The ambiguity of confinement

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A non-trivial example

- Consider the following theory

$$L = -\frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} + m^2 W^a_\mu W_a^\mu$$

$$+ \left(D^{ij}_\mu h \right)^+ D^\mu_{ij} h + \lambda \left(h^2 - v^2 \right)^2 + \ln \left(h \right)$$

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Non-trivial tree-level structure
defects or large λ

Well-defined theory, can be simulated on the lattice

[Jersak et al.'85, Evertz et al.'86]

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  - But particles are elementary
  - Integration variables of the path integral

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[Fröhlich et al.'81]

- Theory can be covariantized
- Just a gauge-fundamental Higgs theory

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Detailed correspondence

[Maas '17]

- States do have a one-to-one correspondence in both theories
- Elementary states in ungauged theories can be described by gauge-invariant states in the gauge theory
- Confinement equates to gauge-invariance
- Different substructure mapped to dominance of different composite operators in the gauged theory
    - Not always in one-to-one correspondence with the number of gauged fields
    - No simple interpretation as 'constituents'

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  - But confinement is meaningless in the ungauged theory – there are no substructure particles
- But really are only auxiliary degrees of freedom for a simple tree-level form
- Apparent substructure in the ungauged form is an emergent feature
  - Essentially a dressing of the bare states

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  - Note: Gauge-invariance implies positivity, but positivity not necessarily implies being physical [Seiler ‘82]

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Discussed possibilities

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No generally satisfied criterion

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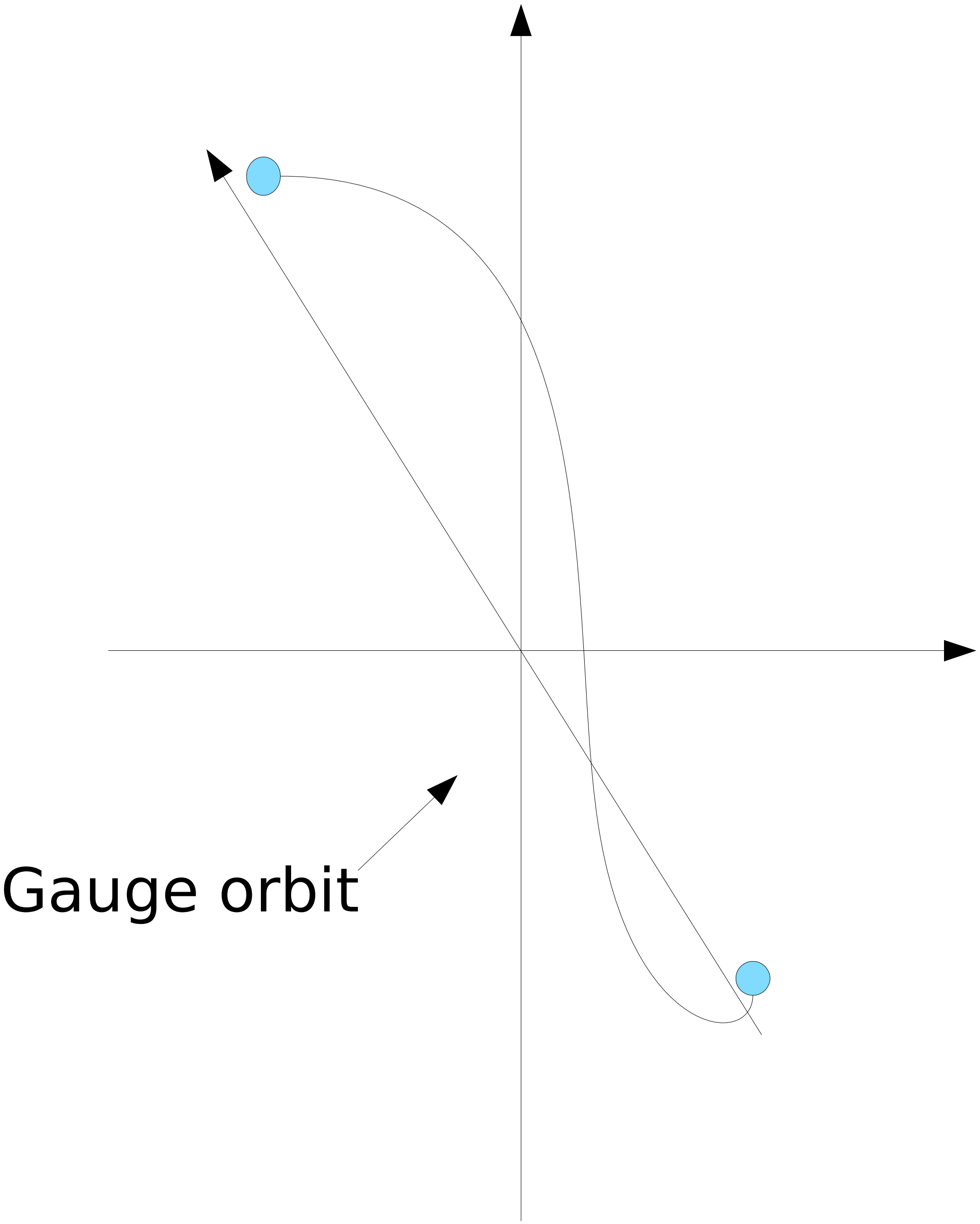
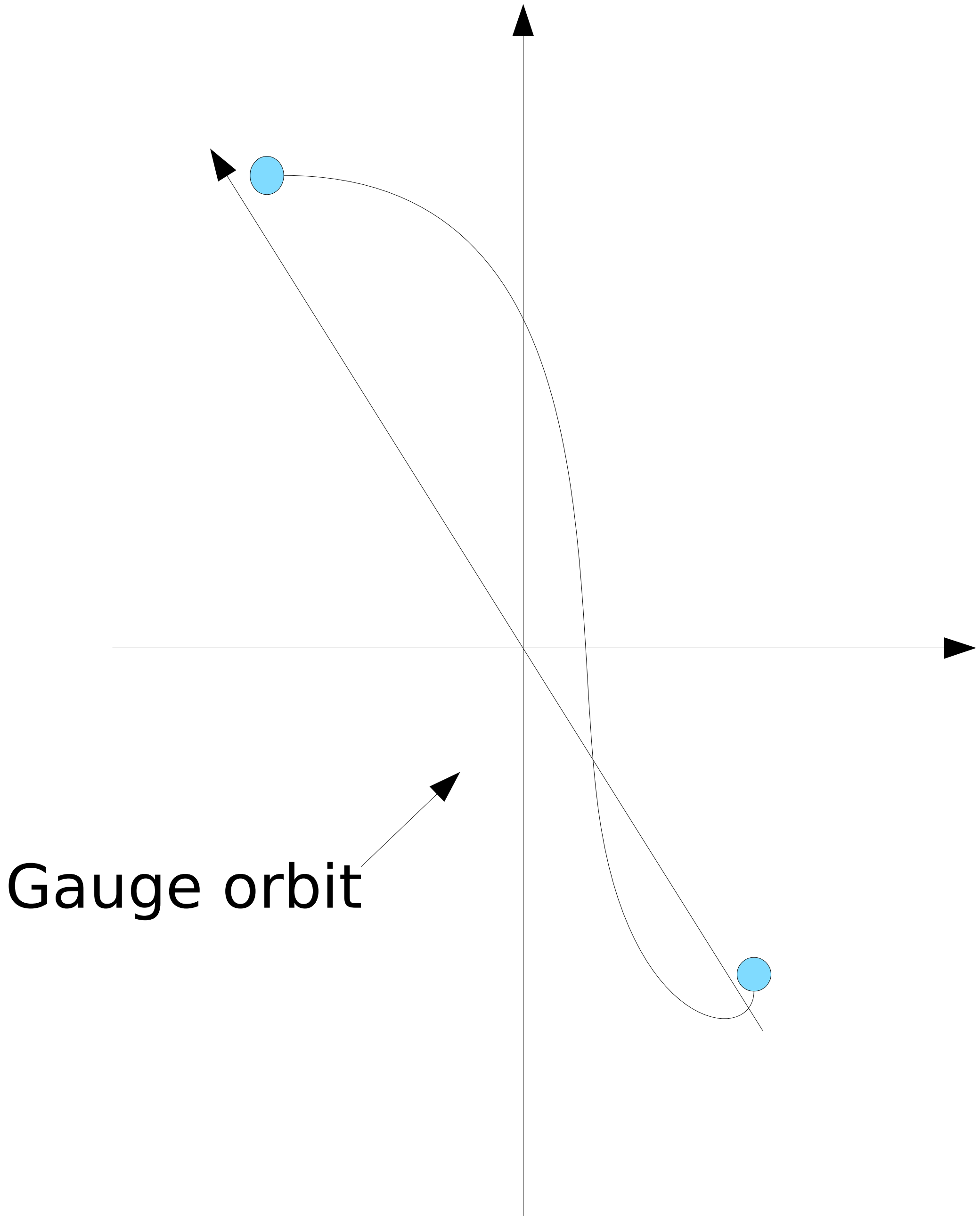
- Gauge symmetry is the existence of equivalent field configurations along gauge orbits



[Maas '11,'12]

What about BRST and the state space?

