Parton Quasi Distributions

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(with Craig Roberts)

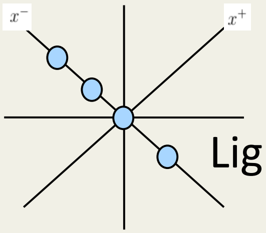
Nankai University

From Correlation Functions to QCD Phenomenology
2017/04/06, Physikzentrum, Bad Honnel, Germany

Quark Distribution

$$\langle P|\bar{\psi}(0)\gamma_\mu U[0,x]\psi(x)|P\rangle = P_\mu h(P\cdot x, x^2) + x_\mu g(P\cdot x, x^2)$$

- Hadron state of four-momentum P
- U link operator providing the gauge invariance
- Proportional to x contains sub-leading twist pieces



Light Front

$$q(u) = P_+ \int \frac{dx_-}{2\pi} \mathrm{e}^{iuP_+x_-} h(P_+x_-, 0)$$

Quark Distribution

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- Hadron state of four-momentum P
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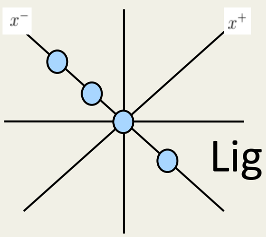
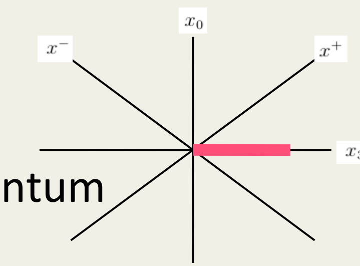
- Proportional to z contains sub-leading twist pieces



Light Front

Finite momentum



$$\infty \leftarrow P_3$$

$$q(u) = P_+ \int \frac{dx_-}{2\pi} \mathrm{e}^{iuP_+x_-} h(P_+x_-, 0)$$

$$\tilde{q}(y, P_3) = P_3 \int \frac{dx_3}{2\pi} \mathrm{e}^{-iyP_3x_3} h(-P_3x_3, -x_3^2)$$

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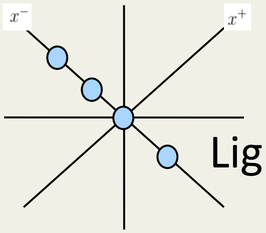

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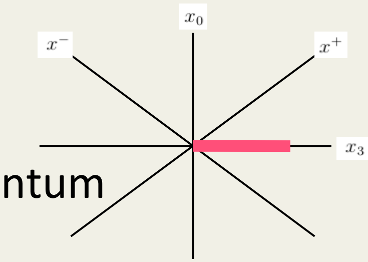

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The transverse-momentum unintegrated parton distribution(TMD)

$$q(u,k_1,k_2) = P_+\int \frac{dx_-}{2\pi}\mathrm{e}^{iuP_+z_-}\int \frac{dx_1}{2\pi}\mathrm{e}^{ik_1x_1}\int \frac{dx_2}{2\pi}\mathrm{e}^{ik_2x_2}h(P_+x_-,-x_1^2-x_2^2)$$

Radyushkin relation

$$\tilde{q}(y,P_3) = P_3\int dk_1\int du\, q(u,k_1^2+(u-y)^2P_3^2)$$

Similar in Distribution Amplitude

Quark Distribution

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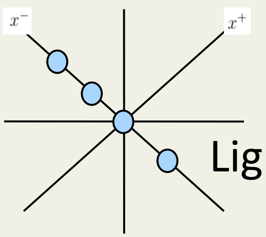
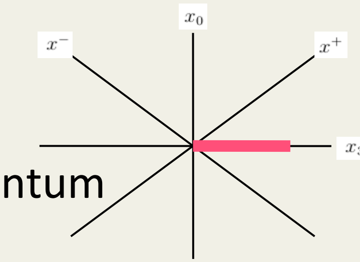


Light Front

Finite momentum



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- It not possible to provide infinite P_3(Lattice and DSEs)
- How can one then recover infinite limit from a moderately-large-P_3?
- LaMEF…idea from HQEF

$$O(m_b/\Lambda) = \underline{Z(m_b/\Lambda, \Lambda/\mu)} o(\mu) + \mathcal{O}(1/m_b) + \ldots \Rightarrow \tilde{q}(P_3/\Lambda) = \underline{Z(P_3/\Lambda, \Lambda/\mu)} q(\mu) + \mathcal{O}(1/P_3^2) + \ldots$$

Matching coefficients


References (41) Citations (168) Files Plots

Parton Physics on a Euclidean Lattice

Xiangdong Ji (Maryland U. & Shanghai Jiaotong U. & Shanghai Jiaotong U

May 7, 2013 - 4 pages

Phys.Rev.Lett. 110 (2013) 262002

Quark Quasi Distribution----lQCD Progress

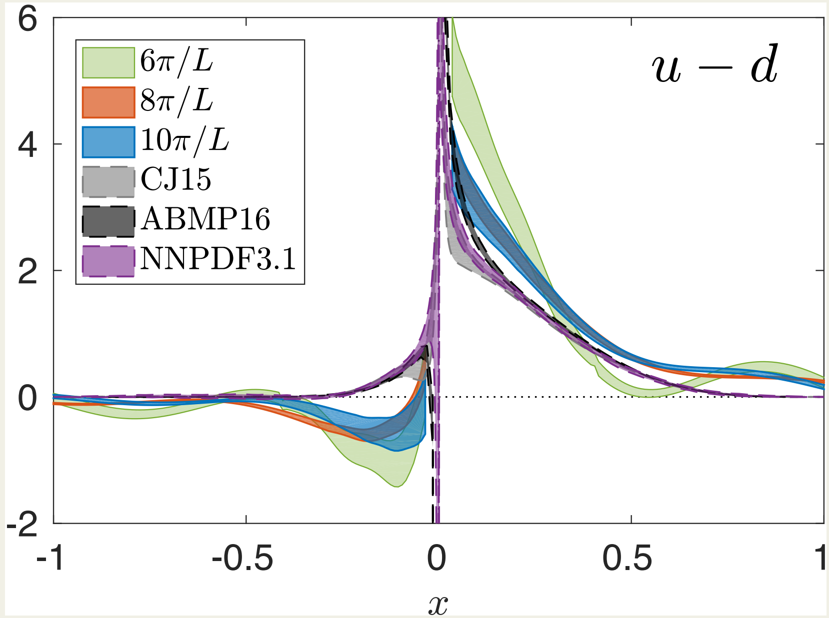
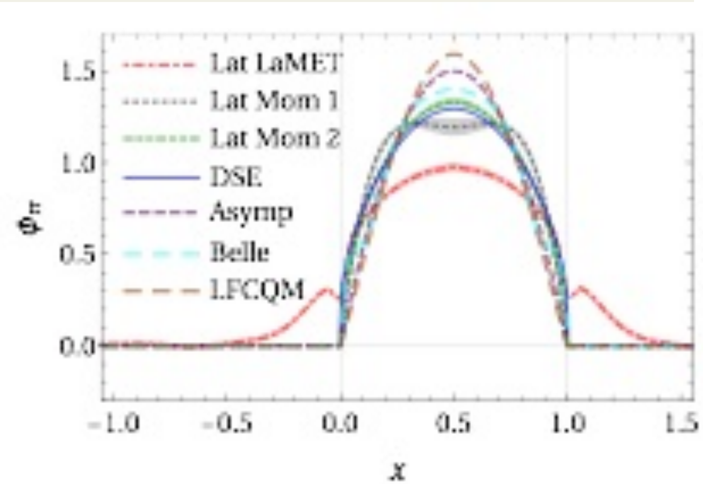


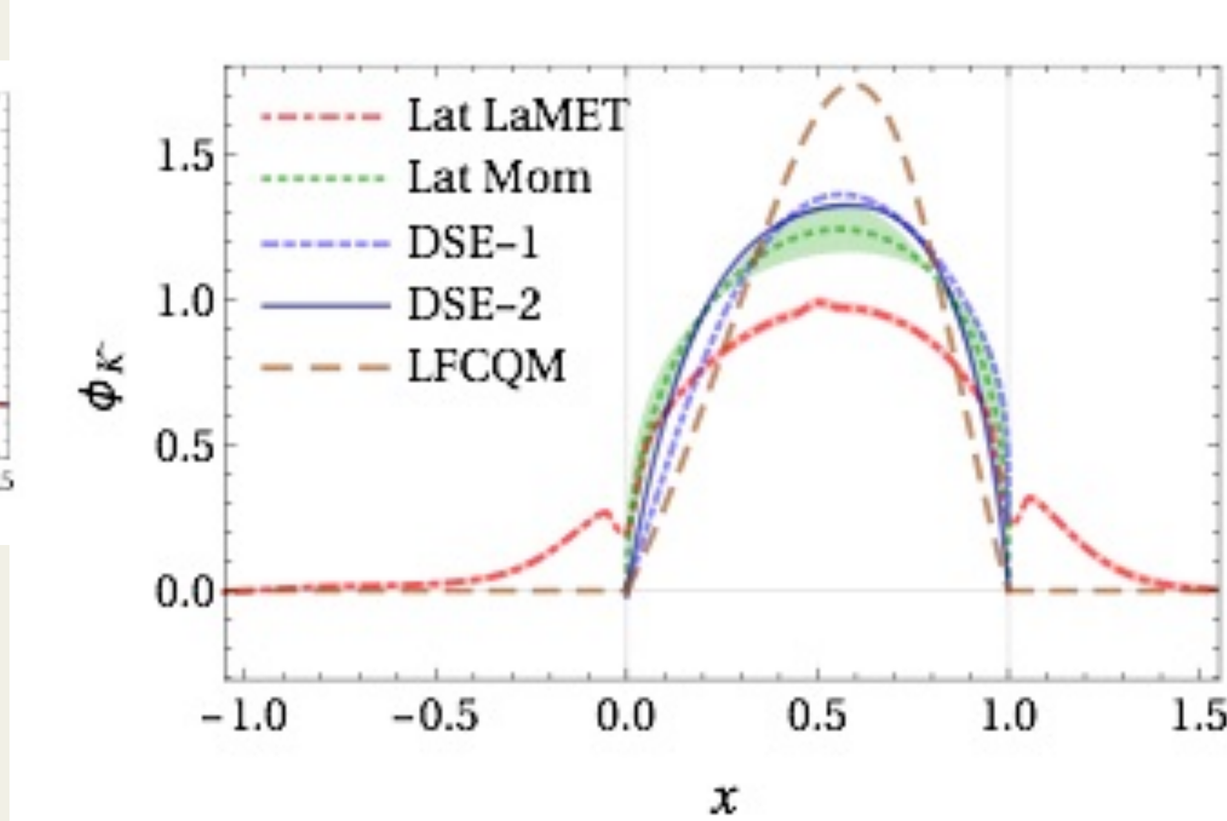
FIG. 4: Comparison of unpolarized PDF at momenta $\frac{6\pi}{L}$ (green band), $\frac{8\pi}{L}$ (orange band), $\frac{10\pi}{L}$ (blue band), and ABMP16 [39] (NNLO), NNPDF [40] (NNLO) and CJ15 [38] (NLO) phenomenological curves.

