

QCD from quark, gluon, and meson correlators

Mario Mitter

cf. poster Anton K. Cyrol

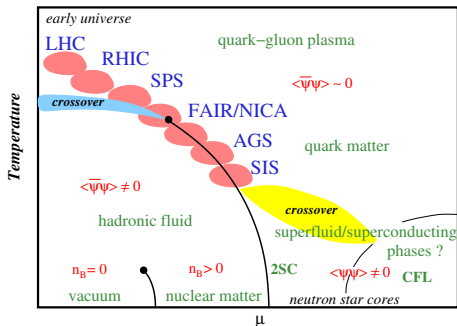
Brookhaven National Laboratory

Bad Honnef, April 2018



fQCD collaboration - QCD (phase diagram) with FRG:

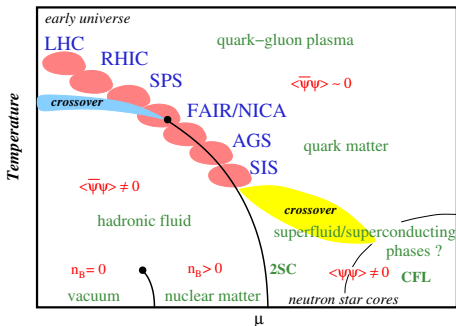
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[Schaefer, Wagner, '08]

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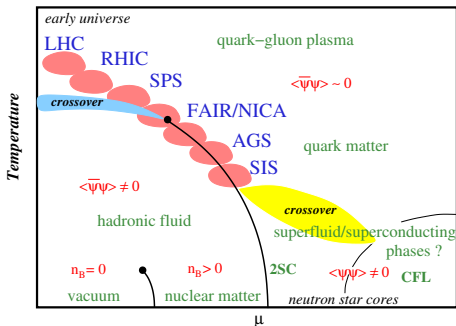


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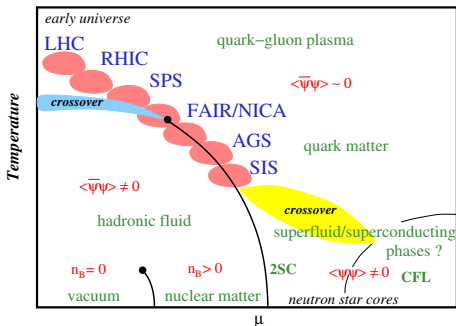


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hard road towards first-principles QCD from FRG

QCD from the effective action (gauge fixing necessary)

$$\Gamma[\Phi] = \sum_n \int_{\{p_i\}} \Gamma_{\Phi_1 \dots \Phi_n}^{(n)}(p_1, \dots, p_{n-1}) \Phi^1(p_1) \dots \Phi^n(-p_1 - \dots - p_{n-1})$$

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 - ▶ bound state spectrum: pole structure of the $\Gamma^{(n)}$ cf. talk N. Wink, ...
e.g. [Roberts, Williams, '94], [Alkofer, Smekal, '00], [Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, '16]
 - ▶ form factors: photon-particle correlators
e.g. [Cloet, Eichmann, El-Bennich, Klahn, Roberts, '08], [Sanchis-Alepuz, Williams, Alkofer, '13]
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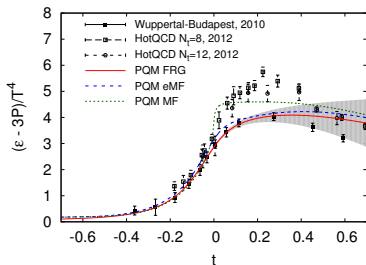
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 - ▶ further quantities: $\Gamma[\Phi] \propto$ eff. potential, propagators, 't Hooft determinant
 - ★ chiral condensate(s)/ $\langle \sigma \rangle$
e.g. [Schaefer, Wambach '04], [Fischer, Luecker, Mueller '11], [MM, Schaefer, '13]
 - ★ (dressed) Polyakov loop
e.g. [Fischer, '09], [Braun, Haas, Marhauser, Pawłowski, '09], [MM, et al., '17]
 - ★ axial anomaly e.g. [Grahl, Rischke, '13], [MM, Schaefer, '13], [Fejos, '15], [Heller, MM, '15]
 - ★ spectral functions e.g. [Tripolt, Strodthoff, Smekal, Wambach, '14]

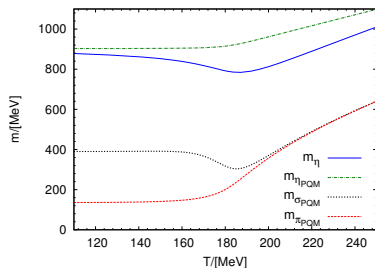
e.g. EOS and axial anomaly in “QCD-enhanced” models

