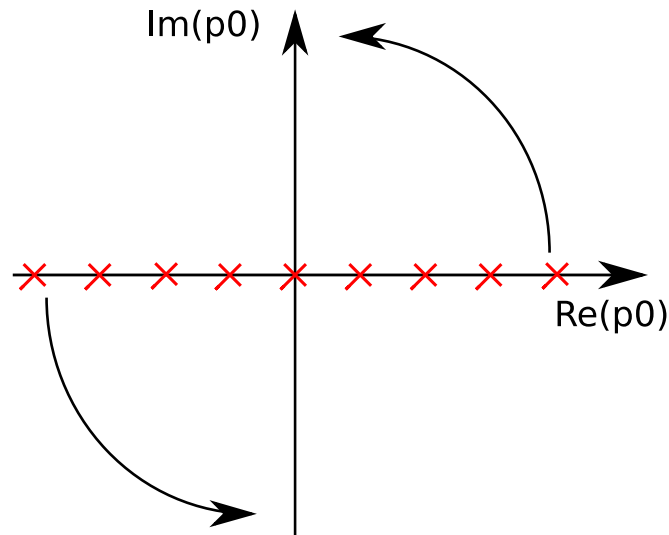


# Real time correlation functions at finite temperature



**Nicolas Wink**

In collaboration with

N. Strodthoff

J. M. Pawłowski

Based on

Formalism + Vacuum

[Pawłowski, Strodthoff, PhysRevD.92.094009](#)

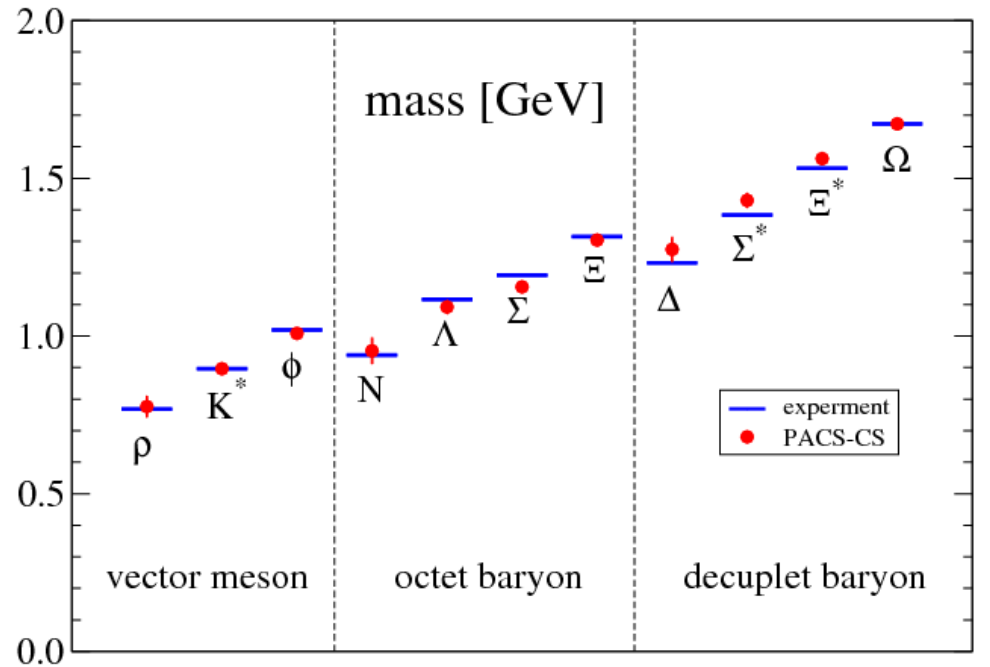
Self-consistent Vacuum

[Strodthoff, arXiv:1611.05036](#)