Reconstruction of light-cone parton distribution functions from lattice QCD simulations at the physical point

C. Alexandrou, et al, arXiv:1803.02685

Quark quasi Distribution Amplitudes----lQCD Progress





Kaon Distribution Amplitude from Lattice QCD and the Flavor SU(3) Symmetry
J-W Chen, et al (LP^3), arXiv:1712.10025

No Report Yet!

Pion quasi Distribution Function----lQCD Progress

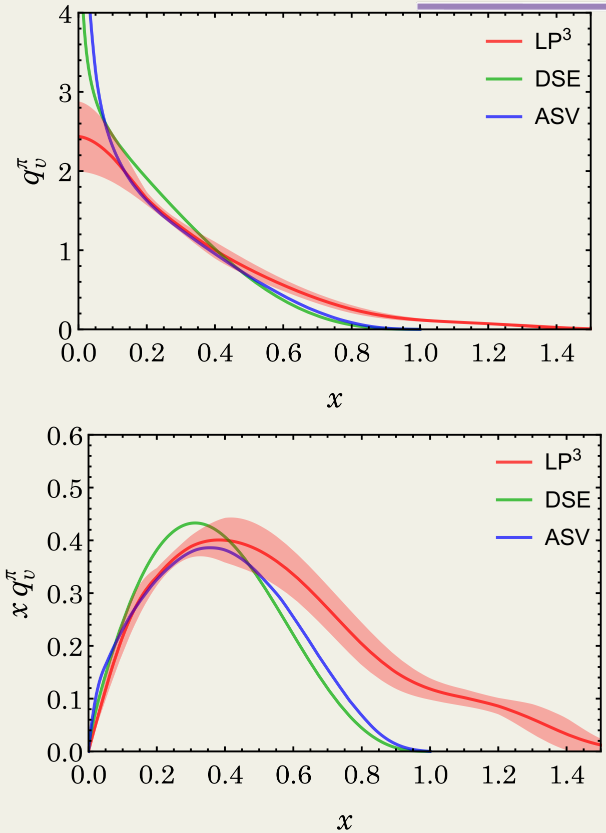


FIG. 5. Our pion valence-quark PDF result (LP^3, red) at the scale μ = 4 GeV, contrasted with analysis from the Dyson-Schwinger equation [14] (DSE, green) at the scale 5.2 GeV and from a fit to Drell-Yan data from Ref. [7] (ASV, blue) at 4 GeV.

- Box size L=3fm and pion mass 310MeV
- Distribution is comparable quantitatively with the results extracted from experimental data as well as from the Dyson-Schwinger equation

First direct lattice-QCD calculation of the x-dependence of the pion parton distribution function

J-W Chen, et al (LP^3), arXiv:1804.01483

Pion/Kaon quasi DA/DF----Phenomenology

- Quasi-parton distribution functions: a study in the diquark spectator model
  L. Gamberg, et al, arXiv: 1412.3401
- Pion distribution amplitude and quasi-distributions
  A. Radyushkin, arXiv: 1701.02688
- Quasi-distribution amplitudes for pion and kaon via the nonlocal chiral-quark model
  S. Nam, arXiv: 1704.03824
- Nonperturbative partonic quasidistributions of the pion from chiral quark models
  W. Broniowski, E. R. Arriola, arXiv: 1707.09588

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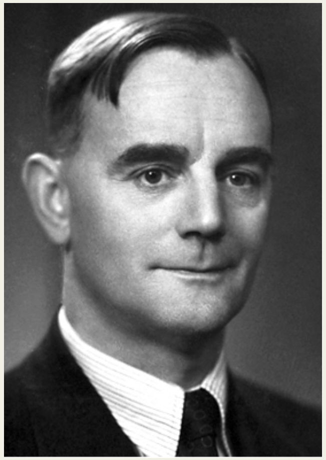
  W. Broniowski, E. R. Arriola, arXiv: 1707.09588
- Pion and kaon valence-quark quasiparton distributions
  S. Xu, LC, C. D. Roberts, H. Zong, arXiv: 1802.09552

What is the lowest value of P_3 for which the quasi-distribution provides a realistic sketch of the true distribution?
It is possible to provide information about endpoint behavior?

Why Pion-----Messager of QCD

- Pion is Massless...



• In October 1934**, Hideki Yukawa** predicated the existence of a "heavy quantum" meson, exchanging nuclear force between neutrons and protons.
• It was discovered by **Cecil Powel** in 1949 in cosmic ray tracks in a photographic emulsion.
• Pion was nicely accommodated in the Eight Fold way of **Murray Gell-Mann** in 1961.
• **Yoichiro Nambu** associated it with CSB in 1960.

Pion's dichotomy
Goldstone boson and Bound State

Maris, Roberts and Tandy, Phys. Lett. **B420**(1998) 267-273

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails(chiral limit)

$$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2\frac{d^2B(k^2)}{d^2k^2} + \ldots$$

- Given the dichotomy of pion the fine-tuning should not play any role in an explanation of pion properties;
- Descriptions of pion within frameworks that cannot faithfully express symmetries and their breaking patterns(such as constituent-quark models) are unreliable;
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.

Pion/Kaon PDF----endpoint behavior

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- One of the earliest predictions of the QCD parton model (1974,1975):

  $q^{\pi}(x) \sim (1-x)^{2}$

- Owing to the validity of factorisation in QCD,
  $q^{\pi}(x)$ is directly measurable in πN Drell-Yan experiments
- E615 @ FNAL (Conway:1989fs): leading-order analysis of πN Drell-Yan

  $q^{\pi}(x) \sim (1-x)^{1}$

Numerous "explanations"

- Nambu – Jona-Lasinio model, translationally invariant regularisaion

  $q^{\pi}(x) \sim (1-x)^{0}$,

  which becomes "*1*" after evolving from a low resolution scale
- NJL models with a hard cutoff & also some duality arguments:

  $q^{\pi}(x) \sim (1-x)^{1}$
- Relativistic constituent quark models:

  $q^{\pi}(x) \sim (1-x)^{0\ldots2}$

  depending on the form of model wave function
- Instanton-based models

  $q^{\pi}(x) \sim (1-x)^{1\ldots2}$

Pion/Kaon PDF----endpoint behavior

- One of the earliest predictions of the QCD parton model (1974,1975):

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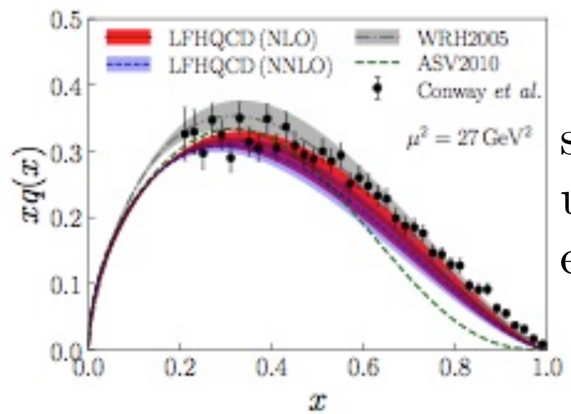


FIG. 3. Model comparison (red band) for $xq(x)$ in the pion with the NLO fits [81, 82] (gray band and green curve) and the LO extraction of Fermilab E615 Drell-Yan data [83]. NNLO results are also included (light blue band).

$$q_\tau(x) \sim (1-x)^{2\tau-3},$$

the contribution from higher Fock components was determined from the analysis of the time-like region [80]. Up to twist-4

$$q_v^{u,d}(x) = (1-\gamma)q_{\tau=2}(x) + \gamma q_{\tau=4}(x), \qquad (22)$$

structure. The falloff of the pion PDF at large-x is an unresolved issue [85] which requires a new generation of experiments.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD
Guy F. de Téramond, et al, arXiv:1801.09154

Pion/Kaon PDF----DSEs story

- Valence-quark distributions in the pion
  M. Hecht, C. D. Roberts, S. M. Schmidt, arXiv: nucl-th/0008049
- Pion and Kaon valence-quark parton distribution functions
  T. Nguyen, et al, arXiv: 1102.2448
- Valence-quark distribution functions in the kaon and pion
  C. Chen, arXiv: 1602.01502

summary

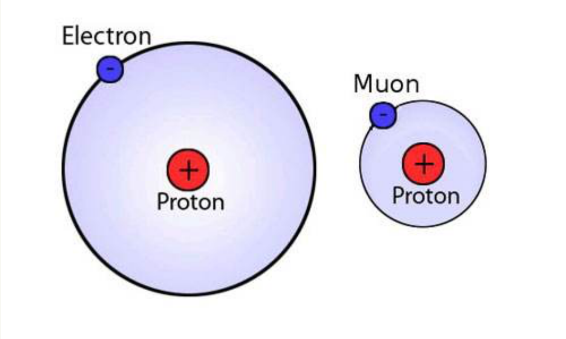
- Dressed-quark basis and symmetry-preserving (beyond-handbag) expressions used to analyse π & K valence-quark PDFs … guarantee that at hadronic scale

  $q_V(x;\zeta_H) \propto (1-x)^2$ on $x \simeq 1$

- Flavour-dependence of DCSB modulates the strength of SU(3)-flavour symmetry breaking in meson PDFs, as it does in every other nonperturbative quantity

- At hadronic scale ζ_H:
  - valence dressed-quarks carry roughly two-thirds of pion's light-front momentum, with the bulk of the remainder carried by glue … sea-quarks carry roughly 5%
  - contrast, valence dressed-quarks carry approximately 95% of the kaon's light-front momentum, with the remainder lying in the gluon distribution … sea-quarks carry ≃ 0 %
    - heavier *s*-quarks radiate soft gluons less readily than lighter quarks and momentum conservation subsequently constrains gluons associated with the kaon's *u*-quark

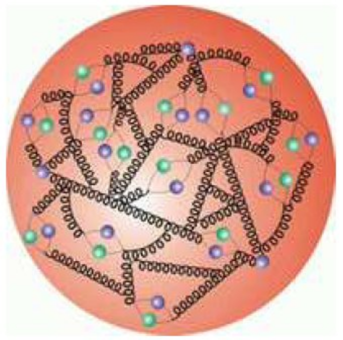
Bound state and quantum field theory



QED

Trace anomaly

- All renormalisable four-dimensional theories possess a trace anomaly;
- The size of the trace anomaly in QED must be great deal smaller than that in QCD.