- equation of state
- $N_f = 2 + 1$ PQM model with FRG
- unquenched Polyakov-loop potential from [Braun, Haas, Marhauser, Pawłowski, '11]
- η' -meson screening mass
- $N_f = 2$ PQM model, extended MF
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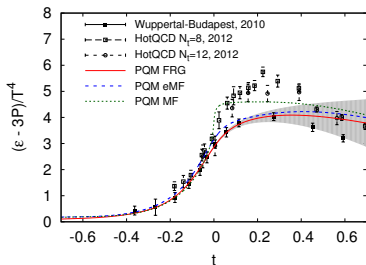
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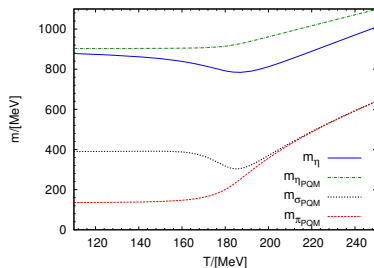
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- phase diagrams
- spectral functions
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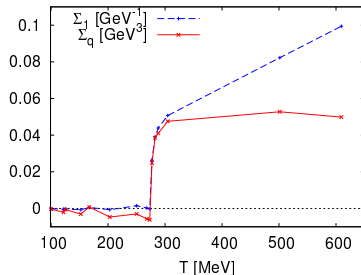
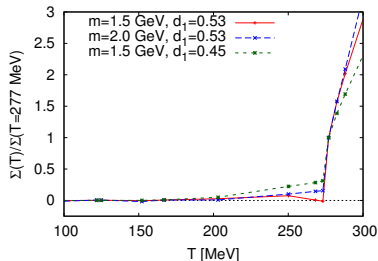
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e.g. Center symmetry order parameters [MM, Hopfer, Schaefer, Alkofer, 2017]

- quenched (scalar) Quantum Chromodynamics
- correlators from Dyson-Schwinger equation (DSE)
- lattice gluon input and vertex models



from scalar propagator:

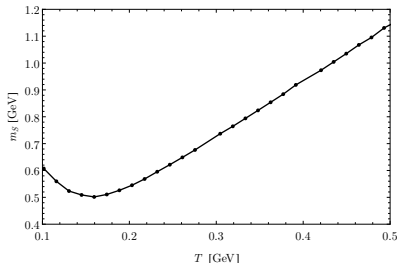
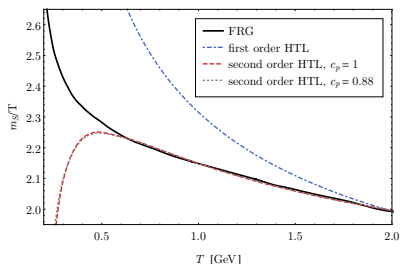
$$\int_0^{2\pi} d\varphi T \sum_n D_\varphi^2(\vec{p} = \vec{0}, \omega_n(\varphi))$$

from quark propagator:

$$\int_0^{2\pi} d\varphi T \sum_n \left[\frac{1}{4} \text{tr}_D S(\vec{0}, \omega_n(\varphi)) \right]^2$$

- cf. e.g. [Fischer '09], [Braun, Haas, Marhauser, Pawłowski, '09],...

- correlators from Functional Renormalisation Group (FRG)
- screening mass:
fit to exponential decay of chromo-electric gluon propagator



- excellent agreement with beyond leading order Debye mass of [Arnold, Yaffe, '95] for $T \gtrsim 0.6$ GeV
- smooth transition to nonperturbative regime

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hard problem: quantitatively reliable first-principles method @ large $\frac{\mu}{T}$

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crawling towards QCD at finite temperature and density:

- quenched matter part [MM, Strodthoff, Pawłowski, 2014]
- pure $SU(N)$ YM-theory: 4D and 3D [Cyrol, Fister, MM, Pawłowski, Strodthoff, 2016]
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main finding: very sensitive to small quantitative errors in α_{pert} :

\Rightarrow need “BRST-consistency”!

(Euclidean) Correlation functions with the FRG

- mass-like IR regulator:

- ▶ $S[\Phi] \rightarrow S[\Phi] + \langle \Phi, R_k \Phi \rangle$

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[Wetterich '93]

$$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} \right)$$


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- gauge-fixed approach (Landau gauge): ghosts appear
- aim for “apparent convergence” of $\Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$

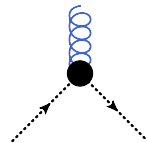
classical tensors with momentum dependent-dressings:



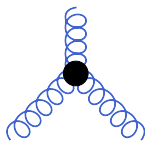
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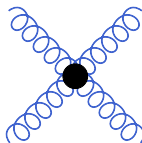
$$1/Z_c(p)$$



$$\lambda_{\bar{c}cA}(p, q, z)$$



$$\lambda_{A^3}(p, q, z)$$



$$\lambda_{A^4}(\bar{p})/\lambda_{A^4}(p, q, z)$$

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- functional derivatives of Wetterich equation
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$$\partial_t \text{---} \text{---}^{-1} = \text{---} \text{---} \text{---} - 2 \text{---} \text{---} \text{---} + \frac{1}{2} \text{---} \text{---} \text{---}$$

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- set of coupled equations: cf. DoFun [Huber, Braun, '11], FormTracer [Cyrol, MM, Strodthoff, '16]
 all propagators/vertices dressed and momentum-dependent

