A trajectory of meson's PDA corresponding to current quark mass

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Outline

Quantum ChromoDynamics

Challenges in QCD:

QCD is a non-abelian gauge theory which describes the strong interaction between hadrons and quarks, gluons inside.

- *Asymptotic free behavior at high energy*
- *Dynamical chiral symmetry breaking (DCSB) and confinement at low energy*

  - **DCSB** is a complex phenomenon. It occurs in QCD because the **effective coupling runs**, becoming large at infrared momenta. However, the mechanisms are sophisticated and can be understood via the gap equation.

The running quark mass describes the almost-massless pseudoscalar meson states and the massive baryon states.

- **Confinement** is actually also a dynamical phenomenon of QCD, which is related to the gluon mass dynamical generation. (arXiv:1802.08184)
  If we define color charge Q^a as the Noether charge under global gauge transformation, then we have:

$$Q^a = \int d^3x J_0^a(x) \tag{1}$$

with $J_\mu^a = g\bar{\psi}(x)\gamma_\mu t^a \psi(x) + gf^{abc}A_\nu^b(x)F_{\nu\mu}^c(x) = \partial_\nu F_{\nu\mu}^a(x)$
Then we can get the relation:

$$\left\{1 - \int d^4k\delta^4(k)k^2 D(k^2)\right\}\left\langle \text{phys}\middle|\hat{Q}^a\middle|\text{phys}\right\rangle = 0\,. \tag{2}$$

leading twist PDAs

One can approach these phenomena of QCD by studying the properties and inside structures of strong-interaction bound-states (in vacuum)

I here introduce our results of leading twist parton distribution amplitudes (PDAs) of mesons:

- Light front parton distribution amplitude is related to the light front wave function which is a counterpart of the wave function in quantum mechanics.
- In the theory of strong interactions, the cross-sections for many hard exclusive hadronic reactions can be expressed in terms of the leading twist PDAs of the hadrons involved.

leading twist PDAs

There is only one independent PDA at leading twist for pseudoscalar meson.
Pseudoscalar mesons' leading twist parton distribution amplitudes:

$$\begin{aligned} & < 0|\bar{\psi}(0)\gamma_5\gamma_\mu\psi(n)|p > \\ = & \; p_\mu f_{PS} \int_0^1 dx e^{-ixp\cdot n} \phi_{PS}(x,\zeta)\,. \end{aligned} \quad (3)$$

leading twist PDAs

Vector mesons' leading twist parton distribution amplitudes:

$$
\begin{aligned}
& < 0|\bar{\psi}(0)\sigma_{\mu\nu}\psi(n)|p,\lambda > \\
= \quad & i(e^{\lambda}_{\mu}p_{\nu} - e^{\lambda}_{\nu}p_{\mu})f^{\perp}_{V}\int_0^1 dx e^{-ixp\cdot n}\phi_{V\perp}(x,\zeta)\,, \qquad (4) \\
& < 0|\bar{\psi}(0)\gamma_{\mu}\psi(n)|p,\lambda > \\
= \quad & p_{\mu}\frac{e^{\lambda}\cdot n}{p\cdot n}f_V m_V\int_0^1 dx e^{-ixp\cdot n}\phi_{V\parallel}(x,\zeta) \\
+ \quad & (e^{\lambda}_{\mu} - p_{\mu}\frac{e^{\lambda}\cdot n}{p\cdot n})f_V m_V\int_0^1 dx e^{-ixp\cdot n}g^{(\nu)}_{V\perp}(x,\zeta)\,, \qquad (5) \\
& < 0|\bar{u}(0)\gamma_{\mu}\gamma_5 d(n)|p,\lambda > \\
= \quad & -\frac{1}{4}\epsilon_{\mu\nu\tau\sigma}e^{\lambda}_{\nu}p^{\tau}n^{\sigma}f_V m_V\int_0^1 dx e^{-ixp\cdot n}g^{(a)}_{V\perp}(x,\zeta)\,, \qquad (6)
\end{aligned}
$$

where $\phi_{\perp}(x,\zeta)$ and $\phi_{\parallel}(x,\zeta)$ is the transversally and longitudinally polarized amplitude respectively.