QCD

Field theory Successful:

- Nonrelativistic quantum mechanics to handle bound state;
- Perturbation theory to handle relativistic effects

Field theory not Successful yet:

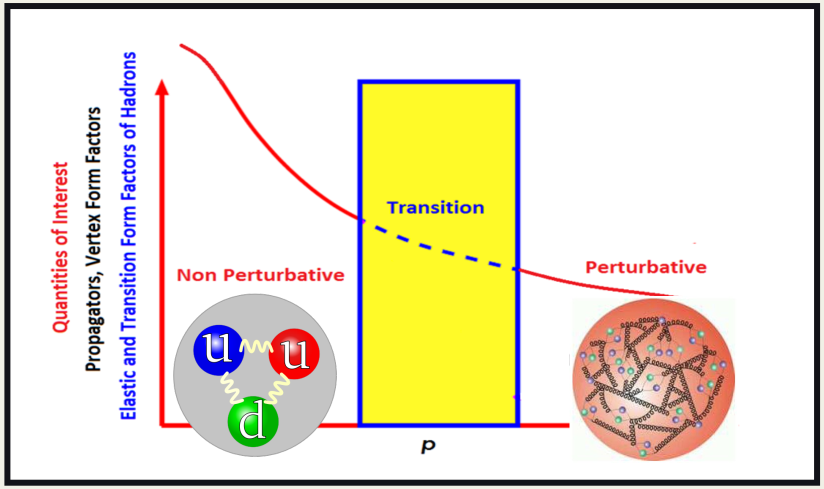
- Growth of the running coupling constant in the infrared region;
- Confinement;
- Dynamical Chiral Symmetry Breaking;
- Possible nontrivial vacuum structure in hadron

It is not at all clear that renormalizable field theories possess any bound states.

Hadron bound state problem

Constituent quark model-> intuitive understanding of many low energy observables.

Minimum number of constituents required



Feynman's parton model-> intuitive understanding of high-energy phenomena.

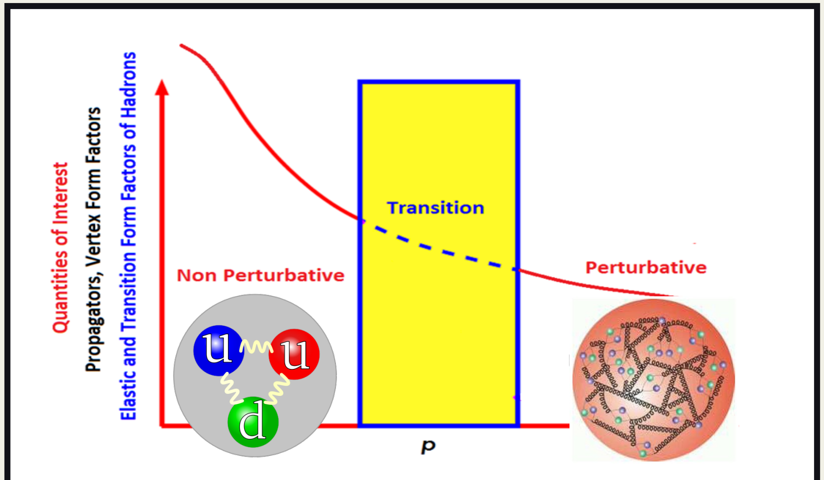
Constituent picture; Probabilistic interpretation of distribution functions

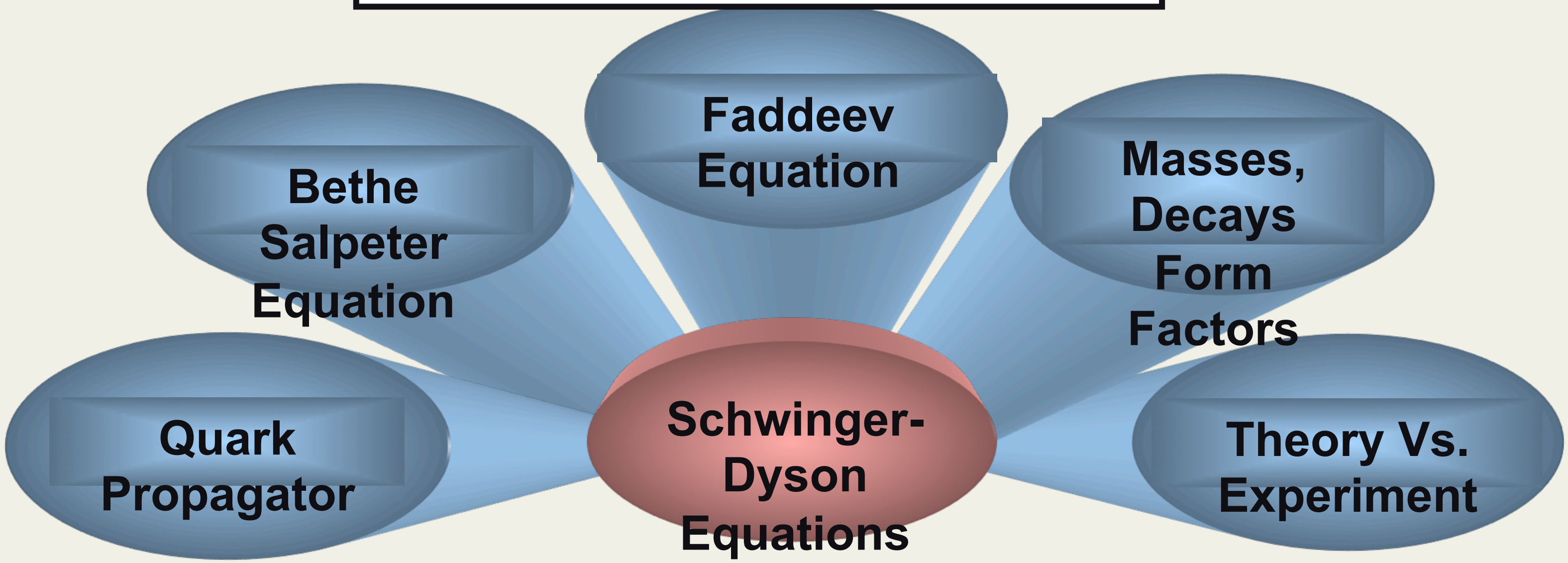
QCD vacuum in the hadron is very complicated medium
Individual quarks and gluons are lost in the sea

Both the constituent quark model and the parton model are put in peril by QCD with a possible complicated vacuum structure.

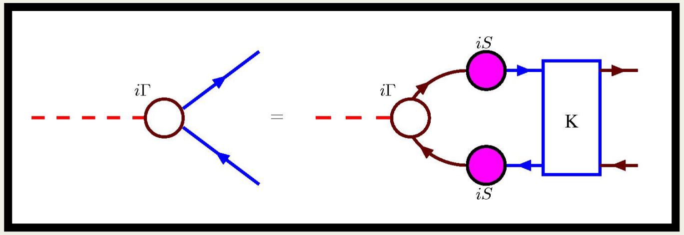
Dyson-Schwinger Equation scope

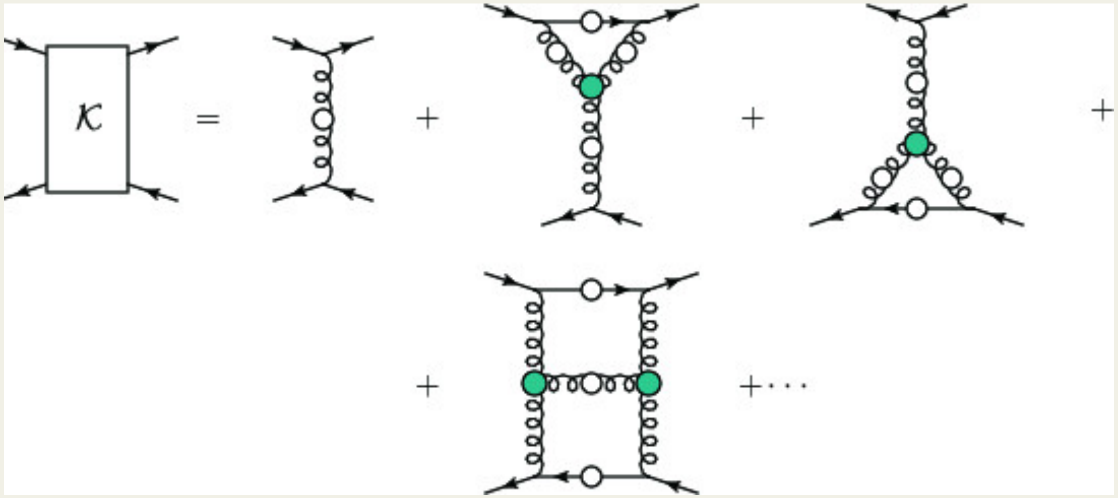
Study bound state problem within an continuum field theory



