Heavy quark QCD phase diagram at two-loop order in perturbation theory

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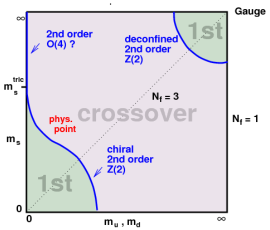
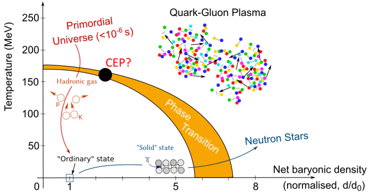
Motivation





Several approaches on the market:

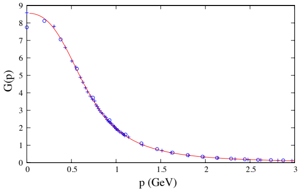
- Lattice QCD [de Forcrand, Philipsen, Rodriguez-Quintero, Mendes, ...]
- Dyson Schwinger Equations [Alkofer, Fischer, Huber, ...]
- Functional Renormalization Group [Pawlowski, Mitter, Schaefer...]
- Variational Approach [Reinhardt, Quandt, ...]
- Gribov-Zwanziger Action [Dudal, Oliveira, Zwanziger...]
- Matrix-, QM-, NJL-Model,... [Pisarski, Dumitru, Schaffner-B., Stiele, ...]
- Curci-Ferrari Model [Reinosa, Serreau, Tissier, Wschebor, ...]

Curci-Ferrari and gluon mass term

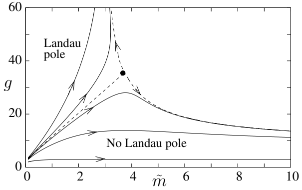
$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 + \bar\psi(\not{D} + M + \mu\gamma_0)\psi \right\} + S_{FP} + \int_x \left\{ \frac{1}{2} m^2 (A^a_\mu)^2 \right\}$$

This gluon mass term can be motivated in several ways

- phenomenologically from lattice data of the Landau gauge gluon propagator saturating in the IR
- Residual ambiguity after non-complete gauge-fixing in Fadeev-Popov procedure due to presence of Gribov copies, see talk by Mathieu Tissier!



one-loop gluon propagator against lattice data,
from [Tissier, Wschebor (2011)]
[Bogolubsky et al. (2009), Dudal, Oliveira, Vandersickel (2010)]



YM one-loop RG flow,
from [Serreau, Tissier (2012)]

Polyakov loops as order parameters

At the YM point, a relevant order parameter for the deconfinement transition is the (anti-)Polyakov loop. It is related to the free energy F_q necessary to bring a quark into a "bath" of gluons.

$$\ell \equiv \frac{1}{3}\mathrm{tr}\left\langle P\exp\left(ig\int_0^\beta d\tau A_0^a t^a\right)\right\rangle \sim e^{-\beta F_q} \qquad \bar{\ell} \sim e^{-\beta F_{\bar{q}}}$$

Hence

$$\ell = 0 \leftrightarrow F_q = \infty \leftrightarrow \text{confinement} \qquad \ell \neq 0 \leftrightarrow F_q < \infty \leftrightarrow \text{deconfinement}$$

Introducing quarks, center symmetry is explicitly broken. For heavy quarks, this breaking is "soft", thus:

$$\ell \approx 0 \leftrightarrow F_q \approx \infty \leftrightarrow \text{confinement} \qquad \ell \not\approx 0 \leftrightarrow F_q < \infty \leftrightarrow \text{deconfinement}$$

→ It is thus very important to work in a choice of gauge which doesn't explicitly "strongly" break center symmetry (any more)!

Landau-DeWitt gauge [Braun, Pawlowski, Gies (2010)]

$$A_\mu^a = \bar{A}_\mu^a + a_\mu^a$$

In practice, at each temperature, the background field $\bar{A}_\mu^a$ is chosen such that the expectation value $\langle a_\mu^a \rangle$ vanishes in the limit of vanishing sources.

