Mass Sensitivity of the QCD Phase Structure

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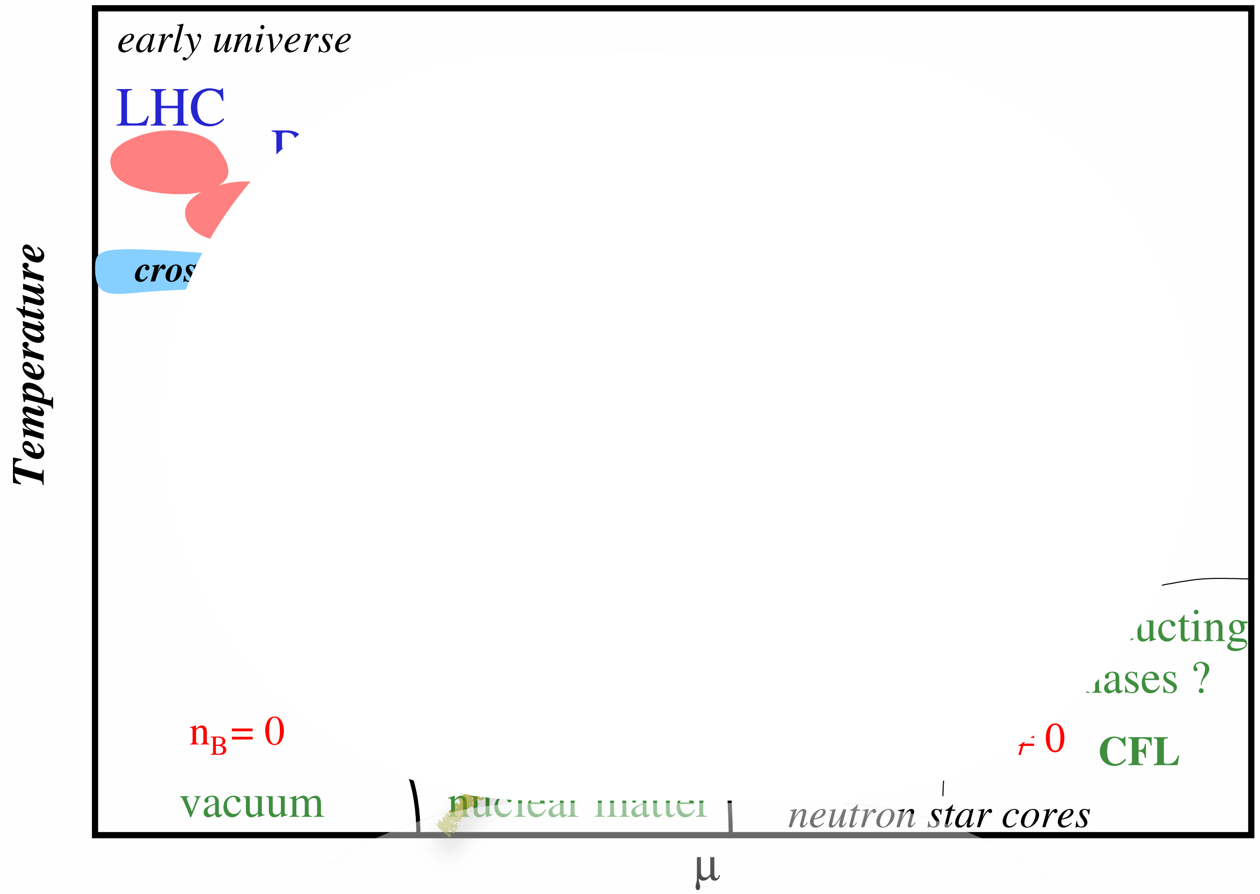
HIC for FAIR
Helmholtz International Center

Germany

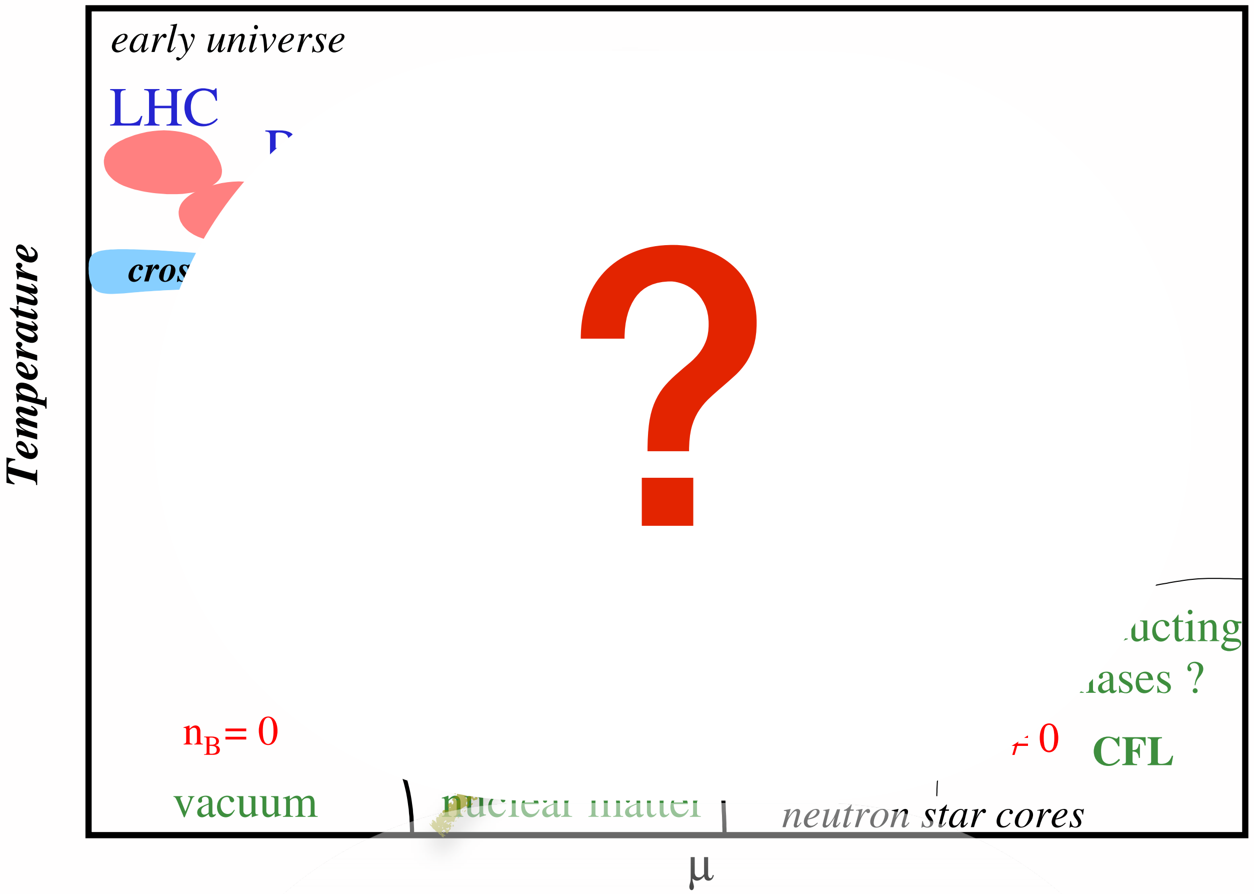
April 6th, 2018

666. WE-Heraeus-Seminar
From correlation functions to QCD phenomenology
03rd of April 2018 - 06th of April 2018
Physikzentrum Bad Honnef, Germany