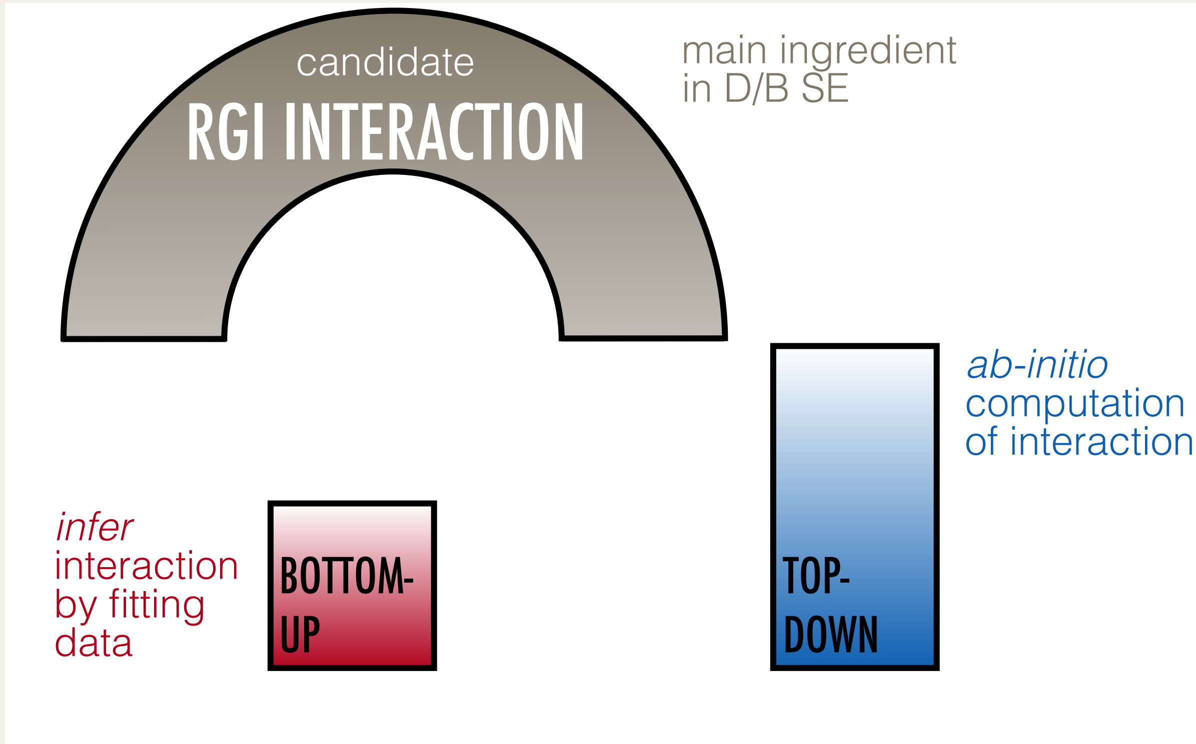


Bethe-Salpeter Equations for meson bound state



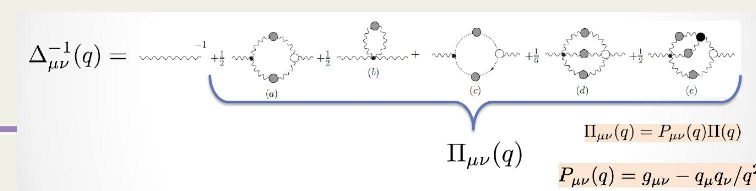


RGI interaction



- Bottom-up scheme – infer interaction by fitting data within a truncation of the matter sector DSEs that are relevant to bound-state properties.

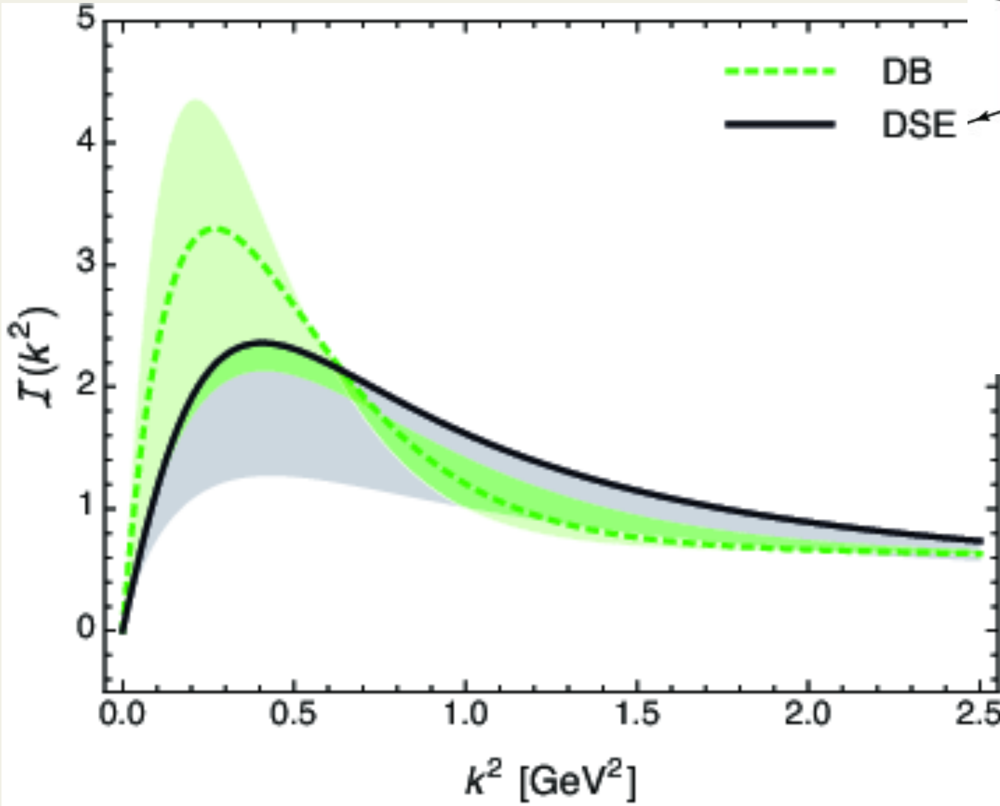
- Top-down approach – *ab initio* computation of the interaction via direct analysis of the gauge-sector gap equations

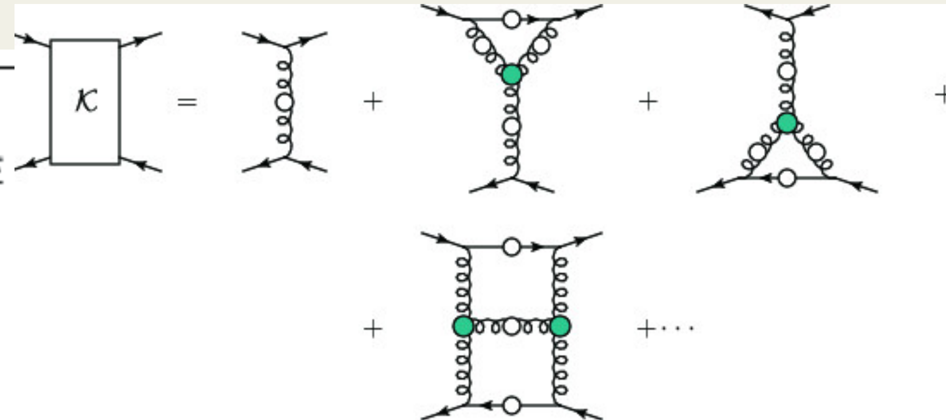


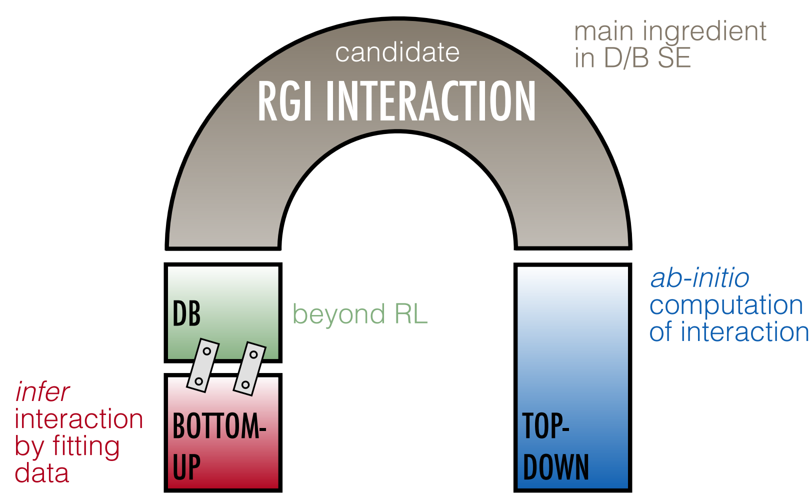
D.Binosi

Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain),
C. D. Roberts (US), arXiv:1412.4782 [nucl-th] ,
Phys. Lett. B 742 (2015) 183





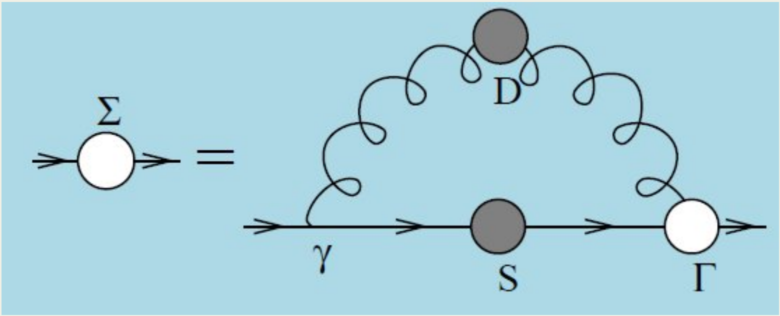


Lei Chang and C. D. Roberts, Phys. Rev. Lett.103 (2009) 081601;
Lei Chang, Yu-xin Liu and C. D. Roberts, Phys. Rev. Lett.106 (2011) 072001

– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using the sophisticated matter-sector ANL-PKU DSE truncation

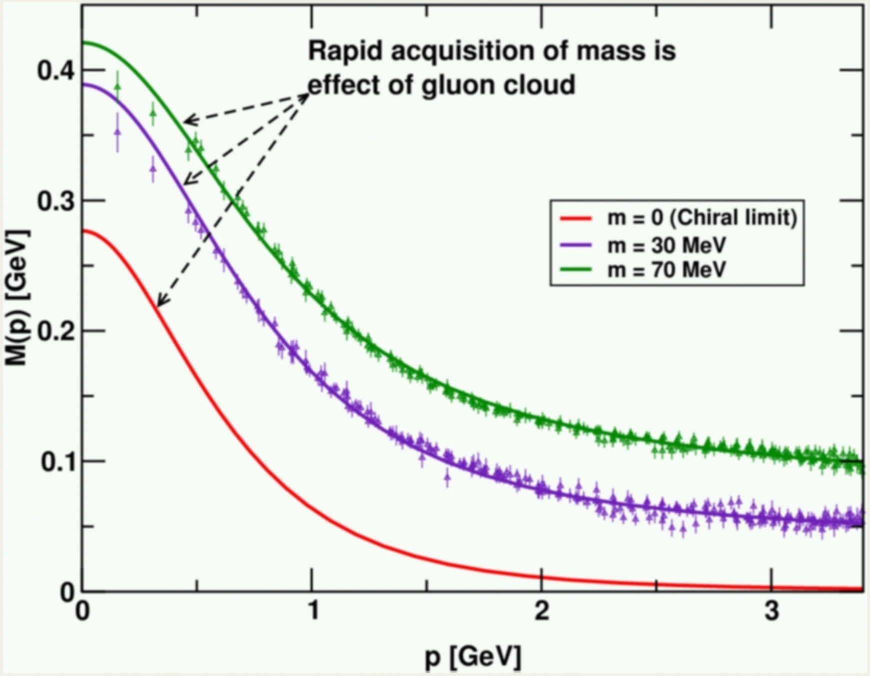
Dynamical Chiral Symmetry Breaking

- Is a crucial emergent phenomenon in QCD
- Expressed in hadron wave functions not in vacuum condensates
- Contemporary theory indicates that it is responsible for more than 98% of the visible mass in the Universe; namely, given that classical massless-QCD is a conformally invariant theory, then DCSB is the origin of *mass from nothing*.