This corresponds to finding the absolute minimum of $\tilde{\Gamma}[\bar{A}] \equiv \Gamma[\bar{A}, \langle a \rangle = 0]$, where $\Gamma[\bar{A}, \langle a \rangle]$ is the effective action for $\langle a \rangle$ in the presence of $\bar{A}$.

Seek the minima in the subspace of configurations $\bar{A}$ that respect the symmetries of the system at finite temperature.
$\longrightarrow$ One restricts to temporal and homogenous backgrounds:

$$\bar{A}_\mu(\tau, \mathbf{x}) = \bar{A}_0 \delta_{\mu 0}$$

$\longrightarrow$ functional $\tilde{\Gamma}[\bar{A}]$ reduces to an effective potential $V(\bar{A}_0)$ for the constant matrix field $\bar{A}_0$.

One can always rotate this matrix $\bar{A}_0$ into the Cartan subalgebra:

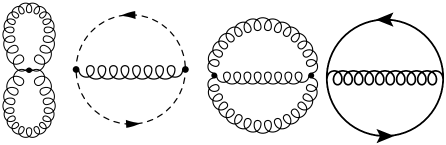
$$\beta g \bar{A}_0 = r_3 \frac{\lambda_3}{2} + r_8 \frac{\lambda_8}{2}$$

Then $V(\bar{A}_0)$ reduces to a function of 2 components $V(r_3, r_8)$.

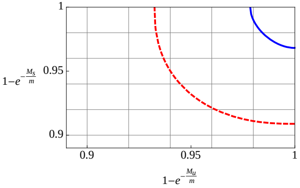
	r_3	r_8
$\mu = 0$	$\mathbb{R}$	0
$\mu \in i\,\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
$\mu \in \mathbb{R}$	$\mathbb{R}$	$i\,\mathbb{R}$

Two-loop Expansion

$$
\begin{aligned}
V(r_3, r_8) \quad = \quad & -\mathrm{Tr}\,\mathrm{Ln}\left(\not{\partial} + M + \mu\gamma_0 - ig\gamma_0 \bar{A}^k t^k\right) \\
& + \frac{3}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2 + m^2\right) - \frac{1}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2\right) \\
& +
\end{aligned}
$$



Vanishing chemical potential



$$R_{N_f} \equiv \frac{M_c(N_f)}{T_c(N_f)}$$

$$\mathcal{O}(1): \quad M_{\text{bare}} = M_{\text{ren.}}$$
$$\mathcal{O}(g^2): \quad M_{\text{bare}} = Z_M \, M_{\text{ren.}}$$

$\longrightarrow$ hard to compare between different approaches!

However, Z_M is independent of N_f at $\mathcal{O}(g^2)$, and observing

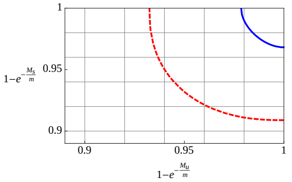
$$\frac{T_c(N_f = 3) - T_c(N_f = 1)}{T_c(N_f = 1)} \approx 0.2\%$$

allows for:

$$R_{N'_f}/R_{N_f} \approx M_c(N'_f)/M_c(N_f)$$

is scheme indep. & comparable to other approaches up to higher order corrections.

Vanishing chemical potential



$$R_{N_f} \equiv \frac{M_c(N_f)}{T_c(N_f)}$$

$$R_{N'_f}/R_{N_f} \approx M_c(N'_f)/M_c(N_f)$$

R_{N_f}	$N_f = 1$	$N_f = 2$	$N_f = 3$	R_2/R_1	R_3/R_1
1-loop [1]	6.74	7.59	8.07	1.13	1.20
2-loop	7.53	8.40	8.90	1.12	1.18
Lattice [2]	7.23	7.92	8.33	1.10	1.15
DSE [3]	1.42	1.83	2.04	1.29	1.43
Matrix [4]	8.04	8.85	9.33	1.10	1.16

$\longrightarrow$ The overall good agreement seems to suggest that the underlying dynamics is well-described within perturbation theory.

