top quark Yukawa coupling



Bednyakov et al, ’15

Vacuum is unstable at 1.3σ





Where is new physics?

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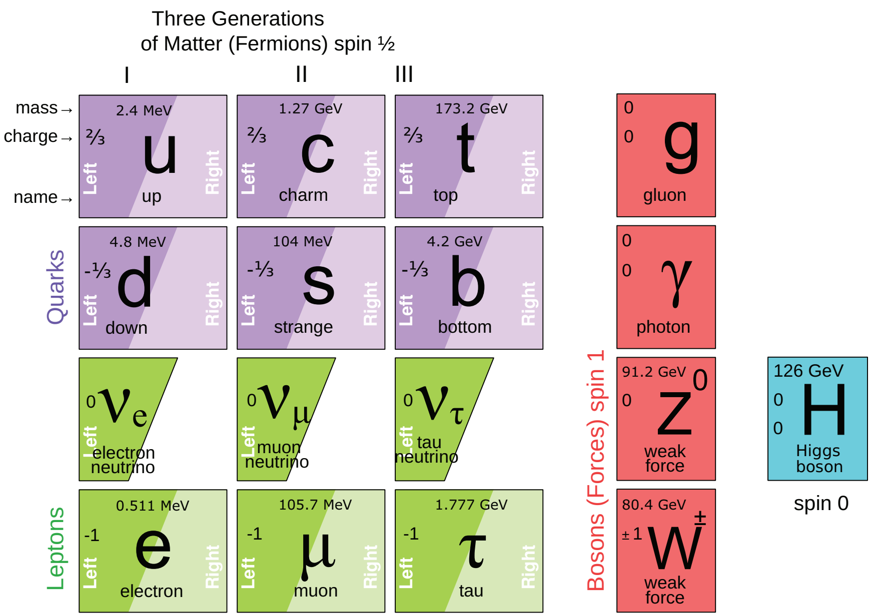
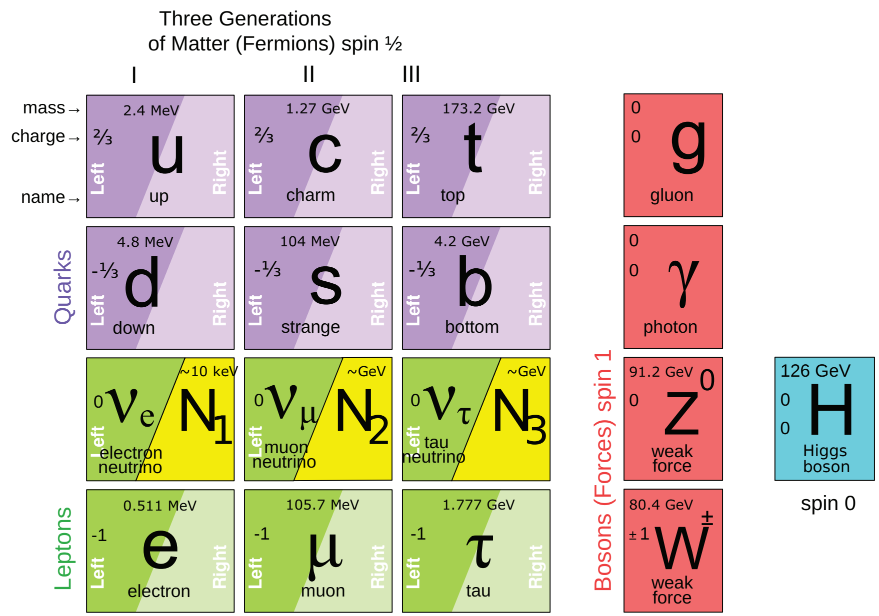
Below the Fermi scale

New physics below the Fermi scale: the νMSM





Role of the Higgs: EW symmetry breaking, inflation

Role of N_1 with mass in keV region: dark matter.

Role of $N_2, \ N_3$ with mass in 100 MeV – GeV region: “give” masses to neutrinos and produce baryon asymmetry of the Universe.

All fermions can be embedded in SO(10)

Scale and conformal invariance. FRG?

Why scale invariance?

If the mass of the Higgs boson is put to zero in the SM, the Lagrangian has a wider symmetry: it is scale and conformally invariant.

Dilatations - global scale transformations ($\sigma = const$)

$$\Psi(x) \rightarrow \sigma^n \Psi(\sigma x) \, ,$$

$n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

It is tempting to use this symmetry for solution of the hierarchy problem

Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a} ,$$

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The way out: scale independent subtraction of divergences Englert, Truffin '76; Wetterich '88; MS, Zenhausern, '08

Scale-invariant SU(5) construction

Extra field - dilaton χ. Also appears as a normalisation point in renormalisation procedure. Constraint for Σ should be replaced by

$$\sigma_1 = \sigma_2 = \sigma_3 = \alpha\chi^2 \,, \quad \sigma_4 = \sigma_5 = \frac{9\alpha}{4}\chi^2 \,,$$

where α is a dimensionless constant. The remaining two conditions for vectors and H remain the same.