QC₃D phase structure



QC₃D phase structure

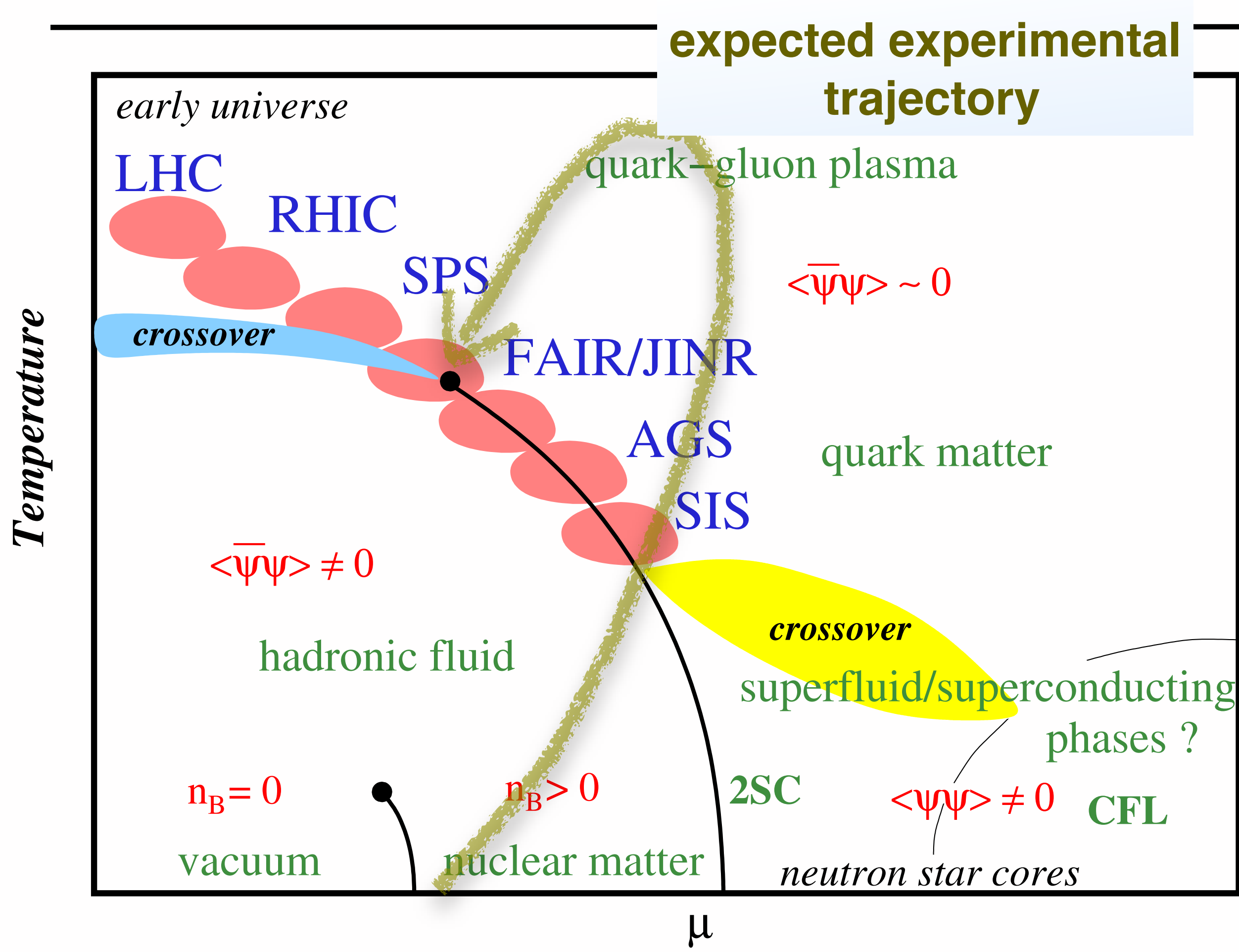


vacuum/nuclear matter/transition &

only corners of the phase diagram are known

from “first principles”

conjectured QC$_3$D phase structure



knowledge so far

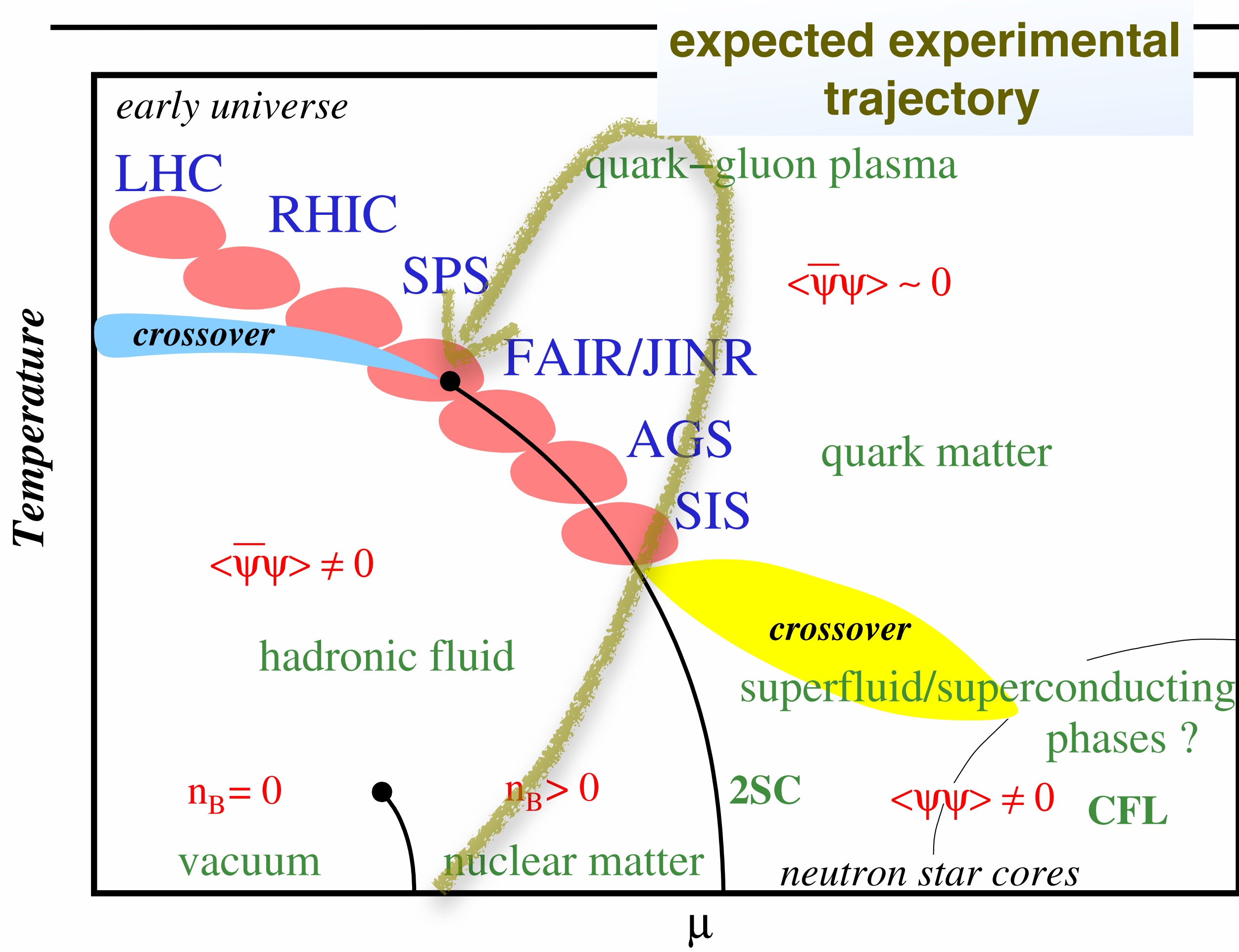
mostly based on model calculations

assumptions:

equilibrium, homogeneous phases,

infinite volume, ...

conjectured QC_3D phase structure



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Open/unclear questions:

- **CEP:** existence/location/number
- relation between chiral & deconfinement?