(Some) numerical results

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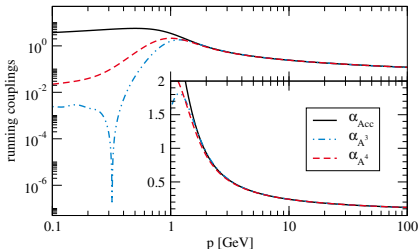
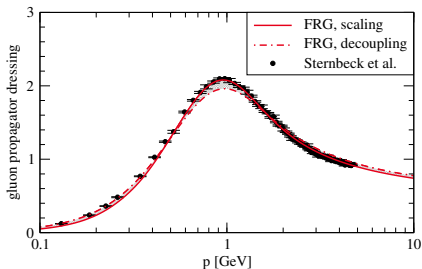
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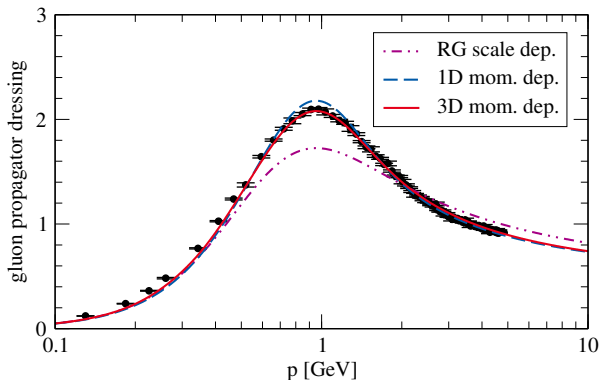
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- difference to decoupling not important for all practical purposes
- $\Gamma_{AA}^{(2)}(p) \propto Z_A(p) p^2 (\delta^{\mu\nu} - p^\mu p^\nu / p^2)$
- IR-suppression \Leftrightarrow “confinement”
- smooth transition to perturbation theory
- running couplings
- degeneracy at large $p \Rightarrow$ “BRST-consistency”



lattice data: A. Sternbeck, E. M. Ilgenfritz, M. Müller-Preussker, A. Schiller, and I. L. Bogolubsky, PoS LAT2006, 076.

lattice data: [Sternbeck, Ilgenfritz, Muller-Preussker, Schiller, Bogolubsky, '06]

- different approximations for vertex dressing functions:



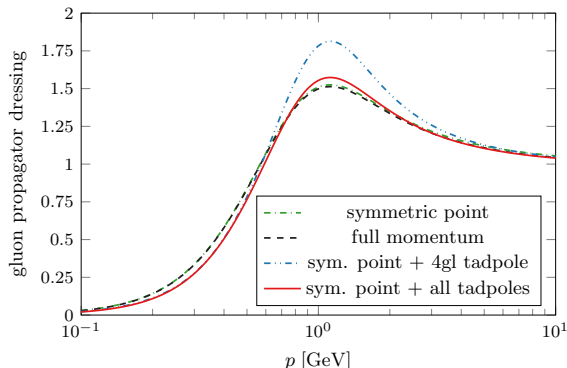
- RG scale dep.: $\lambda_X(k; p, q, z) \equiv \lambda_X(k)$
- 1D mom dep.: $\lambda_X(k; p, q, z) \equiv \lambda_X(k; \bar{p})$
- 3D mom dep.: $\lambda_{\bar{c}A/A^3}(k; p, q, z)$ and $\lambda_{A^4}(k; \bar{p})/\lambda_{A^4}(k; p, q, z)$

[Corell, Cyrol, Fister, MM, Pawlowski, Strodthoff, '18]

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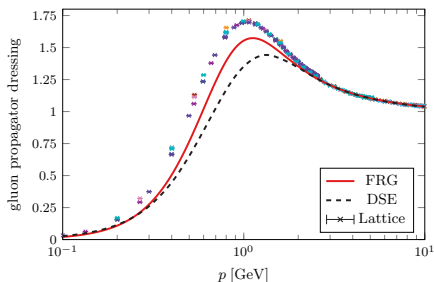


- symmetric point: $\lambda_X(k; p, q, z) \equiv \lambda_X(k; \bar{p})$
- full momentum: $\lambda_{\bar{c}cA/A^3}(k; p, q, z)$ and $\lambda_{A^4}(k; \bar{p})/\lambda_{A^4}(k; p, q, z)$
- +4gl tadpole: all tensors of 4-gl vertex in propagator tadpoles
- all tadpoles: all tensors of all vertices in propagator tadpoles

Two more observations in 3D YM theory

[Corell, Cyrol, Fister, MM, Pawłowski, Strodthoff, '18]

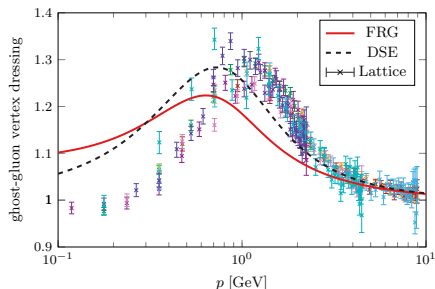
- gluon propagator dressing:
does not agree as well with lattice



lattice data: [Cucchieri, Maas, Mendes, '06 and '08], [A. Maas '15 and in preparation]

DSE data: [M. Huber 2016]

- ghost-gluon vertex dressing:
shows a scale mismatch



What do we learn from the comparison of 3D and 4D YM?

[Corell, Cyrol, Fister, MM, Pawlowski, Strodthoff, '18]

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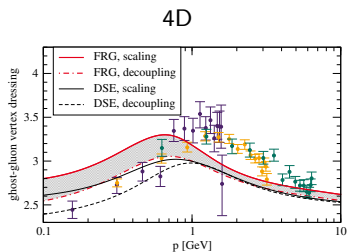
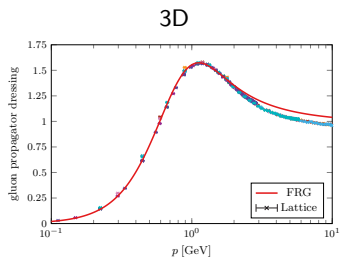
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- 4D: “classical truncation” seems to work well (so why care?)