Finite temperature

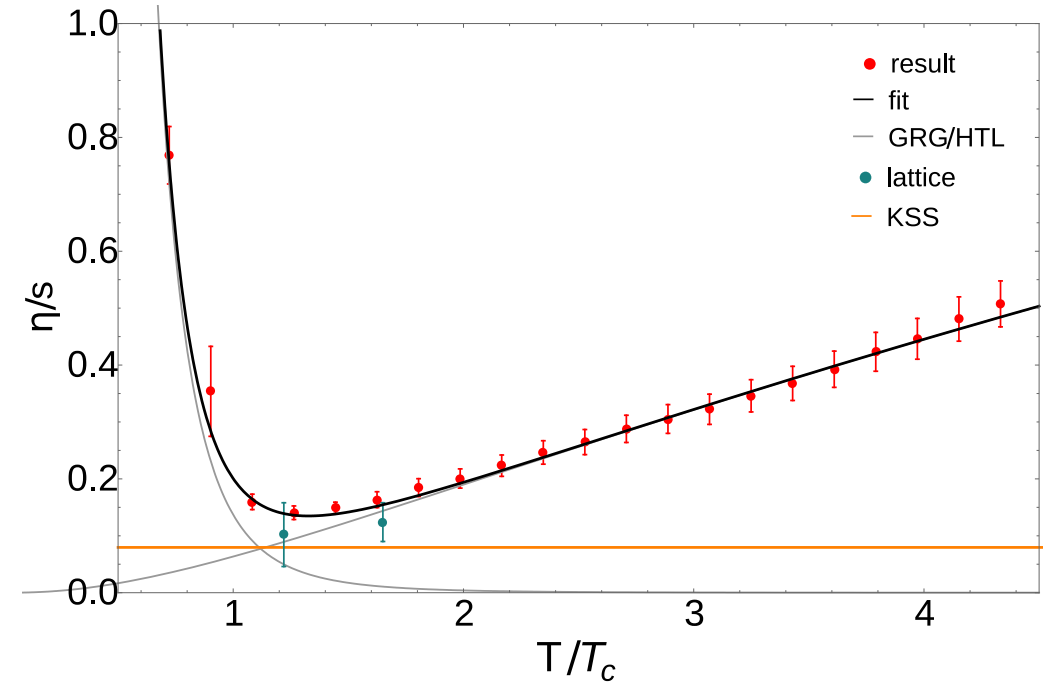
[Pawłowski, Strodthoff, NW, in prep](#)

# Why real time correlation functions?



PACS-CS collaboration

Bound state spectrum

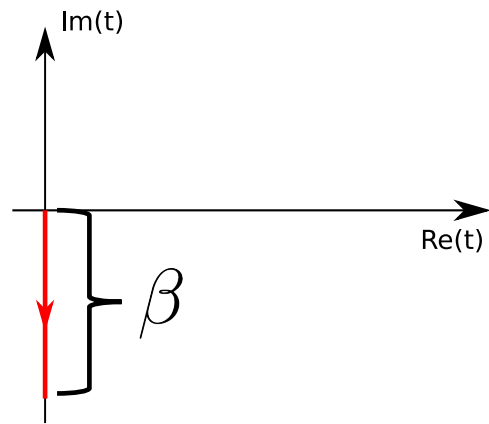


PRL, 115 (2015) no.11, 112002

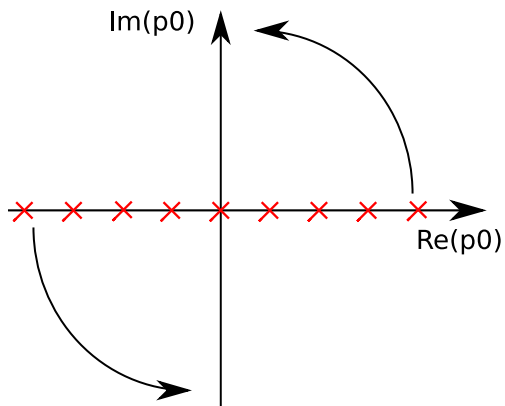
Christiansen, Haas, Pawłowski, Strodthoff