- ***Dynamical***, not spontaneous
  - Add nothing to QCD ,

    No Higgs field, nothing!
    Effect achieved purely through quark+gluon dynamics.



Quark, das ist alles Quark！（夸克［胡扯］，全是夸克［胡扯］！）

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$$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2 \frac{d^2B(k^2)}{d^2k^2} + \ldots$$

- Analytical structure of mass function in the whole momentum region needed;
- Chebyshev polynomials of BSE amplitude does work

$$E(k;P|P^2=0) = \sum_{n=0}^{\infty} E_n(k^2)(k\cdot P)^n$$

Consider $B(k^2) \propto \frac{1}{k^2}$ up to logarithm in the ultraviolet region

$$E_n(k^2) \propto \frac{1}{k^{2+n/2}}$$

Power suppress of amplitude ensure the light front physics safe

Pion's LFWF definition

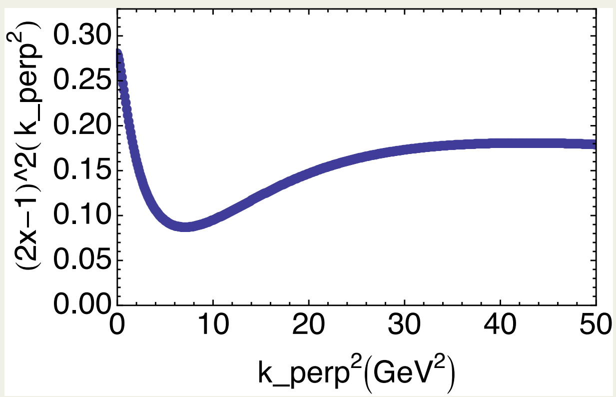
$$\psi_{\uparrow\downarrow}(x,k_\perp^2)=\frac{1}{f_\pi}\int\frac{d^2k_\parallel}{\pi}\delta(n\cdot k_+-xn\cdot P)\mathrm{tr}_{\mathrm{CD}}[\gamma_5\gamma\cdot n\chi(k_+,k_-)] \quad (1)$$

and

$$k_\perp^i\psi_{\uparrow\uparrow}(x,k_\perp^2)=\frac{1}{f_\pi}\int\frac{d^2k_\parallel}{\pi}\delta(n\cdot k_+-xn\cdot P)\mathrm{tr}_{\mathrm{CD}}[\gamma_5\sigma_{ni}\chi(k_+,k_-)] \quad (2)$$

with $k_\pm=k\pm\frac{P}{2}$, $\sigma_{ni}=\frac{I}{2}(\gamma\cdot n\gamma_i-\gamma_i\gamma\cdot n)$ and χ is the pion BS wave function. Where $\psi_{\uparrow\downarrow}(x,k_\perp^2)$ denotes the pion light front wave function with anti parallel quark helicity and $\psi_{\uparrow\uparrow}(x,k_\perp^2)$ the parallel quark helicity. For finite x and $k_\perp$ the above integration is convergent

$x^{0.28}(1-x)^{0.28}$

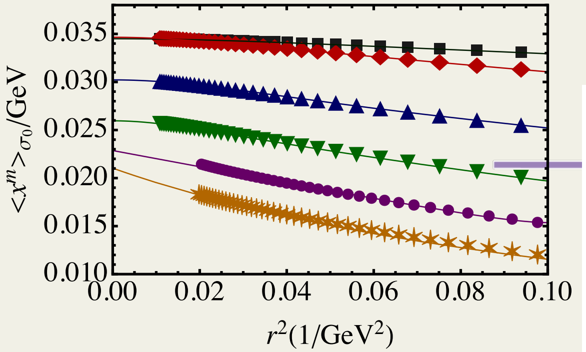


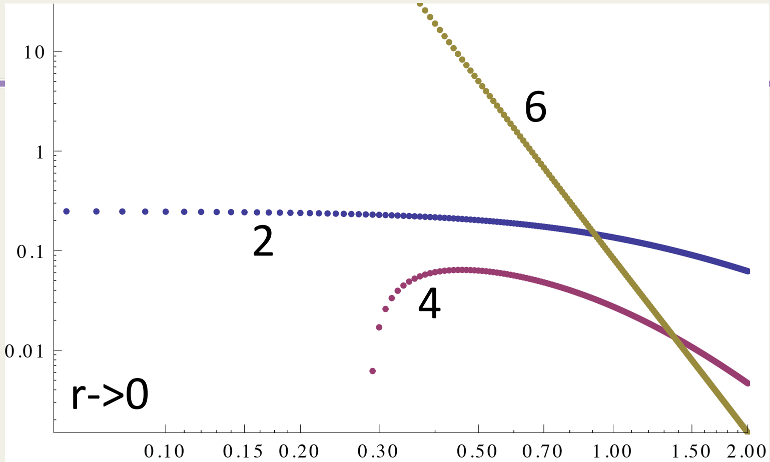
$x(1-x)$

- Non-factorization
- Higher moments is not stable
- Pointwise unreachable

Infrared soft……Ultraviolet hard

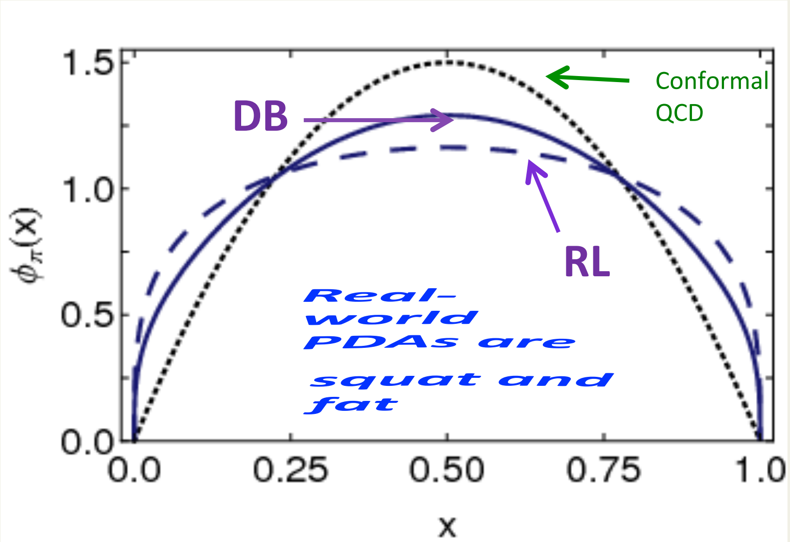
BS wave function tells more good story about the light front wave function

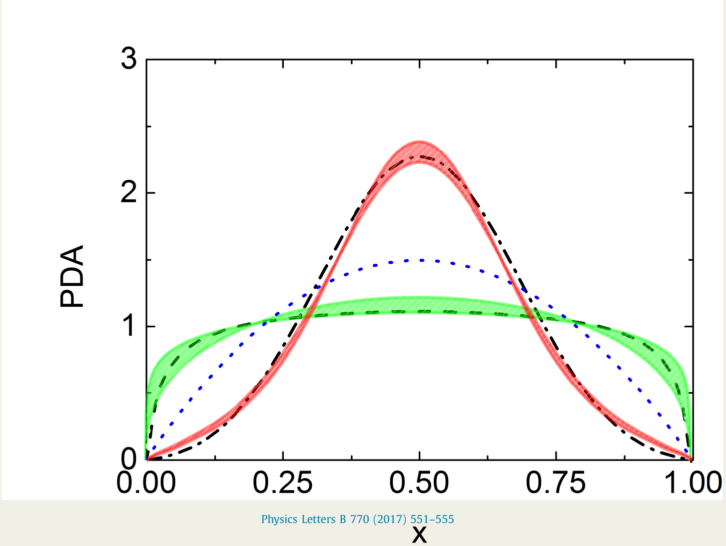




Pion's PDA moments

Direct Computation Difficulty!!!!





$$
\begin{aligned}
\Gamma_\pi(k;P) &= \gamma_5\{\gamma\cdot PF(k;P)+\gamma\cdot kG(k;P)+iE(k;P)+\sigma_{\mu\nu}k_\mu P_\nu H(k;P)\} \\
&= \gamma_5\left\{\int_{-1}^{1}dz\int_0^\infty d\gamma\frac{\gamma\cdot Pf(\gamma,z)+\gamma\cdot kg(\gamma,z)+ie(\gamma,z)+\sigma_{\mu\nu}k_\mu P_\nu h(\gamma,z)}{(k^2+zk\cdot P+M^2+\gamma)^2}\right\}
\end{aligned}
$$

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Bayesian extraction of the parton distribution amplitude from the Bethe–Salpeter wave function

Fei Gao [b,c], Lei Chang [a,*], Yu-xin Liu [b,c,d]

Quark propagator:

$$S^{-1}(k) = i\gamma \cdot k + M$$

Bethe-Salpeter Amplitude:

$$\begin{aligned}\Gamma_\pi(k, k-P) \;=\;& i\gamma_5 \mathcal{N} \int_{-1}^{1} dz \left\{ \rho_0(z) \frac{M^2}{(k - \frac{1}{2}P + \frac{z}{2}P)^2 + M^2} + \rho_1(z) \frac{1}{2} \frac{M^4}{((k - \frac{1}{2}P + \frac{z}{2}P)^2 + M^2)^2} + \ldots \right\} \\ & + i\gamma_5 \gamma \cdot k \gamma \cdot (k - P) \mathcal{N} \frac{1}{2} \int_{-1}^{1} dz \rho_2(z) \left\{ \frac{M^2}{((k - \frac{1}{2}P + \frac{z}{2}P)^2 + M^2)^2} + \ldots \right\} \\ & + \gamma_5\text{-odd terms}\end{aligned}$$

rho(z) is weight function accounting relative motion between the partons
rho(z) is kind of even function;
rho(z) take no any singularity;

Model Way---conformal limit

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$$
\begin{aligned}
\Gamma_\pi(k,k-P) &= i\gamma_5\mathcal{N}\int_{-1}^{1}dz\left\{\rho_0(z)\frac{M^2}{(k-\frac{1}{2}P+\frac{z}{2}P)^2+M^2}+\rho_1(z)\frac{1}{2}\frac{M^4}{((k-\frac{1}{2}P+\frac{z}{2}P)^2+M^2)^2}+\ldots\right\}\\
&+i\gamma_5\gamma\cdot k\gamma\cdot(k-P)\mathcal{N}\frac{1}{2}\int_{-1}^{1}dz\rho_2(z)\left\{\frac{M^2}{((k-\frac{1}{2}P+\frac{z}{2}P)^2+M^2)^2}+\ldots\right\}\\
&+\gamma_5\text{-odd terms}
\end{aligned}
$$

