Dynamical gluon mass generation: Theory and Applications

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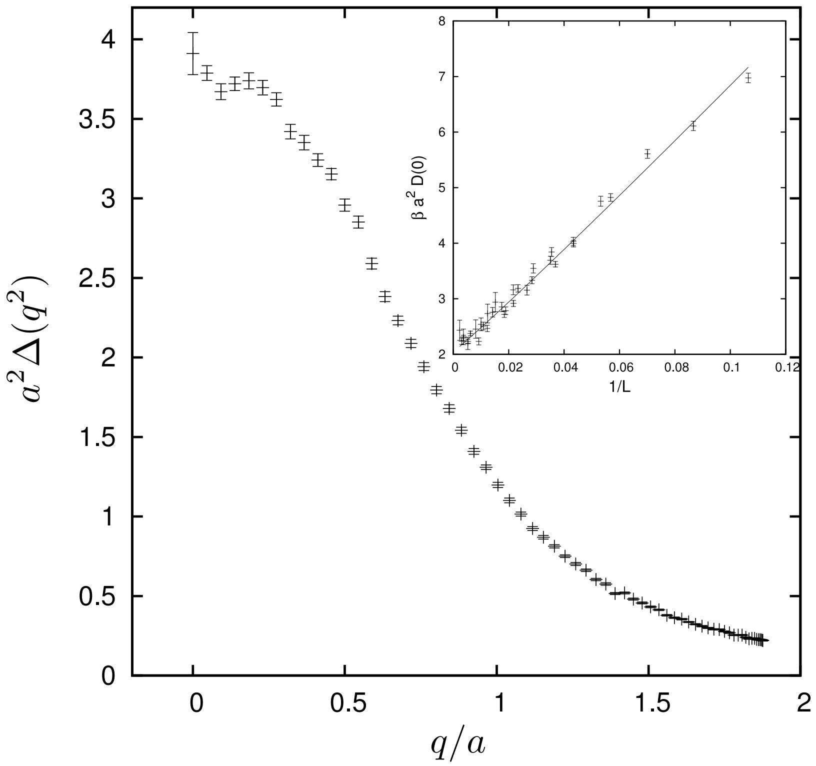
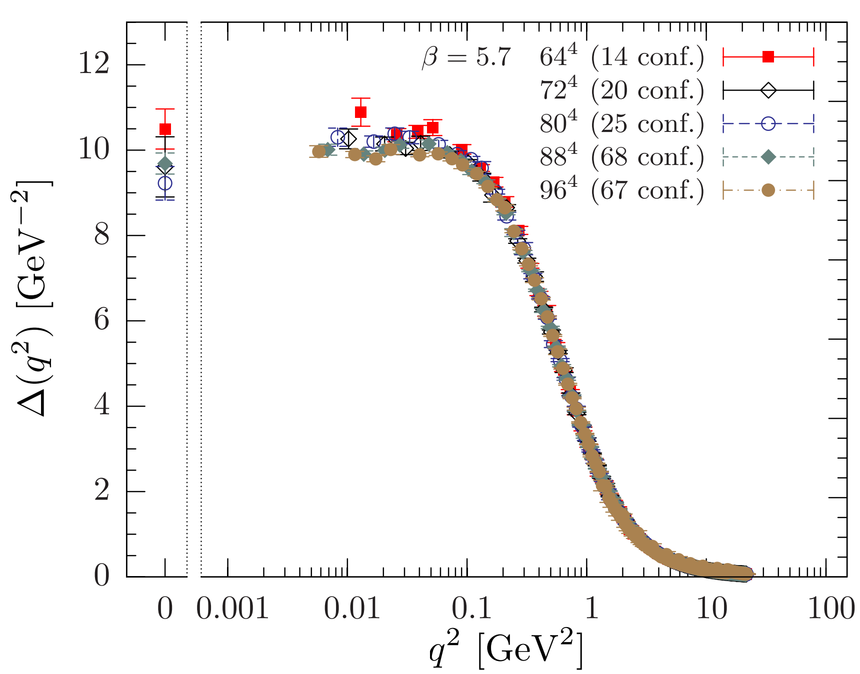
Gluon propagator (lattice)

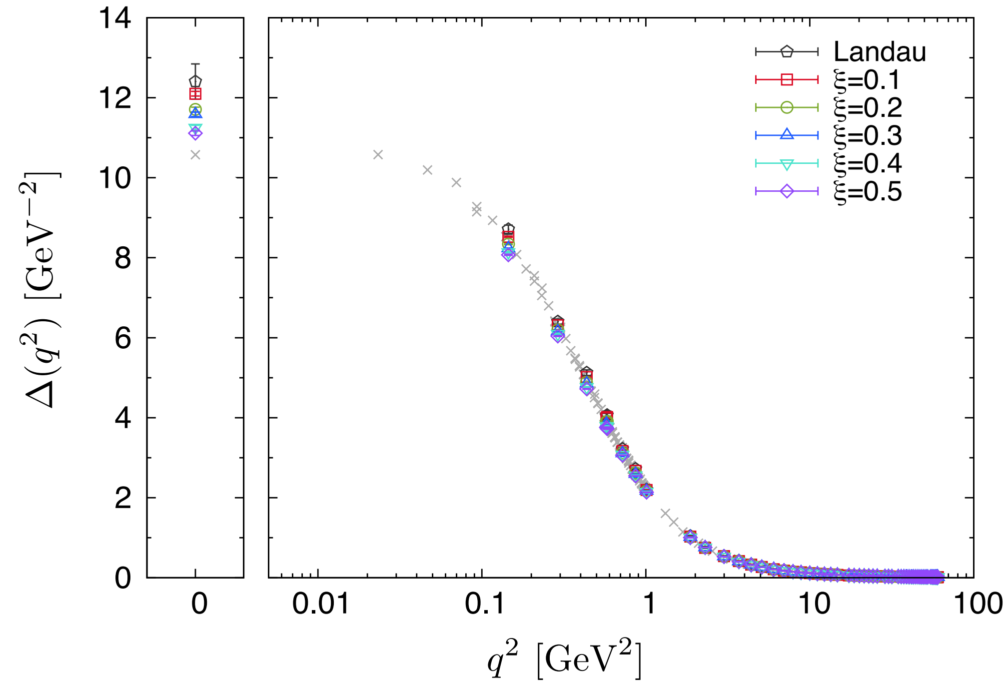
- **Landau gauge** (quenched)
  Cucchieri, Mendes POS LATTICE (2007)



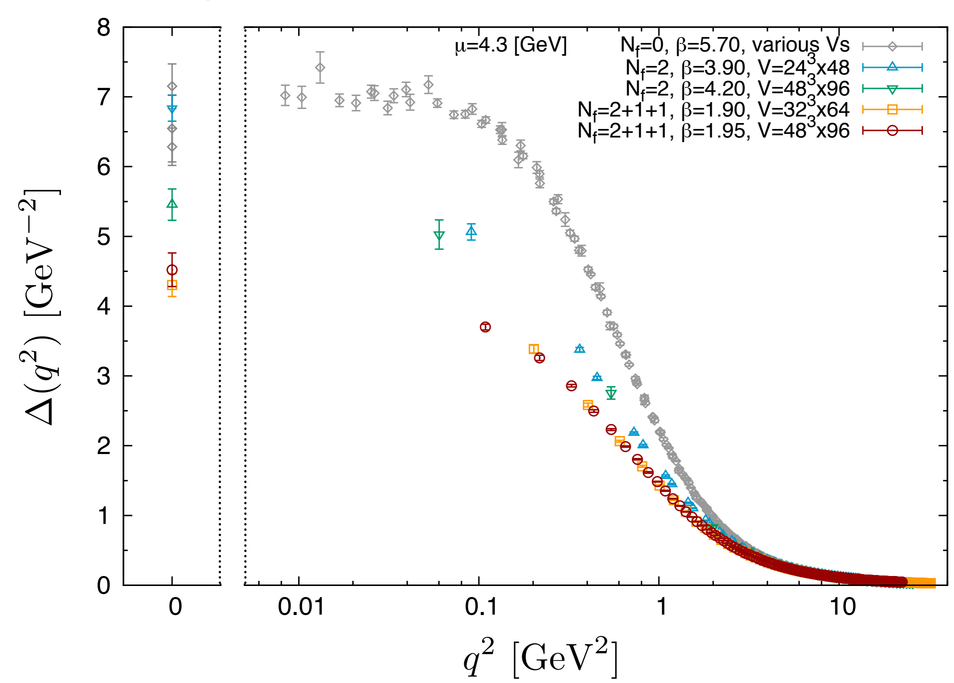
Bogolubsky, Ilgenfritz, Muller-Preussker, Sternbeck, PLB 676 (2007/2009)



- **Linear gauges** (quenched)
  Bicudo, DB, Cardoso, Oliveira, Silva, PRD 92 (2015)



- **Landau gauge** (unquenched)
  Ayala, Bashir, DB, Cristoforetti, Rodriguez-Quintero, PRD 86 (2012)



Gluon propagator (continuum)

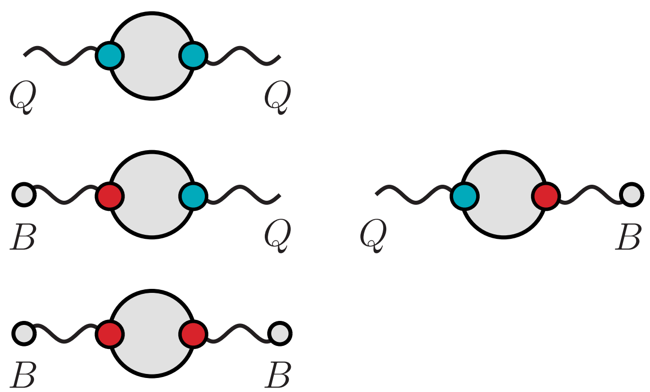
Cornwall, PRD 26 (1982)

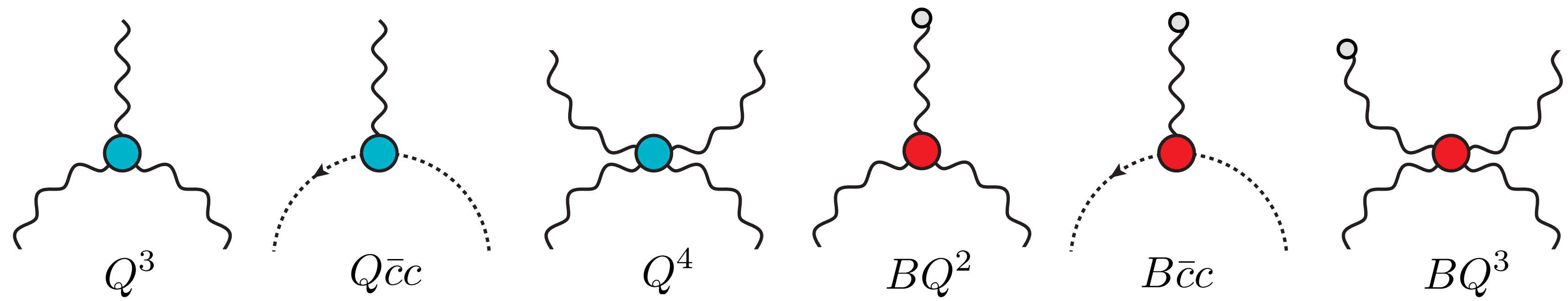
Lavelle, PRD 44 (1991)
Halzen, Krein, Natale, PRD 47 (1993)
Sczepaniak, Swanson, PRD 65 (2002)
Aguilar, DB, Papavassiliou, PRD 78 (2008)
Boucaud et al., JHEP 06 (2008)
Dudal, Gracey, Sorella, Vandersickel, Verschelde, PRD 78 (2008)
Fischer, Maas, Pawlowski, AP 324 (2009)
Braun, Gies, Pawlowski, PLB 684 (2010)
Tissier, Wschebor, PRD 82 (2010)
Pennington, Wilson, PRD 84 (2011)
Kondo, PRD 84 (2011)
Strauss, Fischer, Kellermann, PRL 109 (2012)
Watson, Reinhardt, PRD 85 (2012)
Serreau, Tissier, PLB 712 (2012)
Huber, PRD 91 (2015)
Gao, Qin, Roberts, Rodriguez-Quintero, PRD 97 (2018)
…

