The extended Linear Sigma Model as low-energy model for QCD

Dirk H. Rischke

Institut für Theoretische Physik

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Department of Modern Physics USTC Hefei

CRC–TR 211 Strong-interaction matter under extreme conditions

with:

Florian Divotgey, Jürgen Eser, Phillip Lakaschus, Justin Mauldin, Lisa Olbrich,

Susanna Gallas, Denis Parganlija, Stanislaus Janowski,

Mario Mitter (Brookhaven National Laboratory),

Francesco Giacosa (Jan Kochanowski University, Kielce),

Peter Kovacs, Gyuri Wolf, Miklós Zétényi
(Wigner Research Center for Physics, Budapest)

An effective chiral approach: the extended Linear Sigma Model (eLSM)

Chiral symmetry of QCD (classically): global $U(N_f)_r \times U(N_f)_\ell$ symmetry

$\Longrightarrow$ dynamically broken in vacuum by nonzero quark condensate $\langle \bar{q}q \rangle \neq 0$

$\Longrightarrow$ restored at nonzero temperature T and chemical potential μ

$\Longrightarrow$ degeneracy of hadronic chiral partners in the chirally restored phase

$\Longrightarrow$ for this application: chiral symmetry must be linearly realized

$\Longrightarrow$ Linear Sigma Model, extended by (axial-)vector mesons $\Longrightarrow$ eLSM

Disclaimer: No attempt to fit precision data for hadron vacuum phenomenology!

(No attempt to compete with chiral perturbation theory)

Nevertheless: achieve reasonable description of hadron vacuum phenomenology!

Moreover: strong statement on the nature of the scalar mesons!

scalar-meson puzzle: too many scalar states to fit into a $q\bar{q}$ meson nonet

$$f_0(500),\ f_0(980),\ f_0(1370),\ f_0(1500),\ f_0(1710)$$

$\Longrightarrow$ Jaffe's conjecture: R.L. Jaffe, PRD 15 (1977) 267, 281

light scalars $f_0(500),\ f_0(980)$ are (predominantly) $[qq][\bar{q}\bar{q}]$ tetraquark states

$\Longrightarrow$ fifth scalar meson $f_0(1710)$ could be (predominantly) glueball state

Scalar and pseudoscalar mesons

Assume mesons to be $\bar{q}q$ states: $\Phi \sim \bar{q}_r q_\ell \;, \quad \Phi^\dagger \sim \bar{q}_\ell q_r$

$\Longrightarrow \; \Phi \in (N_f^*, N_f)$ irrep of $U(N_f)_r \times U(N_f)_\ell$

$\Longrightarrow \; \Phi \longrightarrow \Phi' = U_L \Phi U_R^\dagger \,, \quad \Phi^\dagger \longrightarrow \Phi^{\dagger\prime} = U_R \Phi^\dagger U_L^\dagger$

$\Longrightarrow \; \Phi \equiv \phi_a T_a, \quad T_a$ generators of $U(N_f), \quad \phi_a \equiv \sigma_a + i\pi_a$

$$\mathcal{L}_S = \mathrm{Tr}\left(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2\, \Phi^\dagger \Phi\right) - \lambda_1 \left[\mathrm{Tr}\left(\Phi^\dagger\Phi\right)\right]^2 - \lambda_2 \mathrm{Tr}\left(\Phi^\dagger\Phi\right)^2 + c\left(\det\Phi - \det\Phi^\dagger\right)^2 + \mathrm{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right] + \mathrm{Tr}\left[E\,\Phi^\dagger\Phi\right]$$

Spontaneous symmetry breaking (SSB): $m^2 < 0 \quad \Longrightarrow \langle \Phi \rangle = \begin{pmatrix} \phi_N & 0 & 0 \\ 0 & \phi_N & 0 \\ 0 & 0 & \phi_S \end{pmatrix}$

$U(1)_A$ anomaly: $c \neq 0$

Explicit symmetry breaking due to different non-zero quark masses:

$H \equiv h_a C_a \;, \quad E \equiv \epsilon_a C_a \;, \quad h_a \,, \; \epsilon_a \neq 0 \,, \quad C_a \equiv T_a, \; a = 3, 8$

Vector and axial-vector mesons

Introduce left- and right-handed vector fields $\mathcal{L}_\mu \sim \bar{q}_\ell \gamma_\mu q_\ell \ , \ \ \mathcal{R}_\mu \sim \bar{q}_r \gamma_\mu q_r \ ,$

$\Longrightarrow \ \mathcal{L}_\mu \in (1, N_f^2)$ irrep of $U(N_f)_r \times U(N_f)_\ell \ \Longrightarrow \ \mathcal{L}_\mu \longrightarrow \mathcal{L}'_\mu = U_L \mathcal{L}_\mu U_L^\dagger$

$\Longrightarrow \ \mathcal{R}_\mu \in (N_f^2, 1)$ irrep of $U(N_f)_r \times U(N_f)_\ell \ \Longrightarrow \ \mathcal{R}_\mu \longrightarrow \mathcal{R}'_\mu = U_R \mathcal{R}_\mu U_R^\dagger$