Introducing the weight functions

$$\rho_0(z)=\rho_1(z)=\rho_2(z)=\frac{3}{4}(1-z^2)$$

Two particle leading twists DA

$$
\begin{aligned}
&\langle 0|\bar\psi_f(-x)\gamma_5\gamma\cdot n\psi_g(x)|P_{\bar g f}(q)\rangle\\
&= f_P\, n\cdot q\int_0^1 du\, \mathrm{e}^{-ix\cdot q\,(2u-1)}\varphi_P^{(2)}(u,\zeta)\,,\\
-&\langle 0|\bar\psi_f(-x)i\gamma_5\psi_g(x)|P_{\bar g f}(q)\rangle\\
&= i\rho_P^\zeta\int_0^1 du\, \mathrm{e}^{-ix\cdot q\,(2u-1)}\omega_P^{(3)}(u,\zeta)\,,\\
&\langle 0|\bar\psi_f(-x)i\gamma_5\sigma_{\mu\nu}q_\mu n_\nu\psi_g(x)|P_{\bar g f}(q)\rangle\\
&= \frac{1}{4}\rho_P^\zeta\, n\cdot q\int_0^1 du\, \mathrm{e}^{-ix\cdot q\,(2u-1)}\frac{d}{du}\upsilon_P^{(3)}(u,\zeta)\,.
\end{aligned}
$$

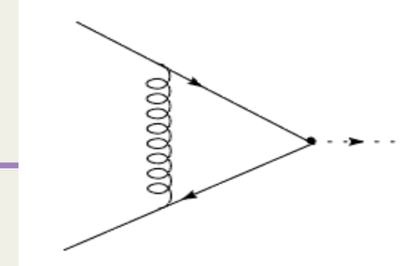
$$\Rightarrow$$

$$\phi^{(2)}(x)=6x(1-x)$$

$$\omega^{(3)}(x)=1$$

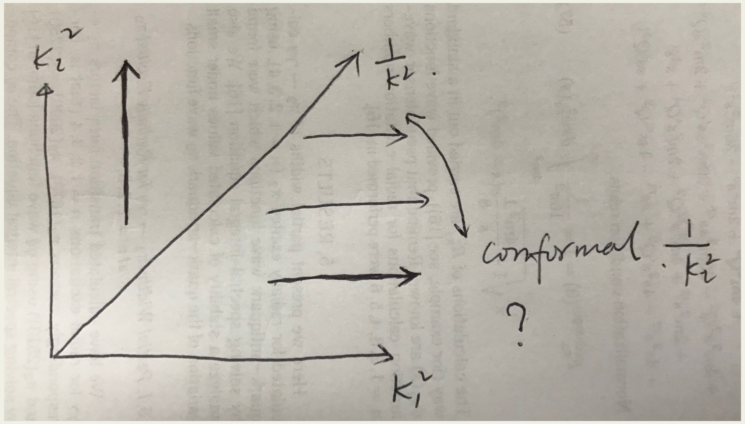
$$\upsilon^{(3)}(x)=6x(1-x)$$

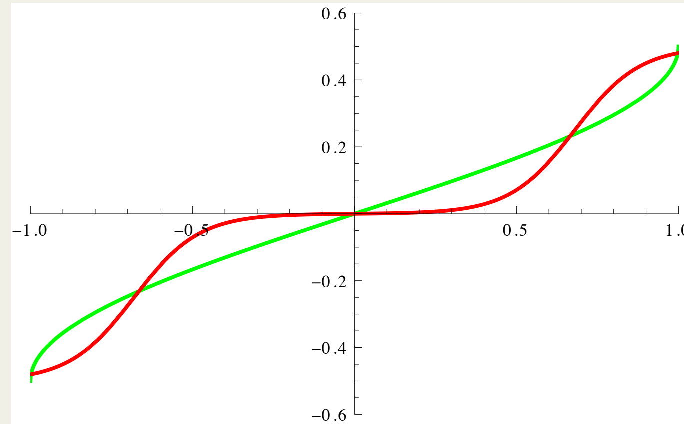
Model Way---Asymptotic behavior of BS amplitude



$$\Gamma_\pi(k, k-P) \;=\; i\gamma_5 \mathcal{N} \int_{-1}^{1} dz \left\{ \rho(z) \frac{M^2}{(k - \frac{1}{2}P + \frac{z}{2}P)^2 + M^2} \right\}$$

$$\rho(z) = \frac{d\omega(z)}{dz}$$





Model-I:A smooth approximation to the step function

$$\omega_I(z) = \frac{1}{2}\left(\frac{1}{1 + \mathrm{e}^{\frac{-z+z0}{t}}} - \frac{1}{1 + \mathrm{e}^{\frac{z+z0}{t}}} \right)$$

Model-II:An integrable singularity form at boundary

$$\omega_{II}(z) = \frac{1}{\pi}\mathrm{ArcSin}[z]$$

Model-I: the Bethe-Salpeter amplitude of Pion decreases as

$$\Gamma_\pi(k_1^2, k_2^2) \to \frac{1}{k_1^2}$$

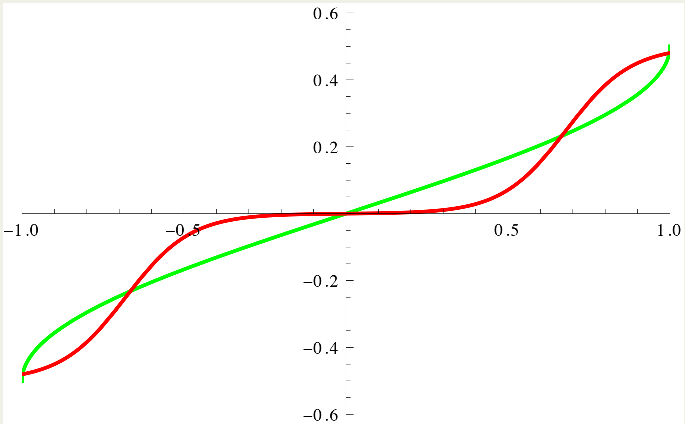
as $k_1^2 \to \infty$ with k_2^2 fixed finite, or $k_1^2 \to \infty$ with k_1^2/k_2^2 fixed finite.

Model-II: the Bethe-Salpeter amplitude of Pion decreases as

$$\Gamma_\pi(k_1^2, k_2^2) \to \frac{1}{\sqrt{k_1^2}}, \frac{1}{k_1^2}$$

as $k_1^2 \to \infty$ with k_2^2 fixed finite and as $k_1^2 \to \infty$ with k_1^2/k_2^2 fixed finite.

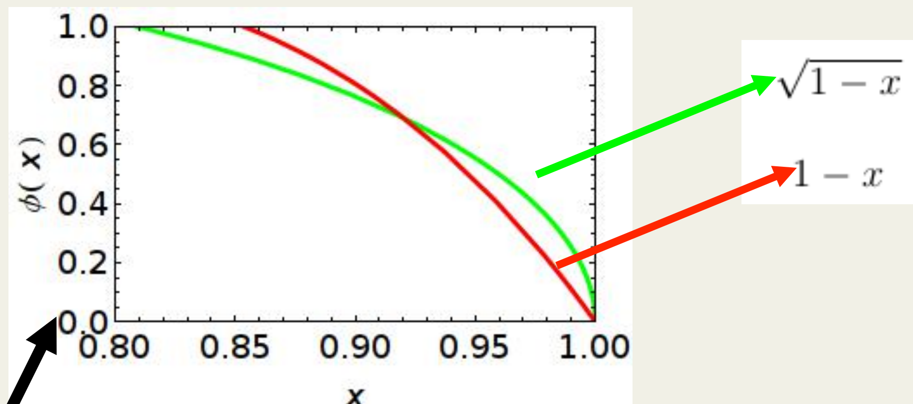
Parton Distribution Amplitude

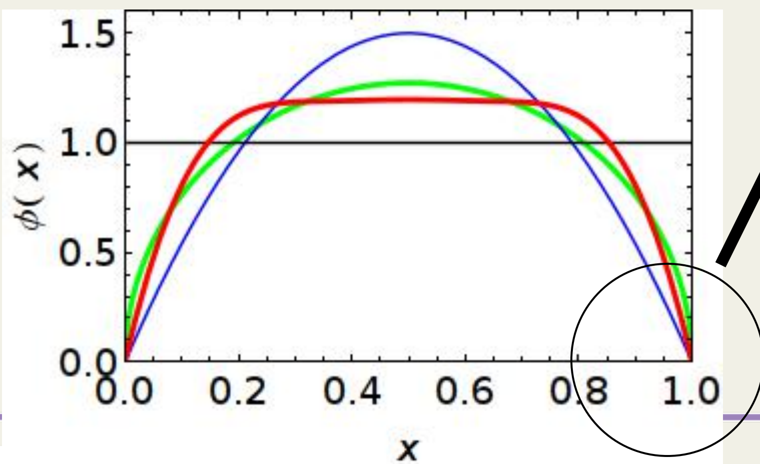


1,Second moment $\int dx\varphi(x)(2x-1)^2$

NJL-like	Model-I	Model-II	Asymptotic
1/3	1/4	1/4	1/5

2,x \to 1 limit





3,Inverse moment $\int dx\frac{\varphi(x)}{x}$

NJL-like	Model-I	Model-II	Asymptotic
Infinity	3.45	4	3

Quasi Distribution Amplitudes

$$f_K\,\tilde{\varphi}_K(x) = \mathrm{tr}_{\mathrm{CD}} \int_{dk} \delta^x_{\tilde{n}}(k_K)\,\gamma_5\gamma\cdot\tilde{n}\,\chi^{(2)}_K(k^K_-;P_K)\,,$$

with

$$\tilde{n} = (0,0,1,0)$$

- **From BSA to BSW**

$$\chi^{(2)}_K(k^K_-;P_K) = \mathcal{M}(k;P_K)\int_0^1 d\alpha\, 2\,\mathcal{X}_K(\alpha;\sigma^3(\alpha))\,,$$

with $\sigma = (k-\alpha P_K)^2 + \Omega_K^2$,

$$\begin{aligned}\Omega_K^2 &= vM_u^2 + (1-v)\Lambda^2 \\ &\quad + (M_s^2 - M_u^2)\left(\alpha - \tfrac{1}{2}[1-w][1-v]\right) \\ &\quad + (\alpha[\alpha-1] + \tfrac{1}{4}[1-v][1-w^2])M_K^2\,,\end{aligned}$$

$$\begin{aligned}\mathcal{X}_K(\alpha;\sigma^3) = &\left[\int_{-1}^{1-2\alpha} dw \int_{1+\frac{2\alpha}{w-1}}^{1} dv\right. \\ &\left. + \int_{1-2\alpha}^{1} dw \int_{\frac{w-1+2\alpha}{w+1}}^{1} dv\right] \frac{\rho_K(w)}{n_K}\frac{\Lambda^2}{\sigma^3}\,.\end{aligned}$$

w and alpha different;
alpha and k_{perp} nonfactorized

- **One can obtain Radyushkin relation**

$$\tilde{\varphi}_K(x) = \frac{P_z}{16\pi^3}\int_0^1 d\alpha \int_{-\infty}^{\infty} dk\, \psi_K^{\uparrow\downarrow}(\alpha, k^2 + (x-\alpha)^2 P_z^2)\,.$$