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[Corell, Cyrol, Fister, MM, Pawłowski, Strodthoff, '18]

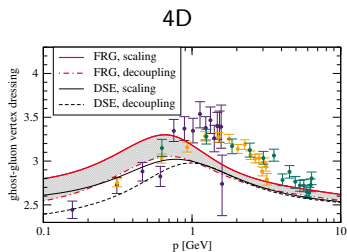
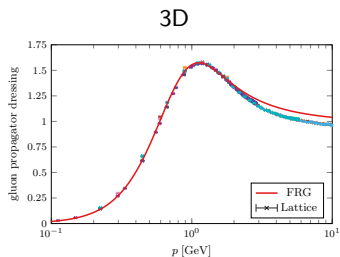
- effect of non-classical contributions seems more important in 3D
- large individual contributions possible, even for small overall effect
cf. talk Huber
- scale mismatch between glue and ghost
⇒ need consistent extensions of truncation
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- QCD? important higher-order contributions
cf. talk Serreau

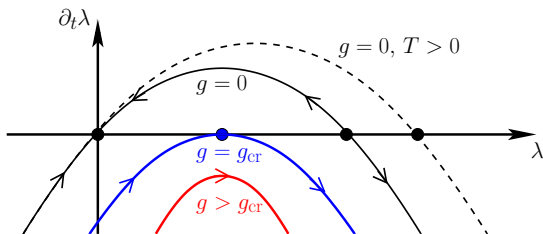
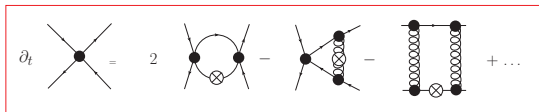
Chiral symmetry breaking

- χ SB \Leftrightarrow resonance in 4-Fermi interaction λ (pion pole):

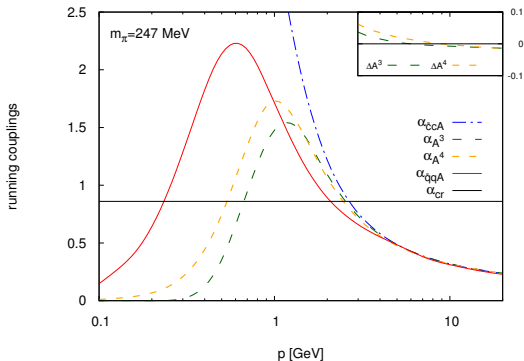
Chiral symmetry breaking

- χ SB \Leftrightarrow resonance in 4-Fermi interaction λ (pion pole):
- β -function of momentum independent 4-Fermi interaction:

$$\partial_t \lambda = 2\lambda + a\lambda^2 + b\lambda\alpha + c\alpha^2, \quad b > 0, \quad a, c \leq 0$$

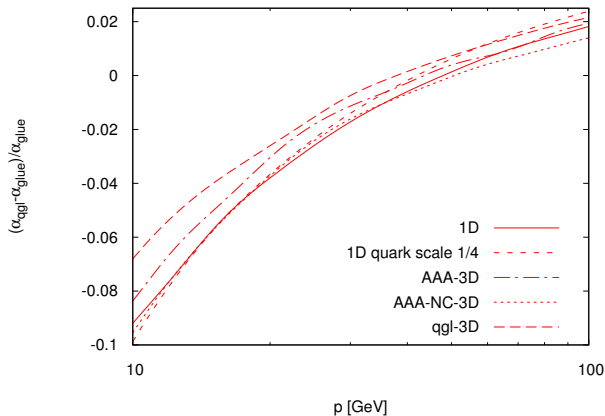


[Braun, 2011]



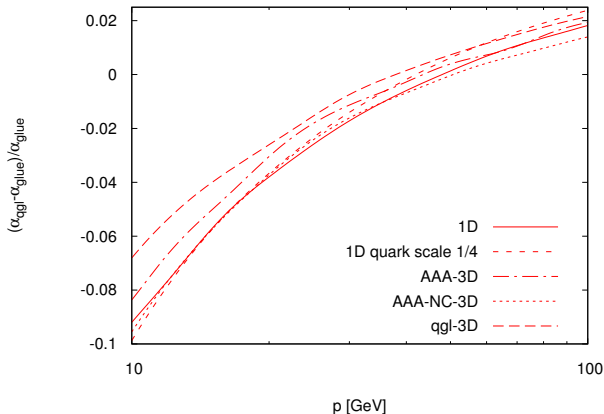
- agreement in perturbative regime required by Slavnov-Taylor identities
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{\bar{q}Aq} > \alpha_{cr}$: necessary for chiral symmetry breaking
- area above α_{cr} very sensitive to errors
 \Rightarrow have use STI in perturbative regime