leading twist PDAs

According to the definitions above, consider the following projection of the meson's Bethe-Salpeter wave function onto the light front .

$$
\begin{aligned}
& f_{PS}\phi_{PS}(x,\zeta) \\
=& tr_{CD} Z_2(\zeta,\Lambda)\int_{dq}^{\Lambda}\delta(n\cdot q_+ - x n\cdot P)\gamma_5\gamma\cdot n\chi_5(q;P)\,, \qquad \text{(7a)} \\
& f_V^{\perp}\phi_{V\perp}(x,\zeta) m_V^2 \\
=& n\cdot P tr_{CD} Z_T(\zeta,\Lambda)\int_{dq}^{\Lambda}\delta(n\cdot q_+ - x n\cdot P)\sigma_{\mu\lambda}P_{\mu}\chi_{\lambda}(q;P)\,, \qquad \text{(7b)} \\
& f_V n\cdot P\phi_{V\parallel}(x,\zeta) \\
=& m_V tr_{CD} Z_2(\zeta,\Lambda)\int_{dq}^{\Lambda}\delta(n\cdot q_+ - x n\cdot P) n\cdot\gamma n_{\lambda}\chi_{\lambda}(q;P)\,, \qquad \text{(7c)}
\end{aligned}
$$

where $\chi_\lambda(q;P) = S(q+\eta P)\Gamma_\lambda S(q-\bar{\eta}P)$.

heavy-flavour meson

It's more convinient to calculate the moments $< x^n >$:

$$< x^n > = \int dx\, x^n \phi(x) \tag{8}$$

$$\begin{aligned} & f_{PS} < x^n > \\ = \;& \frac{n_\mu}{n\cdot P} tr_{CD} Z_2 \int_{dq}^{\zeta} (\frac{n\cdot q_+}{n\cdot P})^n \gamma_5 \gamma_\mu \chi_5(q;P)\,, \end{aligned} \tag{9}$$

$$\begin{aligned} & f_V^{\perp} < x^n > \\ = \;& \frac{\eta^{\lambda\mu} P_\nu}{P^2} tr_{CD} Z_2 \int_{dq}^{\zeta} (\frac{n\cdot q_+}{n\cdot P})^n \sigma_{\mu\nu} \chi_\lambda(q;P)\,, \end{aligned} \tag{10}$$

$$\begin{aligned} & f_V \frac{-(n\cdot P)^2}{P^2} < x^n > \\ = \;& tr_{CD} Z_2 \int_{dq}^{\zeta} (\frac{n\cdot q_+}{n\cdot P})^n n\cdot\gamma n_\lambda \chi_\lambda(q;P)\,, \end{aligned} \tag{11}$$

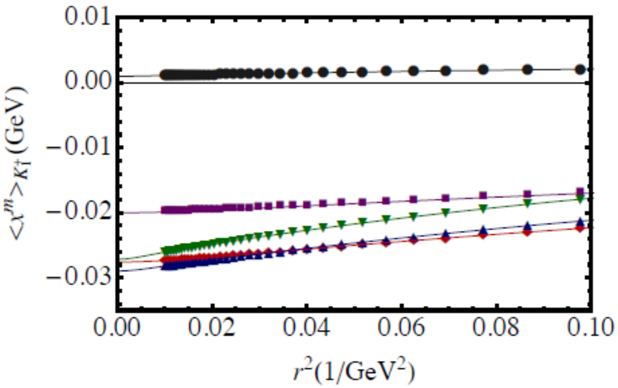
leading twist PDAs

The moments contain the factor $\left(\frac{n \cdot q_+}{n \cdot P}\right)$, the solution of DSEs in Euclidean space cannot be directly used because of this oscillator.