- Gauge-fixing is the introduction of a non-flat weight along a gauge orbit, such that all copy-independent quantities remain unchanged



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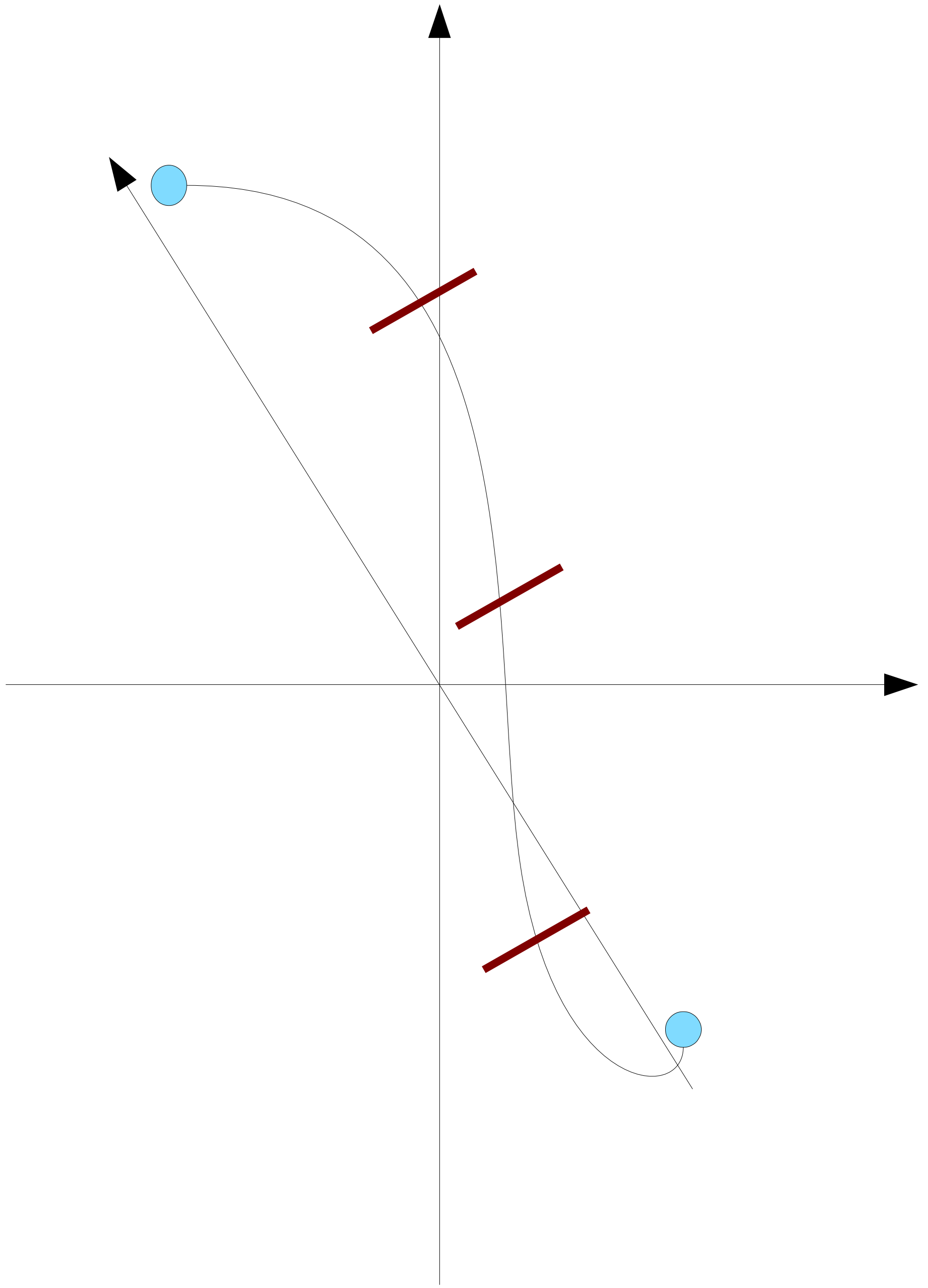
  - Averaging over all or some copies



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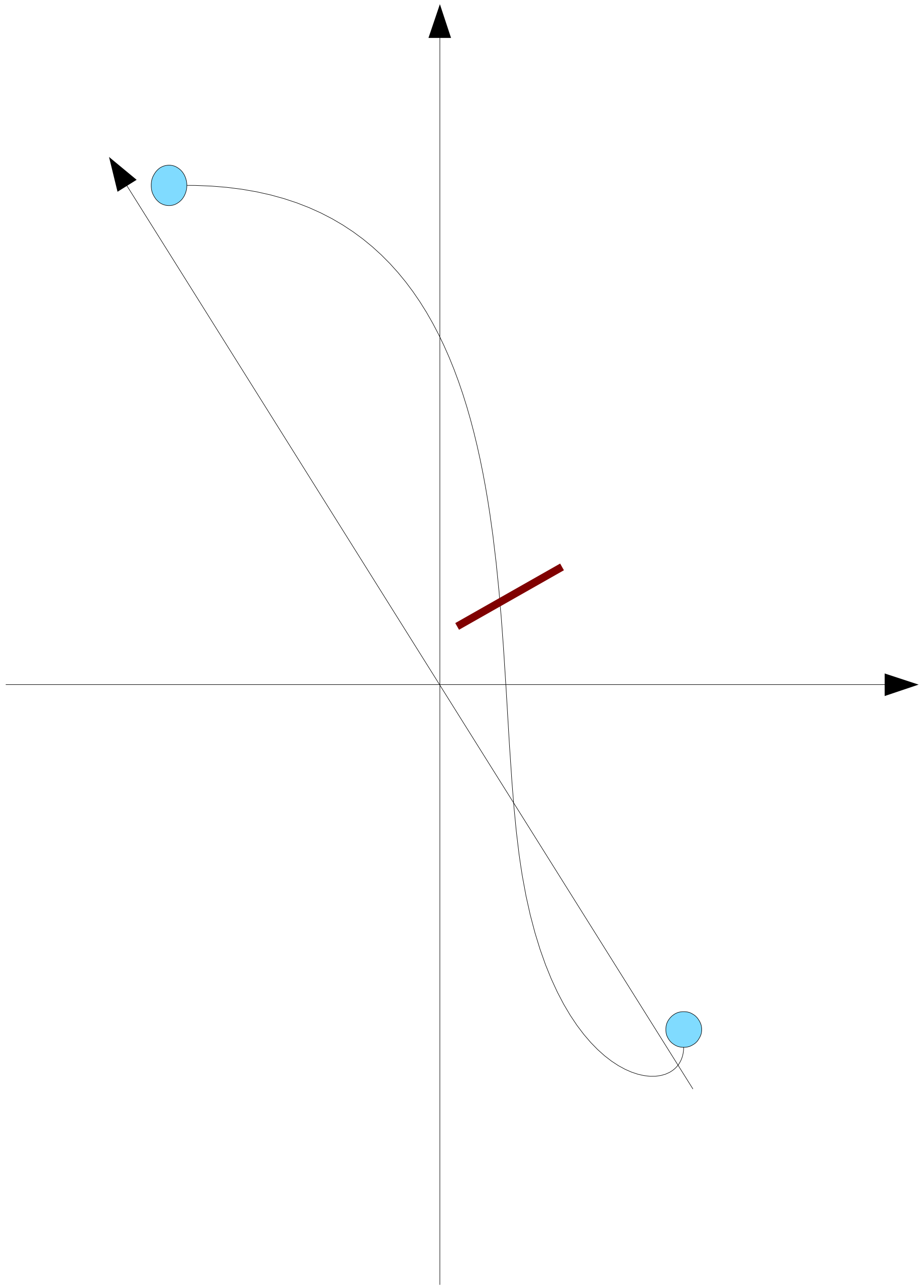
- Two possibilities
  - Averaging over all or some copies
  - Single out one copy as representative
    - Limiting case of an average