PT-BFM framework

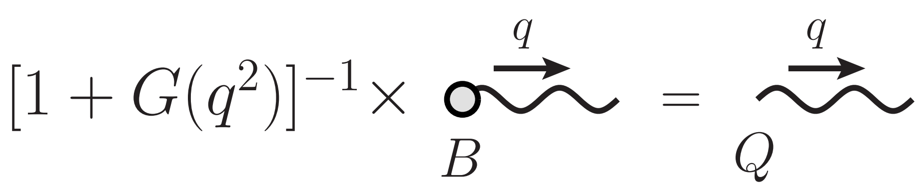
- **Split gauge field**
  into background (B) and quantum fluctuating (Q) parts
  Abbott, NPB 185 (1981)

- **Proliferation of Green's functions**
  three possibilities in two-point gluon sector





- **Symmetry induced identities**
  relate B and Q functions; in 2-point sector:
  DB, Papavassiliou, PRD 66 (2002)



- **G function known**
  constrained by antiBRST symmetry
  DB, Quadri, PRD 88 (2013)

$$1 + G(0) = F^{-1}(0)$$

PT-BFM framework

- **Resumed gluon SDE**
  expressed in terms of QB self-energy

$$\Delta^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i\widetilde{\Pi}_{\mu\nu}(q)}{1+G(q^2)}$$

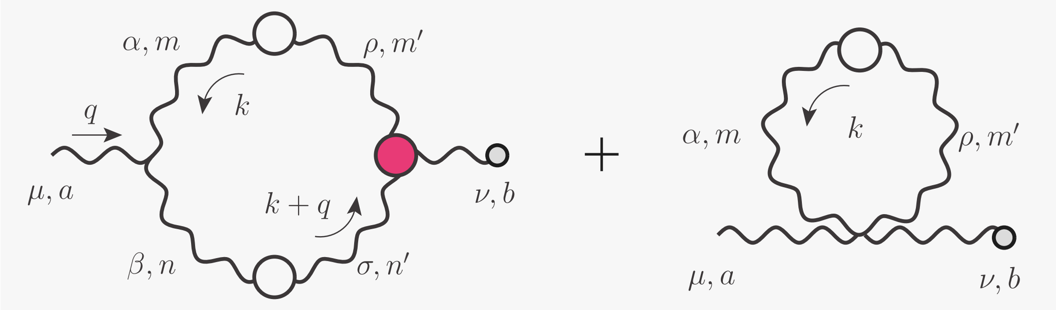
- **Divergence of B legs**
  gives rise to Abelian STIs

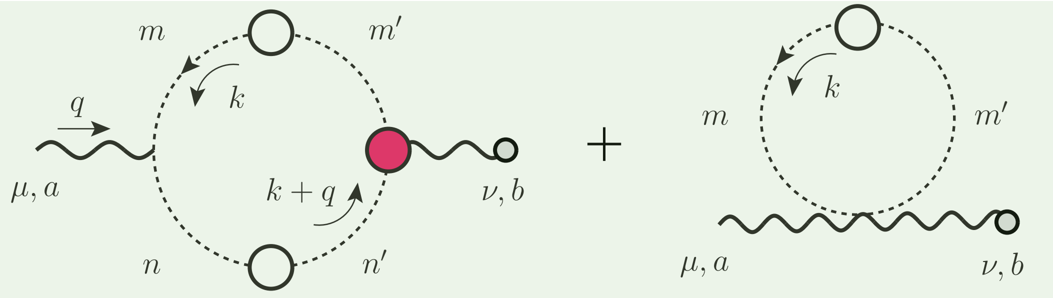
$$q^\nu \widetilde{\Gamma}_{\nu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

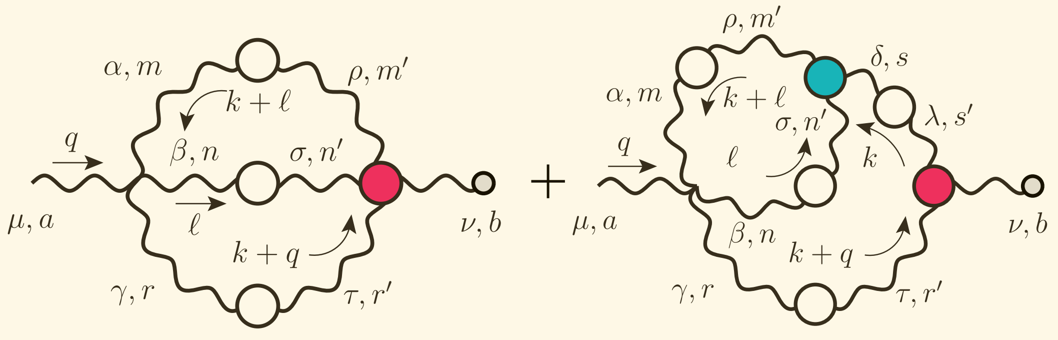
$$q^\nu \widetilde{\Gamma}_{\nu}(q,r,p) = iD^{-1}(r^2) - iD^{-1}(p^2)$$

$$q^\nu \widetilde{\Gamma}^{mnrs}_{\nu\alpha\beta\gamma}(q,r,p,t) = f^{mse}f^{ern}\Gamma_{\alpha\beta\gamma}(r,p,q+t) + f^{mne}f^{esr}\Gamma_{\beta\gamma\alpha}(p,t,q+r) + f^{mre}f^{ens}\Gamma_{\gamma\alpha\beta}(t,r,q+p)$$

- **Stronger version of transversality**
  allows gauge invariant truncations

$q^\nu \times$  $= 0$

$q^\nu \times$  $= 0$

$q^\nu \times$  $= 0$

$\Delta(0)$ in the absence of poles

- **Abelian STI for BQ^2 and BQ^3 vertices:**

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

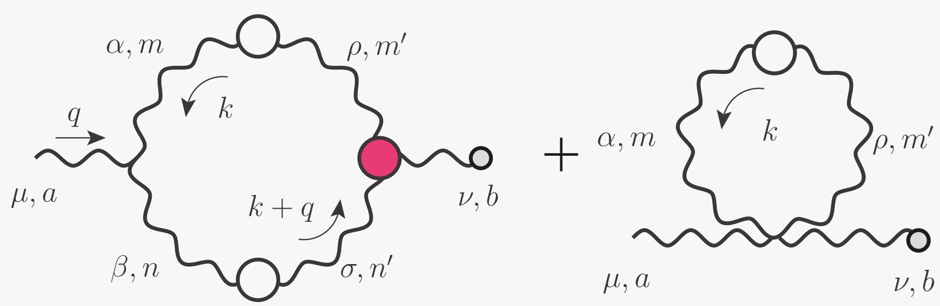
$$\begin{aligned} q^\nu \widetilde{\Gamma}^{mnrs}_{\nu\alpha\beta\gamma}(q,r,p,t) &= f^{mse}f^{ern}\Gamma_{\alpha\beta\gamma}(r,p,q+t) \\ &+ f^{mne}f^{esr}\Gamma_{\beta\gamma\alpha}(p,t,q+r) \\ &+ f^{mre}f^{ens}\Gamma_{\gamma\alpha\beta}(t,r,q+p) \end{aligned}$$

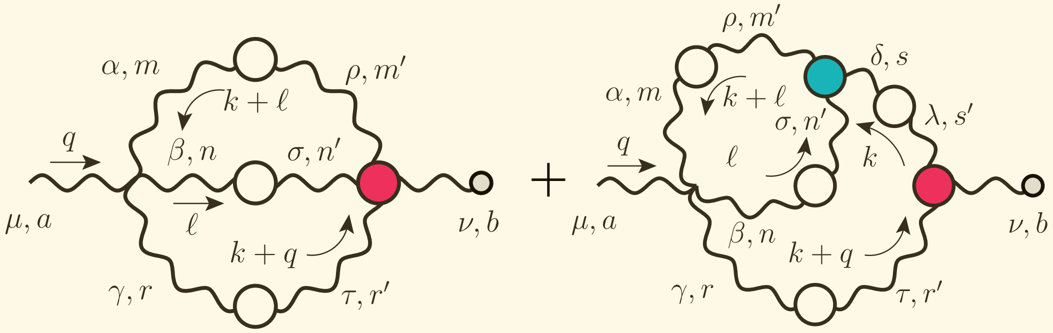
- **Taylor expand around q=0**
  assuming no $1/q^2$ poles are present

$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial}{\partial r^\mu}\Delta^{-1}_{\alpha\beta}(r)$$

$$\begin{aligned} \widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma}(0,-r,-p,r+p) &= -f^{mne}f^{esr}\frac{\partial}{\partial r^\mu}\Gamma_{\alpha\beta\gamma}(-r,-p,r+p) \\ &+ f^{mre}f^{ens}\frac{\partial}{\partial p^\mu}\Gamma_{\alpha\beta\gamma}(-r,-p,r+p) \end{aligned}$$

- **Plug into BQ gluon self-energy:**

$\Delta^{-1}(0) = \lim_{q\to 0} \mathrm{Tr}$ 

$+$ 

$$\sim \quad \alpha_s \int_k \frac{\partial}{\partial k_\mu}\left[k_\mu \Delta(k^2)\right] \quad + \quad \alpha_s^2 \int_k \frac{\partial}{\partial k_\mu}\left[k_\mu \Delta(k^2) Y(k^2)\right]$$

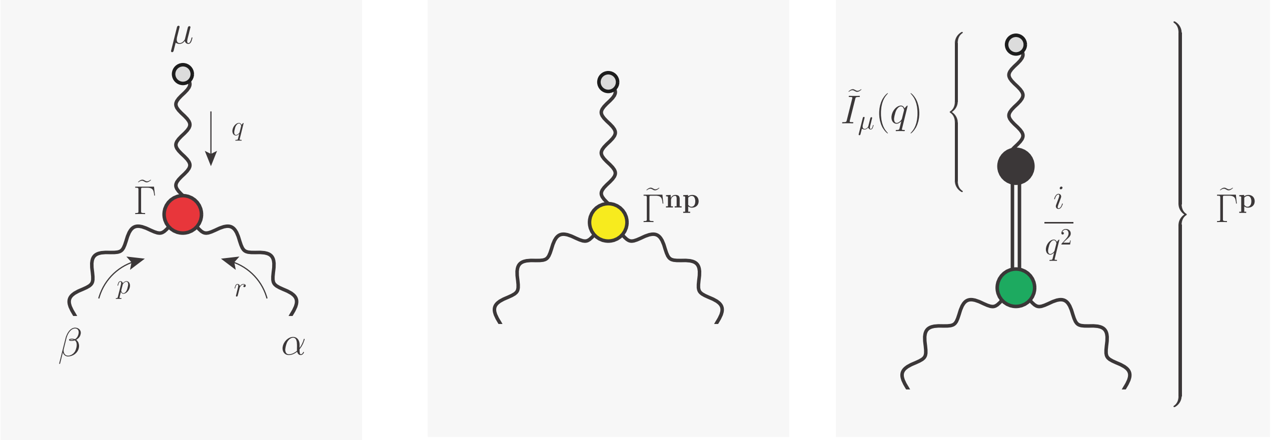
- **Seagull identity $\Delta^{-1}(0) = 0$**
  valid independently for each set of diagrams
  Aguilar, DB, Figueiredo, Papavassiliou, PRD 94 (2016)

- **No gluon mass scale generation**
  need to relax one fo the underlying assumptions…

Schwinger mechanism

- **Derivation of the $q \to 0$ Abelian WI**
  hinges on the absence of massless poles
- **Assume now that massless poles are dynamically generated in the vertex**
  Schwinger, PR 125 (1962); PR 128 (1962)
  Jackiv, Johnson, PRD 8 (1973)
  Eichten, Feinberg, PRD 10 (1973)

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p)$$



- **Colored composite massless**
  bound state excitations

- **Poles are longitudinally coupled**
  enforced by Lorenz invariance

$$\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p) = \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p)$$

- **5 possible form factors**
  in Landau gauge only $\widetilde{C}_1(q,r,p)g_{\alpha\beta}$ survives
- **Bose symmetry**
  Implies $\widetilde{C}_{\alpha\beta}(0,r,-r) = 0$

Evading the seagull identity

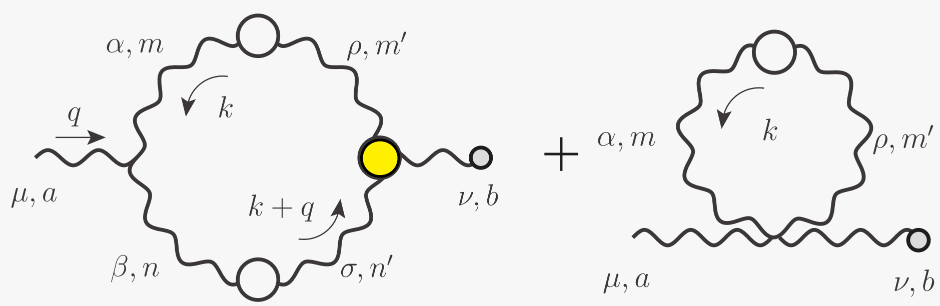
- **Vertex satisfies the same Abelian STI**