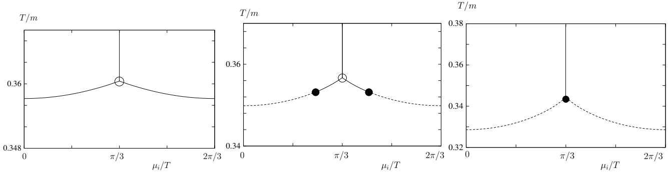
[1] U. Reinosa, J. Serreau, M. Tissier (2015)

[2] M. Fromm, J. Langelage, S. Lottini and O. Philipsen (2012)

[3] C. S. Fischer, J. Luecker and J. M. Pawlowski (2015)

[4] K. Kashiwa, R. D. Pisarski and V. V. Skokov (2012)

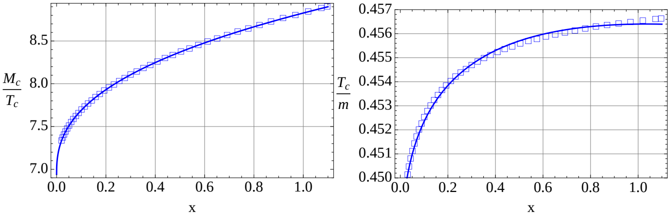
Imaginary chemical potential $\mu = i\mu_i$



The vicinity of the tricritical point is approximately described by the mean field scaling behavior

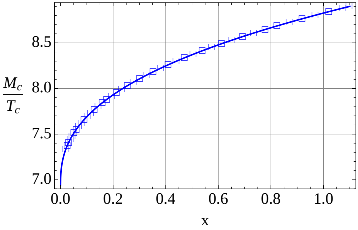
$$\frac{M_c(\mu_i)}{T_c(\mu_i)} = \frac{M_{\text{tric.}}}{T_{\text{tric.}}} + K\left[\left(\frac{\pi}{3}\right)^2 - \left(\frac{\mu_i}{T_c}\right)^2\right]^{\frac{2}{5}}$$

[de Forcrand, Philipsen (2010); Fischer, Luecker, Pawlowski (2015)]



$$x \equiv (\pi/3)^2 + (\mu/T_c)^2 = (\pi/3)^2 - (\mu_i/T_c)^2$$

Imaginary chemical potential $\mu = i\mu_i$



$$\frac{M_c}{T_c}(x) \approx 6.939 + 1.888\, x^{2/5}$$

$$\frac{M_c(N_f,\mu_i)}{M_c(N_f=1,\mu_i)} \approx \frac{R_{N_f}(\mu_i)}{R_1(\mu_i)}$$

at $\mu = \mu_i i = i\pi/3$

$R_{N_f}(\pi/3)$	$N_f = 1$	$N_f = 2$	$N_f = 3$	R_2/R_1	R_3/R_1
1-loop [1]	4.74	5.63	6.15	1.19	1.30
2-loop	5.47	6.41	6.94	1.17	1.27
Lattice [2]	5.56	6.25	6.66	1.12	1.20
DSE [3]	0.41	0.85	1.11	2.07	2.70
Matrix [4]	5.00	5.90	6.40	1.18	1.28

[1] Reinosa et al. (2015), [2] Fromm et al. (2012), [3] Fischer et al. (2015), [4] Kashiwa et al.(2012)

Real chemical potential

- $V(r_3, r_8) \in \mathbb{C}$
- $V(\ell, \bar{\ell}) \in \mathbb{C}$ $\quad\longrightarrow\quad$ physical point $\hat{\neq}$ absolute minimum

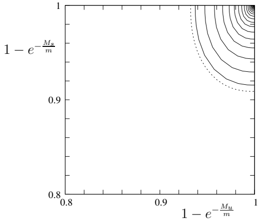
Common fix: $V = \mathrm{Re}\, V + \cancel{i\,\mathrm{Im}\, V}$ $\rightarrow$ No explicit breaking of charge conjugation, ie $r_8 \equiv 0$ or $q \hat{=} \bar{q}$!