Transport coefficients

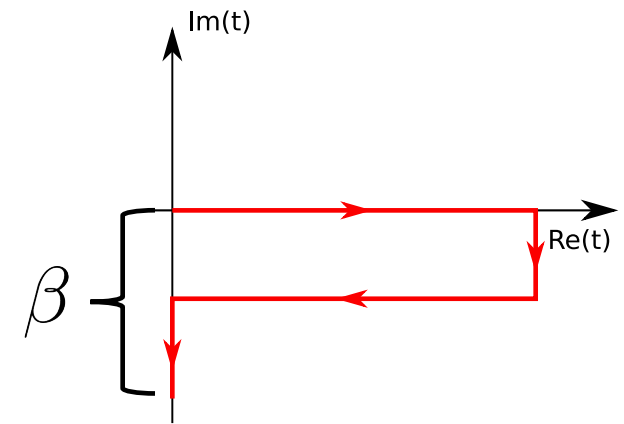
# From imaginary to real times



Matsubara contour



Continuation from Matsubara frequencies



Schwinger-Keldysh contour

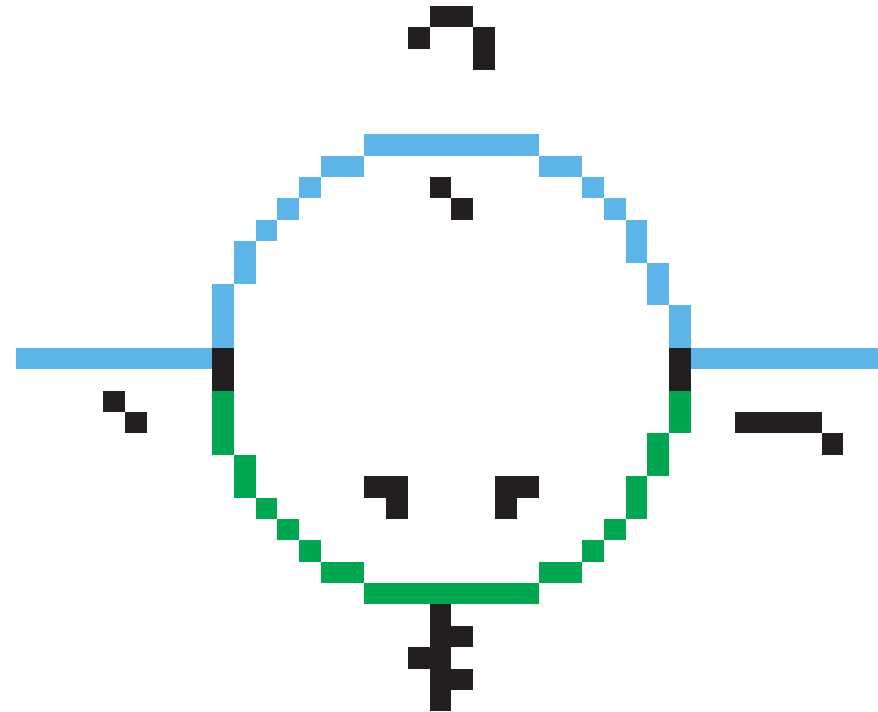
Use analyticity constraints and KMS condition to obtain real time correlation functions from Matsubara formalism