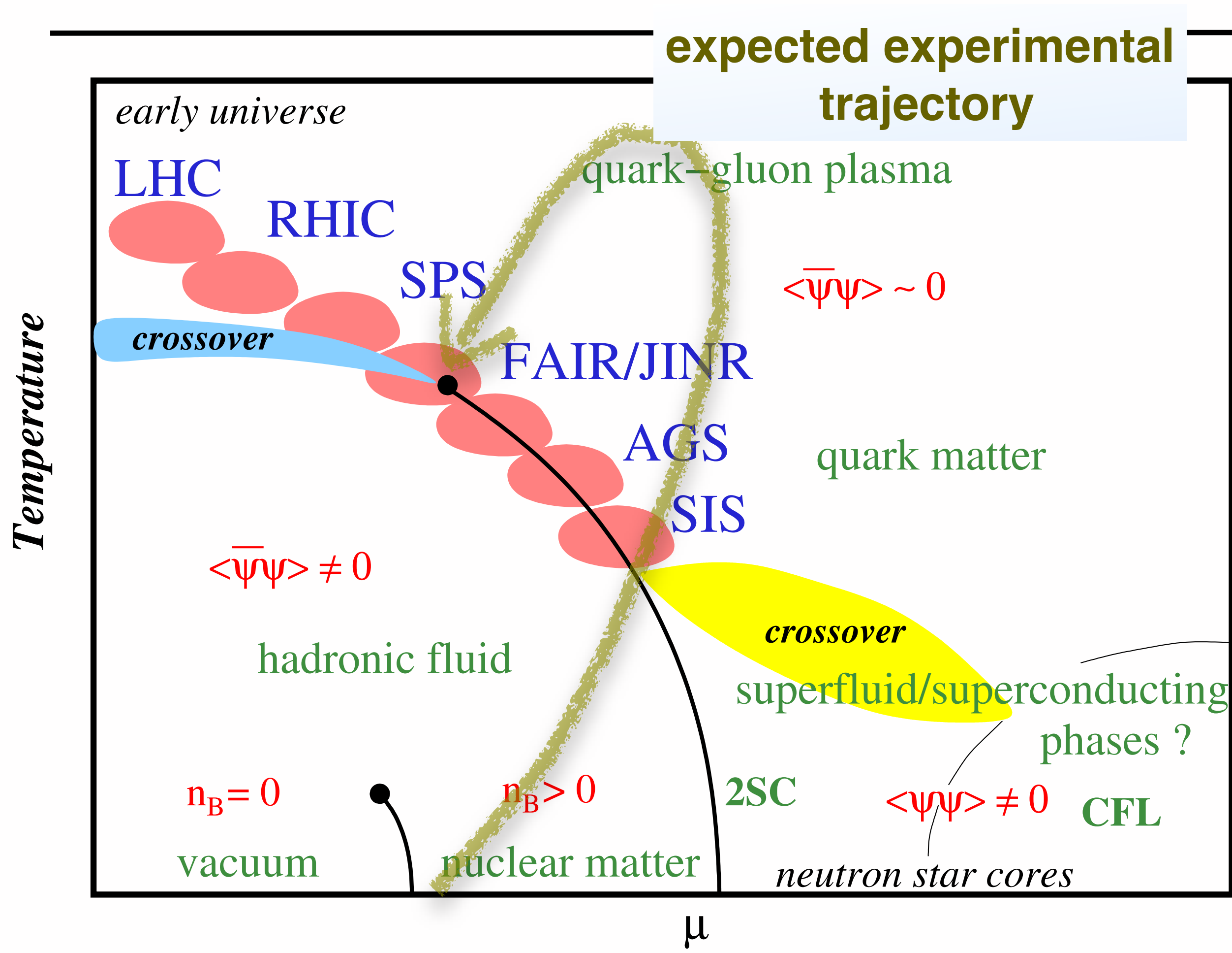
  chiral ⇔ deconfinement CEP?
- **Quarkyonic phase:** coincidence of both transitions at μ=0 & μ>0?
- **inhomogeneous phases?** ➜ more favored?
- **axial anomaly restoration** around chiral transition?
- **finite volume effects?** ➜ lattice comparison/ influence boundary conditions
- **role of fluctuations?** so far mostly Mean-Field results

  ➜ effects of fluctuations important

  examples: size of crit reg. around CEP
- **What are good experimental signatures?**

  ➜ cumulants?

conjectured QC$_3$D phase structure



Theory:

➜ Lattice: but simulations restricted to small μ

➜ Models: effective theories parameter dependency

➜ Functional QFT methods: FRG,DSE, nPI

Experiment: (non-equilibrium? ➜ most likely thermal equilibrium)

➜ in a finite box (HBT radii: freeze-out vol. ~ 5000 fm^3)

(UrQMD ($\sqrt{s}$): system vol. ~ 50 - 250 fm^3)

knowledge so far

mostly based on model calculations

assumptions:

equilibrium, homogeneous phases,

infinite volume, ...

Theoretical aim:

deeper understanding & more realistic HIC description

➜ existence of critical end point(s)?

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Agenda

- **Role of Fluctuations:**

  from mean-field approximations to the FRG

- **Columbia plot**

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  from mean-field approximations to the FRG

- **Columbia plot**

complementary to NICA

FAIR
Facility for Antiproton and Ion Research in Europe GmbH

2016

March 2018

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Agenda

- **Role of Fluctuations:**

  from mean-field approximations to the FRG

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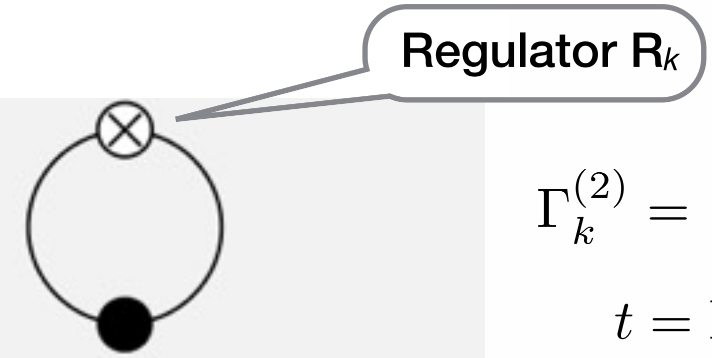
FRG and QCD

- **FRG (average effective action)** [Wetterich 1993]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$

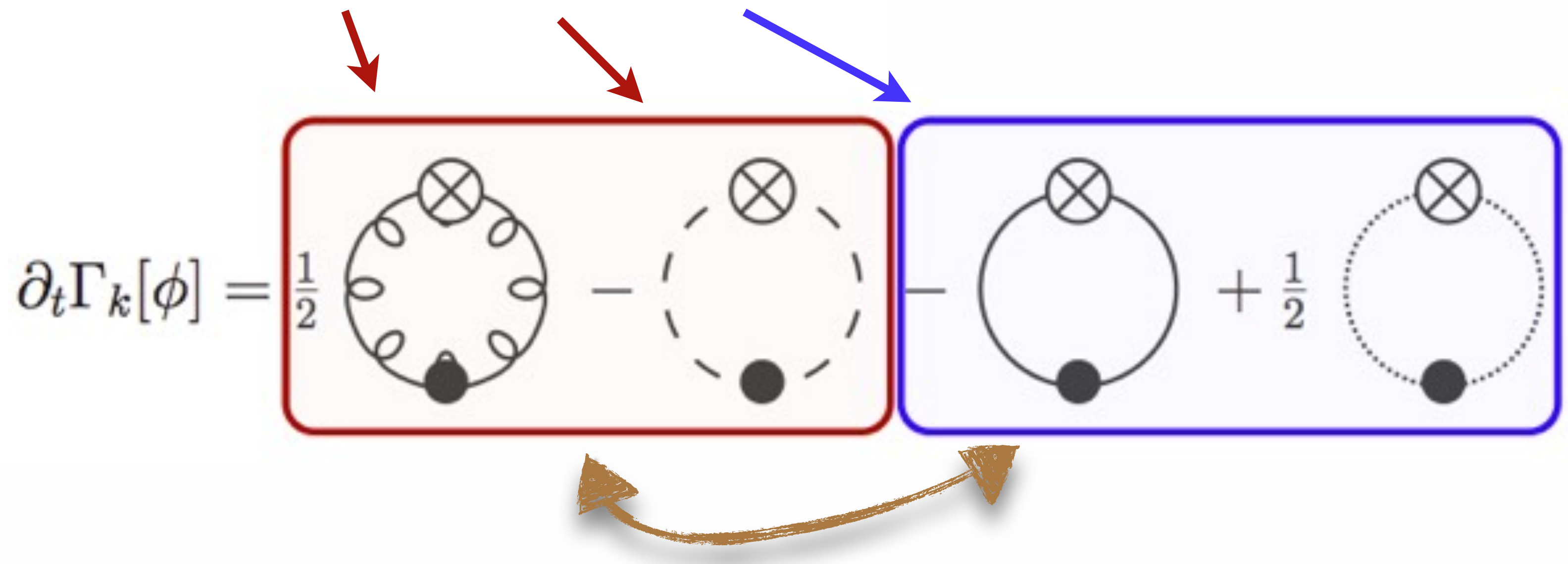


$$\Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

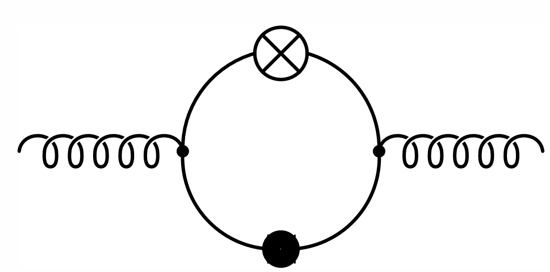
$$t = \ln(k/\Lambda)$$

- **full dynamical QCD flow:**

fluctuations of **gluon**, **ghost**, **quark** and (via hadronization) **meson** [Pawlowski et al. 2009....]