Here we added a factor $1/(1+k^2r^2)^m$ *in the integration of* d^4k *for each moment* $< 2x-1 >^{2m}$

- compute moments as a function of r
- extrapolate to r=0

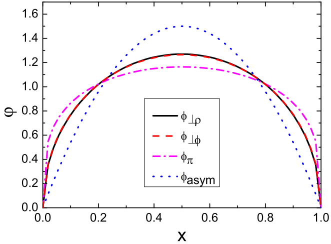
Figure: Extrapolation of moments (Phys. Rev. D 93, 114033,2016)

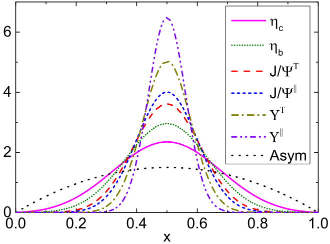


The moments can be obtained directly from the numerical data without other inputs, and then we can fit the PDA with Gegenbauer polynomials.

critical mass scale

We obtain the PDAs after fitting the moments.





- For the same quark mass, the different Lorentz structure affect meson's amplitudes.
- As the quark goes heavier, the PDAs tend to be δ function.

critical mass scale

PDAs for light meson:

- broader than the asymptotic form $\phi^{asy}(x) = 6x(1-x)$;
- the broadest shape of PDA, $\phi(x) = constant$, means the meson is point-like.

PDAs for heavy meson:

- narrower than the asymptotic form $\phi^{asy}(x) = 6x(1-x)$;
- the narrowest shape of PDA, $\phi(x) = \delta(x - 1/2)$, means the meson is like a two-static-particle system.

critical mass scale

There must exit a critical mass at which $\phi(x) = \phi^{asy}(x)$

Table: current quark mass at which the PDA is the asymptotic form (PLB, 11.075, 2015)

$D\omega$	$(0.87\ \text{GeV})^3$	$(0.55\ \text{GeV})^3$
φ_{PS}	0.19 GeV	0.15 GeV
$\varphi_{V,\perp}$	0.13 GeV	0.13 GeV
$\varphi_{V,\parallel}$	0.11 GeV	0.12 GeV

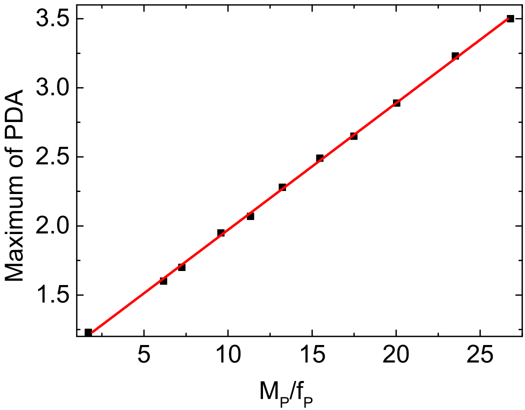
Lies in the neighborhood of the s-quark current-mass defining a transition boundary for internal hadron dynamics.

Noticing that, as the quark mass increases, the maximum of PDA becomes larger.

The maximum of PDA can characterise the shape of PDA and thus, characterise the inside structure of meson.

Focusing on this maximum, we can draw a trajectory as the current quark mass changes, and find the relation between this maximum and the physical observables like decay constant and meson mass.

For the pseudoscalar meson:

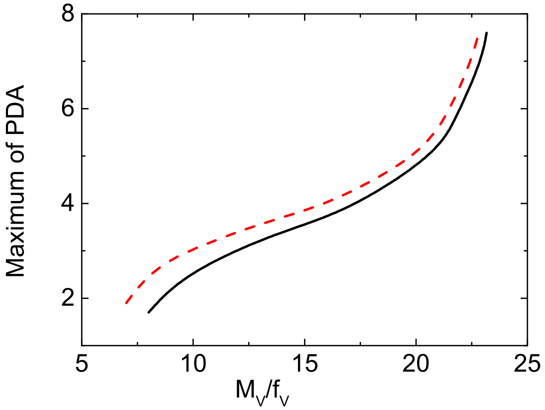


Linear relation for pseudoscalar meson between the maximum and the ratio of M_P/f_P.