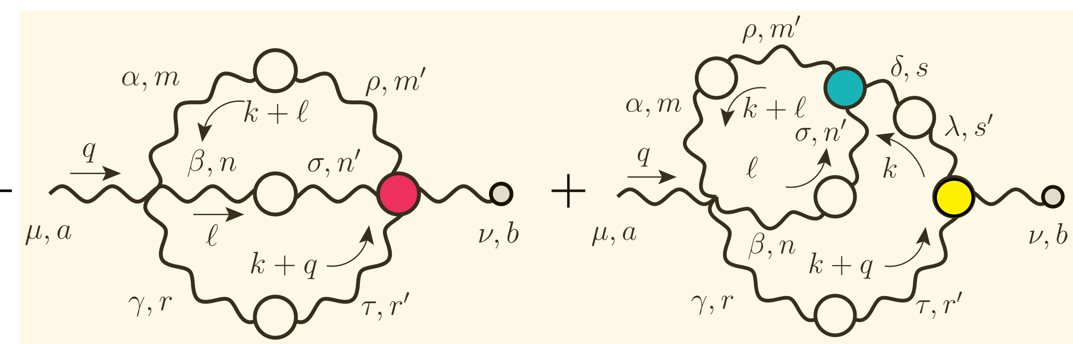
$$q^\mu \Gamma^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{C}_{\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

- **Expand around q=0**
  match orders in q

$$\Gamma^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial}{\partial r^\mu}\Delta^{-1}_{\alpha\beta}(r) - \left\{\frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,-r-q)\right\}_{q=0}$$

- **Plug into BQ gluon self-energy again**
  seagull identity is now deformed
  Aguilar, DB, Figueiredo, Papavassiliou, PRD 94 (2016)

$\Delta^{-1}(0) = \lim_{q\to 0} \mathrm{Tr}$ 

+ 

$$\sim \quad \alpha_s \int_k k^2\Delta^2(k^2)\widetilde{C}_1'(k^2) \quad - \quad \alpha_s^2 \int_k k^2\Delta^2(k^2)Y(k^2)\widetilde{C}_1'(k^2)$$

- **If $\widetilde{C}_1' \neq 0$, a gluon mass scale can be generated**

Gluon mass scale

- **Physically motivated parametrization**

  $$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

- **Insert into STI**

$$\begin{array}{ccc} q^{\mu}\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) & & p^2 J(p^2) P_{\alpha\beta}(p) - r^2 J(r^2) P_{\alpha\beta}(r) \\ + & = & + \\ \widetilde{C}_{\alpha\beta}(q,r,p) & & m^2(p^2) P_{\alpha\beta}(p) - m^2(r^2) P_{\alpha\beta}(r) \end{array}$$

- **Kinetic term**
  associated with np part of the STI
- **Mass term**
  associated with massless poles amplitude

- **Focus on the $g_{\alpha\beta}$ part**
  take limit as $q \to 0$

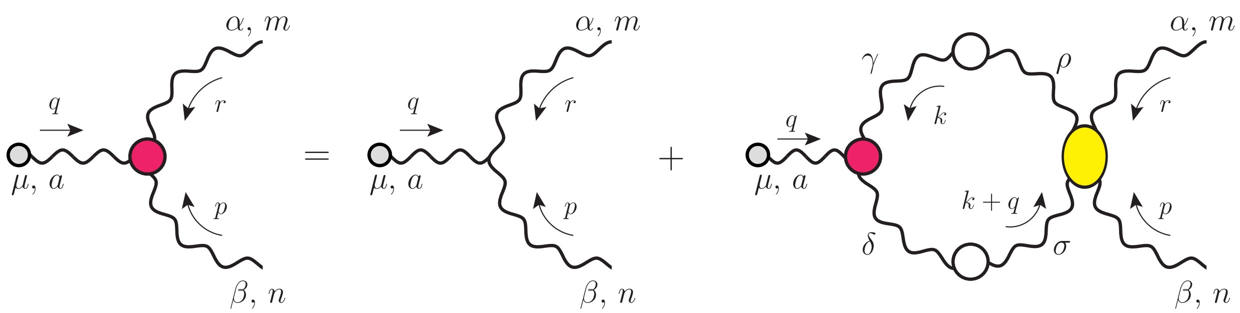
  $$\widetilde{C}_1'(r^2) = \frac{\mathrm{d}m^2(r^2)}{\mathrm{d}r^2} \quad \Rightarrow \quad m^2(x) = \Delta^{-1}(0) + \int_0^x \mathrm{d}y\, \widetilde{C}_1'(y)$$

- **Not yet a running mass**
  must be vanishing in the UV

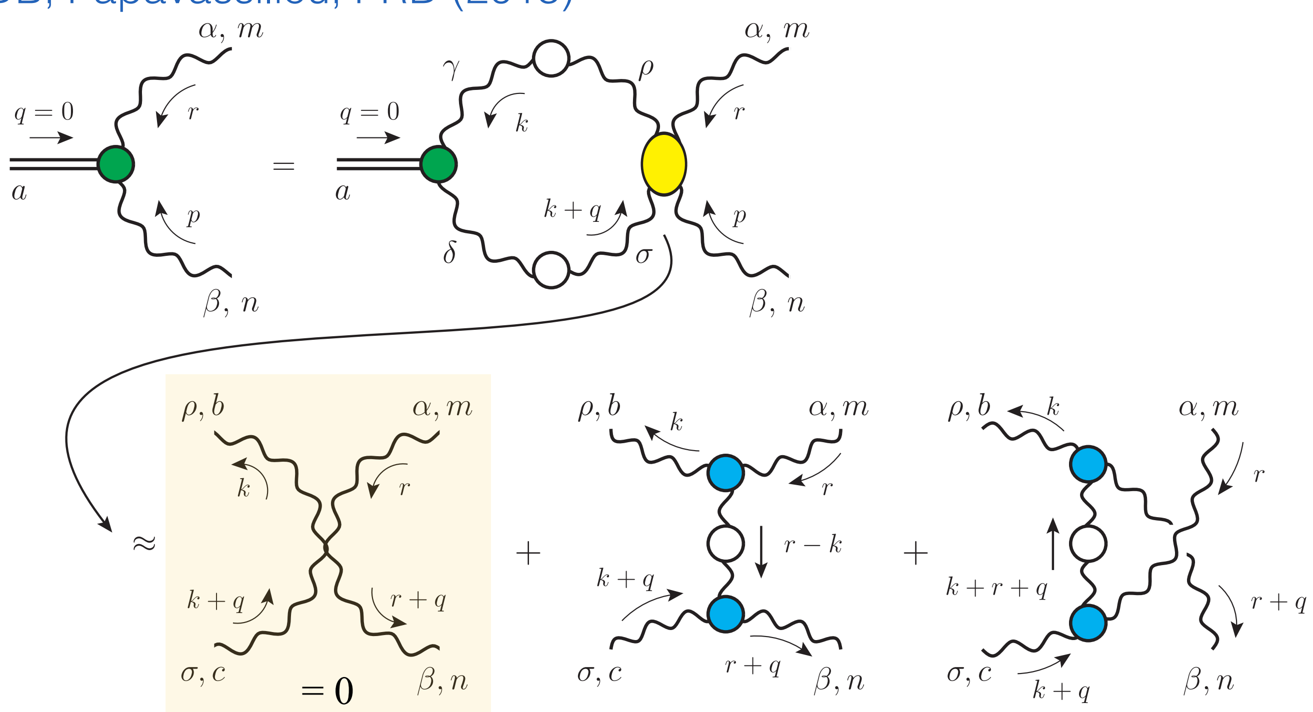
  $$m^2(\infty) = 0 \quad \Rightarrow \quad m^2(x) = -\int_x^{\infty} \mathrm{d}y\, \widetilde{C}_1'(y)$$

BSE for massless poles

- **Consider BSE for the full vertex**



- **Replace vertex:** $\Gamma \to \Gamma^{\mathbf{np}} + \Gamma^{\mathbf{p}}$
  expand and equate terms linear in q
  DB, Papavassiliou, PRD (2018)

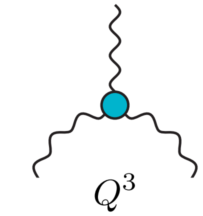


- **Homogeneous BSE**
  eigenvalue proportional to the coupling

$$\widetilde{C}_1'(x) = \alpha_s \int_0^\pi \mathrm{d}\theta \int_0^\infty \mathrm{d}y\, \mathcal{K}(x,y,\theta)\widetilde{C}_1'(y)$$

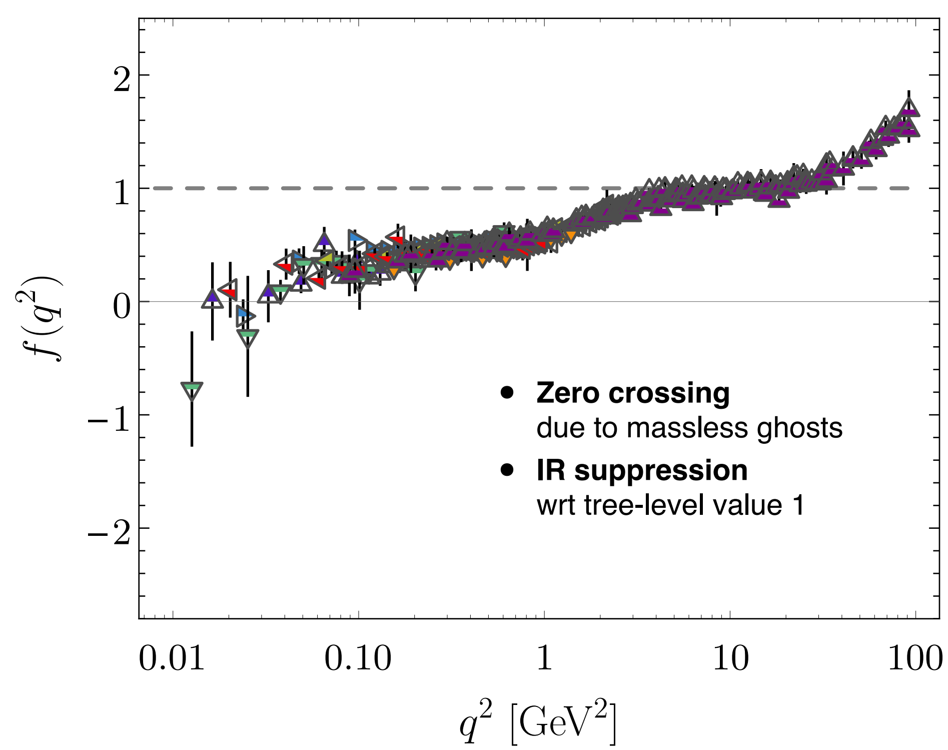
- **Four gluon kernel**
  use one-loop dressed approximation

- **(quantum) Three gluon vertex**
  $\Gamma_{\mu\alpha\beta}(k_1,k_2,k_3) = f(k_2)\Gamma^{(0)}_{\mu\alpha\beta}(k_1,k_2,k_3)$