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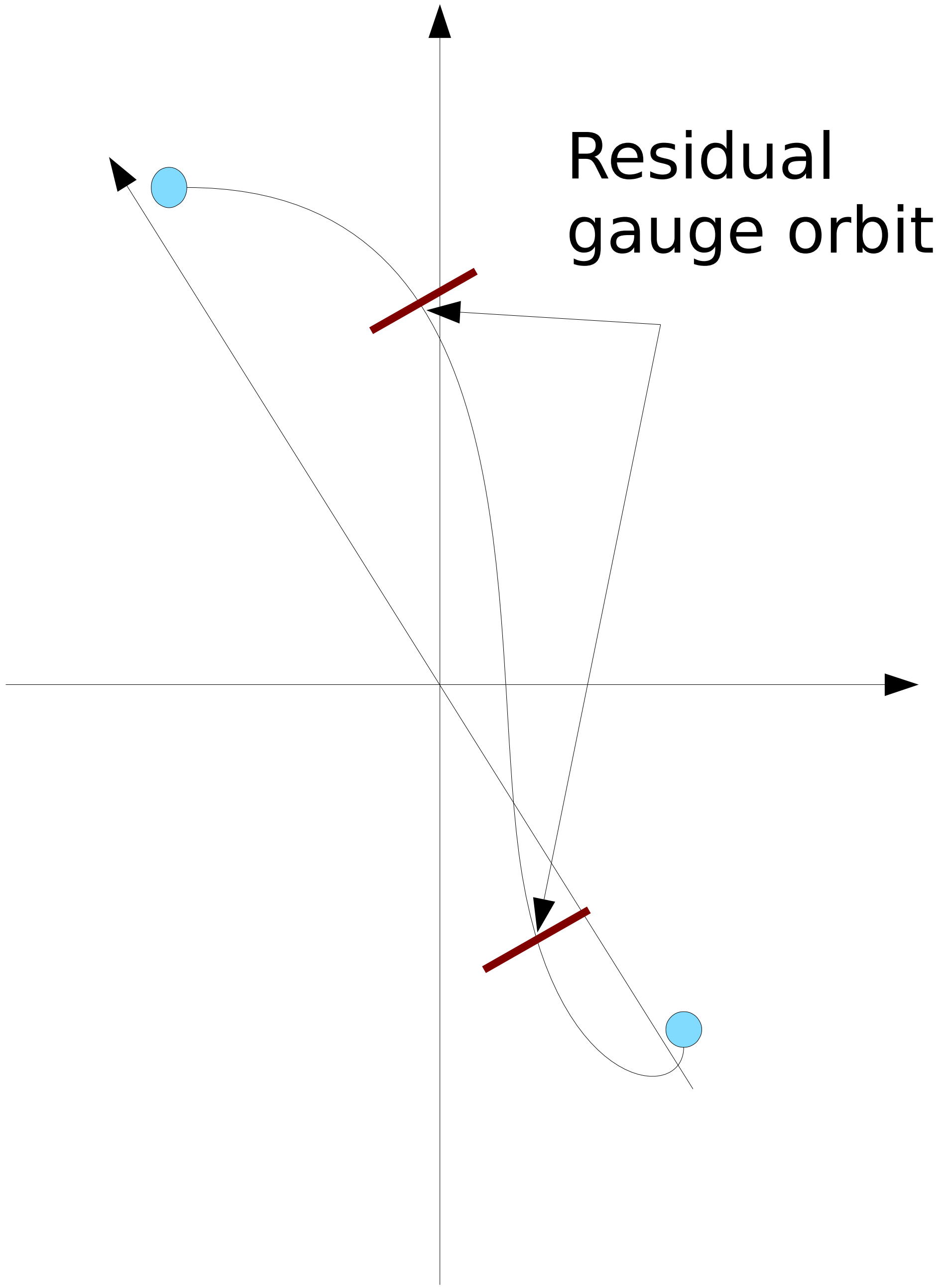
  - May be a single one



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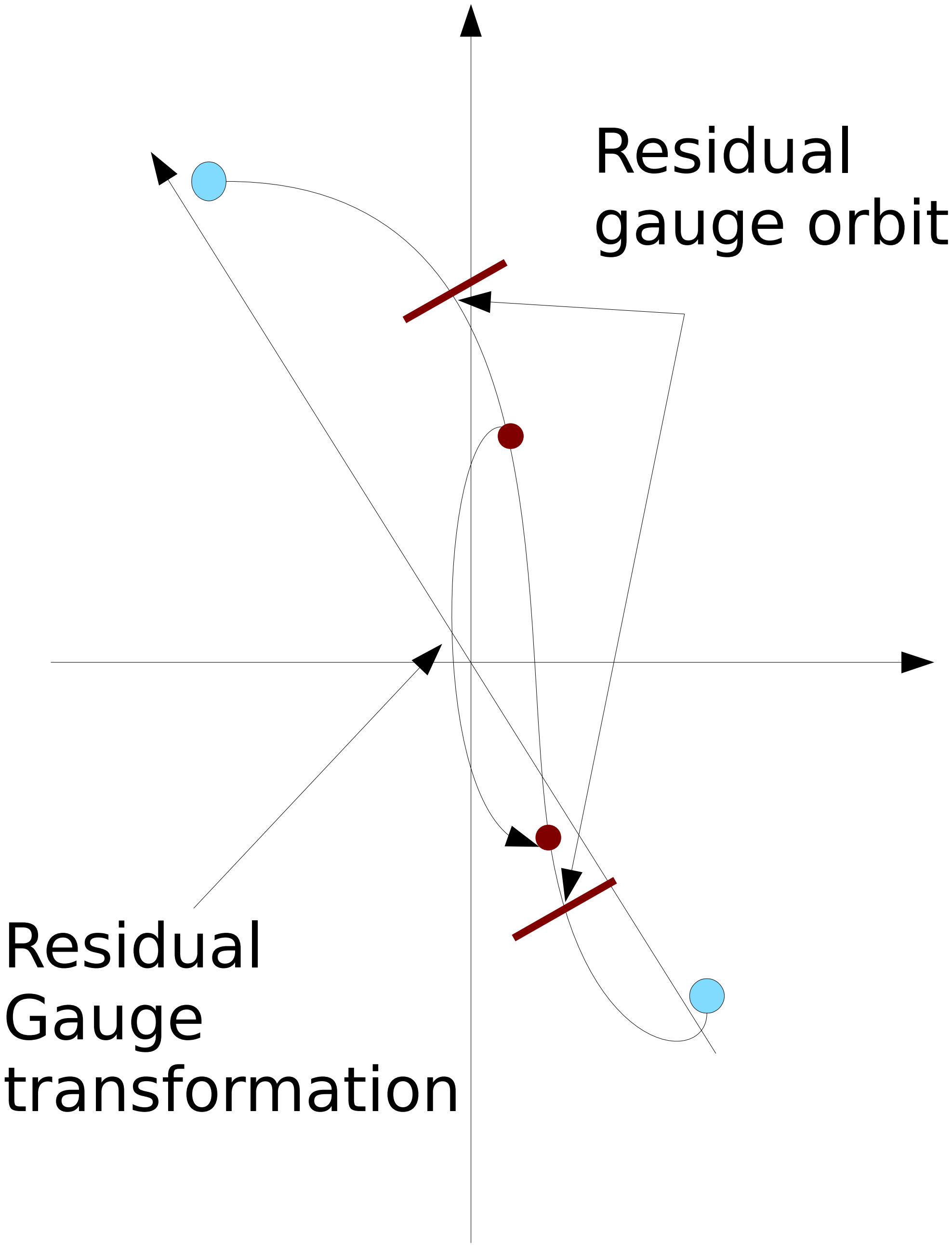