$\Longrightarrow \ \mathcal{L}_\mu \equiv L_\mu^a T_a, \ \ \mathcal{R}_\mu \equiv R_\mu^a T_a$

$$\begin{aligned}\mathcal{L}_V = & -\frac{1}{4} \operatorname{Tr}(\mathcal{L}^0_{\mu\nu} \mathcal{L}_0^{\mu\nu} + \mathcal{R}^0_{\mu\nu} \mathcal{R}_0^{\mu\nu}) + \operatorname{Tr}\left[\left(\frac{1}{2} m_1^2 + \Delta\right) \left(\mathcal{L}_\mu \mathcal{L}^\mu + \mathcal{R}_\mu \mathcal{R}^\mu\right)\right] \\ & + i \frac{g_2}{2} \operatorname{Tr}\left\{\mathcal{L}^0_{\mu\nu}[\mathcal{L}^\mu, \mathcal{L}^\nu] + \mathcal{R}^0_{\mu\nu}[\mathcal{R}^\mu, \mathcal{R}^\nu]\right\} \\ & + g_3 \operatorname{Tr}\left(\mathcal{L}^\mu \mathcal{L}^\nu \mathcal{L}_\mu \mathcal{L}_\nu + \mathcal{R}^\mu \mathcal{R}^\nu \mathcal{R}_\mu \mathcal{R}_\nu\right) - g_4 \operatorname{Tr}\left(\mathcal{L}^\mu \mathcal{L}_\mu \mathcal{L}^\nu \mathcal{L}_\nu + \mathcal{R}^\mu \mathcal{R}_\mu \mathcal{R}^\nu \mathcal{R}_\nu\right) \\ & + g_5 \operatorname{Tr}\left(\mathcal{L}^\mu \mathcal{L}_\mu\right) \operatorname{Tr}\left(\mathcal{R}^\nu \mathcal{R}_\nu\right) \\ & + g_6 \left[\operatorname{Tr}\left(\mathcal{L}^\mu \mathcal{L}_\mu\right) \operatorname{Tr}\left(\mathcal{L}^\nu \mathcal{L}_\nu\right) + \operatorname{Tr}\left(\mathcal{R}^\mu \mathcal{R}_\mu\right) \operatorname{Tr}\left(\mathcal{R}^\nu \mathcal{R}_\nu\right)\right]\end{aligned}$$

$\mathcal{L}^0_{\mu\nu} \equiv \partial_\mu \mathcal{L}_\nu - \partial_\nu \mathcal{L}_\mu, \ \ \mathcal{R}^0_{\mu\nu} \equiv \partial_\mu \mathcal{R}_\nu - \partial_\nu \mathcal{R}_\mu$

vector mesons: $V_\mu^a \equiv \frac{1}{2}\left(L_\mu^a + R_\mu^a\right)$, axial-vector mesons: $A_\mu^a \equiv \frac{1}{2}\left(L_\mu^a - R_\mu^a\right)$

$\Delta = \delta_a C_a$: accounts for different quark masses (like E)

$g_3, \ g_4, \ g_5, \ g_6$: not determined by global fit to masses and decay widths
(mild impact on $\pi\pi$ scattering lengths,
can be determined from LECs of QCD)

Scalar – vector interactions

$$\begin{aligned}\mathcal{L}_{SV} = i\, g_1 \,\mathrm{Tr}\left[\partial_\mu \Phi \left(\Phi^\dagger \mathcal{L}^\mu - \mathcal{R}^\mu \Phi^\dagger\right) - \partial_\mu \Phi^\dagger \left(\mathcal{L}^\mu \Phi - \Phi \mathcal{R}^\mu\right)\right] \\ + \frac{h_1}{2}\,\mathrm{Tr}\left(\Phi^\dagger \Phi\right) \mathrm{Tr}\left(\mathcal{L}_\mu \mathcal{L}^\mu + \mathcal{R}_\mu \mathcal{R}^\mu\right) + (g_1^2 + h_2)\,\mathrm{Tr}\left(\Phi^\dagger \Phi \mathcal{R}_\mu \mathcal{R}^\mu + \Phi \Phi^\dagger \mathcal{L}_\mu \mathcal{L}^\mu\right) \\ - 2(g_1^2 - h_3)\,\mathrm{Tr}\left(\Phi^\dagger \mathcal{L}_\mu \Phi \mathcal{R}^\mu\right)\end{aligned}$$

SSB: induces mass splitting, e.g. $m_{a_1}^2 - m_\rho^2 = (g_1^2 - h_3)\phi_N^2$

$\Longrightarrow$ complete meson Lagrangian: $\boxed{\mathcal{L}_M = \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_{SV}}$

Vacuum phenomenology: Global fit for $N_f = 3$ (I)

$N_f = 3 \Longrightarrow$ two scalar-isoscalar mesons $f_0^L\,,\ f_0^H$ (combinations of $\bar{q}q$ and $\bar{s}s$)

Since nature of these scalar-isoscalar mesons (quarkonium, glueball, four-quark state?) is unclear:

$\Longrightarrow$ at first omit scalar-isoscalar mesons from the fit

$\Longrightarrow$ set large–N_c suppressed parameters $\lambda_1 = h_1 \equiv 0$

$\Longrightarrow$ perform χ^2–fit of $m^2, \lambda_2\,, c\,, h_0\,,\ h_8\,, m_1^2\,, \delta_S\,, g_1\,,\ g_2\,,\ h_2\,,\ h_3$ (11 parameters) to 21 experimental meson masses and decay widths

D. Parganlija, F. Giacosa, P. Kovacs, Gy. Wolf, DHR, PRD 87 (2013) 014011

Constraints: (i) no isospin violation

$\Longrightarrow$ experimental error $= \max(\mathrm{PDG\ error}\,,\ 5\%)$

(ii) $m^2 < 0$ (SSB)

(iii) $\lambda_2 > 0\,,\ \lambda_1 > -\lambda_2/2$ (boundedness of potential)