- **Numerical route**

- Feynman parametrisation to combine denominator products into a single quadratic form
- Cauchy's theorem to evaluate the k_4 integral
- Direct evaluation for k_{perp} integral
- Numerical integration over the Feynman parameters

Quasi Distribution Amplitudes

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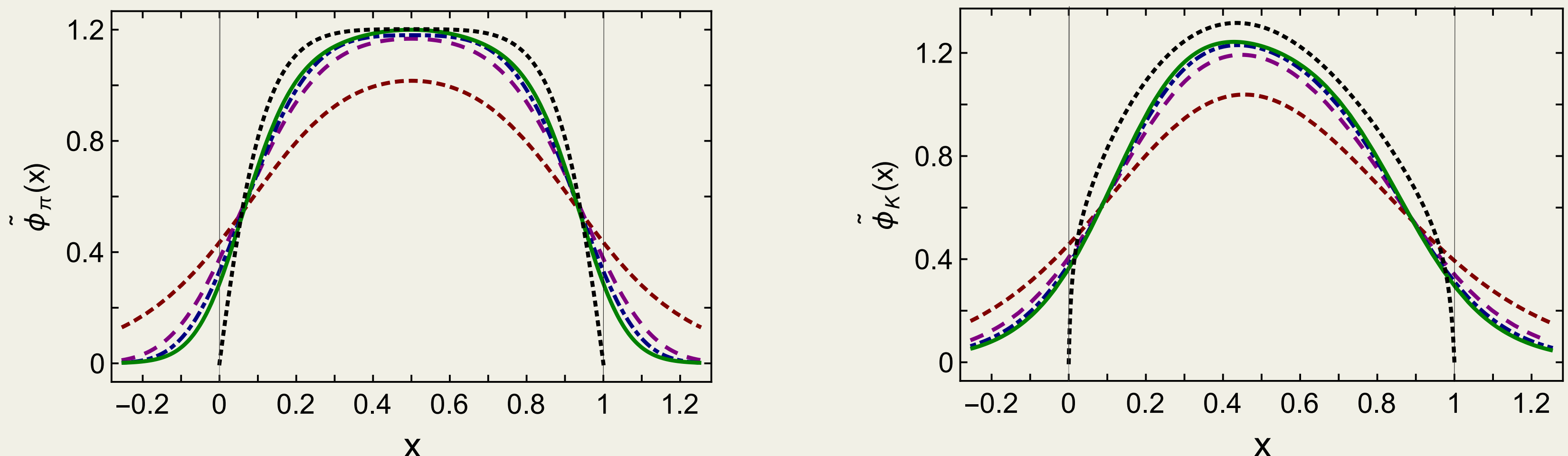
$$f_K\,\tilde{\varphi}_K(x) = \mathrm{tr}_{\mathrm{CD}} \int_{dk} \delta^{x}_{\tilde{n}}(k_K)\,\gamma_5\gamma\cdot\tilde{n}\,\chi^{(2)}_K(k^K_-;P_K)\,,$$



PDA, computed with $P_z/\mathrm{GeV} = 1$ (short-dashed, red), 1.75 (dashed, purple), 2.4 (dot-dashed, blue), 3.0 (solid, green).

- Pz=1.00GeV does not closely resemble true PDA;
- Pz=1.75GeV brings material improvement…a qualitatively sound approximation;
- Further increments in Pz do not bring much improvment...saturation;
- Not possible to provide true endpoint behavior;
- Matching condition need for further consideration.

Pion Parton Distribution Functions

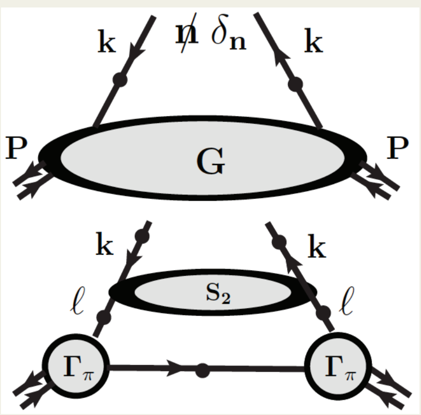
The hadronic tensor relevant to *inclusive deep inelastic lepton-pion scattering* may be expressed in terms of two invariant structure functions. In the deep-inelastic Bjorken limit

$$W_{\mu\nu}(q;P) = F_1(x)\, t_{\mu\nu} - \frac{F_2(x)}{P\cdot q} P^t_\mu P^t_\nu\,,\;\; F_2(x) = 2xF_1(x)\,.$$

F1 is the pion structure function, access to the pion's quark distribution functions

$$F_1(x) = \sum_{q\in\pi} e_q^2\, q^\pi(x)\,,$$

DSE first stage



$$u_\pi(x) = \frac{-J_E}{2(2\pi)^4} \int_{-\infty}^{+\infty} d\beta\, d^2\ell_\perp\, T(n,p;\ell,P)\big|_{\alpha=xP\cdot n},$$

In Fig. 2 we display our DSE result [30] for the valence u-quark distribution evolved to $Q^2 = (5.2\text{ GeV})^2$ in comparison with πN Drell-Yan data [3] at a scale $Q^2 \sim (4.05\text{ GeV})^2$ obtained via a LO analysis. Our distribution at the model scale Q_0 is evolved using leading-order DGLAP. The model scale is fixed to $Q_0 = 0.57$ GeV by matching the x^n moments for $n = 1,2,3$ to the experimental analysis given at $(2\,\text{GeV})^2$ [33]. Our momentum sum rule result $\boxed{2\langle x\rangle = 0.74\text{ (pion)},}$ 0.76 (kaon) at Q_0 shows clearly the implicit inclusion of gluons as a dynamical entity in a true covariant bound-state approach.

Valence partons do not contain all the momentum

T. Nguyen, et al, arXiv: 1102.2448

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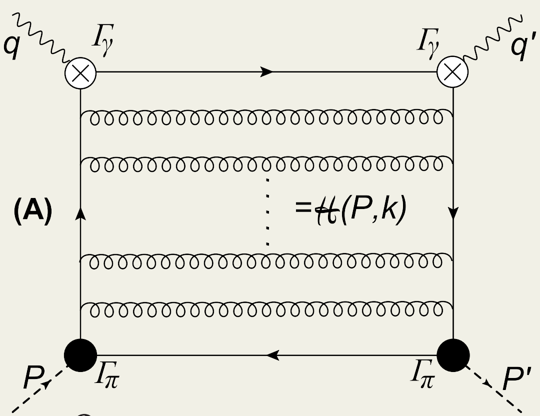
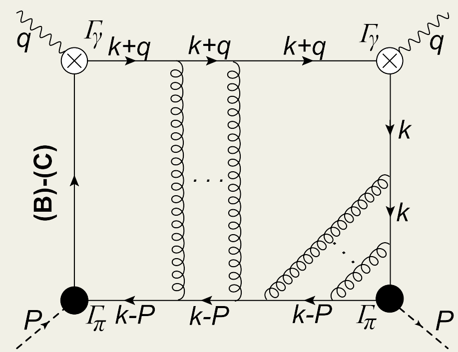
DSE second stage





$$q^\pi_A(x) = N_c \mathrm{tr}\int_{dk} i\Gamma_\pi(k_\eta,-P)$$
$$\times\ S(k_\eta)\,\Gamma^n(k;x)\,S(k_\eta)\,i\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})\,,$$

$$q^\pi_{BC}(x) = N_c \mathrm{tr}\int_{dk} \delta^x_n(k_\eta)\partial_{k_\eta}\Gamma_\pi(k_\eta,-P)S(k_\eta)\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})$$

LC, et al, arXiv: 1406.5450

Pion Parton Distribution Functions

The hadronic tensor relevant to *inclusive deep inelastic lepton-pion scattering* may be expressed in terms of two invariant structure functions. In the deep-inelastic Bjorken limit

$$W_{\mu\nu}(q;P) = F_1(x)\, t_{\mu\nu} - \frac{F_2(x)}{P\cdot q} P^t_\mu P^t_\nu\,,\;\; F_2(x) = 2xF_1(x)\,.$$

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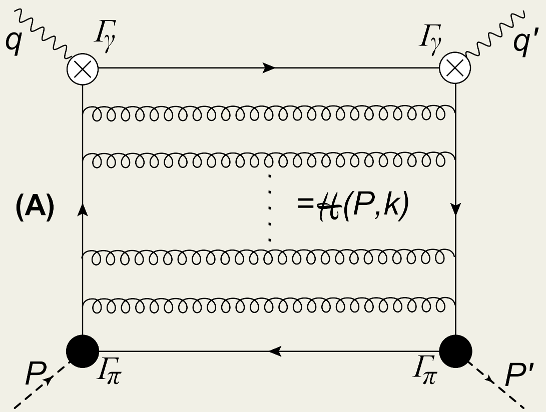
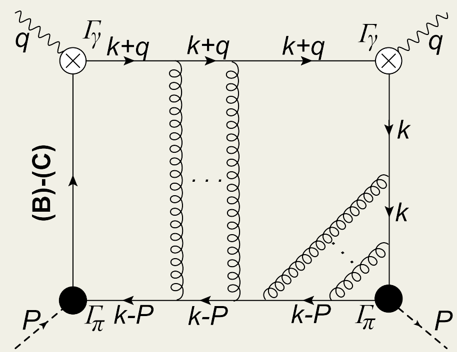
DSE second stage





$$q(x) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta)\, n\cdot\partial_{k_\eta}[\Gamma_\pi(k_\eta,-P)S(k_\eta)]\Gamma_\pi(k_{\bar\eta},P)S(k_{\bar\eta})$$