in presence of **dynamical quarks**:
gluon propagator is modified



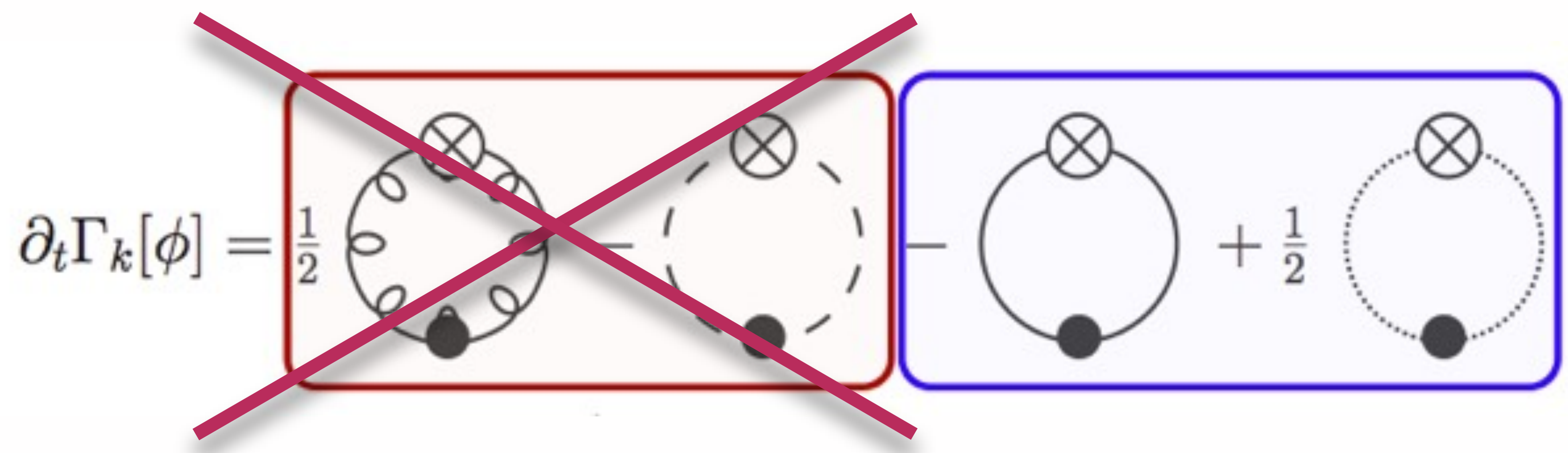
pure Yang Mills flow + matter back-coupling

FRG: quark-meson truncation

chiral phase transition:

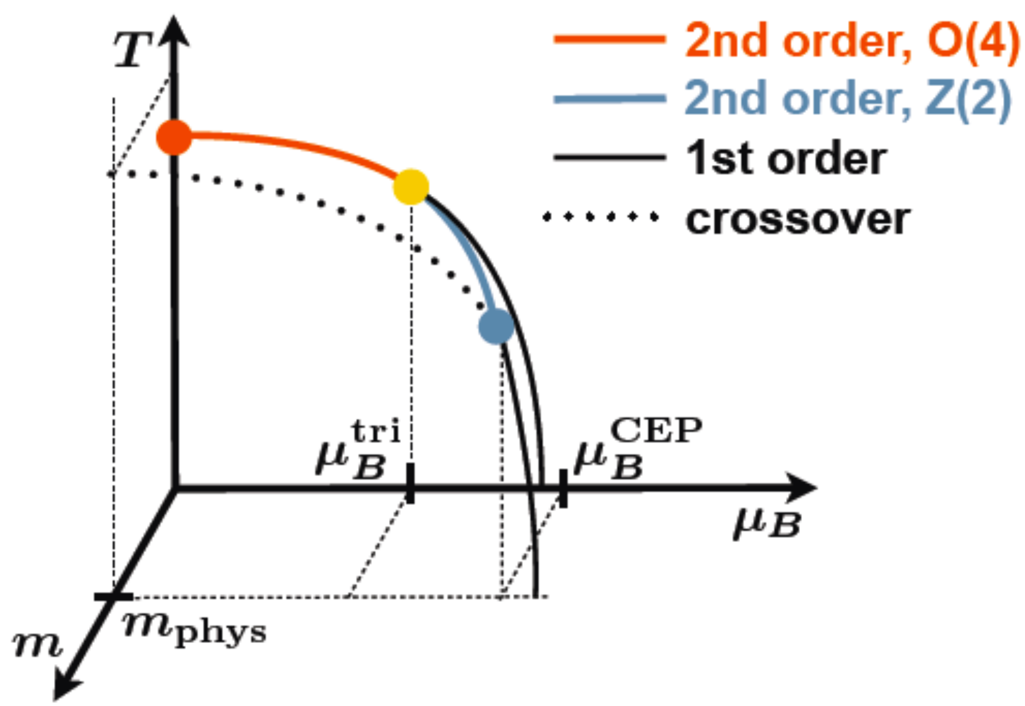
➜ cf. talk by Wei-jie Fu and many others

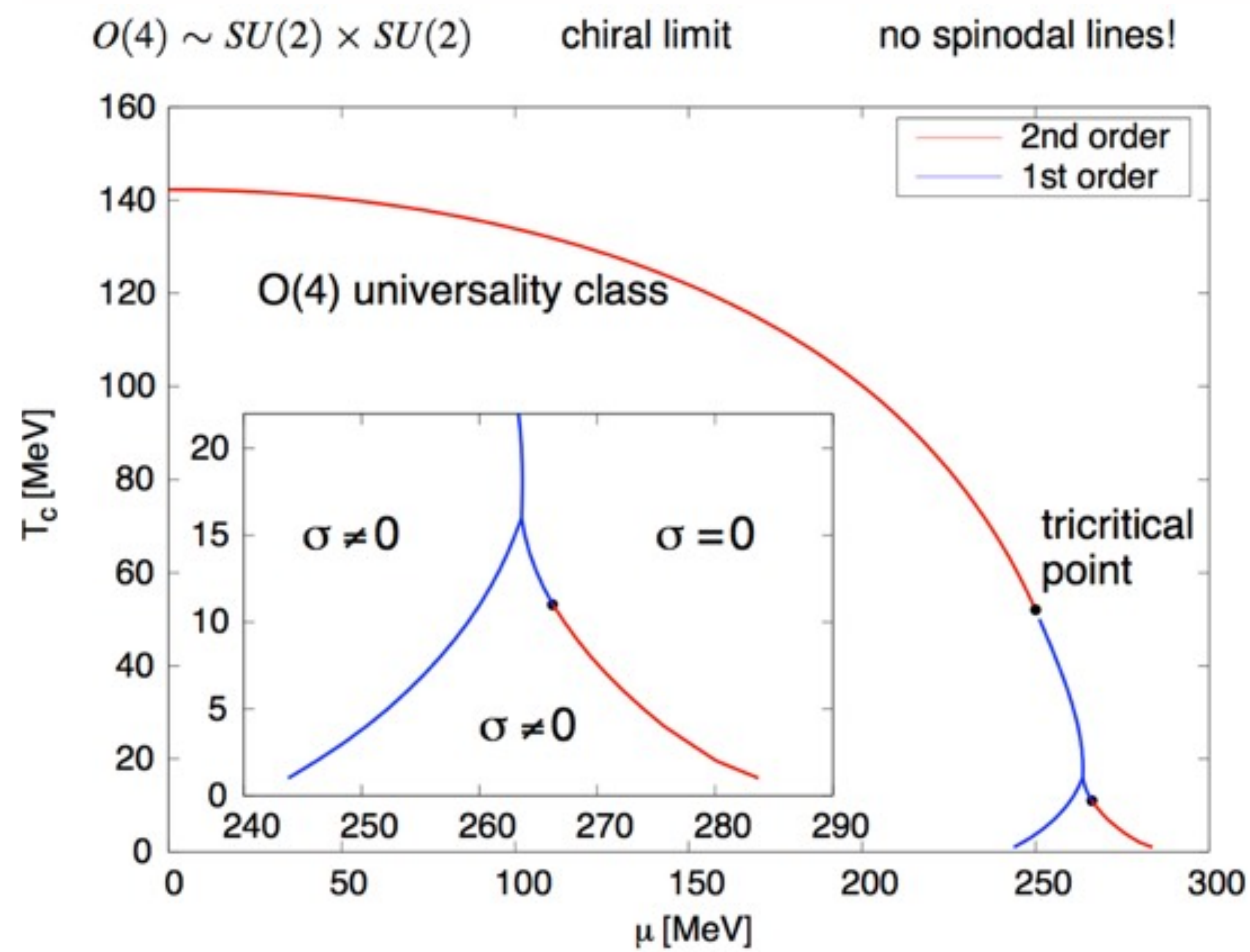
flow for **quark-meson** model truncation:
neglect
YM contributions and bosonic fluctuations