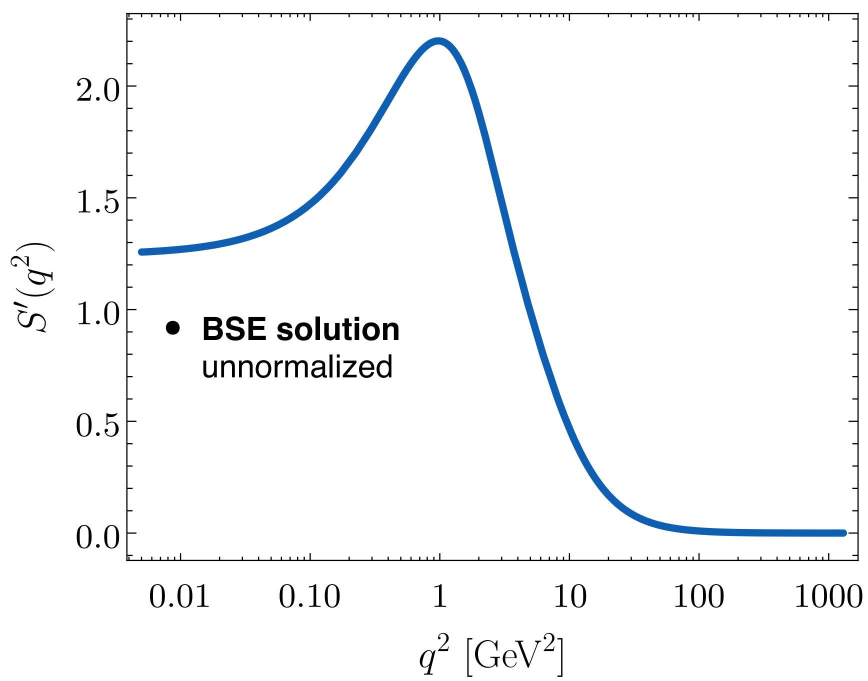


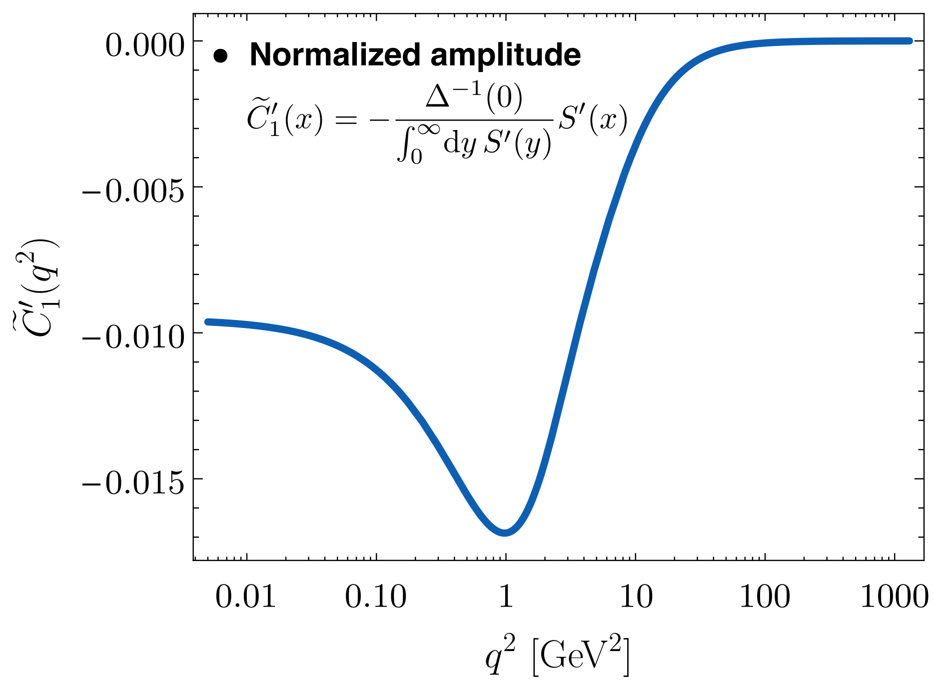
- **Ensures RGI-ness of the BSE**
  and self consistency (more on this later)

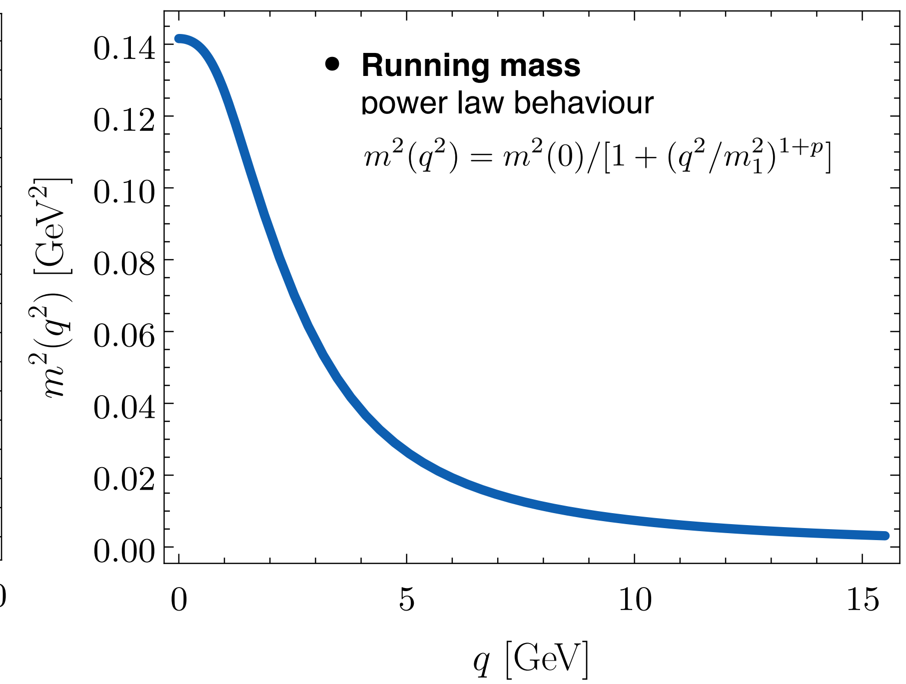
- **Vertex form factor**
  motivated by continuum/lattice studies
  Cucchieri, Maas, Mendes, PRD 74 (2006)
  Pelaez, Tissier, Wschebor, PRD 88 (2013)
  Aguilar, DB, Ibañez, Papavassiliou, PRD 89 (2014)
  Eichmann, Williams, Alkofer, Vujinovic, PRD 89 (2014)
  Blum, Huber, Mitter, von Smekal PRD 89 (2014)
  Athenodorou, DB, Boucaud, De Soto, Papavassiliou, Rodriguez-Quintero, Zafeiropoulos, PLB 761 (2016)

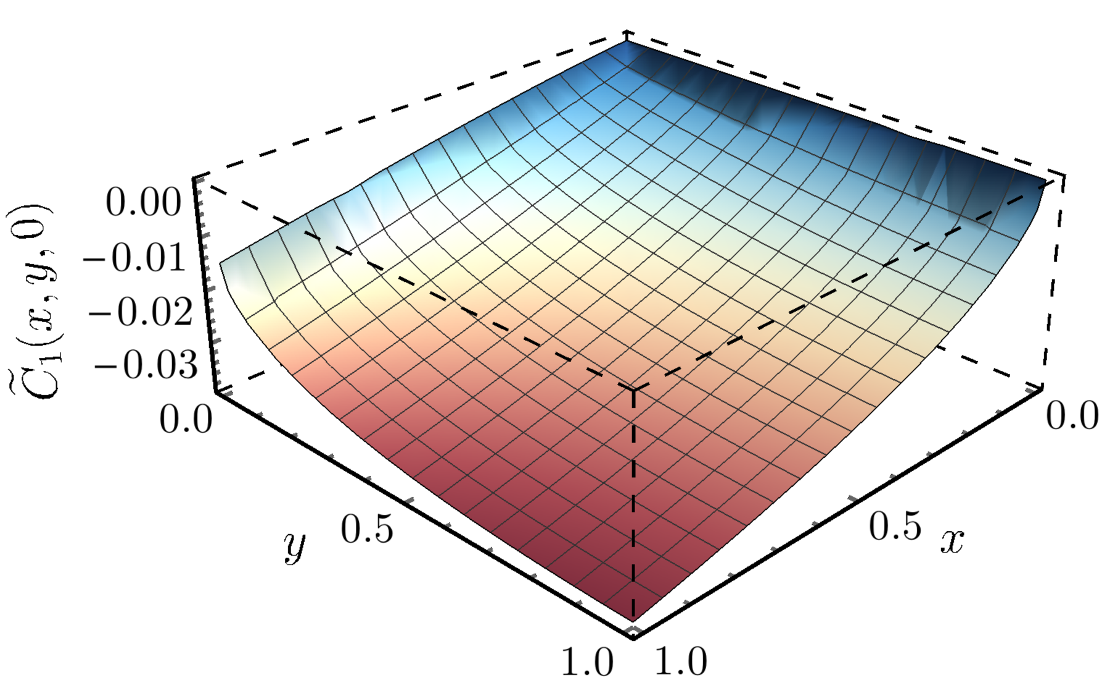


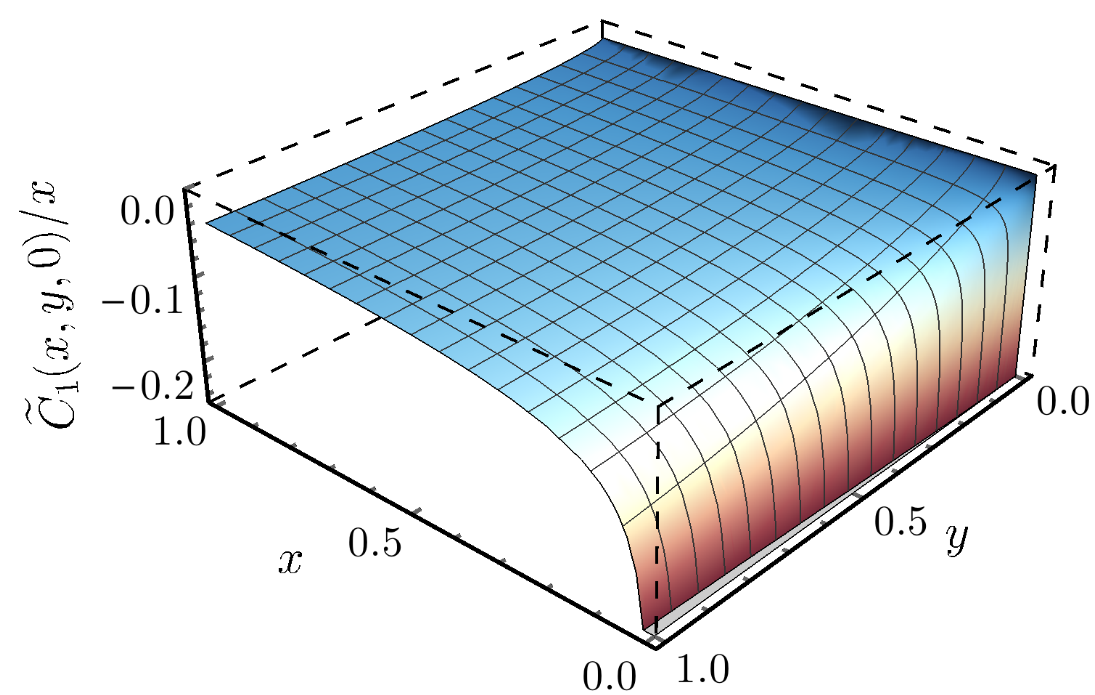
Poles BS amplitude





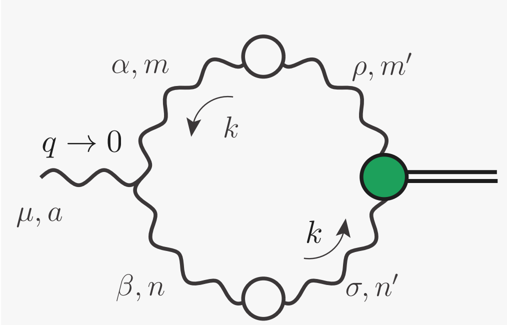
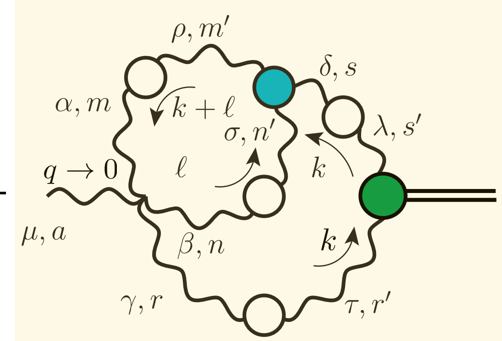






Coupling the gluon SDE

- **Gluon SDE at $q = 0$**
  yields quadratic equation in the coupling
  DB, Papavassiliou, PRD 97 (2018)

$m^2(0) =$  $+$ 

$$-C = B\alpha_s + A\alpha_s^2$$

- **Consistency condition**
  for a given MOM subtraction point μ

$$\alpha_s^{\text{SDE}} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \alpha_s^{\text{BSE}}$$

- **Vertex form factor crucial**
  f=1 implies $\alpha_s^{\text{SDE}} = 0.42$ and $\alpha_s^{\text{BSE}} = 0.27$