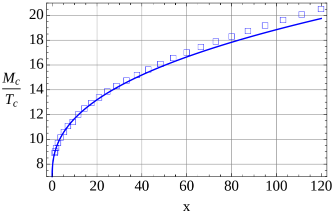
Instead, we can continue the r_8-component via $\boxed{r_8 \rightarrow i r_8}$

$\hat{\equiv}\ \ell \& \bar{\ell} \in \mathbb{R}$ and indep. [Dumitru, Pisarski, Zschiesche (2005)]

Then

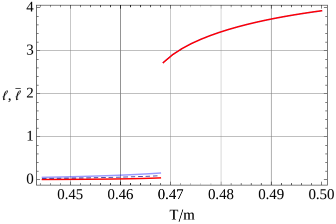
- $V(r_3, r_8) \in \mathbb{C} \longrightarrow V(r_3, i r_8) \in \mathbb{R}$
- min $V(r_3, r_8) \longrightarrow$ saddle point in $\mathbb{R} \times i\mathbb{R}$
- residual ambiguity: Wich saddle $\hat{=}$ physical point?
  $\longrightarrow$ Choose convention to pick the lowest saddle! (well-motivated around $\mu \approx 0$)

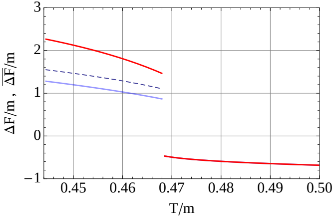


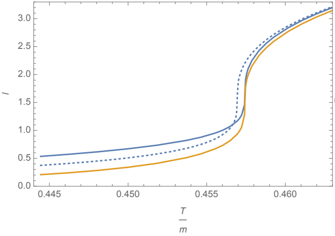


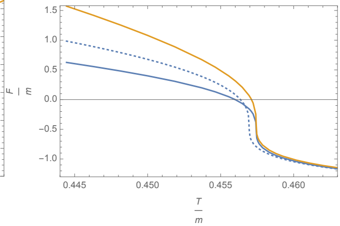
$$\frac{M_c}{T_c}(x) \approx 6.939 + 1.888\, x^{2/5} \qquad x \equiv (\pi/3)^2 + (\mu/T_c)^2 = (\pi/3)^2 - (\mu_i/T_c)^2$$

Explicit breaking of charge-conjugation in Polyakov loops

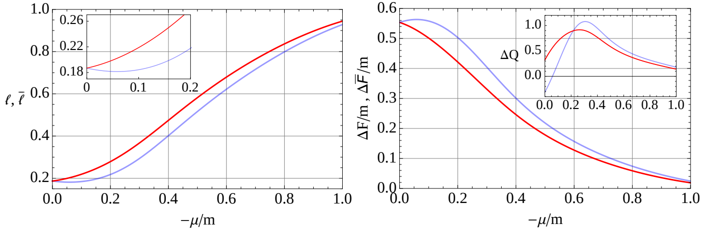








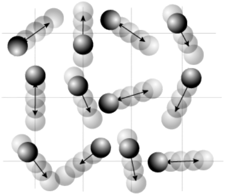
$\ell_{q,\bar{q}}(\hat{\mu})$ and $F_{q,\bar{q}}(\hat{\mu})$



- Trace $\ell_{q,\bar{q}}$ and $F_{q,\bar{q}}$ as functions of $\hat{\mu} = -\mu$

  $\longrightarrow$ ℓ and F_q change monotony, but $\bar{\ell}$ and $F_{\bar{q}}$ don't! Then $\ell, \bar{\ell}$ increase together towards 1 [Dumitru, Hatta, Lenaghan, Orginos, Pisarski (2004)]
- "Free energy must be strictly monotonically decreasing as a function of chemical potential" $\longrightarrow$ contradicts $\ell = e^{-\beta F_q}$?
- Interpretation $\ell \sim e^{-\beta F_q}$ is saved by a simple thermodynamic argument if the charge of the bath at $\hat{\mu} = 0$ is not zero