The scale-invariant potential for the theory: add a quartic self-interaction for the dilaton, $\Lambda'\chi^4$, and replace the mass terms for Σ and H by the dilaton couplings:

$$m_\Sigma^2 = \frac{15\nu\alpha}{4}\chi^2 \,, \quad m_H^2 = \frac{15\mu\alpha}{2}\chi^2 \,,$$

Resulting potential for the Higgs field h:

$$V = \lambda \left(h^\dagger h - \frac{\beta}{2\lambda} \chi^2 \right)^2 + (\Lambda + \Lambda') \chi^4 \ ,$$

and λ, β, Λ are related to the constants appearing in V as

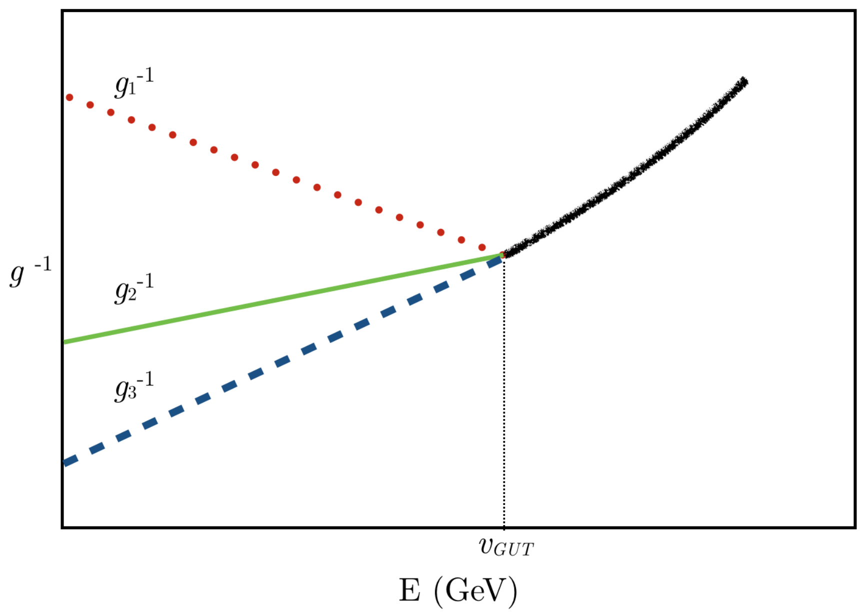
$$\lambda = \frac{\lambda_{HH}}{4} \ , \ \ \beta = \frac{15\alpha}{4} (\mu - \lambda_{\Sigma H} - \lambda'_{\Sigma H}) \ ,$$

$$\Lambda = \left(\frac{15\alpha}{4} \right)^2 \left(\lambda_{\Sigma\Sigma} + \lambda'_{\Sigma\Sigma} - \nu - \lambda^{-1}_{HH} (\mu - \lambda_{\Sigma H} - \lambda'_{\Sigma H})^2 \right) \ .$$

Existence of flat direction (absence of cosmological constant) - unexplained fine-tuning, $\Lambda + \Lambda' = 0$. Gauge hierarchy condition $\frac{\beta}{\alpha} \ll 1$, is a technically natural requirement, since the dilaton has an approximate shift symmetry in the limit $\beta \to 0, \ \Lambda + \Lambda' \to 0$.

UV limit? FRG?

High energy limit, $\boldsymbol{E \gg v_{GUT}}$: equivalent to $\boldsymbol{\chi \to 0}$? $\boldsymbol{\Sigma = 0}$ as a solution to all constraints? If true, the UV degrees of freedom are SU(5) gauge bosons, fermions, dilaton and the Higgs 5-plet. Asymptotically free behaviour?



Inclusion of gravity

Planck scale: through non-minimal coupling of the dilaton to the Ricci scalar.

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + \xi_h h^2\right) \frac{R}{2} ,$$

This term, for $\xi_\chi \sim 1$, does break the shift symmetry. However, this is a coefficient in front of graviton kinetic term. Since the graviton stays massless in any constant scalar background, the perturbative computations of gravitational corrections to the Higgs mass in scale-invariant regularisation are suppressed by M_P. There are no corrections proportional to M_P!

Consequences

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- Since it is a Goldstone boson of spontaneously broken symmetry it has only derivative couplings to matter (inclusion of gravity is essential: it makes scale transformations to be internal symmetry!)
- Fifth force or Brans-Dicke constraints are not applicable to it

Conclusions

- The gauge coupling unification scale may be not related to the mass of any particle
- “Constrained GUTs" provide a specific example of unified theories without leptoquarks
- In these theories the EW scale is stable against radiative corrections

Problems and weak points

- The choice of GUT symmetry is arbitrary
- The choice of scalar multiplets is arbitrary
- Why 3 generations?
- Why the Planck scale is so different from the weak scale?
- Origin of constraints - why the one leading to the SM is the best one?