## Illustrative example

Two bosonic fields with  $\sim \Phi\Phi\varphi$

Calculate  $\Gamma^{(2)}(p)$  for  $p^0 \in \mathbb{C}$

Calculate Matsubara sum  $\sum_{T,q} G_1(q+p)G_2(q)$

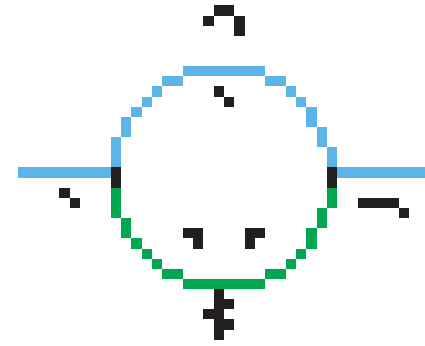


# Illustrative example

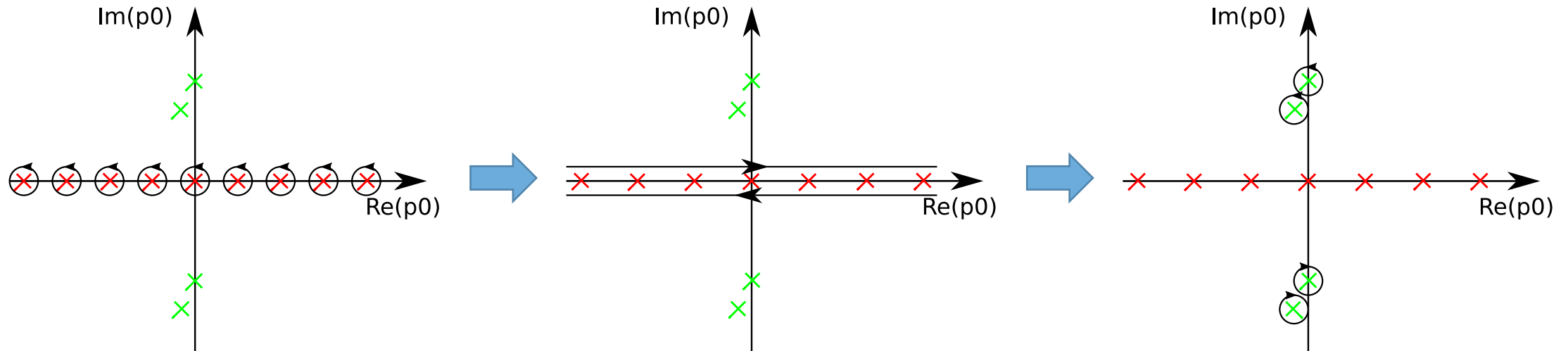
Replace sum by contour integral:

$$T \sum_n f(2\pi nT) = -\frac{1}{2} \int_C dz f(z) [1 + 2n_B(iz)]$$

Bosonic occupation number

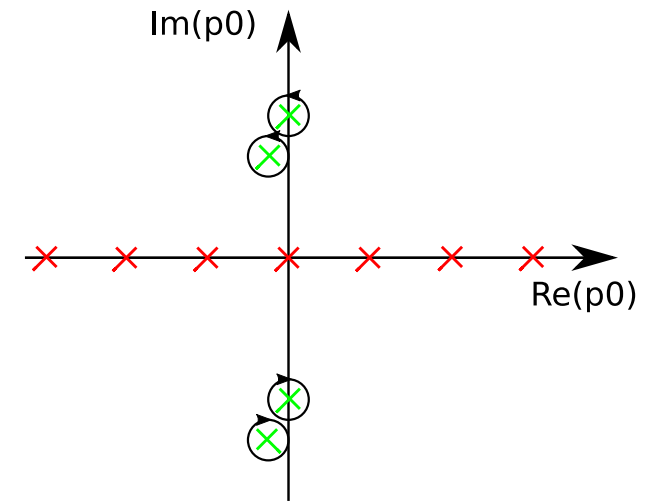


$$\sum_T \frac{1}{(q_0 + p_0)^2 + (\epsilon_{q+p}^1)^2} \frac{1}{(q_0)^2 + (\epsilon_q^2)^2}$$



# Illustrative example

$$\frac{1}{i} \sum_{\pm} (\text{Res}_1^{\pm} \cdot [1 + 2n_B(-i\cancel{p_0} + \epsilon_{q+p}^1)] + \text{Res}_2^{\pm} \cdot [1 + 2n_B(\epsilon_q^2)])$$



Identify ambiguity of the analytic continuation

$$p_0 = 2\pi mT \quad m \in \mathbb{Z}$$



$$n_B(ip_0) = 1$$



Analytic off the imaginary axis

Correct decay behaviour at infinity

Mathematically rigorous

Baym and Mermin, *Journal of Mathematical Physics* 2, 232 (1961)