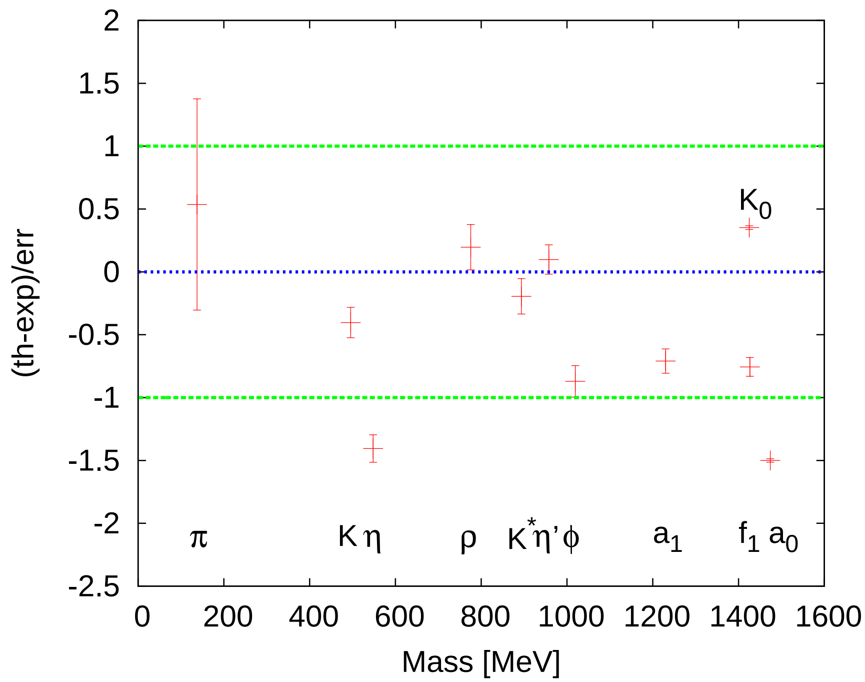
(iv) $m_1 \geq 0$ (boundedness of potential)

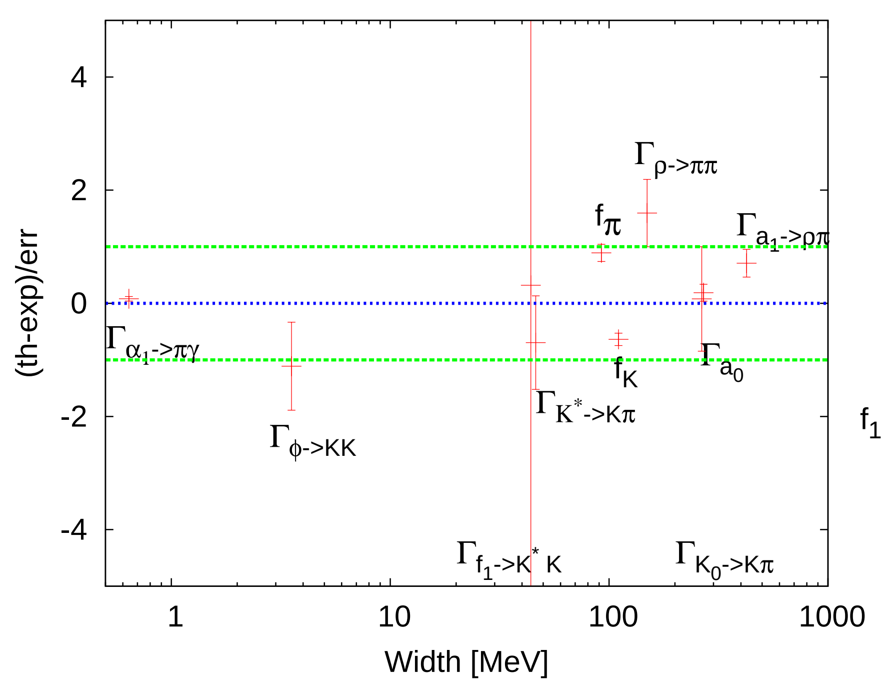
(v) $m_1 \leq m_\rho$ (SSB increases mass of vector mesons)

Vacuum phenomenology: Global fit for $N_f = 3$ (II)

Possible scalar isotriplet/isodoublet combinations:

Combination	χ^2	$\chi^2_{\rm red}$
$a_0(1450)/K_0^\star(1430)$	12.33	1.23
$a_0(980)/K_0^\star(800)$	129.36	11.76
$a_0(980)/K_0^\star(1430)$	22.00	2.00
$a_0(1450)/K_0^\star(800)$	242.27	24.23





Vacuum phenomenology: Global fit for $N_f = 3$ (III)

for $\lambda_1 = h_1 \equiv 0$:

$\Longrightarrow$ prediction for the masses of the isoscalar-scalar states:
$m_{f_0^L} = 1362.7$ MeV, $m_{f_0^H} = 1531.7$ MeV

$\Longrightarrow$ masses are in the range of the heavy scalar states:
$m_{f_0(1370)} = (1350 \pm 150)$ MeV, $m_{f_0(1500)} = (1505 \pm 75)$ MeV,
$m_{f_0(1710)} = 1720 \pm 86$ MeV

$\Longrightarrow$ mass of f_0^L close to mass of $f_0(1370)$

$\Longrightarrow$ mass of f_0^H close to $f_0(1500)$

$\Longrightarrow$ $f_0(1370)\,,\ f_0(1500)$ appear to be (predominantly) $\bar{q}q$–states

$\Longrightarrow$ chiral partners of $\pi,\ \eta'$!

$\Longrightarrow$ light scalar states $f_0(500),\ f_0(980)$ could be (predominantly) $[qq][\bar{q}\bar{q}]$–states,
as suggested by Jaffe R.L. Jaffe, PRD 15 (1977) 267, 281
see, however, W. Heupel, G. Eichmann, C.S. Fischer, PLB 718 (2012) 545

$\Longrightarrow$ light scalars have a dominant $(\bar{q}q)(\bar{q}q)$ component!

$\Longrightarrow$ light scalars are dynamically generated resonances in pseudoscalar scattering continuum!

Low-energy limit (I)

Does the model have the same low-energy limit as QCD?