FRG running couplings: relative deviation



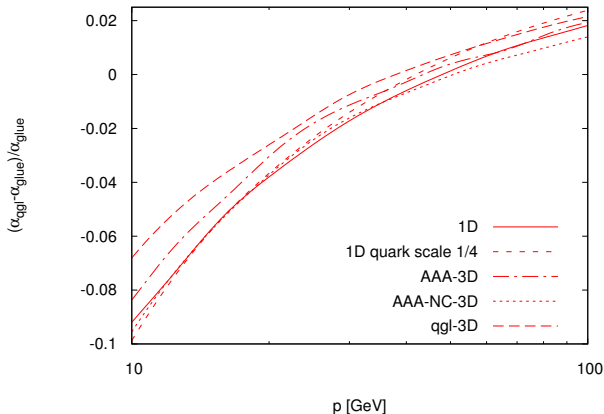
- scale mismatch between matter and ghost-gluon sector \Rightarrow better with $\Gamma_{AA\bar{q}q}$

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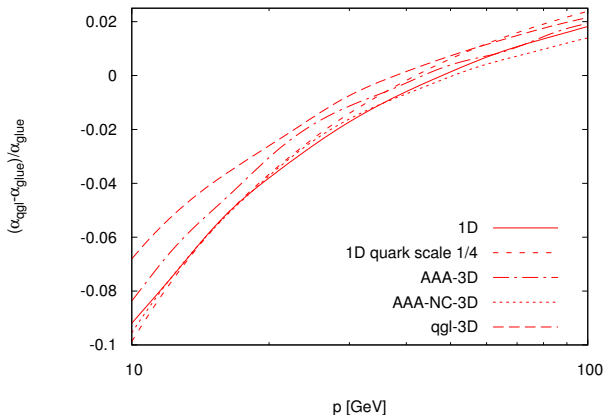
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FRG running couplings: relative deviation

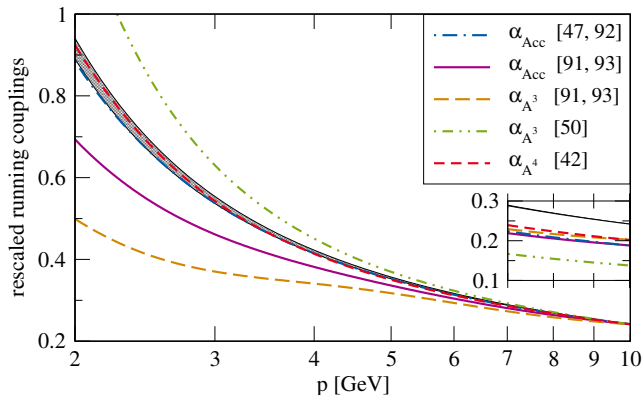


- scale mismatch between matter and ghost-gluon sector \Rightarrow better with $\Gamma_{AA\bar{q}q}$
- importance of non-classical/higher loop contributions
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\Rightarrow similarities to 3D YM (the testing case)

Running couplings: YM theory with FRG and DSE

[Cyrol, Fister, MM, Pawłowski, Strodthoff, 2016]



[42] [A. Cyrol, M. Huber, L. v. Smekal, '15],

[47] [M. Huber, L. v. Smekal, '13], [92] [M. Huber, private communications]

[50] [A. Blum, M. Huber, MM, L. v. Smekal, '14]

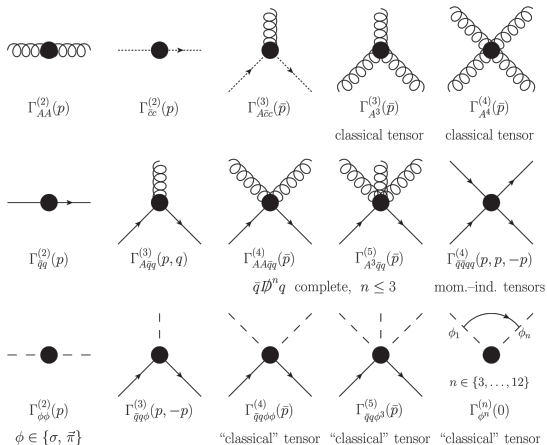
[91] [R. Williams, '14], [93] [R. Williams, private communications]

(maybe most) important
self-consistency check!

$N_f = 2$ Landau-gauge QCD

[Cyrol, MM, Pawłowski, Strodthoff, 2017]

Truncation:

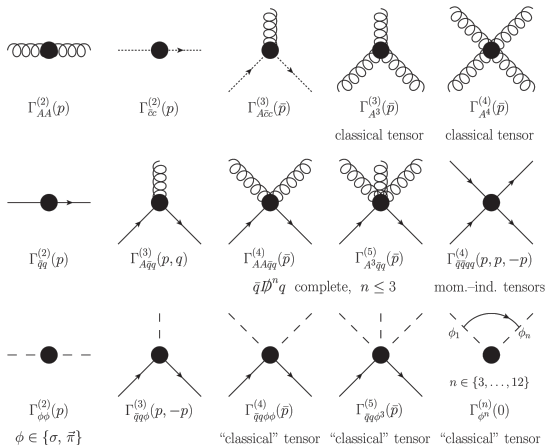


systematics of improving the truncation?

$N_f = 2$ Landau-gauge QCD

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Truncation:



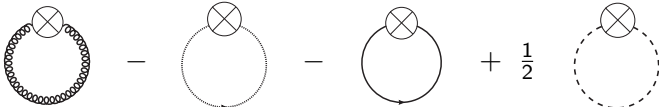
systematics of improving the truncation?