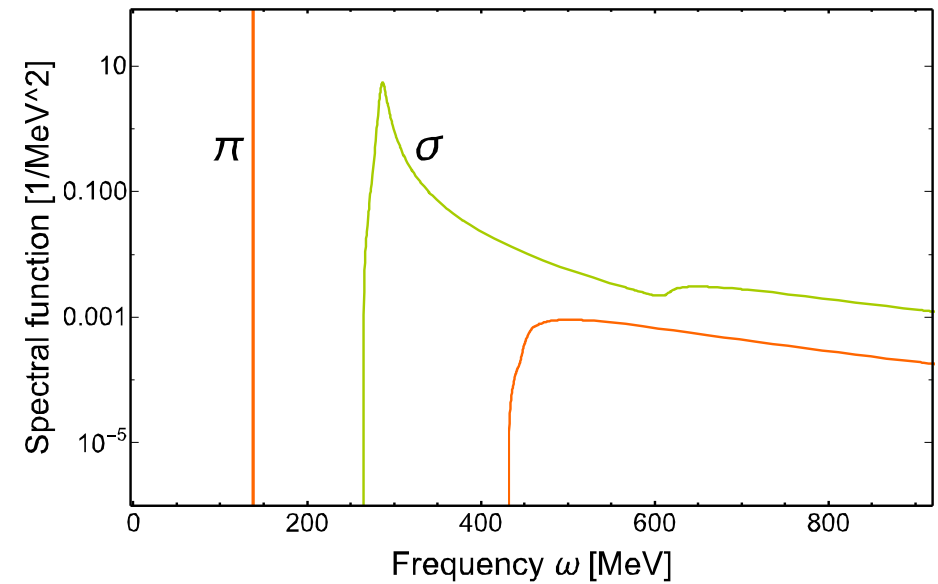
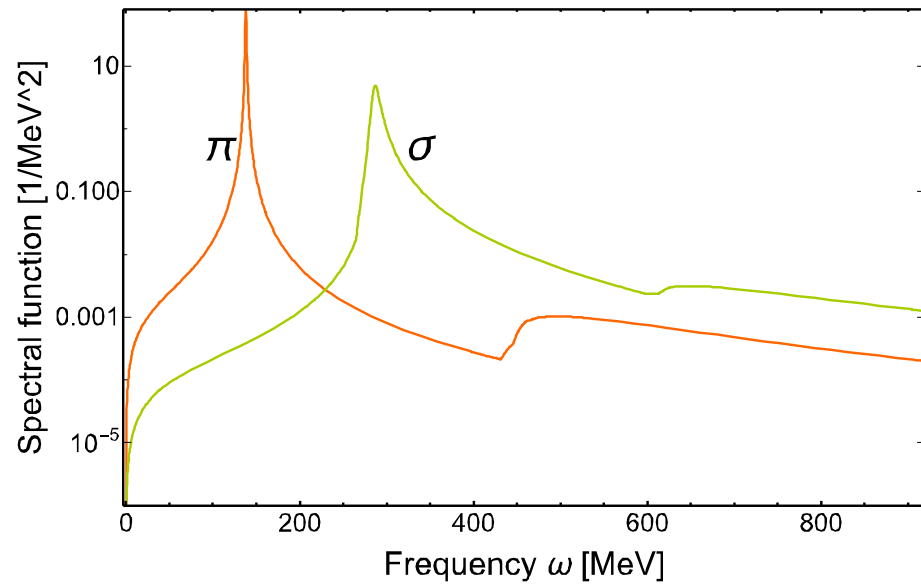
Unique physical analytic continuation identified by setting  $n_B(ip_0) = 1$  everywhere

# Retarded/Advanced Greens function

Retarded Greens function  $\lim_{\varepsilon \rightarrow 0} G(-i(\omega + i\varepsilon))$

Take limit analytically

Numerical extrapolation



# Generalisation to the FRG

No new conceptual problems

Kamikado, Strodthoff, von Smekal, Wambach, Eur.Phys.J. C74, 2806 (2014)

Tripolt, Strodthoff, von Smekal, Wambach, Phys.Rev. D89, 034010 (2014)