$\Longrightarrow$ low-energy limit of QCD: chiral perturbation theory (χPT)

$\Longrightarrow$ take $\mathcal{L}_{\chi PT} = \mathcal{L}_2 + \mathcal{L}_4$

$\Longrightarrow$ use $U = (\sigma + i\vec{\pi}\cdot\vec{\tau})/f_\pi\,,\ \sigma \equiv \sqrt{f_\pi^2 - \vec{\pi}^2}$, and expand $\mathcal{L}_{\chi PT}$ to order π^4, $(\partial\pi)^4$:

$$\mathcal{L} = \frac{1}{2}\,(\partial_\mu\vec{\pi})^2 - \frac{1}{2}\,m_\pi^2\vec{\pi}^2 + C_1\,(\vec{\pi}^2)^2 + C_2\,(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 + C_3\,(\partial_\mu\vec{\pi})^2(\partial_\nu\vec{\pi})^2 + C_4\,[(\partial_\mu\vec{\pi})\cdot\partial_\nu\vec{\pi}]^2$$

Similarly, in eLSM, integrate out all fields except pions, match coefficients:

F. Divotgey, P. Kovacs, F. Giacosa, DHR, EPJ A54 (2018) 5

	χPT	eLSM (tree level!)
C_1	$-M^2/(8f_\pi^2) = -0.28 \pm 1.9$	-0.268 ± 0.021
C_2 [MeV]$^{-2}$	$1/(2f_\pi^2) = (5.882 \pm 0.013)\cdot 10^{-5}$	$(5.399 \pm 0.081)\cdot 10^{-5}$
C_3 [MeV]$^{-4}$	$\ell_1/f_\pi^4 = (-5.61 \pm 0.89)\cdot 10^{-11}$	$(-9.302 \pm 0.591)\cdot 10^{-11} - \dfrac{g_3 - g_4}{4} w_{a_1}^4 Z_\pi^4$
C_4 [MeV]$^{-4}$	$\ell_2/f_\pi^4 = (2.51 \pm 0.41)\cdot 10^{-11}$	$(9.448 \pm 0.589)\cdot 10^{-11} + \dfrac{g_3}{2} w_{a_1}^4 Z_\pi^4$

χPT: $m_\pi^2 = M^2(1 + 2\ell_3 M^2/f_\pi^2)$

eLSM: results for $C_3\,,\ C_4$ for large-N_c suppressed $g_5 = g_6 = 0$

G. Ecker, J. Gasser, A. Pich, E. de Rafael, NPB 321 (1989) 311

$\Longrightarrow$ resonances saturate LECs in χPT $\Longrightarrow$ pion-loop corrections are small

$\Longrightarrow$ tree-level calculation should suffice

Low-energy limit (II)

$\Longrightarrow$ for $g_3 = g_4 = 0$: numerical values for C_3, C_4 do not agree with χPT

$\Longrightarrow$ use χPT results for C_3, C_4 to determine $g_3 = -74 \pm 33\,,\; g_4 = 5 \pm 52$

$\Longrightarrow$ compute other quantities to check consistency

$\Longrightarrow$ e.g. $\pi\pi$ scattering lengths:

$\Longrightarrow$ varying g_3, g_4 between ± 100 has small effect on $a_0^{0,2}$

$\Longrightarrow$ a_0^2 agrees well with data

$\Longrightarrow$ a_0^0 indicates influence of additional light scalar resonance

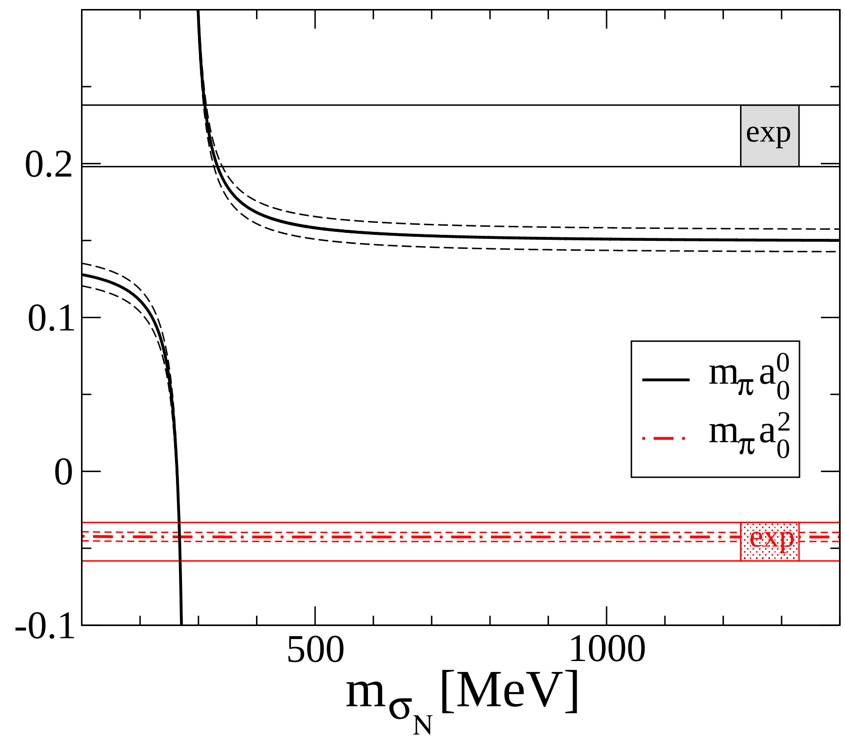
$\Longrightarrow$ $f_0(500)$!

do loop corrections spoil nice agreement at tree level?

$\Longrightarrow$ compute loops to all orders via the Functional Renormalization Group!