- **Lattice data fit**

$$f \sim \lambda \left[1 + b \ln \frac{q^2 + \mathcal{M}^2}{\mu^2 + \mathcal{M}^2} + c \ln \frac{q^2}{\mu^2} + \cdots \right]$$

gluon loops — ghost loops

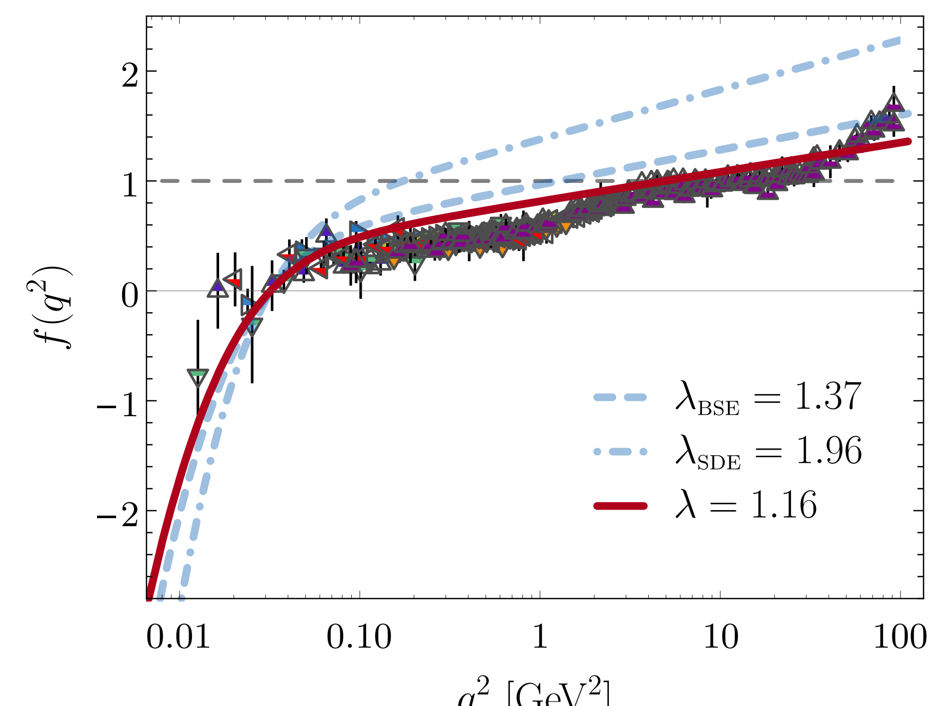
- **Two-loop dressed diagrams fundamental**
  one-loop dressed alone requires negative coupling

$$A = \frac{3C_A^2}{32\pi^3} F(0) \int_0^\infty \mathrm{d}y\, y^2 \Delta^2(y) Y(y) S'(y)$$

$$B = -\frac{3C_A}{8\pi} F(0) \int_0^\infty \mathrm{d}y\, y^2 \Delta^2(y) S'(y)$$

$$C = -\int_0^\infty \mathrm{d}y\, S'(y)$$

- **Start with** $\lambda_0 = 1$
  evaluate A_0, B_0, C_0 and α_0
- **Rescale** f
  track rescaling through BSE/SDE
  $C_0\lambda^3 + \alpha_0 B_0 \lambda + \alpha_0^2 A_0 = 0$
- **Solved @** $\mu = 4.3$ GeV **by**
  $\lambda = 1.16$ $\alpha_s^{\text{BSE}} = \alpha_s^{\text{SDE}} = 0.45$
- **Expected value:** $\alpha_s = 0.32$
  obtained if
  $\lambda_{\text{BSE}} = 1.36$, $\lambda_{\text{SDE}} = 1.97$



Stability under μ changes

- **Consistency condition**
  depends on the MOM subtraction point

- **Changes in the subtraction point**
  amounts to finite renormalizations

$$\Delta(q^2,\overline{\mu}^2) = z_A(\overline{\mu}^2,\mu^2)\Delta(q^2,\mu^2) \qquad z_A^{-1} = \overline{\mu}^2\Delta(\overline{\mu}^2,\mu^2)$$
$$F(q^2,\overline{\mu}^2) = z_c(\overline{\mu}^2,\mu^2)F(q^2,\mu^2) \qquad z_c^{-1} = F(\overline{\mu}^2,\mu^2)$$
$$f(q^2,\overline{\mu}^2) = z_3(\overline{\mu}^2,\mu^2)f(q^2,\mu^2)$$

- **Track changes:**

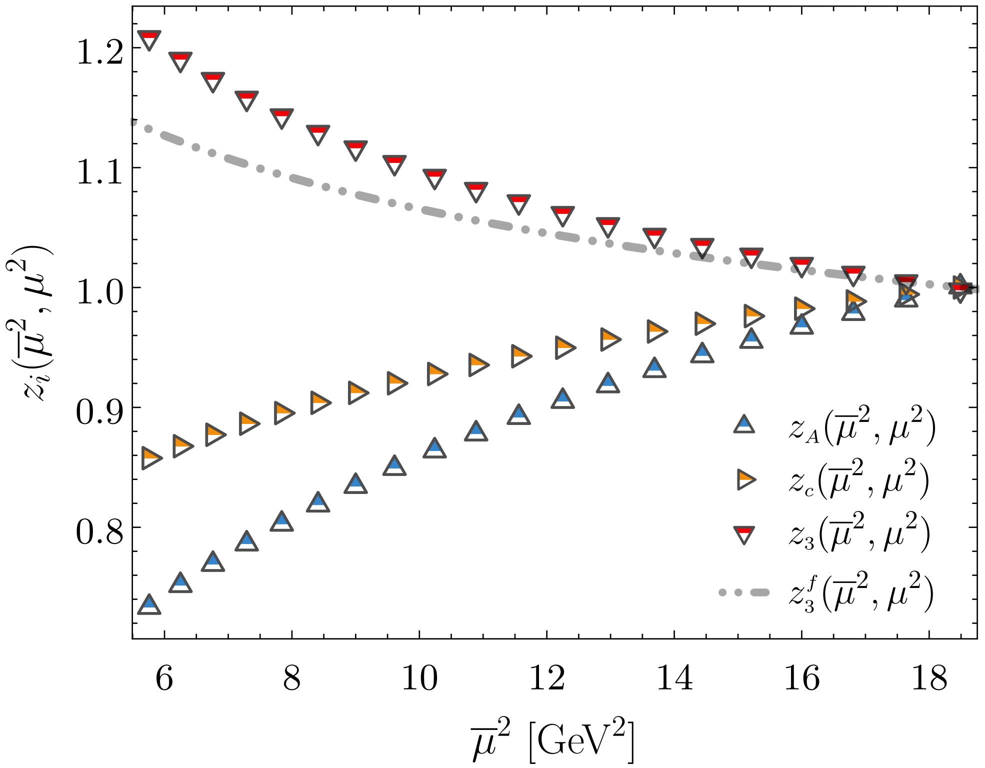
$$\overline{A} = z_A^4 z_c z_3 A; \quad \overline{B} = z_A^2 z_c B; \quad \overline{C} = C$$

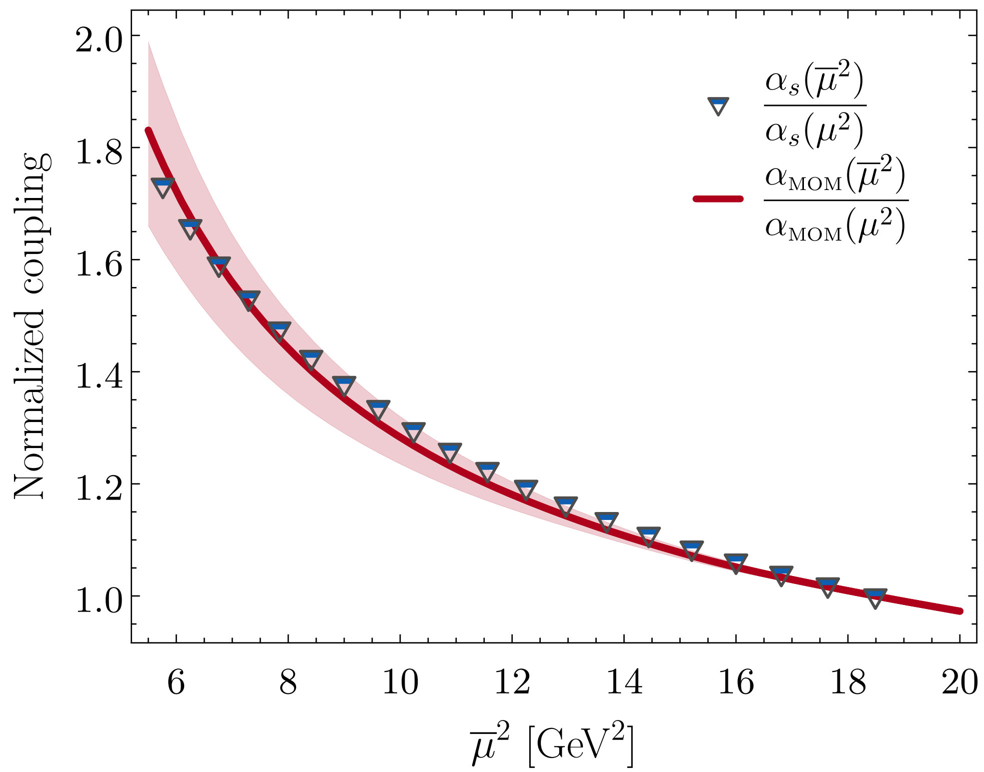
$$\alpha^{\rm BSE}(\overline{\mu}^2) = z_A^{-3} z_3^{-2}\alpha_s \quad \alpha^{\rm SDE}(\overline{\mu}^2) = \frac{-\overline{B}+\sqrt{\overline{B}^2-4\overline{A}\overline{C}}}{2\overline{A}}$$

- **Impose consistency condition**
  will determine z_3

$$C z_3^3 + z_3\left(z_A^{-1}z_c\right)\alpha_s B + \left(z_A^{-2}z_c\right)\alpha_s^2 A = 0$$

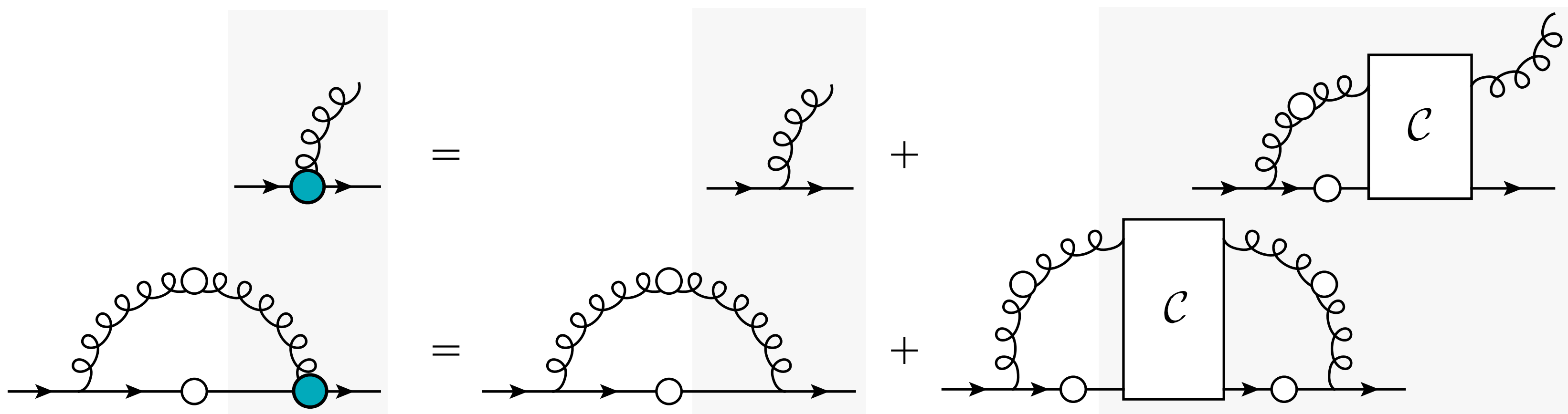
  - **Solve cubic equation for z_3**
    values found will enforce identity
    $\alpha_s^{\rm BSE}(\overline{\mu}^2) = \alpha_s^{\rm SDE}(\overline{\mu}^2)$
  - **Results compare favourably**
    with expected MOM results
    Boucaud, de Soto, Leroy, Le Yaouanc, Micheli, Moutarde, Pene, Rodriguez-Quintero PRD 74 (2006)