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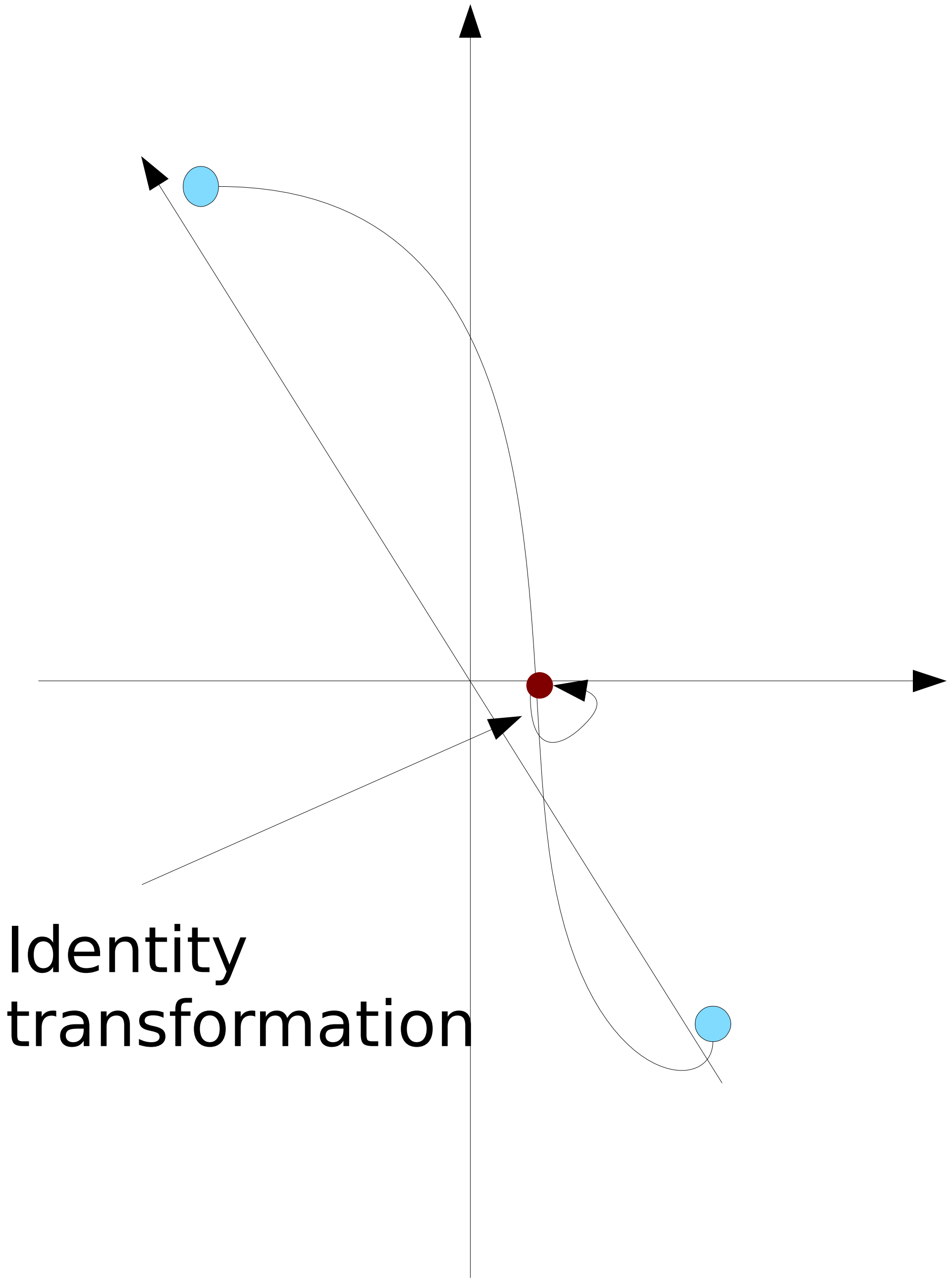

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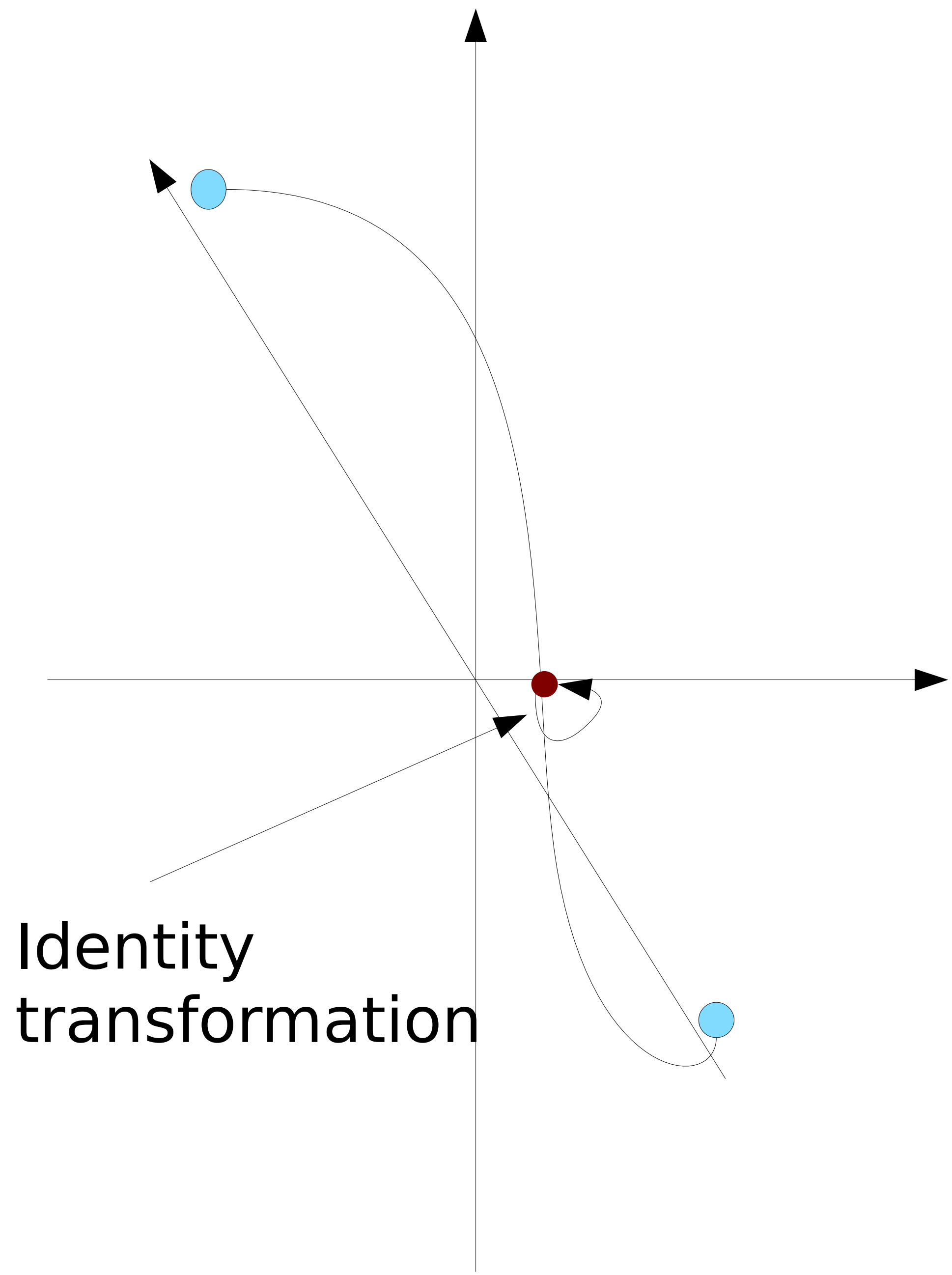
  - May become non-trivial by introduction of ghost fields
  - Gauge field is invariant under ghost transformations
  - E.g. perturbative Landau gauge