J. Eser, F. Divotgey, M. Mitter, DHR, in preparation

$\Longrightarrow$ see J. Eser's talk



F. Divotgey, P. Kovacs, F. Giacosa, DHR, EPJ A54 (2018) 5

Incorporating the scalar glueball

$N_f = 3$: S. Janowski, F. Giacosa, DHR, PRD 90 (2014) 11, 114005

- dilatation symmetry $\Longrightarrow$ dynamical generation of tree-level meson mass parameters through glueball/dilaton field G: $m^2 \rightarrow m^2 \left(\frac{G}{G_0}\right)^2$, $m_1^2 \rightarrow m_1^2 \left(\frac{G}{G_0}\right)^2$
- Note: analyticity prohibits operators with naive scaling dimension > 4
  (would require inverse powers of dilaton field)
  $\Longrightarrow$ effective model is complete!
- add glueball Lagrangian:

$$\mathcal{L}_G = \frac{1}{2}\,(\partial_\mu G)^2 - \frac{1}{4}\,\frac{m_G^2}{\Lambda^2}\,G^4 \left(\ln\left|\frac{G}{\Lambda}\right| - \frac{1}{4}\right)$$

$\Lambda \sim$ gluon condensate $\langle G^a_{\mu\nu} G_a^{\mu\nu}\rangle$

- shift σ_N, σ_S, and G by their v.e.v.'s, $\sigma_{N,S} \rightarrow \sigma_{N,S} + \phi_{N,S}$, $G \rightarrow G + G_0$

$\Longrightarrow$ diagonalize mass matrix

$$M \equiv \begin{pmatrix} m^2_{\sigma_N} & 2\,\lambda_1 \phi_N \phi_S & 2\,m^2 \phi_N G_0^{-1} \\ 2\,\lambda_1 \phi_N \phi_S & m^2_{\sigma_S} & 2\,m^2 \phi_S G_0^{-1} \\ 2\,m^2 \phi_N G_0^{-1} & 2 m^2 \phi_S G_0^{-1} & M_G^2 \end{pmatrix}$$

- χ^2–fit of Λ, λ_1, h_1, m_G, ϵ_S ($\chi^2/\mathrm{d.o.f.} = 0.35$)

$\Longrightarrow$ mixing matrix:

$f_0(1370)$:	$83\%\ \sigma_N$	$6\%\ \sigma_S$	$11\%\ G$
$f_0(1500)$:	$9\%\ \sigma_N$	$88\%\ \sigma_S$	$3\%\ G$
$f_0(1710)$:	$8\%\ \sigma_N$	$6\%\ \sigma_S$	$86\%\ G$

Low-lying scalars

$N_f = 3$: tetraquarks (either $[qq][\bar{q}\bar{q}]$ or $(\bar{q}q)(\bar{q}q)$ configuration) form nonet (just as $\Phi \sim \bar{q}q$)

D. Black, A.H. Fariborz, F. Sannino, J. Schechter, PRD 59 (1999) 074026,

$N_f = 2$: single scalar-isoscalar state $\chi \implies f_0(500)$!!

- incorporate χ as "interpolating field" in the eLSM Lagrangian

  P. Lakaschus, J. Mauldin, F. Giacosa, DHR, in preparation

$$\mathcal{L}_\chi = \frac{1}{2}\left(\partial_\mu \chi \partial^\mu \chi - m_\chi^2 \frac{G^2}{G_0^2}\chi^2\right) + g_\chi \frac{G}{G_0}\chi\left(\sigma^2 + \vec{\pi}^{\,2} - \eta^2 - \vec{a}_0^{\,2}\right)$$
$$+ g_{AV}\frac{G}{G_0}\chi\left(\vec{\rho}_\mu^{\,2} + \vec{a}_{1,\mu}^{\,2} - \omega_\mu^2 - f_{1,\mu}^2\right)$$

- set large–N_c suppressed $\lambda_1 = h_1 = 0$
- express m^2, c, λ_2, g_1, g_2 , h_3, m_1^2 by experimental masses and decay widths
- perform χ^2–fit of h_2, M_G, G_0, m_χ, g_χ, g_{AV} ($\chi^2_{\rm red} = 1.5$)

$\implies$ $m_\pi a_0^0 = 0.207 \pm 0.016$ (exp.: 0.218 ± 0.02)

$m_\pi a_0^2 = -0.028 \pm 0.005$ (exp.: -0.046 ± 0.016)

$\implies$ reasonable description of $\pi\pi$ scattering lengths!

$\implies$ mixing matrix:

$f_0(500)$:	$100\%\,\chi$	$0\%\,\sigma$	$0\%\,G$
$f_0(1370)$:	$0\%\,\chi$	$86\%\,\sigma$	$14\%\,G$
$f_0(1710)$:	$0\%\,\chi$	$14\%\,\sigma$	$86\%\,G$

Baryons and their chiral partners

Inclusion of baryons and their chiral partners ($N_f = 2$):

$\Longrightarrow$ Mirror assignment: C. DeTar and T. Kunihiro, PRD 39 (1989) 2805

$$\Psi_{1,r} \to U_r\, \Psi_{1,r}\,, \quad \Psi_{1,\ell} \to U_\ell\, \Psi_{1,\ell}\,, \quad \text{but: } \Psi_{2,r} \to U_\ell\, \Psi_{2,r}\,, \quad \Psi_{2,\ell} \to U_r\, \Psi_{2,\ell}$$

$\Longrightarrow$ new, chirally invariant mass term:

$$\mathcal{L}_B = \bar\Psi_{1,\ell}\, i\partial\!\!\!/\, \Psi_{1,\ell} + \bar\Psi_{1,r}\, i\partial\!\!\!/\, \Psi_{1,r} + \bar\Psi_{2,\ell}\, i\partial\!\!\!/\, \Psi_{2,\ell} + \bar\Psi_{2,r}\, i\partial\!\!\!/\, \Psi_{2,r} + m_0 \left(\bar\Psi_{2,\ell}\, \Psi_{1,r} - \bar\Psi_{2,r}\, \Psi_{1,\ell} - \bar\Psi_{1,\ell}\, \Psi_{2,r} + \bar\Psi_{1,r}\, \Psi_{2,\ell} \right)$$

Yukawa interaction:

$$\mathcal{L}_{SB} = -\hat g_1 \left(\bar\Psi_{1,\ell}\, \Phi\, \Psi_{1,r} + \bar\Psi_{1,r}\, \Phi^\dagger\, \Psi_{1,\ell} \right) - \hat g_2 \left(\bar\Psi_{2,r}\, \Phi\, \Psi_{2,\ell} + \bar\Psi_{2,\ell}\, \Phi^\dagger\, \Psi_{2,r} \right)$$

$\Longrightarrow$ mass eigenstates:

$$\begin{pmatrix} N \\ N^\star \end{pmatrix} \equiv \begin{pmatrix} N^+ \\ N^- \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5\, e^{-\delta/2} \\ \gamma_5\, e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}\,, \quad \sinh\delta = \frac{\phi}{4\, m_0}(\hat g_1 + \hat g_2)$$

$$m_\pm = \sqrt{m_0^2 + \frac{\phi^2}{16}(\hat g_1 + \hat g_2)^2} \pm \frac{\phi}{4}(\hat g_1 - \hat g_2) \quad \longrightarrow \quad m_0 \quad (\phi \to 0)$$

chiral symmetry restoration: chiral partners become degenerate, but not necessarily massless!