without bosonic fluctuations:
extended Mean-field approximation

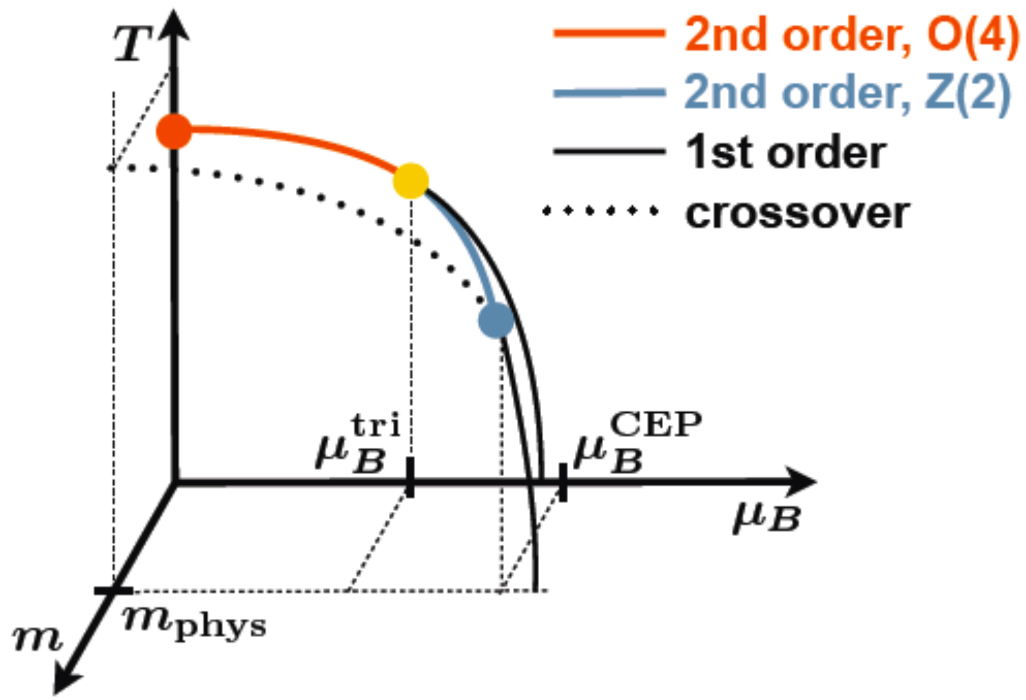
Phase diagram N_f=2 QM

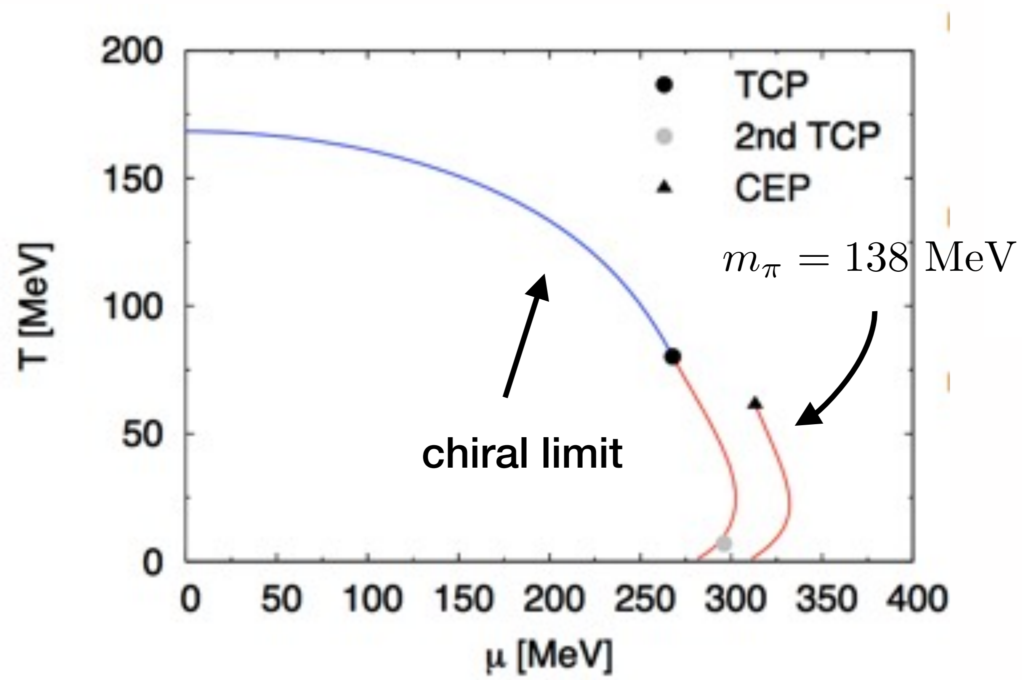


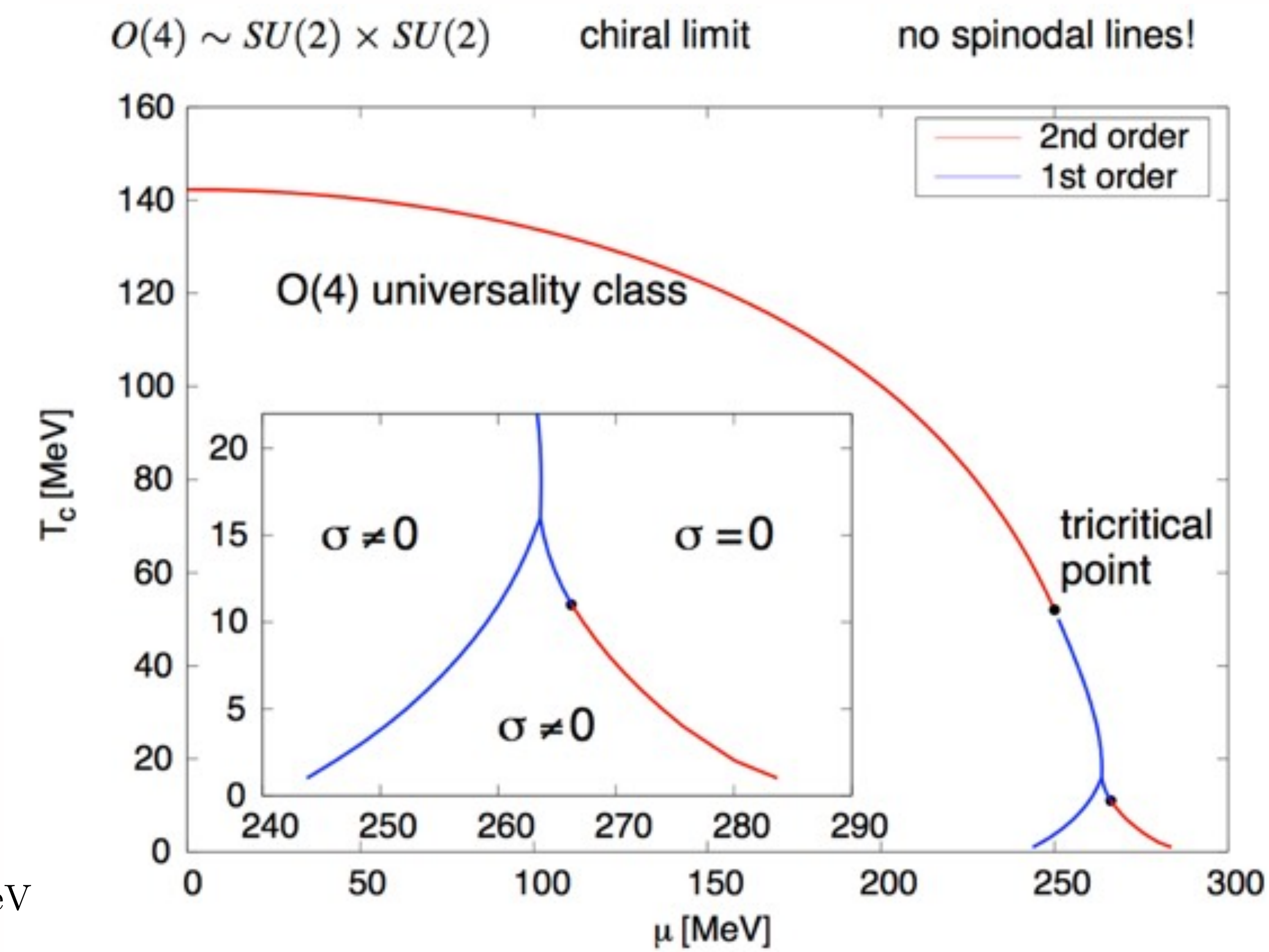


[BJS, J Wambach 2005]

Phase diagram N_f=2 QM







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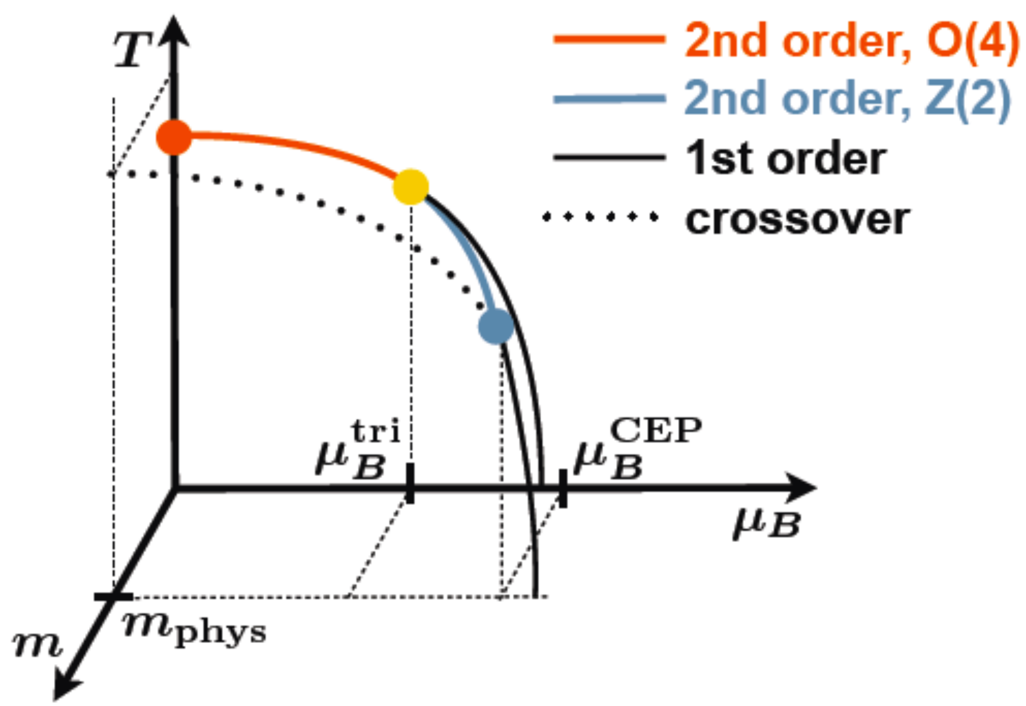
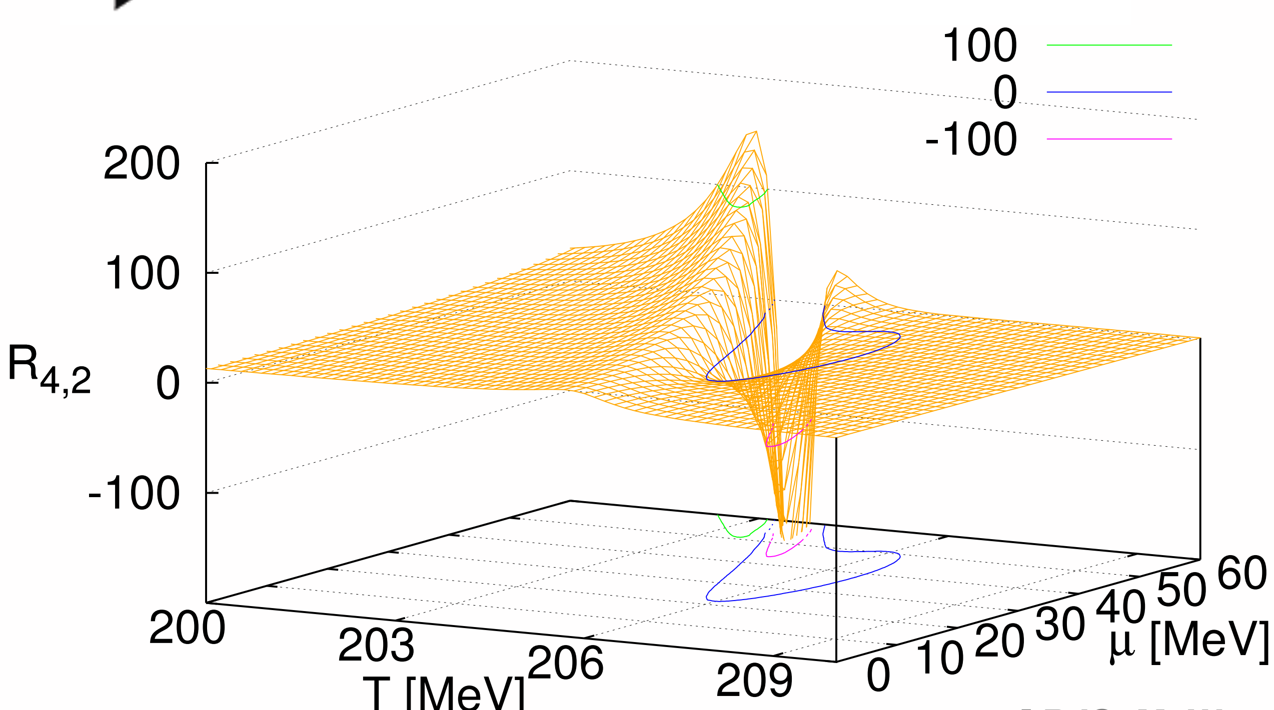
Phase diagram N_f=2 QM





[BJS, M. Wagner 2012]

Fluctuations of oder parameter $\to \infty$ at 2nd order transition

critical fluctuations ➜ phase boundary?

How can we probe a transition? ➜ cumulants

Taylor expansion: **.... more sensitive to criticality**

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n$$

$$c_n(T) = \frac{1}{n!} \frac{\partial^n p(T,\mu)/T^4}{\partial(\mu/T)^n}$$

➜ cf. talk by Marlene Nahrgang et al.