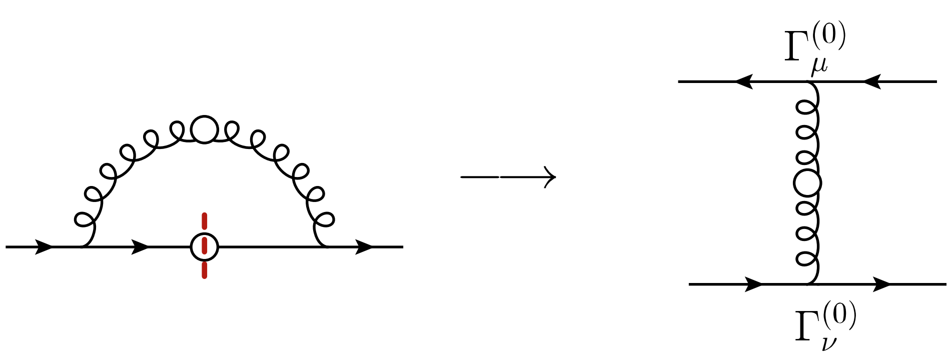
Beyond Maris-Tandy interaction

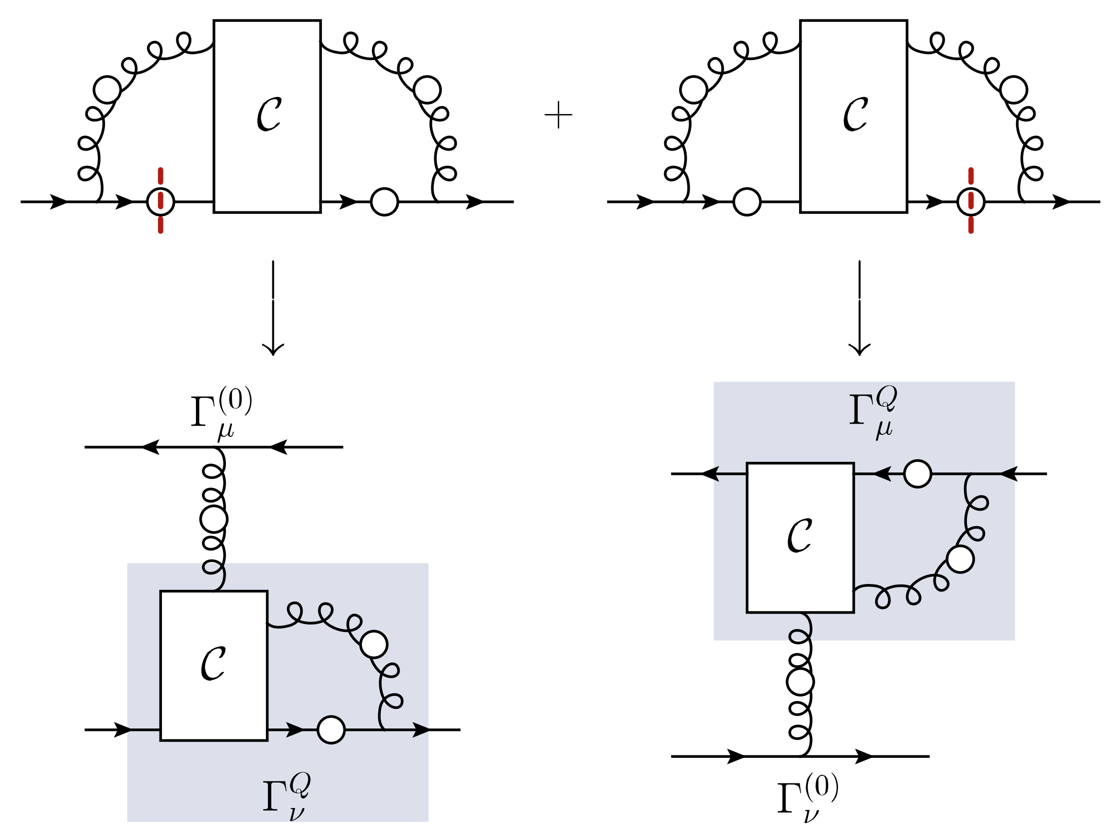
- **Quark-gluon SDE reformulation**
  lead to left-right symmetric gap-equation



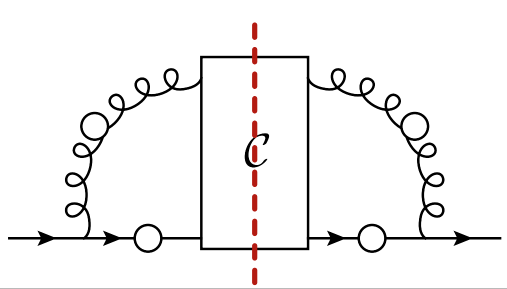
- **Use cut rules**
  to get BSE kernel $\mathcal{K} \sim \frac{\partial \Sigma}{\partial S}$

Munczek, PRD 52 (1995)
Bender, Roberts, von Smekal, PLB 380 (1996)
Heupel, Goecke, Fischer, EPJA 50 (2014)



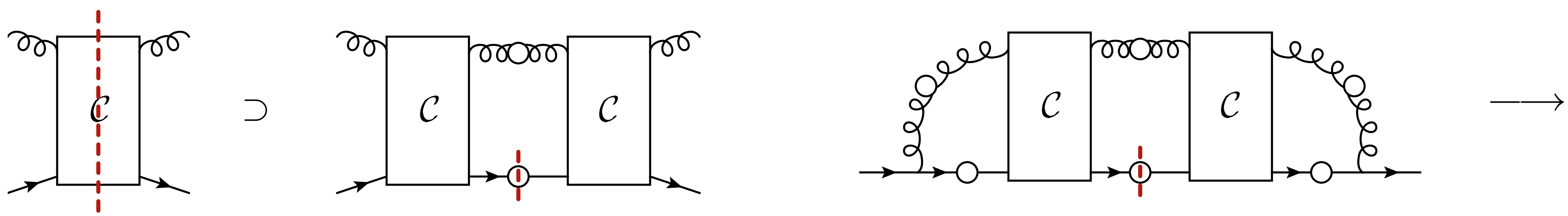


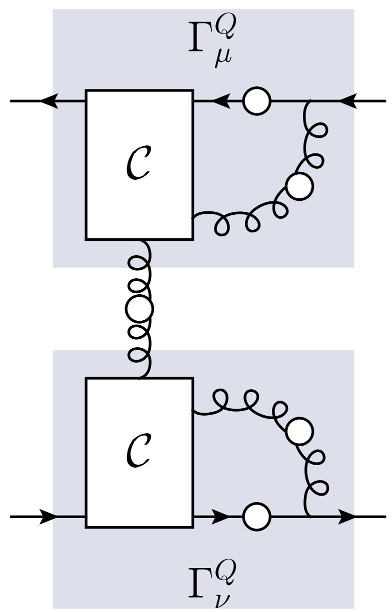
- **Not there yet**
  missing cuts across kernel



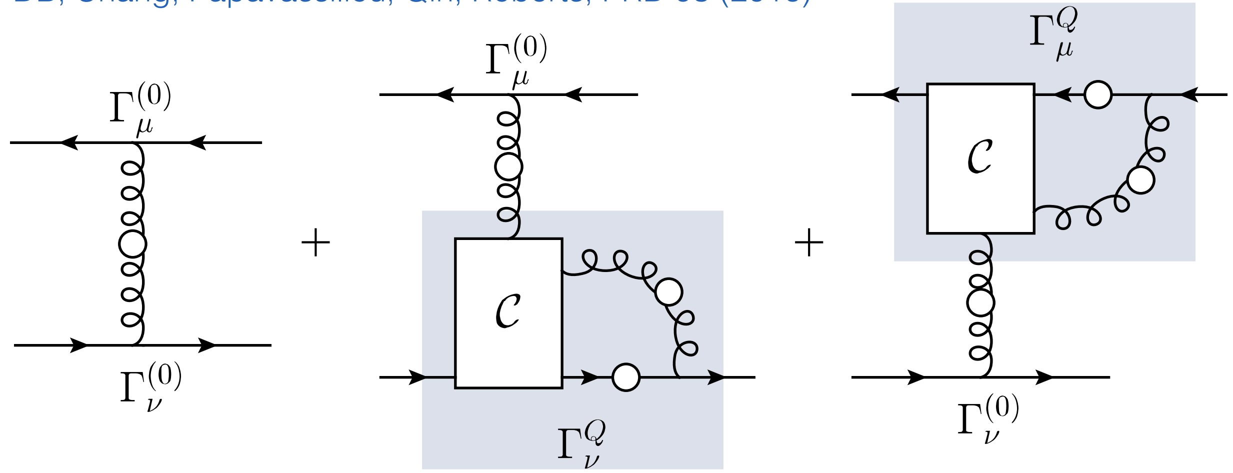
Beyond Maris-Tandy interaction

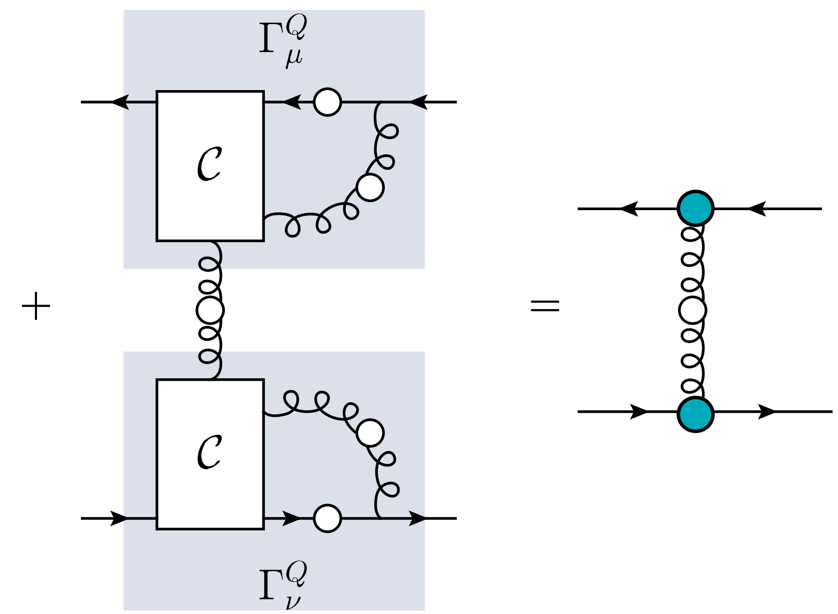
- **Cut through kernel**
  isolate relavant diagram + "boxes"



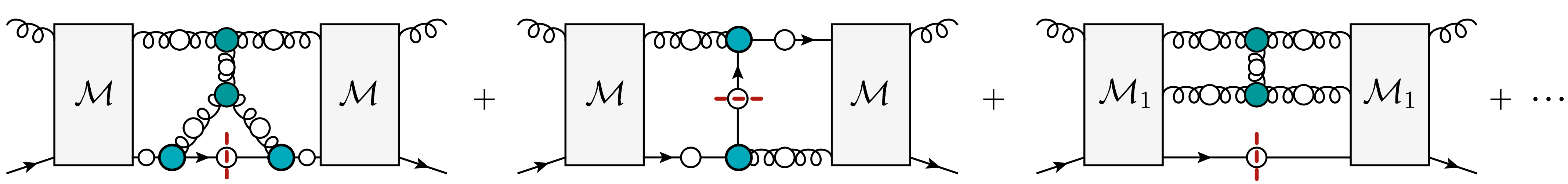


- **Collect terms**
  to generate symmetric kernel
  DB, Chang, Papavassiliou, Qin, Roberts, PRD 93 (2016)