Vector – baryon interactions

$$\mathcal{L}_{VB} = c_1 \left(\bar{\Psi}_{1,\ell} \not{\mathcal{L}} \Psi_{1,\ell} + \bar{\Psi}_{1,r} \not{\mathcal{R}} \Psi_{1,r}\right) + c_2 \left(\bar{\Psi}_{2,\ell} \not{\mathcal{R}} \Psi_{2,\ell} + \bar{\Psi}_{2,r} \not{\mathcal{L}} \Psi_{2,r}\right)$$

Note: in general $c_1 \neq c_2$

$\Longrightarrow$ allows to fit axial coupling constants:

$$g_A = +\tanh\delta \left[1 - \frac{c_1 + c_2}{2\,g_1}\left(1 - \frac{1}{Z^2}\right)\right] - \frac{c_1 - c_2}{2\,g_1}\left(1 - \frac{1}{Z^2}\right)$$
$$g_A^\star = -\tanh\delta \left[1 - \frac{c_1 + c_2}{2\,g_1}\left(1 - \frac{1}{Z^2}\right)\right] - \frac{c_1 - c_2}{2\,g_1}\left(1 - \frac{1}{Z^2}\right) \neq -g_A\,!$$

$\Longrightarrow$ for $c_1 \neq c_2$ compatible with $g_A \simeq 1.26\,,\; g_A^\star \simeq 0\,!$

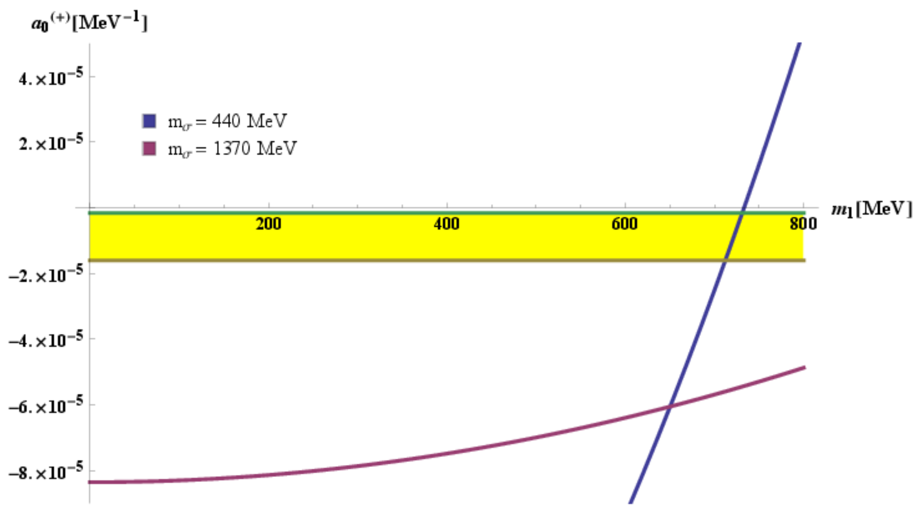
T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

T. Maurer, T. Burch, L.Ya. Glozman, C.B. Lang, D. Mohler, A. Schäfer, arXiv:1202.2834[hep-lat]

Vacuum phenomenology: The chiral partner of the nucleon (I)

S. Gallas, F. Giacosa, DHR, PRD 82 (2010) 014004

- determine m_0, c_1, c_2, $\hat{g}_1$, $\hat{g}_2$ through χ^2–fit to
  $M_N = 940$ MeV, $M_{N^\star} = 1535$ MeV, $\Gamma(N^\star \to N\pi) = (67.5 \pm 23.6)$ MeV,
  $g_A = 1.267 \pm 0.004$, and $g_A^\star = 0.2 \pm 0.3$ T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503
- test validity through comparison to
  - πN scattering lengths:



$\Longrightarrow$ $a_0^{(+)}$ requires a light σ!

$a_0^{(-)} = (6.04 \pm 0.63) \cdot 10^{-4}$ MeV^{-1}

exp.: $(6.4 \pm 0.1) \cdot 10^{-4}$ MeV^{-1}

$\Longrightarrow$ $a_0^{(-)}$ well reproduced

  - decay width $\Gamma(N^\star \to N\eta) = (10.9 \pm 3.8)$ MeV

However: exp.: (78.7 ± 24.3) MeV!

Vacuum phenomenology: The chiral partner of the nucleon (II)

Inclusion of $f_0(500)$ and $f_0(1710)$: P. Lakaschus, J. Mauldin, F. Giacosa, DHR, in preparation

$\Longrightarrow$ mass parameter m_0 generated by interaction Lagrangian:

$$\mathcal{L}_{\chi GN} = -\left[a\chi + bG + c_N\left(\det\Phi + \det\Phi^\dagger\right)\right]\left(\bar{\Psi}_{2,\ell}\,\Psi_{1,r} - \bar{\Psi}_{2,r}\,\Psi_{1,\ell} - \bar{\Psi}_{1,\ell}\,\Psi_{2,r} + \bar{\Psi}_{1,r}\,\Psi_{2,\ell}\right)$$

and condensation $\chi \to \chi_0\,,\ G \to G_0\,,\ \sigma \to \phi$:

$$m_0 = a\chi_0 + bG_0 + \frac{c_N}{2}\phi^2$$

$\Longrightarrow$ new contributions of χ and G to πN scattering lengths:

$\Longrightarrow$ $m_\pi\, a_0^{(+)} = -0.0016$ (exp.: -0.0012 ± 0.0010)

$\Longrightarrow$ description of $a_0^{(+)}$ considerably improved!