Regulator poles

$$R_k(\vec{q}^2)$$

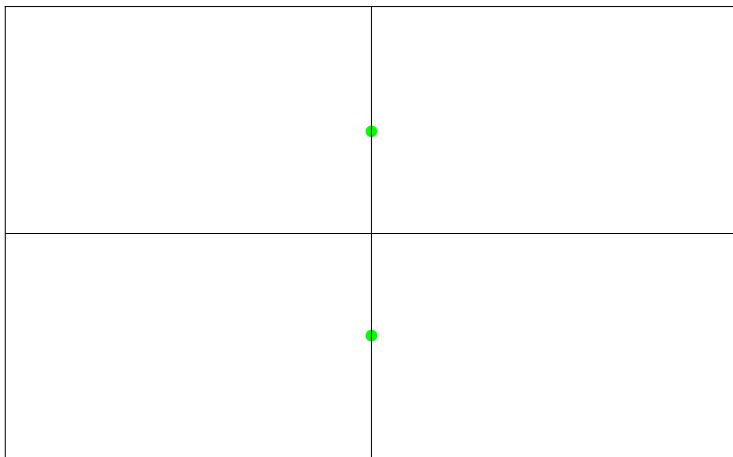


No changes

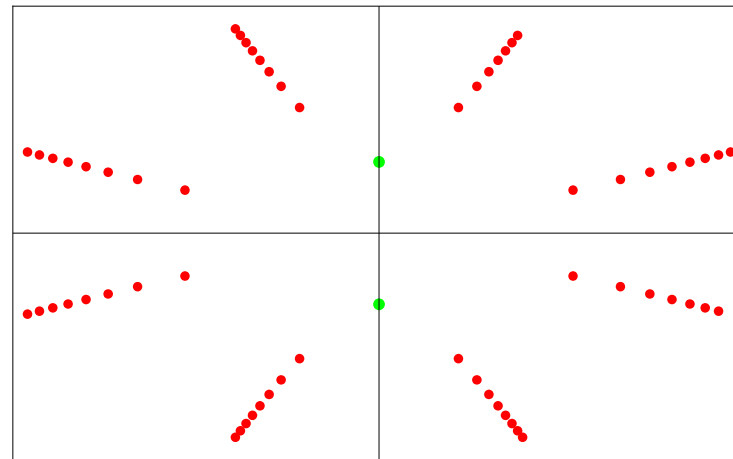
$$R_k(q^2)$$



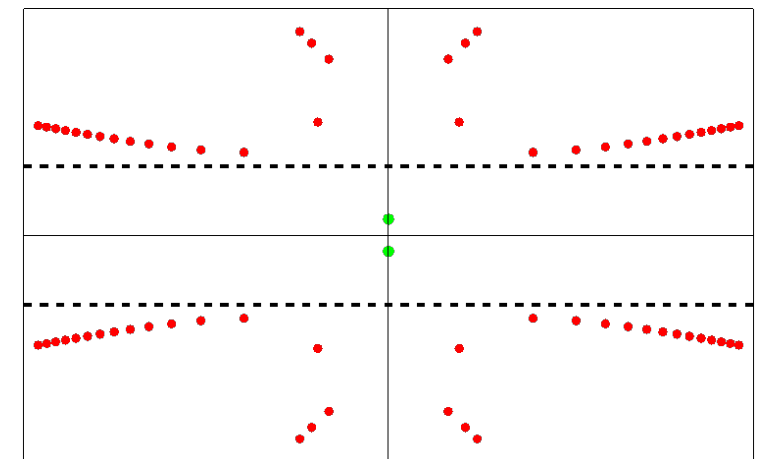
Additional poles



$$\frac{1}{q^2 + m^2}$$



$$\frac{1}{q^2 + m^2 + R_k(q^2)}$$