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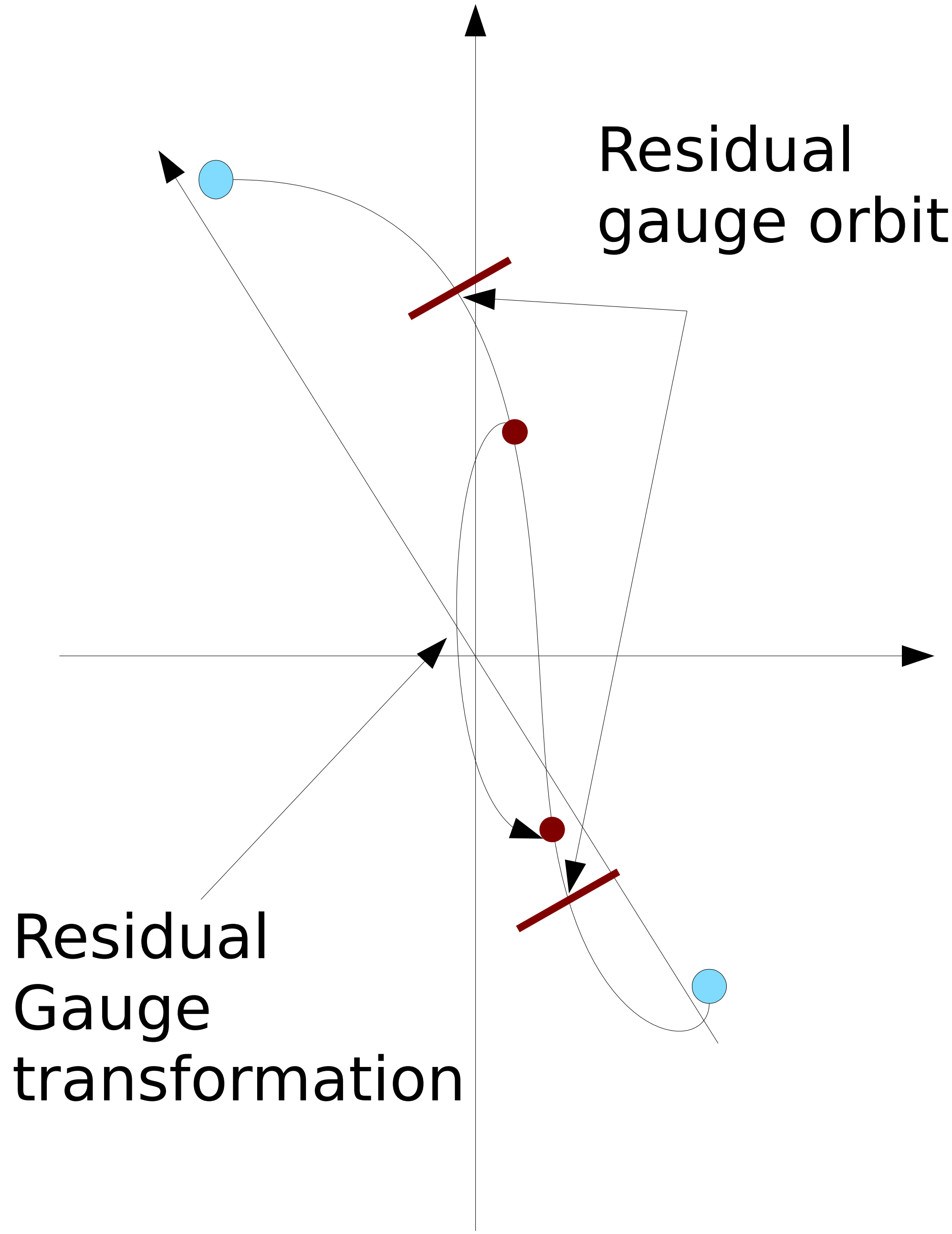

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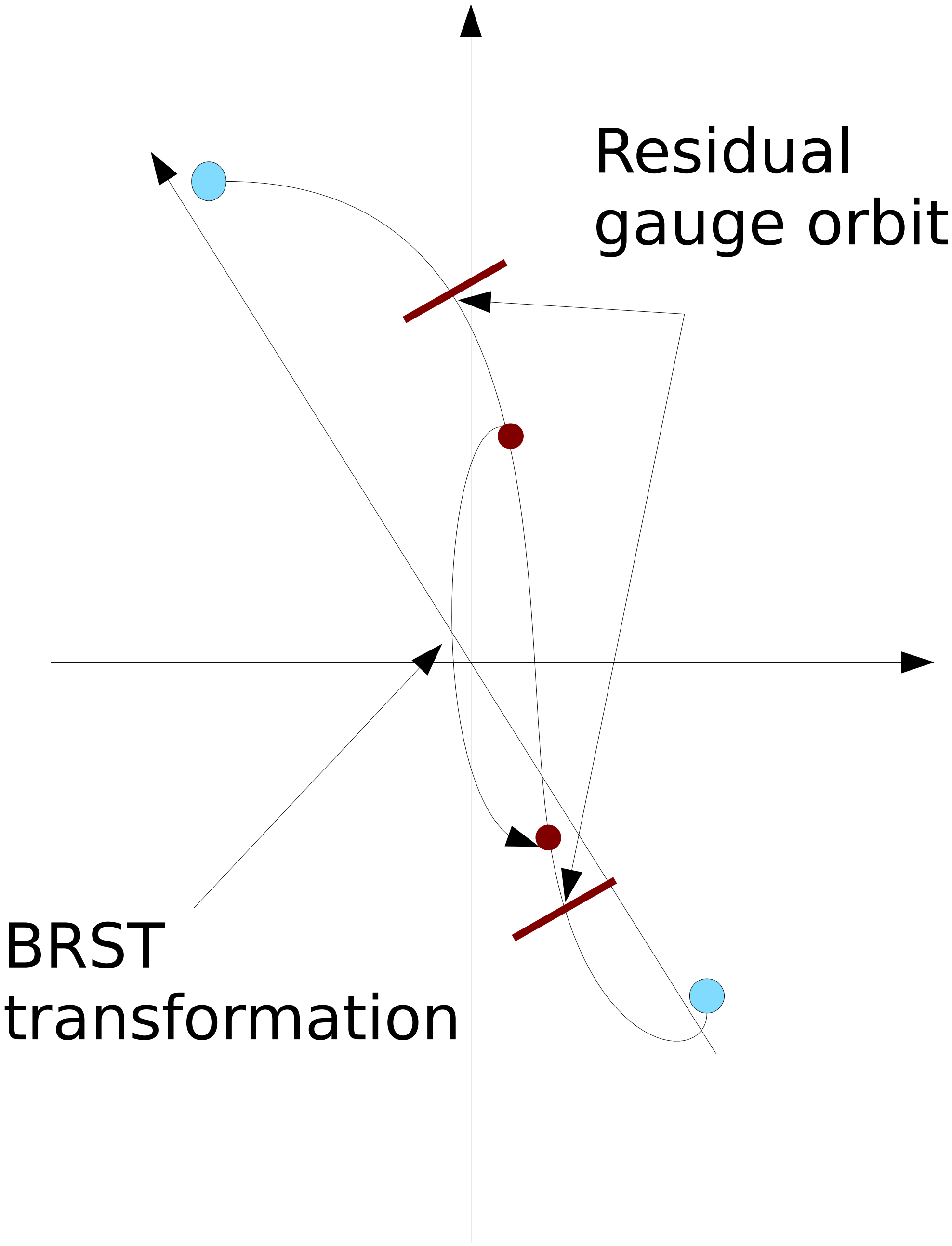
  - Introduce ghost fields
    - Auxilliary fields!
  - Symmetry is BRST
  - Still only gauge transformations for the gauge field



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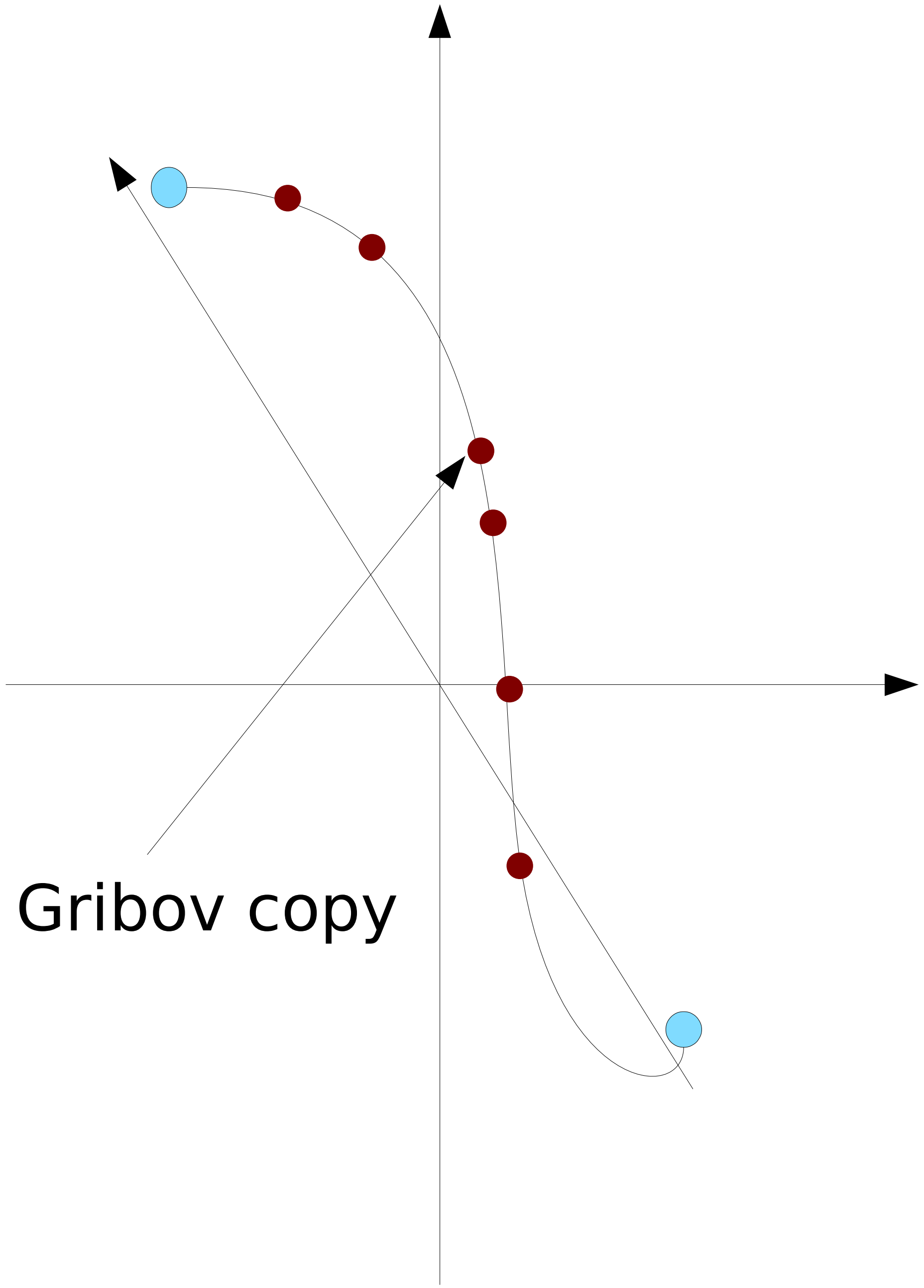