\Rightarrow BRST-invariant operators, e.g. $\bar{\psi}\not{D}^n\psi$

Mesons via dynamical hadronization

[Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels \rightarrow meson exchange
- efficient inclusion of pole structure \Rightarrow no spurious singularities
- low-energy effective model parameters from QCD - range of validity

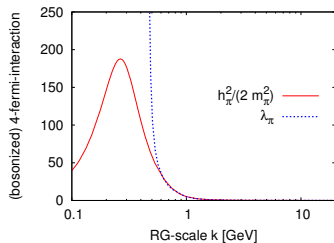
$$\partial_k \Gamma_k = \frac{1}{2} \left(\text{diagram 1} - \text{diagram 2} - \text{diagram 3} + \frac{1}{2} \text{diagram 4} \right)$$


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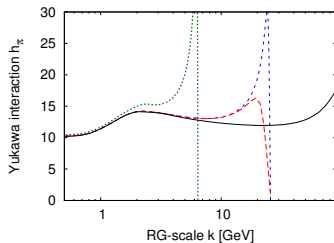
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[MM, Pawłowski, Strodthoff, 2014]



[Braun, Fister, Pawłowski, Rennecke, 2014]

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Some representative equations (numerics-heavy)

[MM, Pawłowski, Strodthoff, 2014],

[Cyrol, Fister, MM, Pawłowski, Strodthoff, 2016]

cf. FormTracer [Cyrol, MM, Strodthoff, 2016]

$$\partial_t \longrightarrow^{-1} = \text{diagram 1} + \text{diagram 2} + \frac{1}{2} \text{diagram 3} + \text{diagram 4} + \text{diagram 5} - \text{diagram 6}$$

$$\partial_t \text{ (triangle)} = - \text{diagram 1} - \text{diagram 2} - \text{diagram 3} - \text{diagram 4} - \frac{1}{2} \text{diagram 5} + 2 \text{diagram 6} - \text{diagram 7} + \text{perm.}$$

$$\partial_t \text{ (cross)} - 2 \text{diagram 1} - \text{diagram 2} - \text{diagram 3} - \text{diagram 4} - \text{diagram 5} - \text{diagram 6} - \text{diagram 7} + \text{perm.}$$

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$$\partial_t \text{---}^{-1} = \text{---} \otimes \text{---} + \text{---} \otimes \text{---} + \frac{1}{2} \text{---} \otimes \text{---} + \text{---} \otimes \text{---} + \text{---} \otimes \text{---} - \text{---} \otimes \text{---}$$

$$\partial_t \text{---} = - \text{---} - \text{---} - \text{---} - \text{---} - \text{---} + 2 \text{---} - \text{---} + \text{perm.}$$

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$$\partial_t \text{---}^{-1} = \text{---} + \text{---}$$

$$\partial_t \text{---}^{-1} = \text{---} - 2 \text{---} + \frac{1}{2} \text{---}$$

$$\partial_t \text{---} = - \text{---} - \text{---} + \text{perm.}$$

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$$\partial_t \text{---}^{-1} = \text{---}^{\circlearrowleft} + \text{---}^{\circlearrowright} + \frac{1}{2} \text{---}^{\circlearrowright} + \text{---}^{\circlearrowleft} + \text{---}^{\circlearrowright} - \text{---}^{\circlearrowright}$$

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$$\partial_t \text{---}^{-1} = \text{---}^{\circlearrowleft} - 2 \text{---}^{\circlearrowright} + \frac{1}{2} \text{---}^{\circlearrowright}$$

$$\partial_t \text{---} = - \text{---}^{\triangleleft} - \text{---}^{\triangleright} + \text{perm.}$$

$$\partial_t \text{---} = - \text{---}^{\triangleleft} - \text{---}^{\triangleright} - \text{---}^{\circlearrowleft} - \text{---}^{\circlearrowright} + 2 \text{---}^{\circlearrowleft} - \text{---}^{\circlearrowright} + \text{perm.}$$

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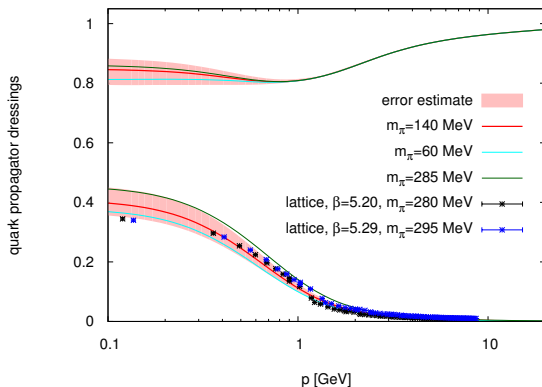
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- $\Gamma_{\bar{q}q}(p) = Z_q(p) (\not{p} + M(p))$



- $S_{\chi SB}: M_q(0) \gg M_q(p \gg \Lambda_{QCD})$
- FRG vs. lattice: bare mass, scale setting, lattice Z_q ?
- very sensitive to $\bar{q}qA$ -interaction, relative scales

lattice data: O. Oliveira, A. Kzlersu, P. J. Silva, J.-I. Skullerud, A. Sternbeck, A. G. Williams, arXiv:1605.09632 [hep-lat].