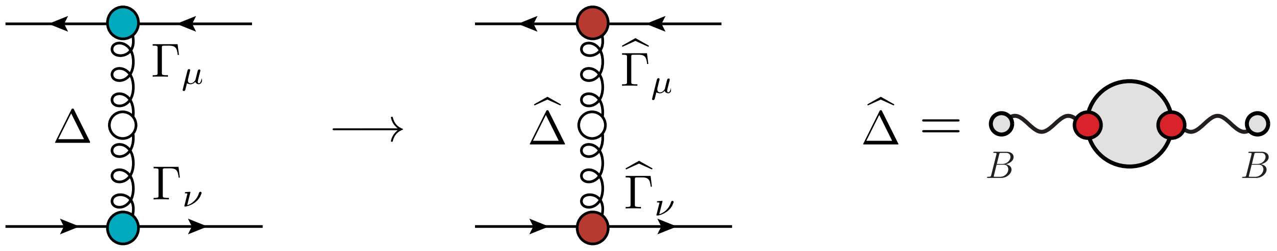


- **Left out are "box-like" when cut**



PT-BFM interaction

- **Isolate process independent part**
  use PT algorithm



- **New structure appears**
  RGI combination

$$\widehat{d}(q^2) = \alpha_s \widehat{\Delta}(q^2)$$

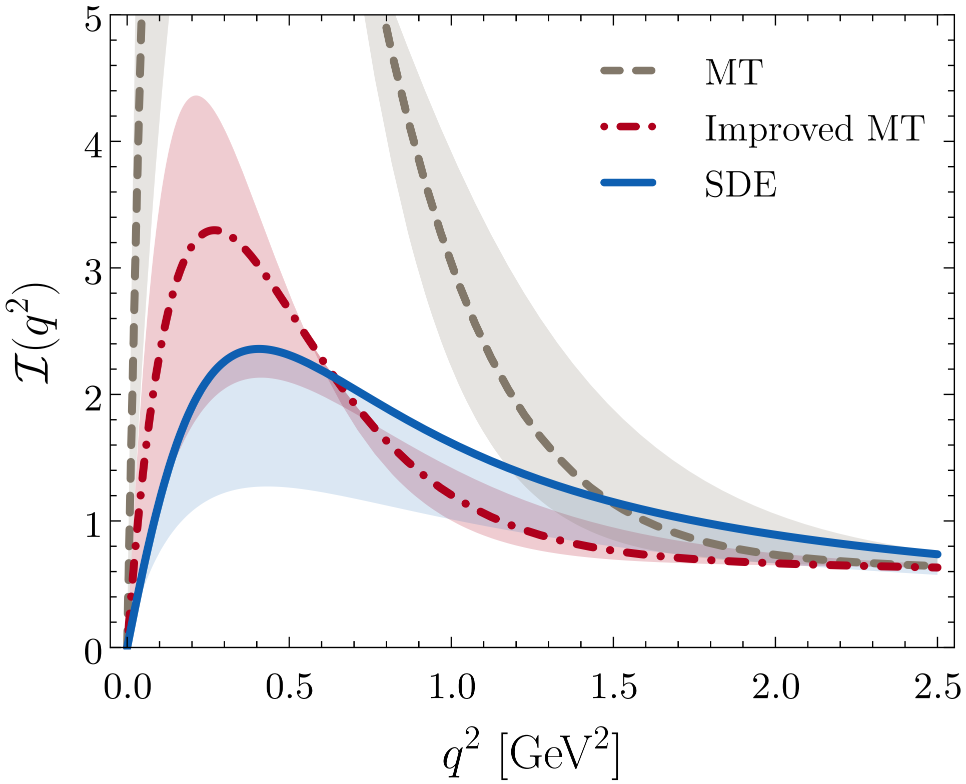
- **Absorbs all the RG logs**
  as the photon in QED
- **Renormalizes as** Z_g^{-2}

$$\widehat{\Delta} \sim \frac{1}{q^2[1 + bg^2 \log q^2/\mu^2]}; \qquad b = 11C_A/48\pi^2$$

- **Candidate interaction strength**
  process independent and RGI
  DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

$$\mathcal{I}(q^2) = q^2 \widehat{d}(q^2) = \frac{\alpha_s \widehat{\Delta}(q^2)}{[1+G(q^2)]^2}$$

$$1 + G(q^2) = Z_c + \frac{g^2 C_A}{3} \int_k \left[2 + \frac{k \cdot q}{k^2 q^2}\right] \Delta(k) D(k+q)$$



Summary

Theory

- **Schwinger mechanism**
  successfully generates gluon mass scale
- **Bound state colored massless poles**
  yield running mass with expected properties
- **Two-loop dressed terms crucial**
  stabilizes IR dynamics
- **Dressed quantum 3-gluon vertex**
  required for consistency of BSE/SDE

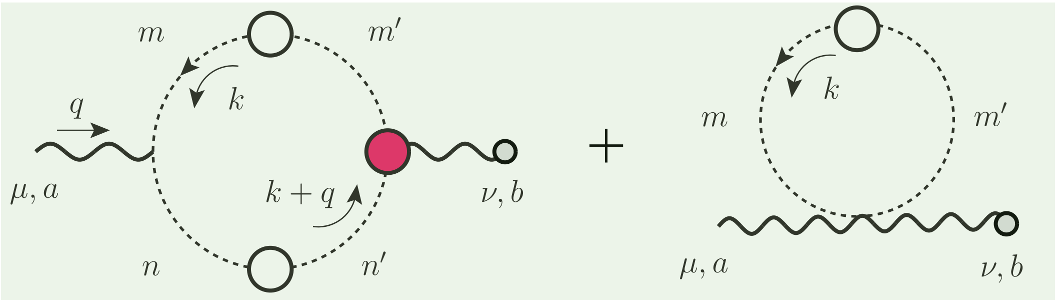
Applications

- **First steps towards BRL analysis**
  within the PT-BFM scheme
- **“Ab-initio” candidate for the interaction**
  definite field theory origins and properties
- **Acts as bridge**
  “bottom-up” vs “top-down” approaches
- **Symmetry preserving BSE scheme**
  work (theoretical+numerical) under way

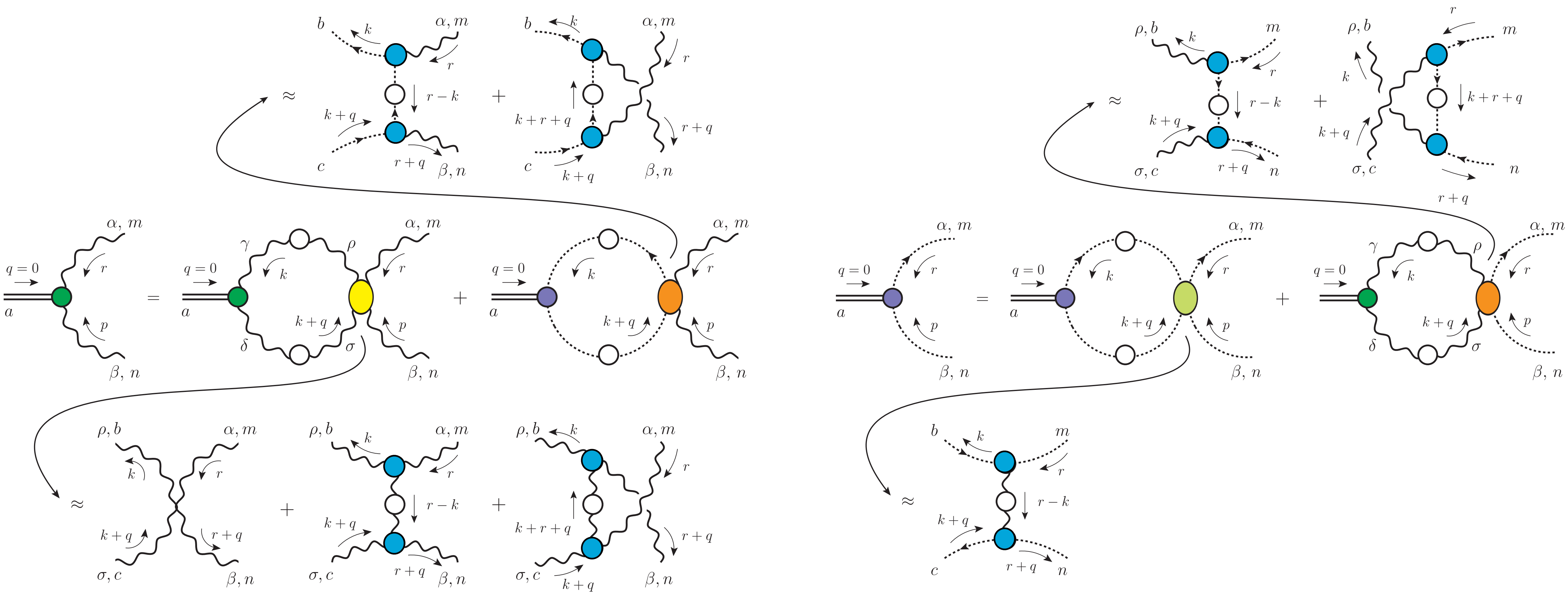
Back up

Ghost contributions

- **Add ghost terms**
  with massless pole in $B\bar{c}c$ vertex
  Aguilar, DB, Figueiredo, Papavassiliou, EPJC 78 (2018)