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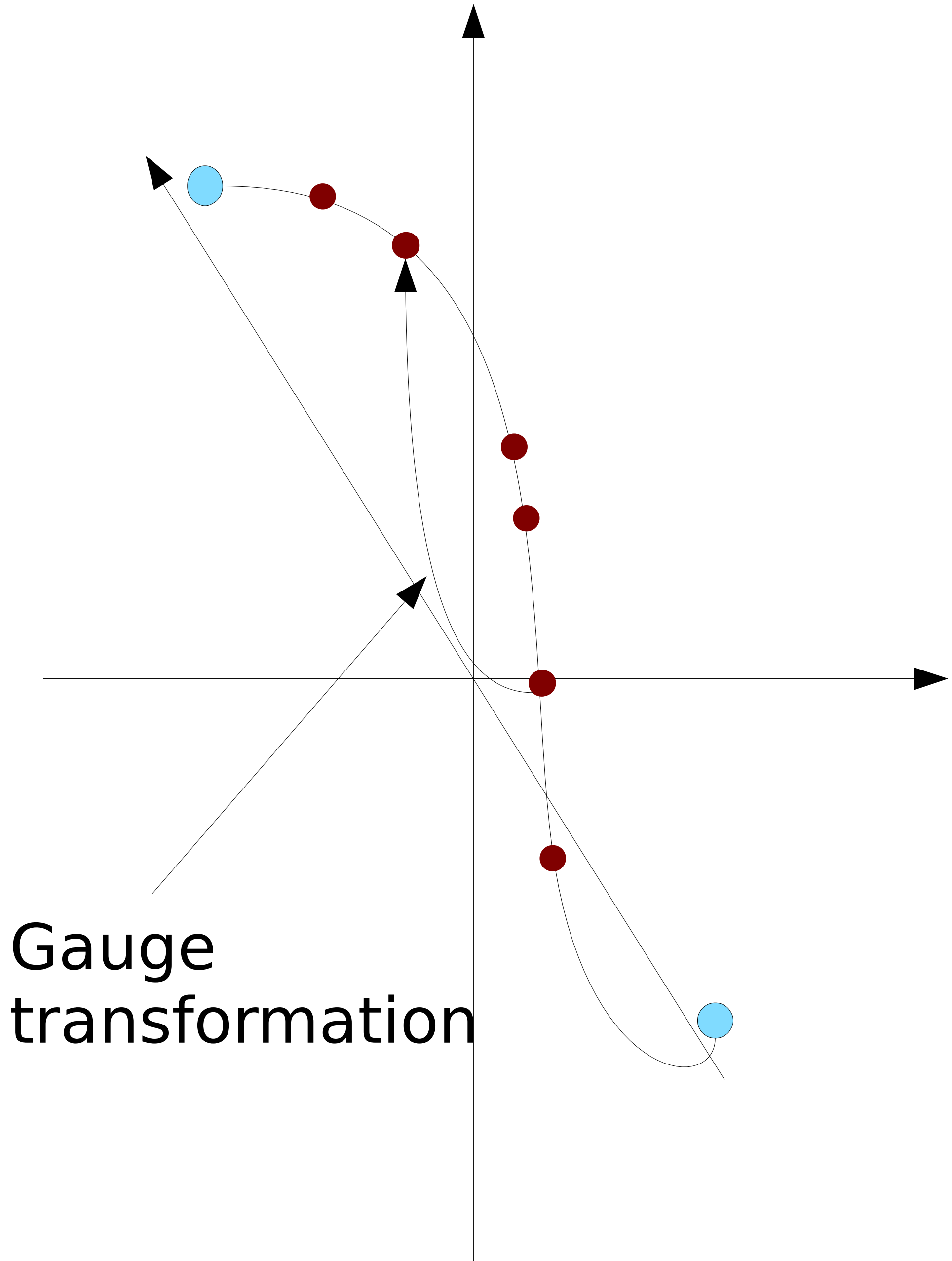

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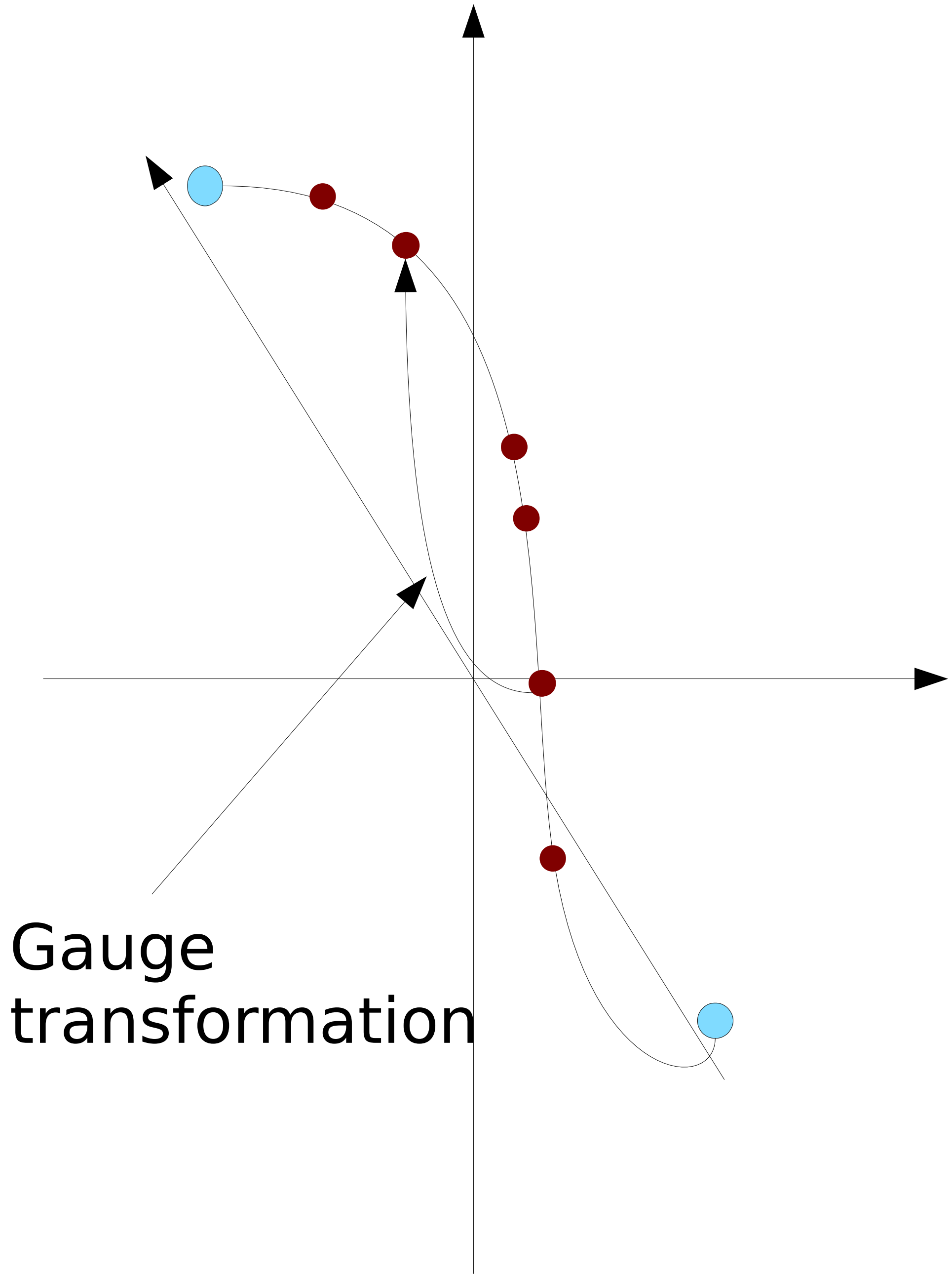