Quark-gluon interactions

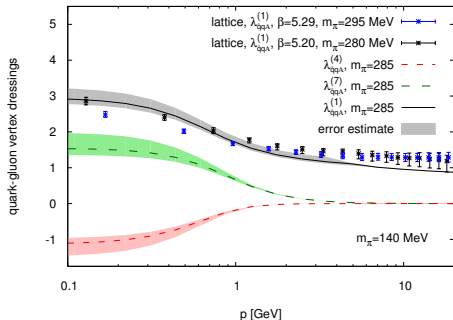
[Cyrol, MM, Pawłowski, Strodthoff, 2017]

- quark-gluon interaction most crucial for chiral symmetry breaking
- transverse tensor basis (8 tensors), e.g. γ^μ , $i(\not{p} + \not{q}) \gamma^\mu$, $\frac{1}{2} [\not{p}, \not{q}] \gamma^\mu$
- $\lambda^{(i)}(p, q) \rightarrow \lambda^{(i)}(p^2, q^2, p \cdot q)$

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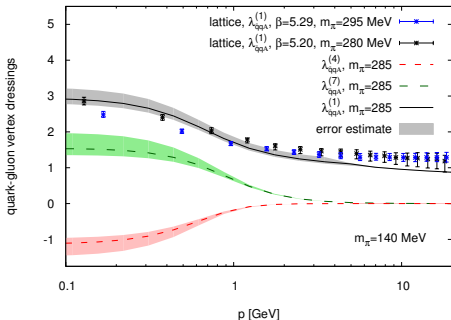


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constrained by STI
at large momenta
 - ▶ chirally symmetric
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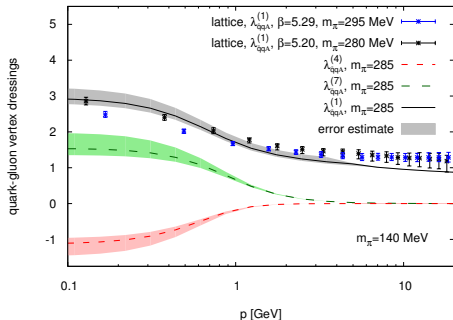
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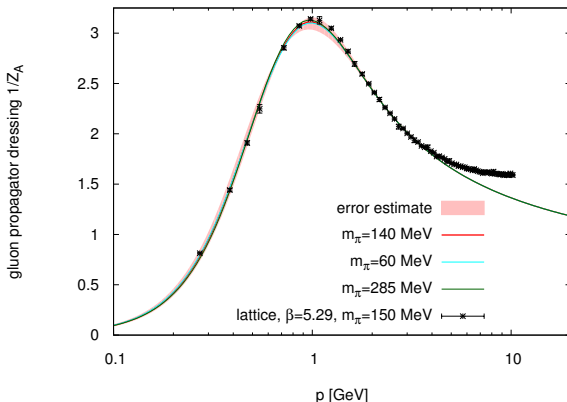
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- ⇒ more evidence for importance of “BRST-connected” operators

lattice data: O. Oliveira, A. Kzlersu, P. J. Silva, J.-I. Skullerud, A. Sternbeck, A. G. Williams, arXiv:1605.09632 [hep-lat].

Gluon propagator

[Cyrol, MM, Pawłowski, Strodthoff, 2017]

- $\Gamma_{AA}(p) = Z_A(p) p^2 (\delta^{\mu\nu} - p^\mu p^\nu / p^2)$

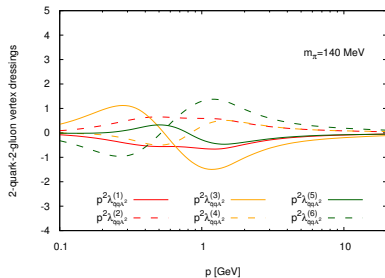
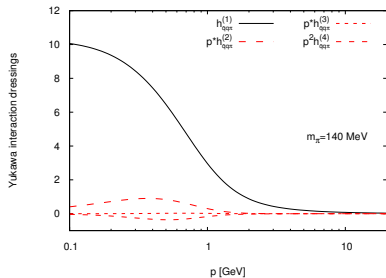
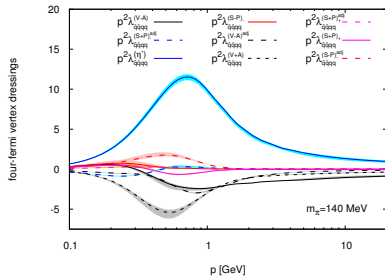
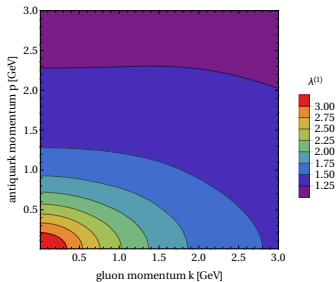


- infrared suppression \Leftrightarrow “confinement”
- insensitive to pion mass
- smooth transition to pert. theory
- scaling solution: lattice comparison?

lattice data: A. Sternbeck, K. Maltman, M. Müller-Preussker, L. von Smekal, PoS LATTICE2012 (2012) 243.