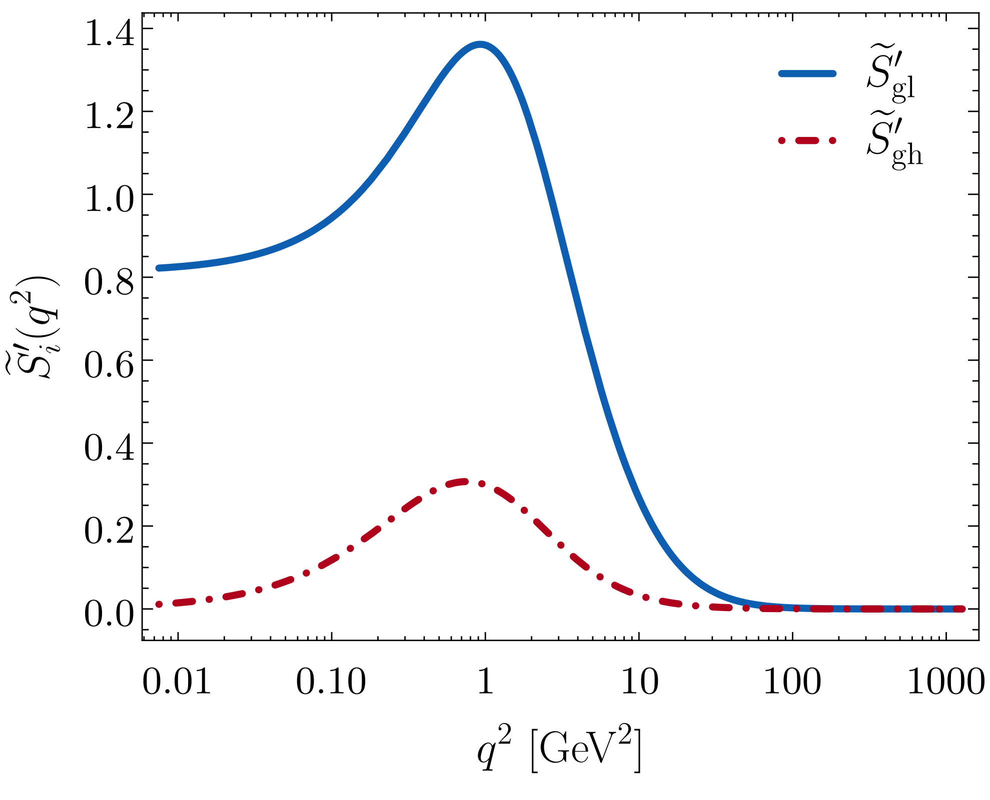


- **Gluon/ghost BSEs**
  coupled in a system

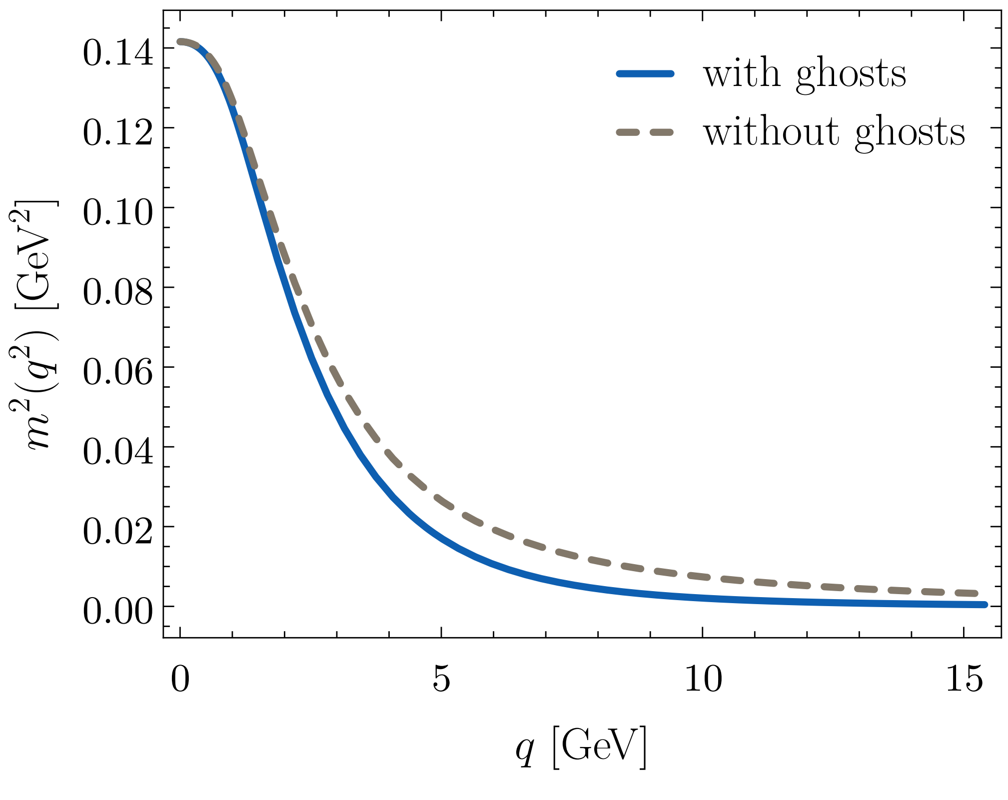


Ghost contributions

- **Ghost pole amplitude suppressed**
  ~ 5 times smaller than gluon at peak

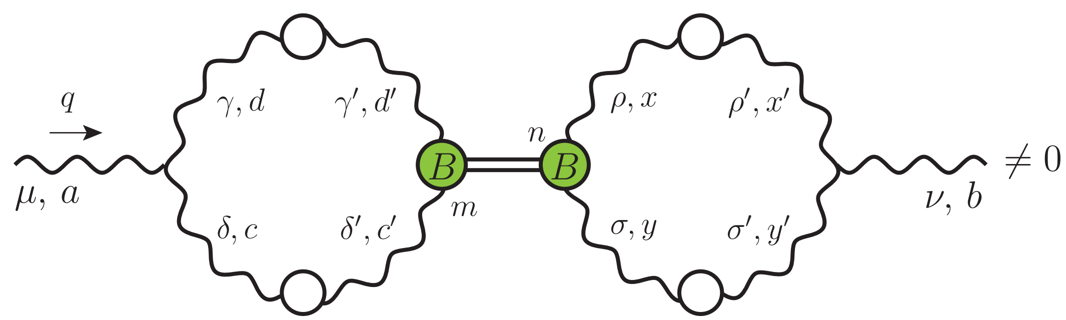


- **Results almost invariate**
  qualitative/quantitative agreement

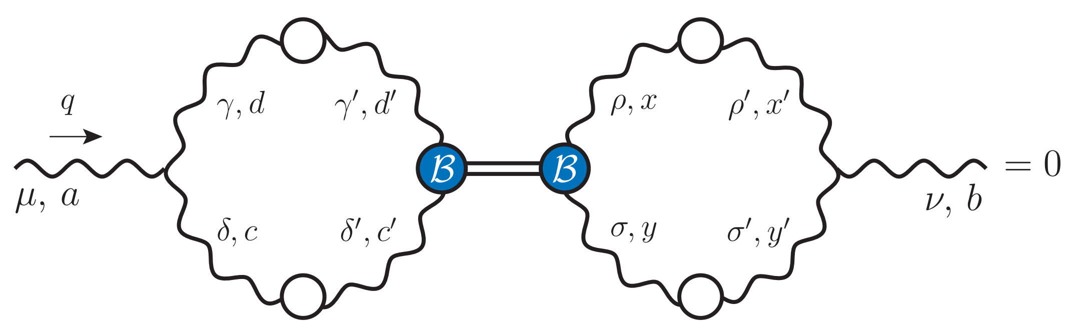


Colored poles vs glueballs

- **Colored bound states differ from glueballs**
  color structure leads to completely different BSEs



$$\left(f^{adc}f^{dmc}\right) \times \delta^{mn} \times \left(f^{nxy}f^{bxy}\right) \neq 0$$



$$\left(f^{adc}\delta^{dc}\right) \times \text{no color} \times \left(f^{bxy}\delta^{xy}\right) = 0$$

QCD Effective charge

- **Remarkable feature of QCD:**
  $\widehat{d}(k^2)$ saturates in the IR
  DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

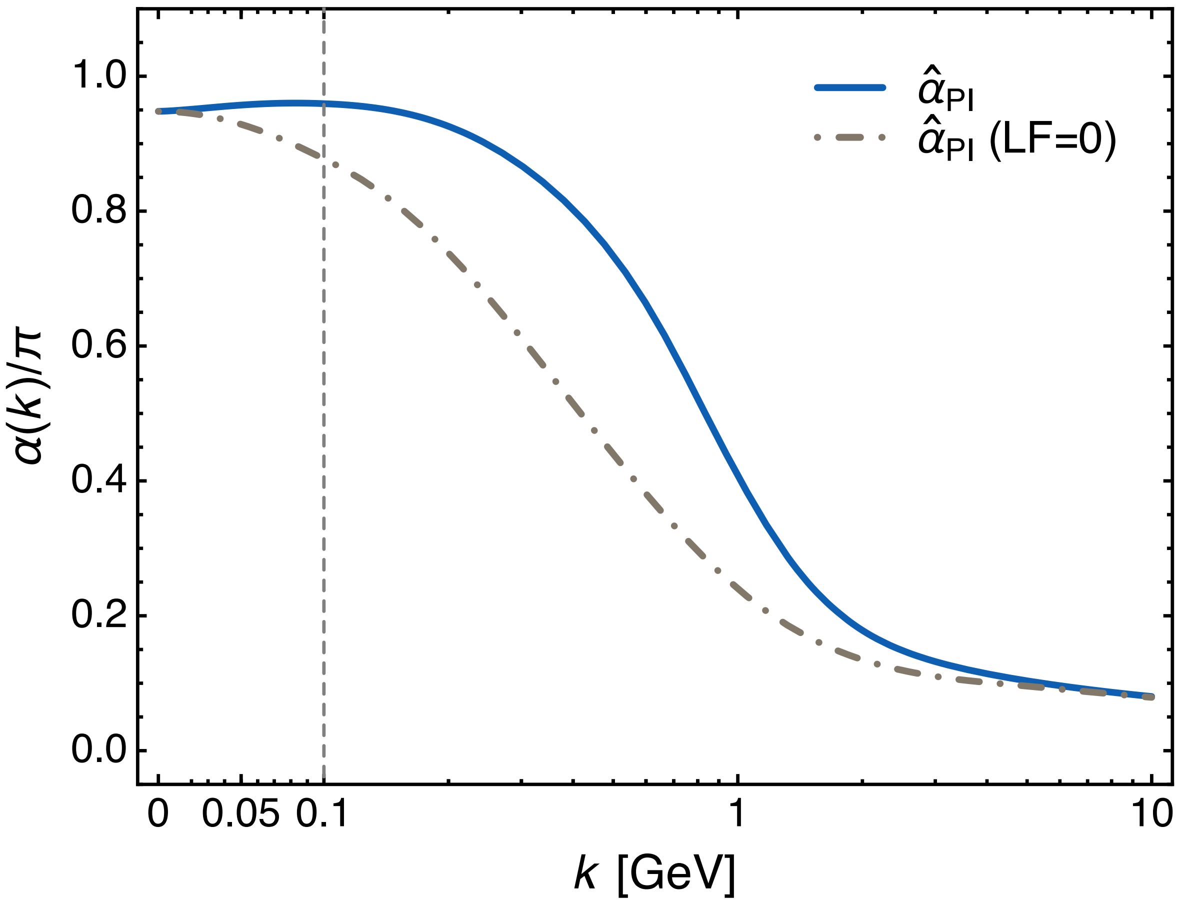
$$\widehat{d}(0) = \frac{\alpha_0}{m_0^2} \approx \frac{0.9\pi}{(m_P/2)^2}$$

- **Define the RG invariant function**

$$\mathcal{D}(k^2) = \frac{\Delta(k^2;\mu^2)}{\Delta(0;\mu^2)m_0^2}$$

- **Extract (process independent) coupling**
  using the quark gap equation
  DB, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero, 1612.04835

$$\widehat{\alpha}(k^2) = \frac{\widehat{d}(k^2)}{\mathcal{D}(k^2)} \underset{k^2 \gg m_0^2}{\longrightarrow} \mathcal{I}(k^2)$$



- **Parameter free**
  completely determined from 2-point sector
- **No Landau pole**
  physical coupling showing an IR fixed point
- **Smoothly connects IR and UV domains**
  no need for matching procedures
- **Essentially non-perturbative result**
  continuum/lattice results plus setting of single mass cale
- **Ghost gluon dynamics critical**
  produces enhancement at intermediate momenta

QCD Effective charge

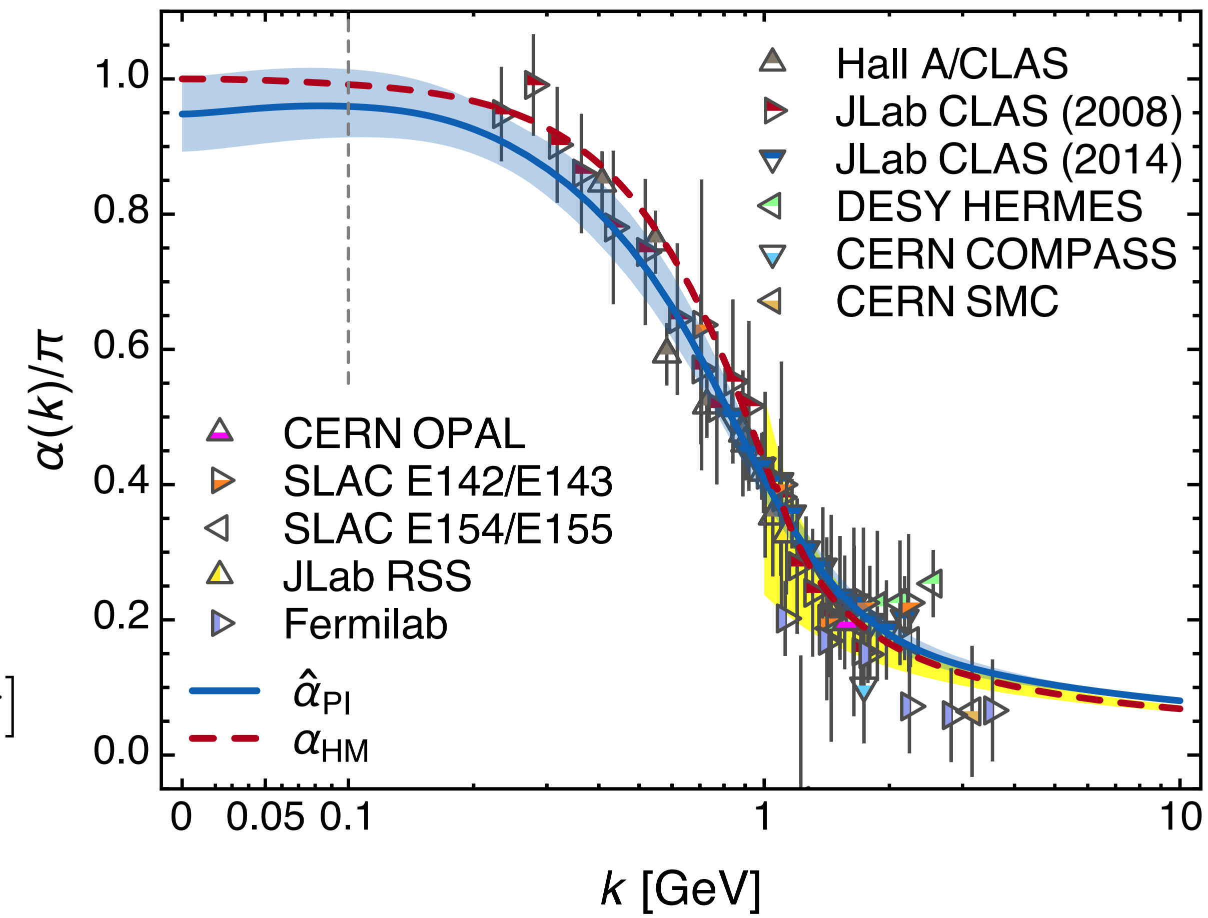
- **Process dependent effective charges**
  fixed by the leading-order term in the expansion of a given observable
  Grunberg, PRD 29 (1984)

- **Bjorken sum rule**
  defines such a charge
  Bjorken, PR 148 (1966); PRD 1 (1970)

$$\int_0^1 \mathrm{d}x\,[g_1^p(x,k^2) - g_1^n(x,k^2)] = \frac{g_A}{6}[1 - \alpha_{g_1}(k^2)/\pi]$$

  - $g_1^{p,n}$ **spin dependent p/n structure functions**
    extracted from measurements using unpolarized targets
  - g^A **nucleon flavour-singlet axial charge**

- **Many merits**
  - **Existence of data**
    for a wide momentum range
  - **Tight sum rules constraints on the integral**
    at IR and UV extremes
  - **Isospin non-singlet**
    suppress contributions from hard-to-compute processes



- **Equivalence in the perturbative domain**
  reasonable definitions of the charge

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\mathrm{MS}}}(k^2)[1 + 1.14\alpha_{\overline{\mathrm{MS}}}(k^2) + \cdots]$$
$$\widehat{\alpha}_{PI}(k^2) = \alpha_{\overline{\mathrm{MS}}}(k^2)[1 + 1.09\alpha_{\overline{\mathrm{MS}}}(k^2) + \cdots]$$

- **Equivalence in the non-perturbative domain**
  highly non-trivial (ghost-gluon interactions)

- **Agreement with light-front holography**
  model for α_{g_1}
  Deur, Brodsky, de Teramond, PPNP 90 (2016)