$$\frac{1}{q^2 + m^2 + R_k(q^2 + m_r)}$$



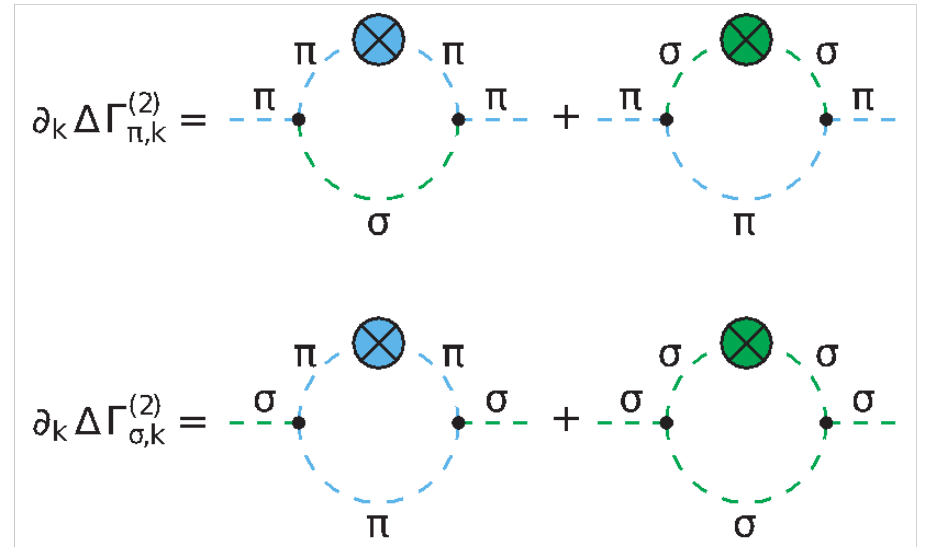
# Application to the O(N)-Model

Effective description of the lightest mesons

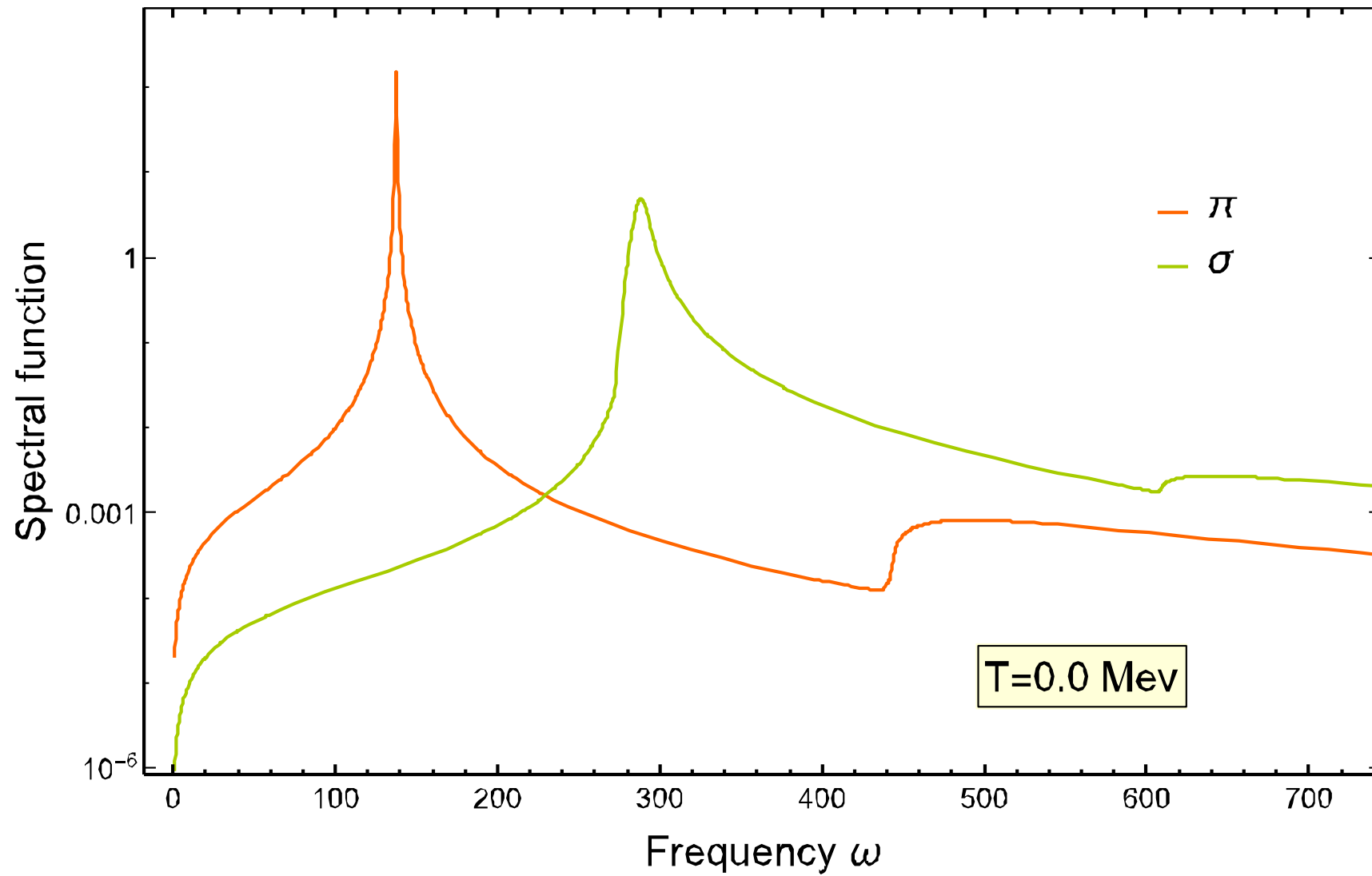
$$\Gamma_k = \int_{T,q} \frac{Z_\sigma}{2} q^2 \sigma \sigma + \frac{Z_\pi}{2} q^2 \pi_a \pi^a + V(\sigma)$$