More correlators

[Cyrol, MM, Pawłowski, Strodthoff, 2017]

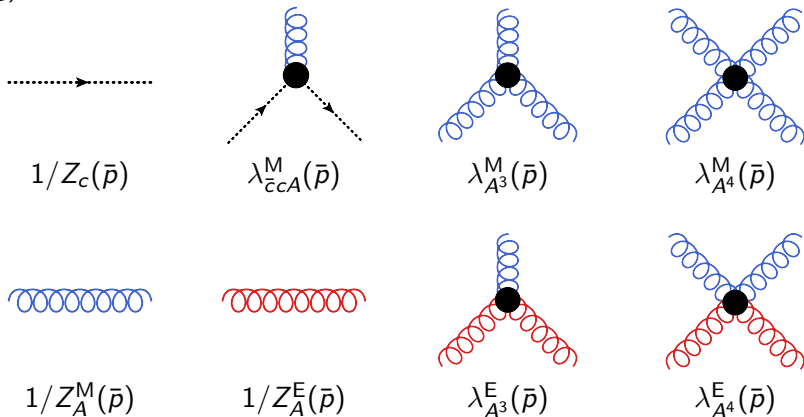


Pure $SU(N)$ YM-theory at $T > 0$

[Cyrol, Fister, MM, Pawłowski, Strodthoff, '16]

[Cyrol, MM, Pawłowski, Strodthoff, '17]

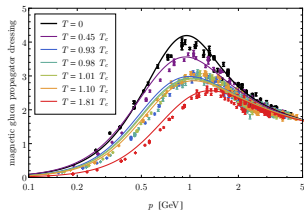
Truncation (blue: magnetic (transverse) leg, red: electric (longitudinal) leg):



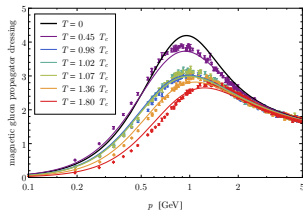
YM Propagators at $T \neq 0$

[Cyrol, MM, Pawłowski, Strodthoff, '17]

Zeroth mode correlation functions



$SU(2)$

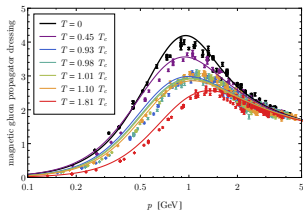


$SU(3)$

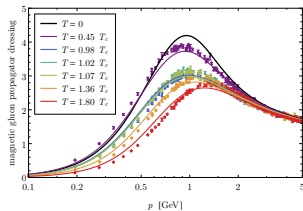
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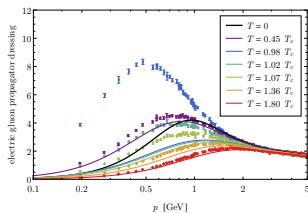
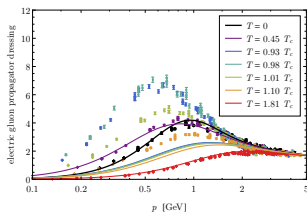
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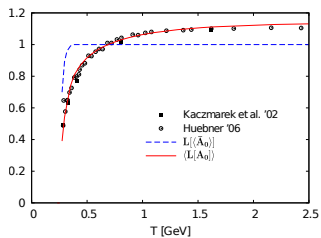
lattice data: A. Maas, J. M. Pawlowski, L. von Smekal, D. Spielmann, Phys.Rev. D85 (2012) 034037. ($SU(2)$)

P. J. Silva, O. Oliveira, P. Bicudo, and N. Cardoso, Phys. Rev. D89, 074503 (2014). ($SU(3)$)

Backgrounds, ghost and zero crossing

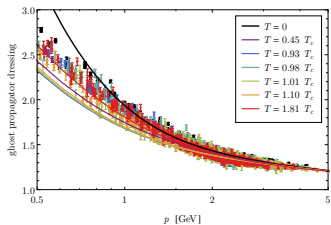
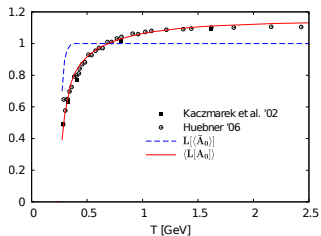
[Cyrol, MM, Pawlowski, Strodthoff, '17]

$\langle \bar{A}_0 \rangle$ important near T_c , cf. [Herbst et al., '15]

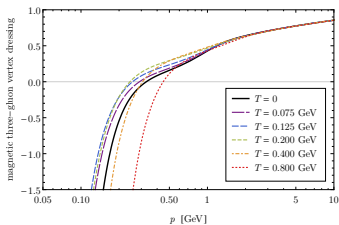
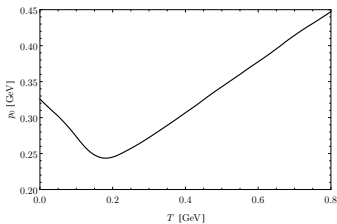


Backgrounds, ghost and zero crossing [Cyrol, MM, Pawlowski, Strodthoff, '17]

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magnetic zero crossing in 3g-vertex



Status and Outlook

- QCD/YM-theory from $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$ and $m_q(\Lambda = \mathcal{O}(10) \text{ GeV})$ with FRG
- in general very good agreement with lattice correlators
- at the stage of improving quantitative reliability
- need BRST-consistency: at least two ways out

- long-term goal: QCD @ $T, \mu > 0$
 - ▶ equation of state
 - ▶ fluctuations of conserved charges
 - ▶ order parameters

- further applications:
 - ▶ input for “QCD-enhanced” models
 - ▶ direct calculation of low-energy constants cf. talk Eser
 - ▶ other strongly-interacting theories
 - ▶ ...