Extension to $N_f = 3$ and four baryon multiplets (I)

L. Olbrich, M. Zetenyi, F. Giacosa, DHR, PRD 93 (2016) 3, 034021

assume baryons to be $q[qq]$ composites $\Longrightarrow$ $B \in (N_f, N_f^*)$:

$$B = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

scalar diquark fields with definite parity:

$$J^P = 0^+ : \qquad \mathcal{D}_{ij} = \frac{1}{\sqrt{2}} \left(q_j^T C \gamma^5 q_i - q_i^T C \gamma^5 q_j \right) \equiv \sum_{k=1}^{3} D_k \epsilon_{kij}$$

$$J^P = 0^- : \qquad \tilde{\mathcal{D}}_{ij} = \frac{1}{\sqrt{2}} \left(q_j^T C q_i - q_i^T C q_j \right) \equiv \sum_{k=1}^{3} \tilde{D}_k \epsilon_{kij}$$

$\Longrightarrow$ right- and left-handed diquark fields:

$$D_k^{r(\ell)} \equiv \frac{1}{\sqrt{2}} \left(\tilde{D}_k \pm D_k \right)$$

$\Longrightarrow$ under chiral transformations:

$$D_k^{r(\ell)} \longrightarrow D_k^{r(\ell)} U_{r(\ell)}^\dagger$$

Extension to $N_f = 3$ and four baryon multiplets (II)

$\Longrightarrow$ right-and left-handed matrix-valued baryon fields $N_{1r(\ell)}$, $N_{2r(\ell)}$:

$$\left(N_{1r(\ell)}\right)_{ij} \equiv D_j^r q_{ir(\ell)} \;, \qquad \left(N_{2r(\ell)}\right)_{ij} \equiv D_j^\ell q_{ir(\ell)}$$

$\Longrightarrow$ under chiral transformations:

$$N_{1r} \longrightarrow U_r N_{1r} U_r^\dagger \;, \quad N_{1\ell} \longrightarrow U_\ell N_{1\ell} U_r^\dagger \;, \quad N_{2r} \longrightarrow U_r N_{2r} U_\ell^\dagger \;, \quad N_{2\ell} \longrightarrow U_\ell N_{2\ell} U_\ell^\dagger$$

$\Longrightarrow$ right-and left-handed "mirror" baryon fields $M_{1r(\ell)}$, $M_{2r(\ell)}$:

$$\left(M_{1r(\ell)}\right)_{ij} \equiv D_j^r \, \partial\!\!\!/ \, q_{ir(\ell)} \;, \qquad \left(M_{2r(\ell)}\right)_{ij} \equiv D_j^\ell \, \partial\!\!\!/ \, q_{ir(\ell)}$$

$\Longrightarrow$ under chiral transformations:

$$M_{1r} \longrightarrow U_\ell M_{1r} U_r^\dagger \;, \quad M_{1\ell} \longrightarrow U_r M_{1\ell} U_r^\dagger \;, \quad M_{2r} \longrightarrow U_\ell M_{2r} U_\ell^\dagger \;, \quad M_{2\ell} \longrightarrow U_r M_{2\ell} U_\ell^\dagger$$

$\Longrightarrow$ form linear combinations with definite positive/negative parity:

$$B_N = \frac{1}{\sqrt{2}}(N_1 - N_2) \;, \quad B_{N\star} = \frac{1}{\sqrt{2}}(N_1 + N_2) \;, \quad B_M = \frac{1}{\sqrt{2}}(M_1 - M_2) \;, \quad B_{M\star} = \frac{1}{\sqrt{2}}(M_1 + M_2)$$

$\Longrightarrow$ assignment to physical particles (zero-mixing limit):

$B_N :\ \{N(939), \Lambda(1116), \Sigma(1193), \Xi(1338)\}$, $B_M : \{N(1440), \Lambda(1600), \Sigma(1620), \Xi(1690)\}$,
$B_{N\star} : \{N(1535), \Lambda(1670), \Sigma(1620), \Xi(?)\}$, $B_{M\star} : \{N(1650), \Lambda(1800), \Sigma(1750), \Xi(?)\}$.

$\Longrightarrow$ construct chirally invariant Lagrangian, reduce it to $N_f = 2$:
$N(939)$, $N(1440)$, $N(1535)$, $N(1650)$

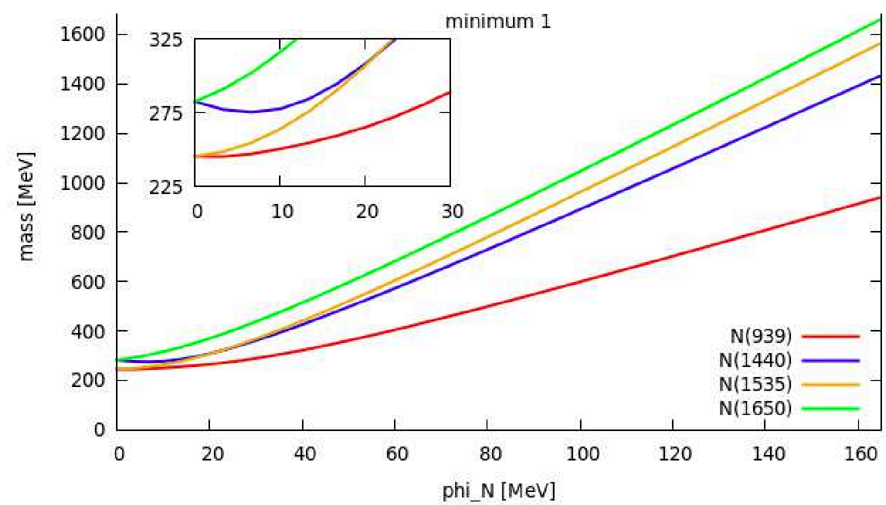
Extension to $N_f = 3$ and four baryon multiplets (III)