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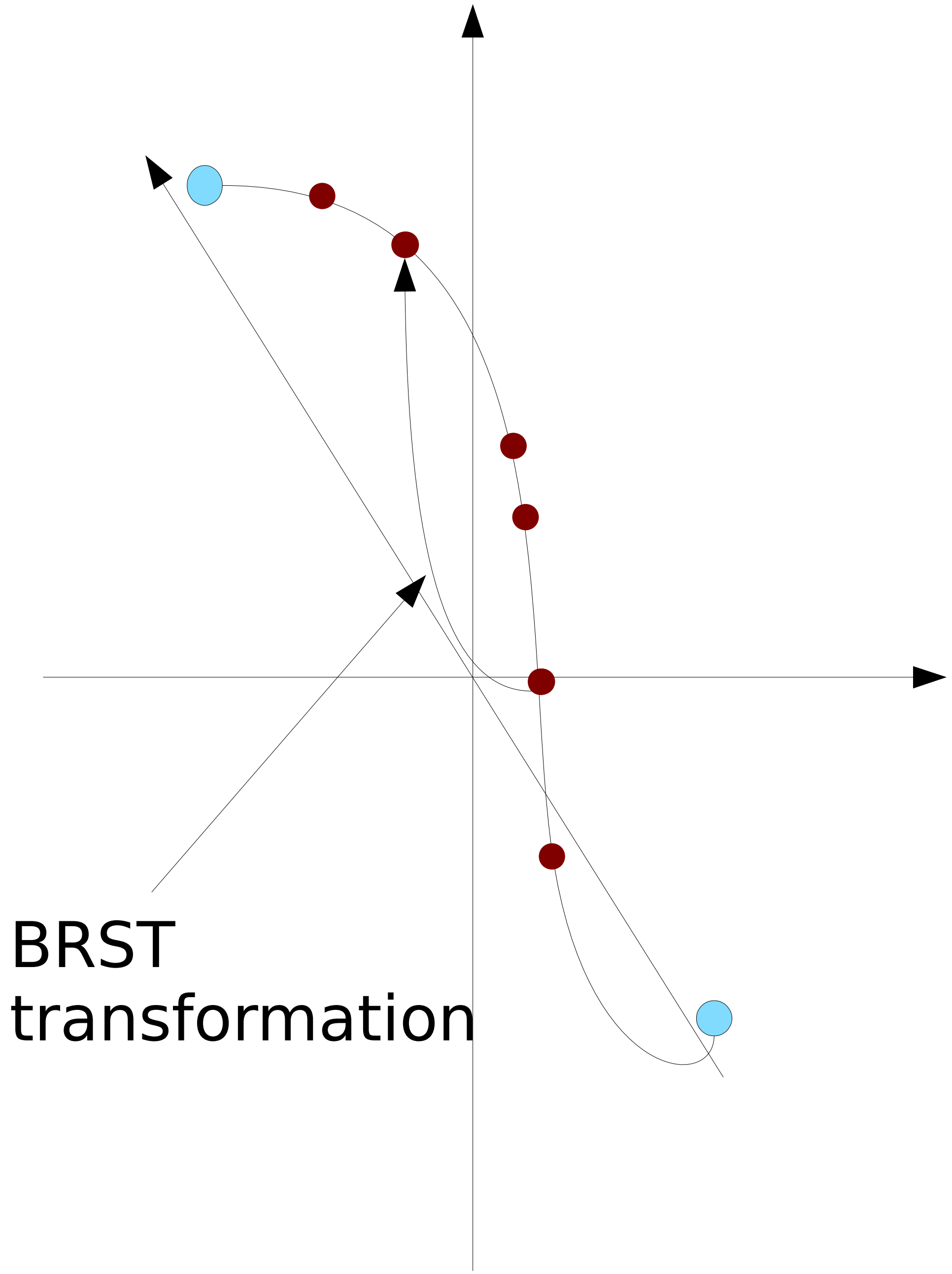

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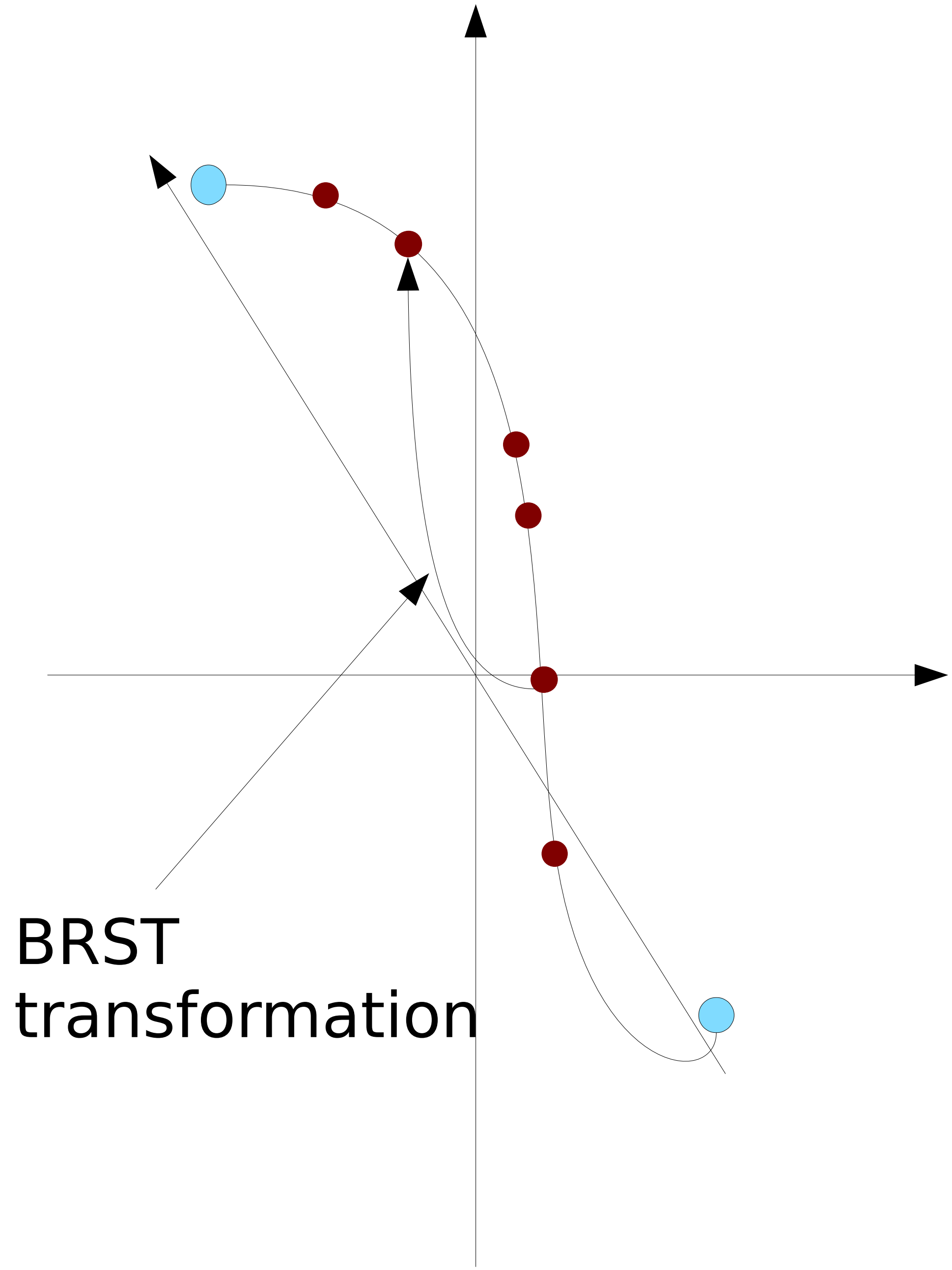