Calculate spectral functions of the O(N) model

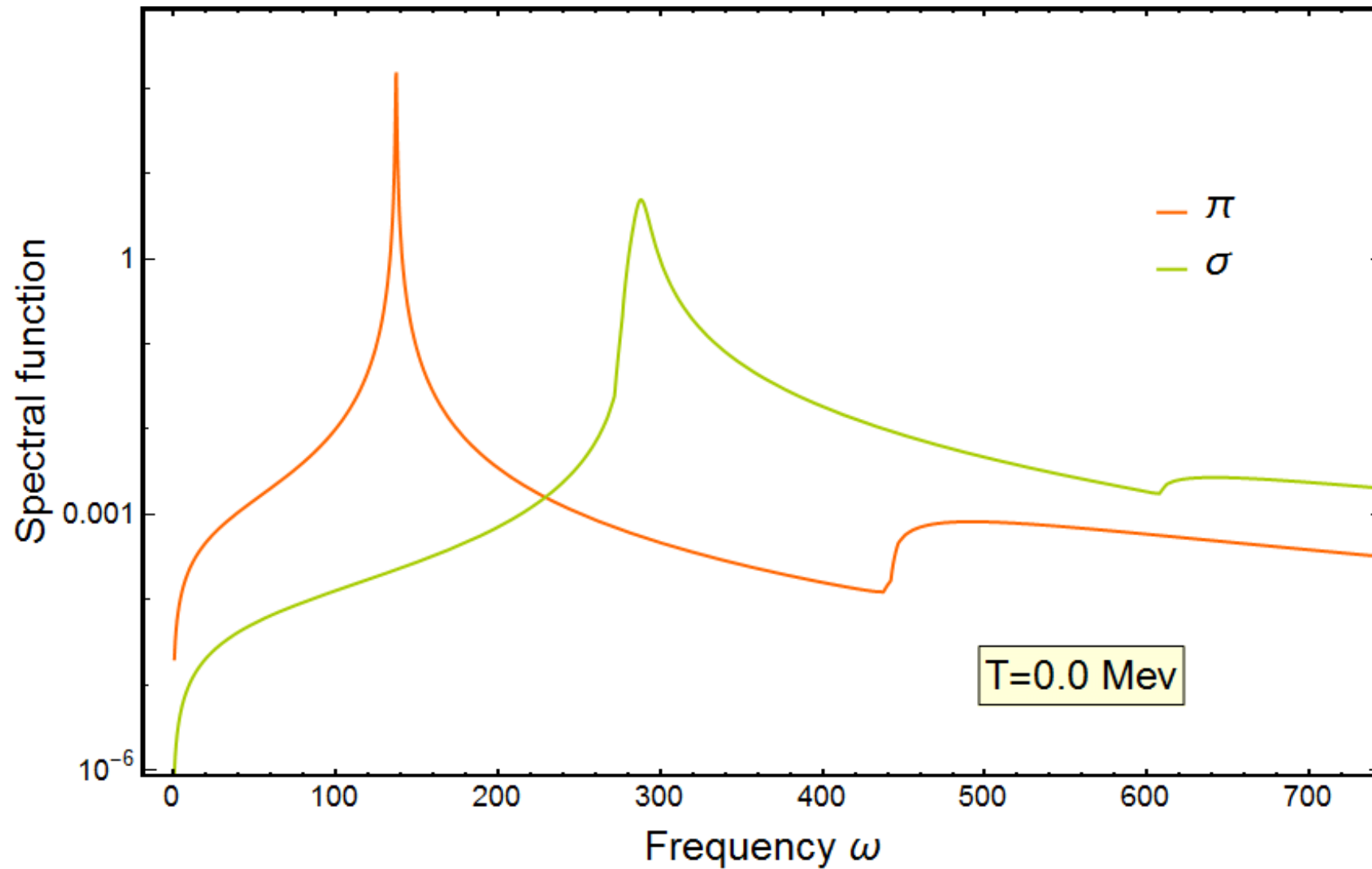
$$\rho(\omega, \vec{p}) = -2 \text{Im} G_R(\omega, \vec{p})$$



# Application to the O(N)-Model



# Application to the O(N)-Model



## Summary & Outlook

- Perform analytic continuation
- Conceptual easy algorithm
- Finite temperature spectral functions

- Fully self-consistent truncation at finite temperature
- Real time representation of vertices
- Application to different model