Extend to Quasi Distribution Functions

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$$q(x) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta) n \cdot \partial_{k_\eta} [\Gamma_\pi(k_\eta, -P) S(k_\eta)] \Gamma_\pi(k_{\bar\eta}, P) S(k_{\bar\eta})$$

$$\Big\downarrow \; n \to \tilde{n}$$

$$\tilde{q}(x) = N_c \mathrm{tr} \int_{dk} \delta_{\tilde{n}}^x(k_\eta) \tilde{n} \cdot \partial_{k_\eta} [\Gamma_\pi(k_\eta, -P) S(k_\eta)] \Gamma_\pi(k_{\bar\eta}, P) S(k_{\bar\eta})$$

$$\begin{aligned}
&\int_{-\infty}^{\infty} d\tilde{x}\, \tilde{u}_V^K(\tilde{x}) = \int_0^1 dx\, u_V^K(x) = 1\,, \\
&\int_{-\infty}^{\infty} d\tilde{x}\, \tilde{s}_V^K(\tilde{x}) = \int_0^1 dx\, s_V^K(x) = 1\,, \\
&\int_{-\infty}^{\infty} d\tilde{x}\, \tilde{x}[\tilde{u}_V^K(\tilde{x}) + \tilde{s}_V^K(\tilde{x})] \\
&\qquad = \int_0^1 dx\, x[u_V^K(x) + s_V^K(x)] \neq 1\,.
\end{aligned}$$

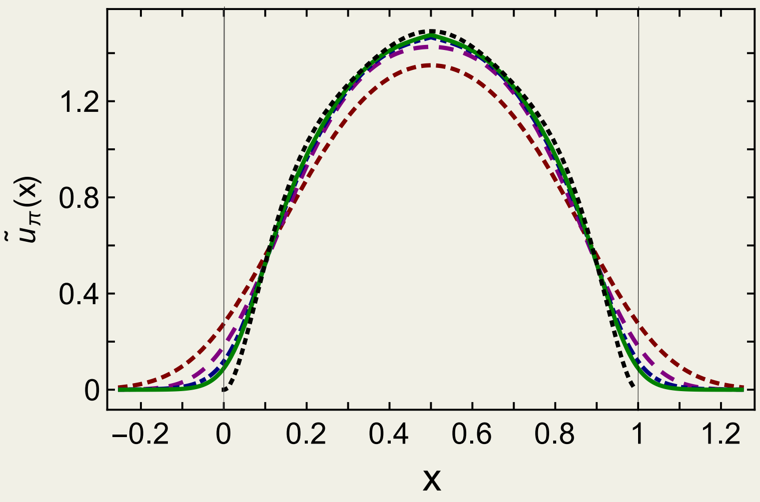
$$\Big\downarrow$$

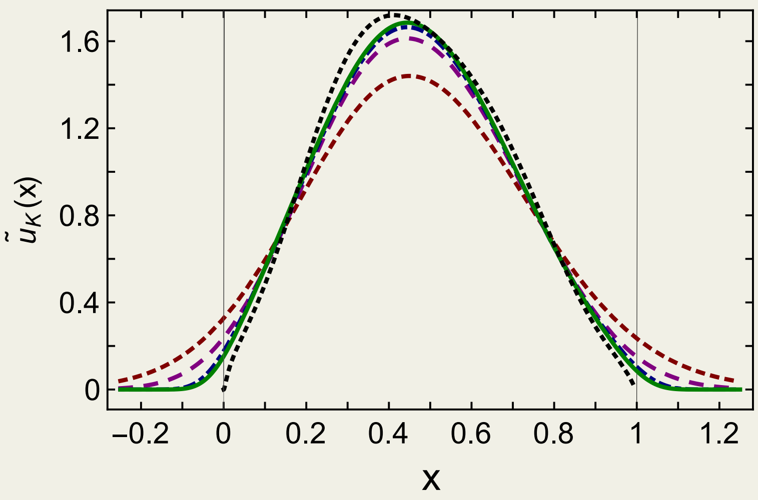
$$\begin{aligned}
\tilde{q}(x) \;=\; & N_c \mathrm{tr} \int_{dk} \delta_{\tilde{n}}^x(k_\eta) \tilde{n} \cdot \partial_{k_\eta} [\Gamma_\pi(k_\eta, -P) S(k_\eta)] \Gamma_\pi(k_{\bar\eta}, P) S(k_{\bar\eta}) \\
& - \frac{1}{2} N_c \mathrm{tr} \int_{dk} \delta_{\tilde{n}}^x(k_\eta) \tilde{n} \cdot \partial_{k_\eta} [\Gamma_\pi(k_\eta, -P) S(k_\eta) \Gamma_\pi(k_{\bar\eta}, P) S(k_{\bar\eta})]
\end{aligned}$$

The subtraction term is ZERO on the light front

Quasi Distribution Functions

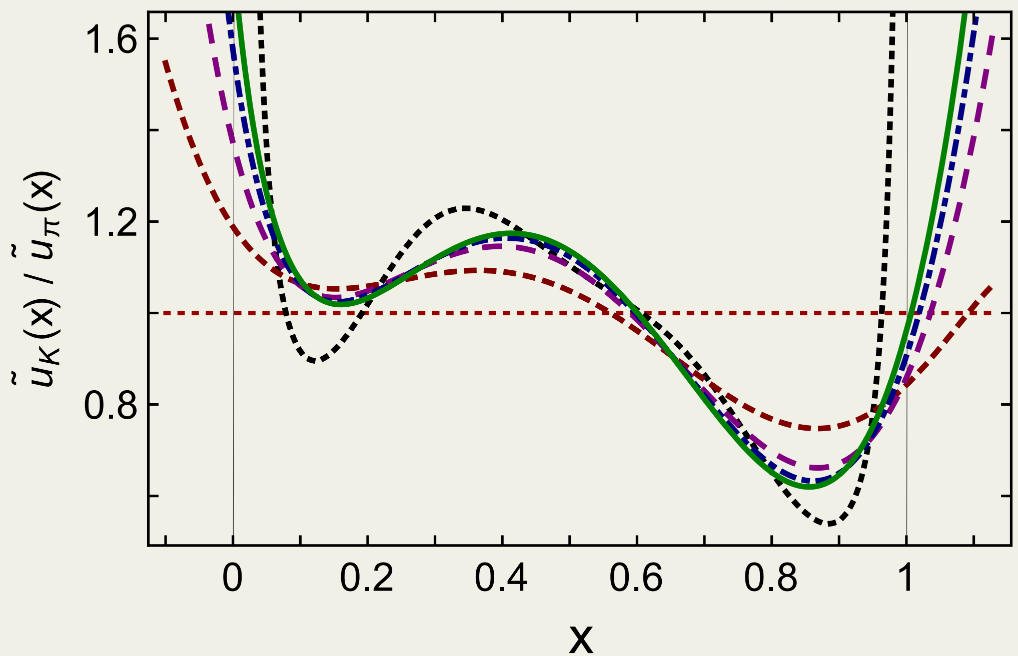
$$\tilde{q}(x) = N_c \mathrm{tr} \int_{dk} \delta^x_{\tilde{n}}(k_\eta)\tilde{n}\cdot\partial_{k_\eta}[\Gamma_\pi(k_\eta,-P)S(k_\eta)]\Gamma_\pi(k_{\bar{\eta}},P)S(k_{\bar{\eta}})$$
$$-\frac{1}{2}N_c \mathrm{tr}\int_{dk} \delta^x_{\tilde{n}}(k_\eta)\tilde{n}\cdot\partial_{k_\eta}[\Gamma_\pi(k_\eta,-P)S(k_\eta)\Gamma_\pi(k_{\bar{\eta}},P)S(k_{\bar{\eta}})]$$





Evolution of Pz similar as the case of Pion

The ratio



- For Pz>1.75GeV the ratio of qPDFs is quantitatively a good approximation to the objective ratio on a material domain, viz. 0.3<x<0.8. This domain almost covers that upon which empirical data is available.
- We anticipate that contemporary lQCD simulations could provide a sound prediction for this ratio before next generation experiments are completed.

- Employing a continuum approach to bound-states we computed the leading twist LFWFs, PDAs, PDFs, qPDAs and qPDFs of pion and Kaon.
- qPDAs and qPDFs cannot be used to determine the objective's large-x behavior.
- We expect that contemporary simulations of lQCD can deliver a reasonable prediction for the Kaon/Pion ratio before next generation experiments are completed.
- Extend this approach to neutron and proton system.

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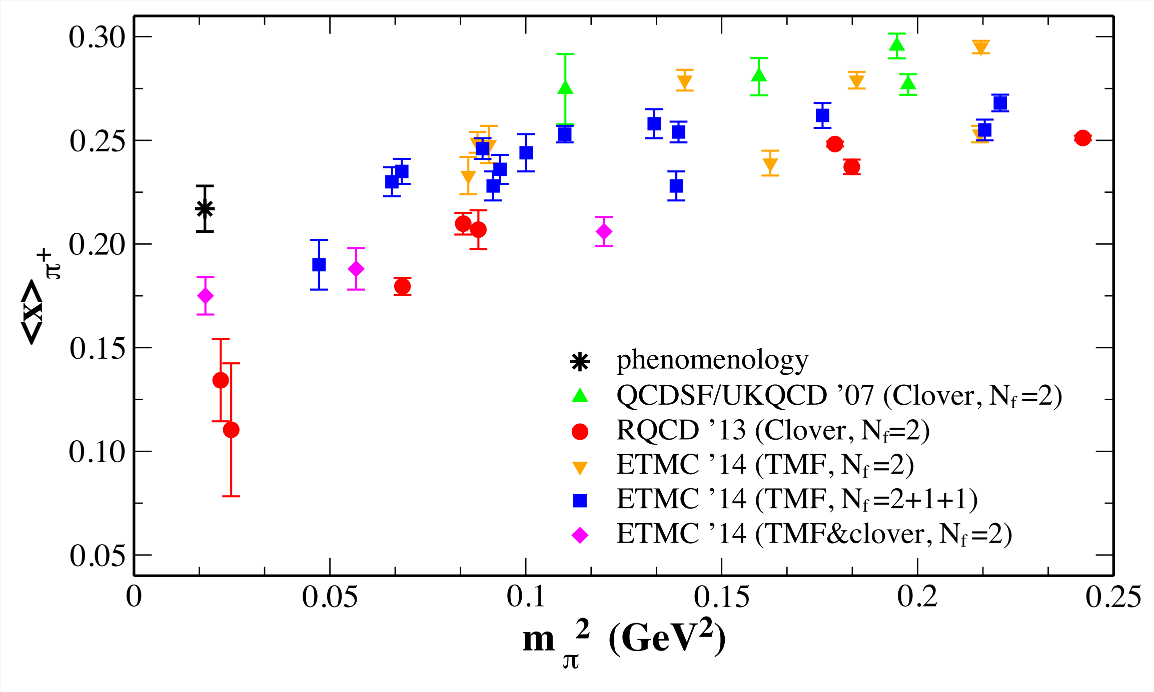
– Extend this approach to neutron and proton system.

Thanks for your attention

Pion PDF----lQCD Progress



Hadron Structure

M. Constantinou, arXiv:1411.0078

A comprehensive study of lattice artifacts is called for in order to understand the observed discrepancies in the lattice data…