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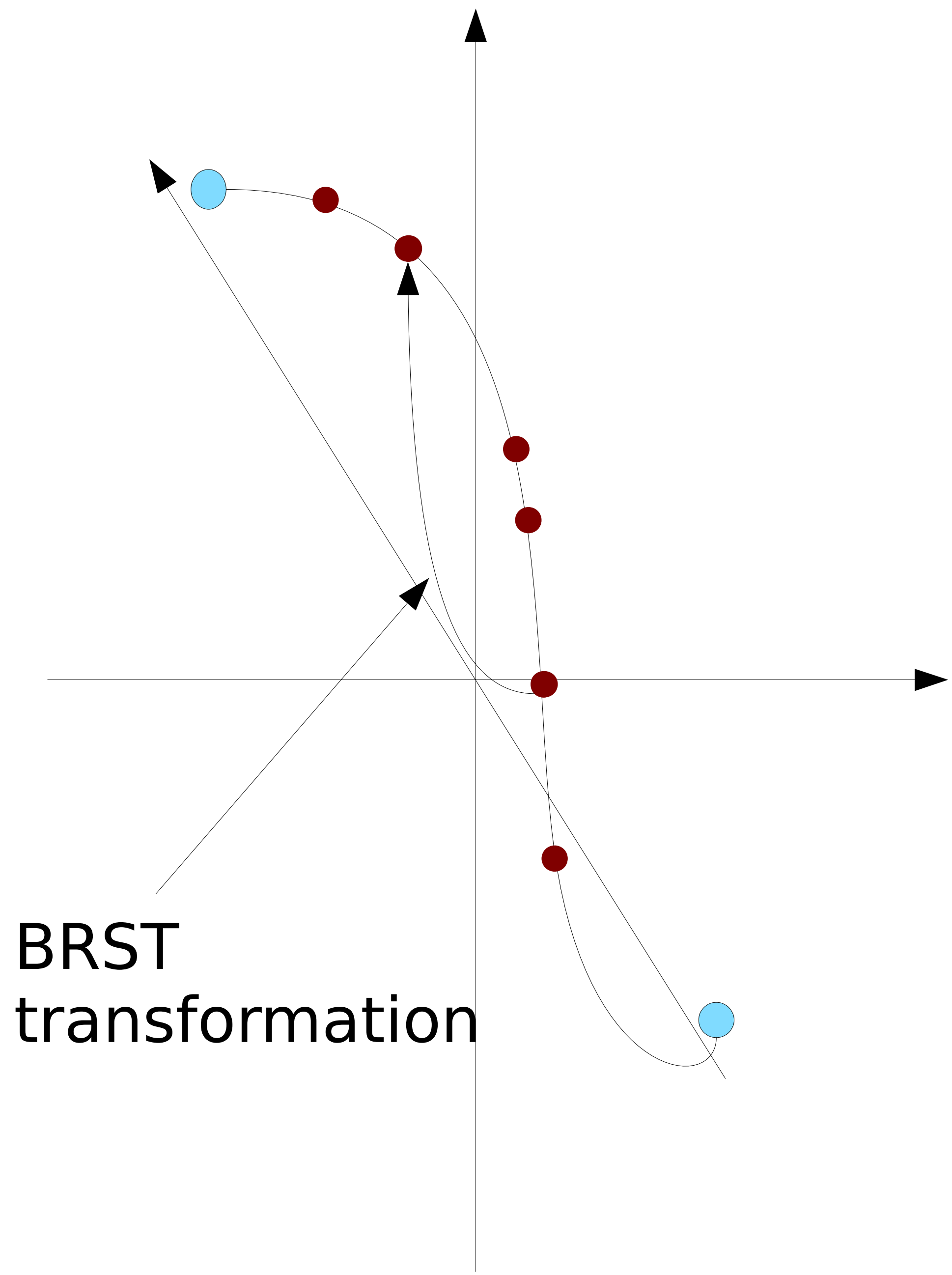

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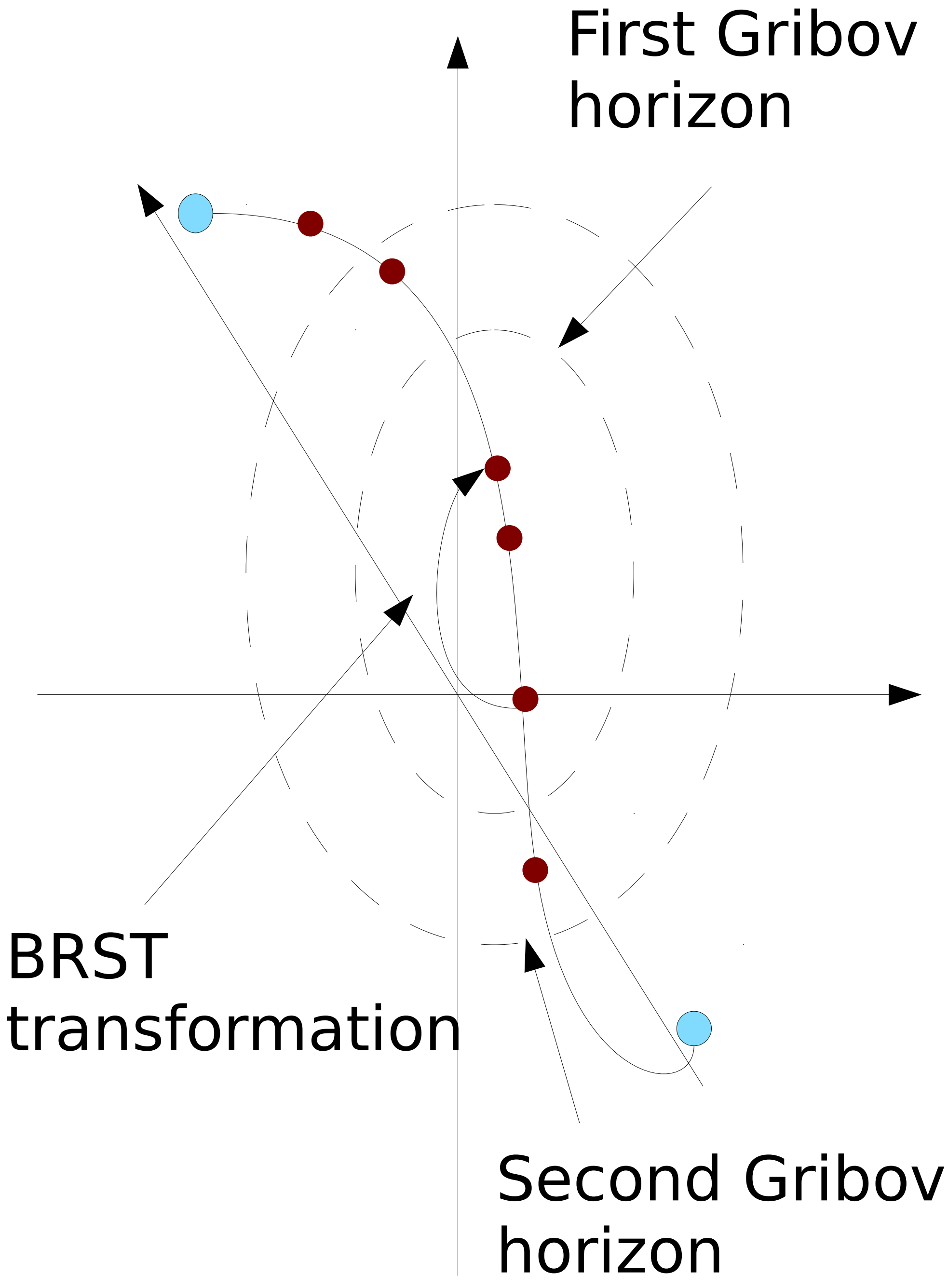
- True for full gauges



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- Conceptually more demanding to create than just gauge invariance
  - But in actual calculations potentially simpler

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- Quantities like the Wilson string tension remain non-trivial, interesting quantities
  - E.g. impact for Regge trajectories

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  - Only separates physical from auxiliary degrees of freedom
  - Like classical mechanics [van Holten '05]
- Quantities like the Wilson string tension remain non-trivial, interesting quantities
  - E.g. impact for Regge trajectories
- But no longer baggage of inexplicable questions attached