■ singular behaviour in $\dfrac{\partial^n p(X)}{\partial X^n}$ with $X = T, \mu, \ldots$

example: Kurtosis

➜ $R_{4,2} = c_4/c_2$ negative region

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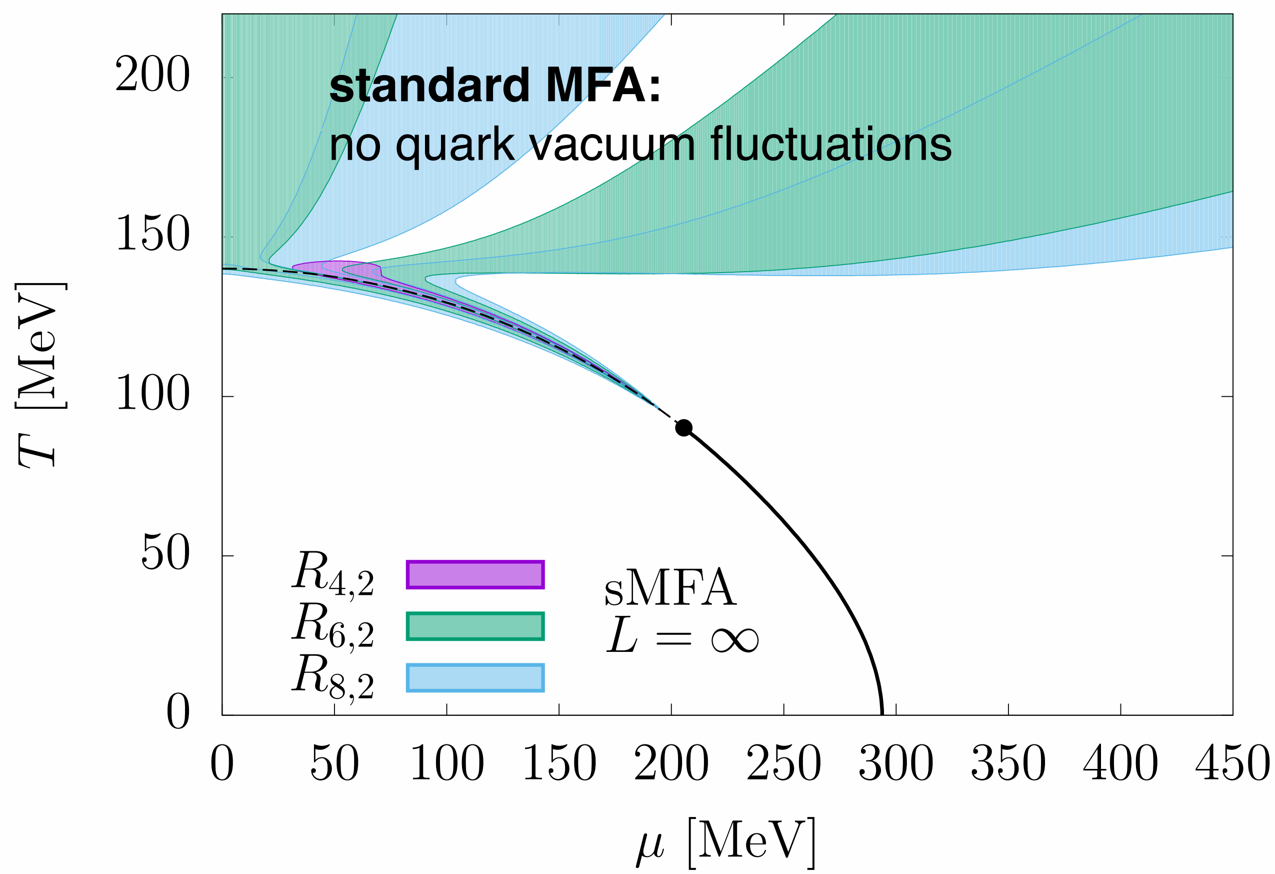
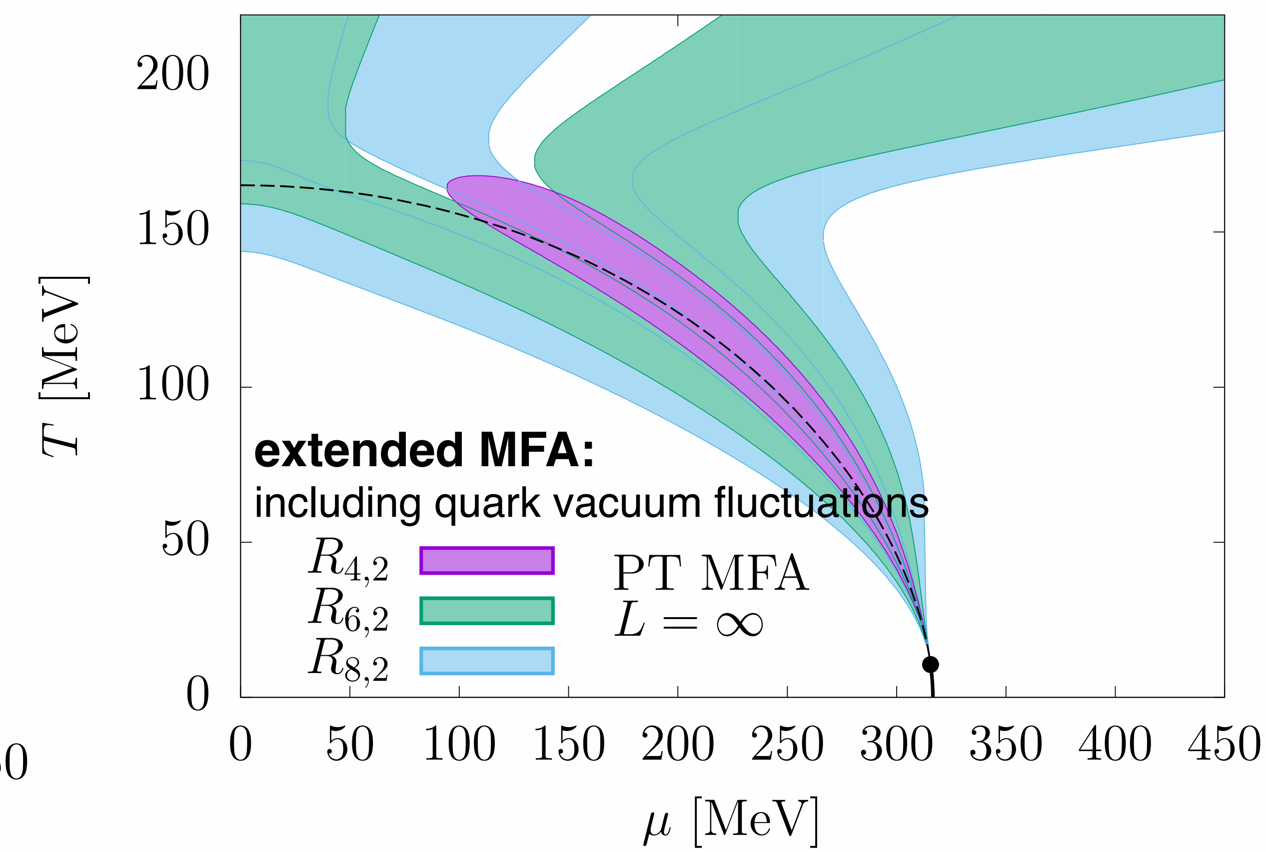
Higher cumulants

[S. Resch, BJS to be published]

infinite volume: influence of fluctuations





findings:

(quark) fluctuations pushes CEP to smaller T and bigger µ

Fluctuations wash out phase transition ➜ broader negative regions

Higher cumulants

[S. Resch, BJS to be published]