The details of the linear relation:

- Scaling behaviour between pseudoscalar meson's inside structure and its respective global properties.
- The range is from chiral limit up to $m_q = 10$GeV.
- Might be some symmetry pattern to preserve this relation.
- Fitting formula for the trajectory: $\phi(x = 1/2) = 0.0918M/f + 1.07$
- In bare vertex approximation, we put too much strength on the coupling, thus the massless pseudoscalar meson becomes too compact.

For vector mesons:



No linear relation for vector meson.

Comparison between pseudoscalar meson and vector meson:

- At large quark mass, M_V/f_V tends to be constant, indicating that the Coulomb-like potential in vector mesons.
- Difference interaction behaviour between pseudoscalar meson and vector meson even at large quark mass:
  1. The meson mass is almost the same between pseudoscalar meson and vector meson,$M_p = M_V = 2M_q$
  2. A quantitative discrepancy for decay constants and also the difference between PDAs.

parametrization independence

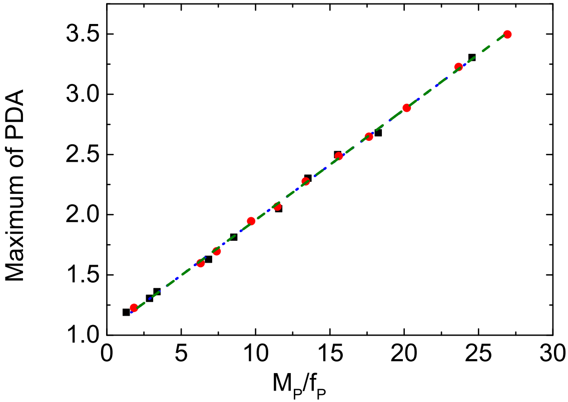
To compute the Bathe-Salpeter wave function, we have employed the gluon model as following:

$$\mathcal{G}_{IR}(k^2) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2} + \frac{4\pi}{k^2}\alpha_{pQCD}(k^2). \tag{12}$$

The interaction strength here we employed can be parametrized as $D\omega$.

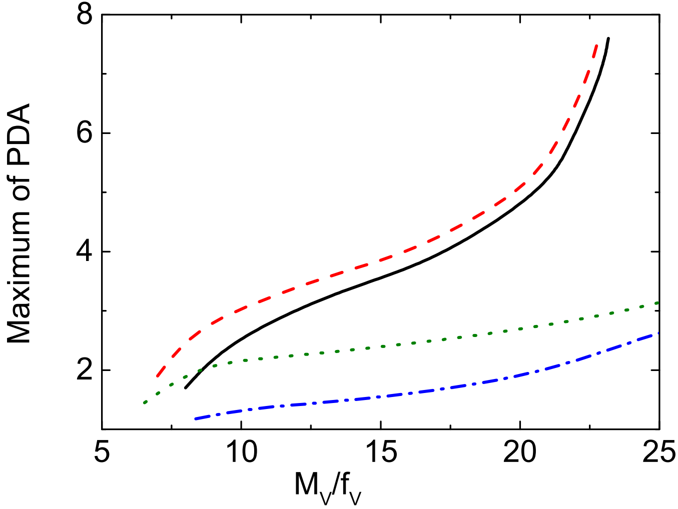
Changing the interaction details by changing ω but keeping the combination $D\omega$ unchanged.

For pseudoscalar meson:



Same linear relation independent of interaction details.

For vector meson:



What we obtained:

- We computed the parton distribution amplitudes of mesons and found a critical mass near above *s* quark mass.
- We found a linear relation between the maximum of pseudoscalar PDA and M_p/f_P, no such relation in vector meson case which reveals the different interaction pattern in these meson even at very large quark mass.
- The linear relation is insensitive to the interaction details in pseudoscalar.

Thank you