Pure Thermal bath



free energy of the bath:

$$F = -T \ln \operatorname{tr} \exp\{-\beta(H - \hat{\mu}Q)\}$$

Q is the baryonic charge and $\hat{\mu} = -\mu$

One easily obtains that

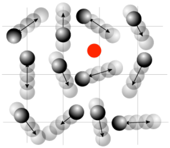
$$\frac{\partial F}{\partial \hat{\mu}} = -\langle Q \rangle \quad \text{and} \quad \frac{\partial \langle Q \rangle}{\partial \hat{\mu}} = \beta \left\langle (Q - \langle Q \rangle)^2 \right\rangle > 0\,.$$

Now, in absence of any external sources, the thermal bath is charge-conjugation invariant for $\hat{\mu} = 0$:

$$\langle Q \rangle_{\hat{\mu}=0} = 0$$

$\longrightarrow$ for any $\hat{\mu} > 0$: $\langle Q \rangle > 0$ and thus $\frac{\partial F}{\partial \hat{\mu}} < 0$, i.e. the free energy of the bath is a decreasing function of $\hat{\mu}$

Thermal bath with charged test source



from before:

$$\frac{\partial F}{\partial \hat{\mu}} = -\langle Q \rangle \qquad \frac{\partial \langle Q \rangle}{\partial \hat{\mu}} = \beta \left\langle (Q - \langle Q \rangle)^2 \right\rangle > 0$$

In the presence of a static quark (q) or antiquark ($\bar{q}$), charge-conjugation invariance is broken s.t.:

$$\langle Q \rangle_{q,\hat{\mu}=0} < 0 \qquad \langle Q \rangle_{\bar{q},\hat{\mu}=0} > 0$$

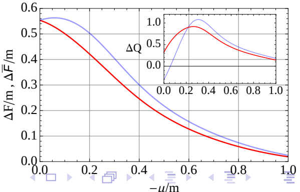
The equations above then imply that

$$\forall \hat{\mu} > 0\,, \quad \langle Q \rangle_{\bar{q}} > 0\,,$$

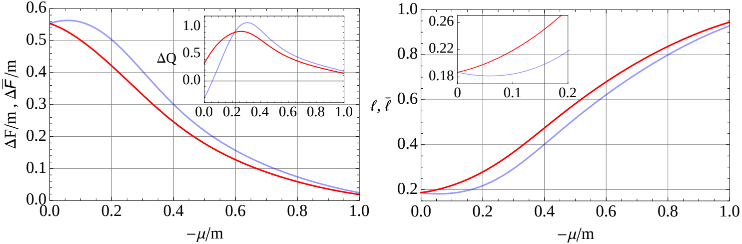
while there exists a certain $\hat{\mu}_0 > 0$ such that,

$$\forall \hat{\mu} \in [0, \hat{\mu}_0]\,,\ \langle Q \rangle_q < 0 \quad \text{and} \quad \forall \hat{\mu} > \hat{\mu}_0\,,\ \langle Q \rangle_q > 0\,.$$

Therefore
$F_{\bar{q}}$ is monotonously decreasing
for $\hat{\mu} > 0$, while
F_q first increases and then decreases



Thermal bath with charged test source



Then

$$\ell \sim e^{-\beta(F_q - F)} \qquad \bar{\ell} \sim e^{-\beta(F_{\bar{q}} - F)}$$

are found by the free energy differences wrt to the bath without any external source.

Since $\frac{\partial F}{\partial \hat{\mu}} = 0\Big|_{\hat{\mu}=0}$, both are dominated for small $\hat{\mu}$ by either F_q or $F_{\bar{q}}$, which explains the different monotony.

$\Delta\langle Q_q \rangle$ and $\Delta\langle Q_{\bar{q}} \rangle$ should approach 0 at large $\hat{\mu}$, which we also observe.

Conclusion

- Improved quantitative reproduction of the phase diagram and Columbia plot at two-loop order, eg critical masses to critical temperature ratios
- suggests that the perturbative description of the phase diagram within the CF model is robust
- Behavior of the Polyakov loops as functions of the chemical potential agrees with their interpretation in terms of quark and anti-quark free energies

OUTLOOK/QUESTIONS:

- Can we describe the chiral transition in the lower left part of the Columbia plot?
- Is there a better way to compare (critical) fermion masses between approaches? Eg give in units of pion masses?