$\Longrightarrow$ χ^2–fit of 12 parameters to 13 experimental quantities:

	our results [GeV]	experiment [GeV]
m_N	0.9389 ± 0.001	0.9389 ± 0.001
$m_{N(1440)}$	1.430 ± 0.0713	1.43 ± 0.0715
$m_{N(1535)}$	1.561 ± 0.0668	1.53 ± 0.0765
$m_{N(1650)}$	1.657 ± 0.0721	1.65 ± 0.087
$\Gamma_{N(1440)\to N\pi}$	0.1948 ± 0.0870	0.195 ± 0.087
$\Gamma_{N(1535)\to N\pi}$	0.0722 ± 0.0188	0.0675 ± 0.0183
$\Gamma_{N(1535)\to N\eta}$	0.0055 ± 0.0026	0.063 ± 0.0183
$\Gamma_{N(1650)\to N\pi}$	0.1121 ± 0.0331	0.105 ± 0.0366
$\Gamma_{N(1650)\to N\eta}$	0.0117 ± 0.0038	0.015 ± 0.008

	our results	experiment/lattice
g_A^N	1.267 ± 0.0025	1.267 ± 0.0025
$g_A^{N(1440)}$	1.2 ± 0.2	1.2 ± 0.2
$g_A^{N(1535)}$	0.2 ± 0.3	0.2 ± 0.3
$g_A^{N(1650)}$	0.5494 ± 0.2	0.55 ± 0.2

Masses as function of φ_N:



mixing matrix:

$$\begin{pmatrix} N(939) \\ N(1535) \\ N(1440) \\ N(1650) \end{pmatrix} = \begin{pmatrix} -0.99 & -0.01 & -0.02 & 0.10 \\ 0.10 & -0.49 & -0.04 & 0.87 \\ -0.01 & 0.09 & 0.99 & 0.10 \\ -0.04 & -0.87 & 0.12 & -0.48 \end{pmatrix} \begin{pmatrix} N \\ N_\star \\ M \\ M_\star \end{pmatrix}$$

$\Longrightarrow$ chiral partners:

$$N(939) \longleftrightarrow N(1535)$$
$$N(1440) \longleftrightarrow N(1650)$$

$\Longrightarrow$ mixing matrix:

$$N(939) \to N\,, \quad N(1535) \to M^\star$$
$$N(1440) \to M\,, \quad N(1650) \to N^\star$$

$U(1)_A$ anomaly and $N(1535) \to N\eta$ decay

L. Olbrich, M. Zetenyi, F. Giacosa, DHR, PRD 97 (2018) 014007

$N_f = 2$: $\det\Phi - \det\Phi^\dagger = -i(\sigma_N \eta_N - \vec{a}_0 \cdot \vec{\pi})$ is parity-odd, $U(1)_A$ violating

$\bar{\Psi}_{2,\ell}\,\Psi_{1,r} + \bar{\Psi}_{2,r}\,\Psi_{1,\ell} - \bar{\Psi}_{1,\ell}\,\Psi_{2,r} - \bar{\Psi}_{1,r}\,\Psi_{2,\ell}$ is parity-odd

$\Longrightarrow$ $$\mathcal{L}_A = \lambda_A \left(\det\Phi - \det\Phi^\dagger\right)\left(\bar{\Psi}_{2,\ell}\,\Psi_{1,r} + \bar{\Psi}_{2,r}\,\Psi_{1,\ell} - \bar{\Psi}_{1,\ell}\,\Psi_{2,r} - \bar{\Psi}_{1,r}\,\Psi_{2,\ell}\right)$$

is parity-even, $U(1)_A$ violating

$\Longrightarrow$ SSB: direct coupling $N(1535)N\eta$

$\Longrightarrow$ adjust λ_A to reproduce $\Gamma(N(1535) \to N\eta)$!

$N_f = 3$: $$\mathcal{L}_A = \lambda_A \left(\det\Phi - \det\Phi^\dagger\right) \mathrm{Tr}\left(\bar{B}_{M\star} B_N - \bar{B}_N B_{M\star} - \bar{B}_{N\star} B_M + \bar{B}_M B_{N\star}\right)$$

$\Longrightarrow$ adjust λ_A to reproduce $\Gamma(N(1535) \to N\eta)$

$\Longrightarrow$ predict $\Gamma(\Lambda(1670) \to \Lambda\eta) = 5.1^{+2.7}_{-2.1}$ MeV

(exp.: (7.5 ± 5) MeV)

Conclusions and Outlook

I. extended Linear Sigma Model (eLSM) with $U(N_f)_r \times U(N_f)_\ell$ symmetry, containing scalar and vector mesons and their chiral partners

II. Vacuum phenomenology:

1. Excellent fit of mesonic vacuum properties for $N_f = 3$
2. Correct low-energy limit of QCD:
   resonance-saturation mechanism (cf. χPT) seems to work also for eLSM
   pion-loop corrections still need to be computed via FRG
3. Scalar-meson puzzle:
   evidence for dominant four-quark component for the light scalar mesons
   glueball is most likely (predominantly) $f_0(1710)$
4. Including $f_0(500)$ as an effective d.o.f. improves description of $\pi\pi$ and πN scattering lengths
   extension to $N_f = 3 \Longrightarrow$ consider whole light scalar nonet
5. Chiral partners: $N(939) \longleftrightarrow N(1535)$, $N(1440) \longleftrightarrow N(1650)$
6. Anomalously large $\Gamma(N(1535) \rightarrow N\eta)$ is most likely due to axial anomaly