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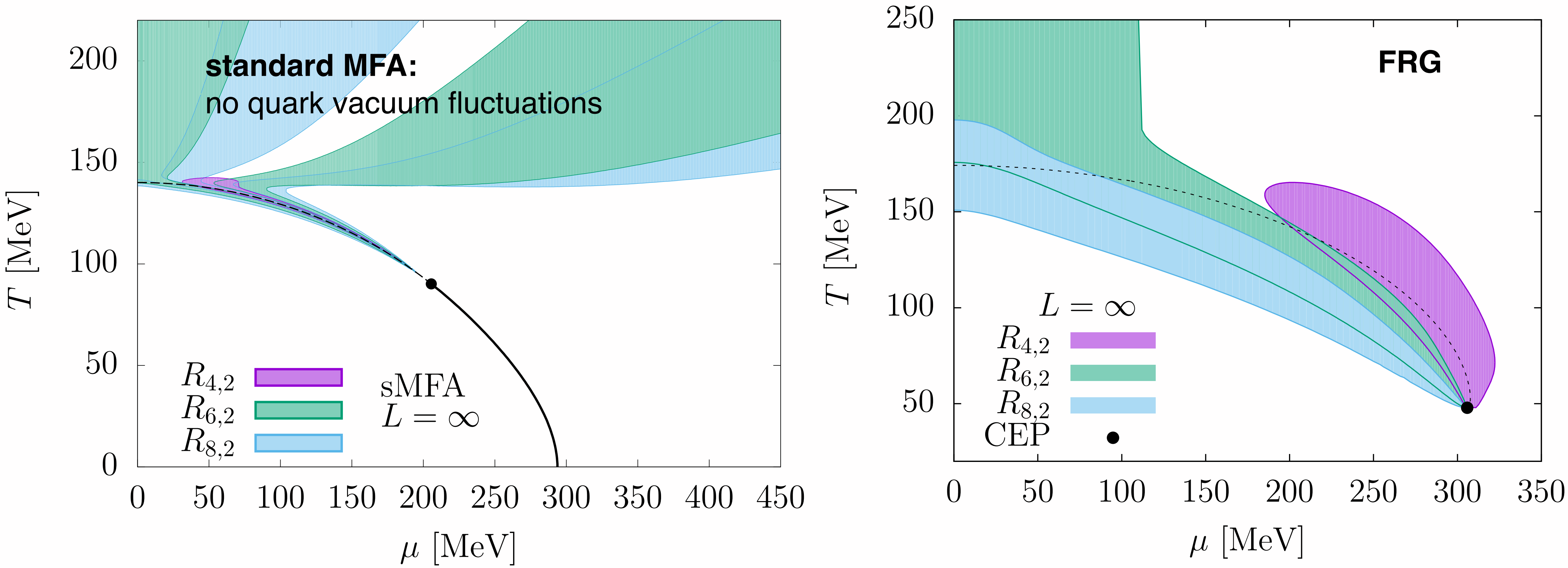


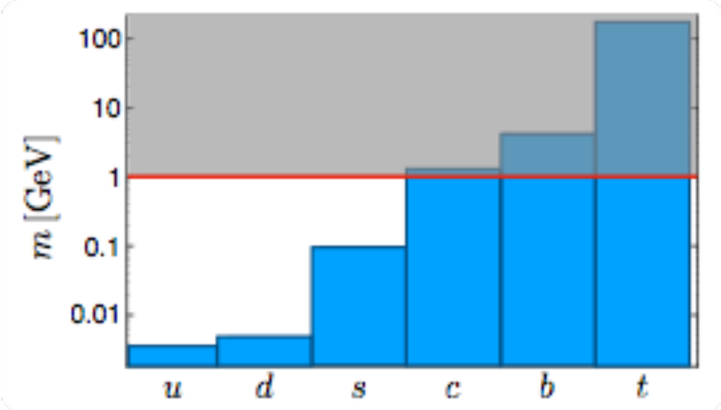
findings:

(quark) fluctuations pushes CEP to smaller T and bigger µ

Fluctuations wash out phase transition ➜ broader negative regions

$N_f = 2+1$ quark flavor

$$k \lesssim 2\pi T_c \approx 1\,\mathrm{GeV}$$



- relevant current quarks: u,d,s $\quad m_u \approx m_d < m_s \;\Rightarrow\; N_f = 2+1$

- dominant 4-quark channel with chiral U(N)xU(N) symmetry

$$\lambda_{\mathrm{S-P}}\left[\left(\bar{q}\,T^a q\right)^2 + \left(\bar{q}\,i\gamma_5 T^a q\right)^2\right]$$

[J. Braun, M. Leonhardt, M. Pospiech 2018]
[M. Mitter, J. Pawlowski, N. Strodthoff 2015]

- (Pseudo)scalar meson nonet via bosonization

$$\Sigma = T^a(\sigma^a + i\pi^a) = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\left(\sigma_l + a_0^0 + i\eta_l + i\pi^0\right) & a_0^- + i\pi^- & \kappa^- + iK^- \\ a_0^+ + i\pi^+ & \frac{1}{\sqrt{2}}\left(\sigma_l - a_0^0 + i\eta_l - i\pi^0\right) & \kappa^0 + iK^0 \\ \kappa^+ + iK^+ & \bar{\kappa}^0 + i\bar{K}^0 & \frac{1}{\sqrt{2}}\left(\sigma_s + i\eta_s\right) \end{pmatrix}$$

- (Pseudo)scalar mixing angles

mass eigenstates

$$\begin{pmatrix} f_0 \\ \sigma \end{pmatrix} = \begin{pmatrix} \cos\varphi_s & -\sin\varphi_s \\ \sin\varphi_s & \cos\varphi_s \end{pmatrix}\begin{pmatrix} \sigma_l \\ \sigma_s \end{pmatrix}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\varphi_p & -\sin\varphi_p \\ \sin\varphi_p & \cos\varphi_p \end{pmatrix}\begin{pmatrix} \eta_l \\ \eta_s \end{pmatrix}$$

flavor eigenstates

N_f = 2+1 initial action

[F. Rennecke, BJS 2017]

$$\Gamma_k = \int_x \Big\{ \bar{q}\, Z_{q,k} \big(\gamma_\mu \partial_\mu + \gamma_0 \mu\big) q + \bar{q} h_k \cdot \Sigma_5 q + \mathrm{tr}\big(Z_{\Sigma,k}\, \partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger\big) + \tilde{U}_k(\Sigma) \Big\}$$

$$q^T = (l, l, s) \qquad \Sigma_5 = T_a(\sigma_a + i\gamma_5 \pi_a)$$

•effective potential $\tilde{U}_k = U_k(\rho_1, \tilde{\rho}_2) - j_l \sigma_l - j_s \sigma_s - c_k\, \xi$ $\qquad \xi = \det(\Sigma + \Sigma^\dagger)$

U(3) x U(3) sym. potential
(two chiral invariants)
$\rho_i = \mathrm{tr}(\Sigma \cdot \Sigma^\dagger)^i$

explicit chiral symmetry breaking:
finite light & strange current quark masses

anomalous U(1)A breaking
via 't Hooft determinant

•wave function renormalizations $p^2 + m^2 \;\to\; Z_k\, p^2 + m^2$

$$Z_{q,k} = \begin{pmatrix} Z_{l,k} & 0 & 0 \\ 0 & Z_{l,k} & 0 \\ 0 & 0 & Z_{s,k} \end{pmatrix}$$

$$Z_{\Sigma,k} = \begin{pmatrix} Z_{\phi,k} & 0 & 0 \\ 0 & Z_{\phi,k} & 0 \\ 0 & 0 & Z_{\phi,k} \end{pmatrix}$$

•Yukawa couplings

$$h_k = \begin{pmatrix} h_{l,k} & h_{l,k} & h_{ls,k} \\ h_{l,k} & h_{l,k} & h_{ls,k} \\ h_{sl,k} & h_{sl,k} & h_{s,k} \end{pmatrix}$$

Different FRG truncations

[F. Rennecke, BJS 2017]

- different truncations:

$$\Gamma_k = \int_x \Big\{ \bar{q}\, Z_{q,k} \big(\gamma_\mu \partial_\mu + \gamma_0 \mu\big) q + \bar{q} h_k \cdot \Sigma_5 q + \mathrm{tr}\big(Z_{\Sigma,k}\, \partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger \big) + \tilde{U}_k(\Sigma) \Big\}$$

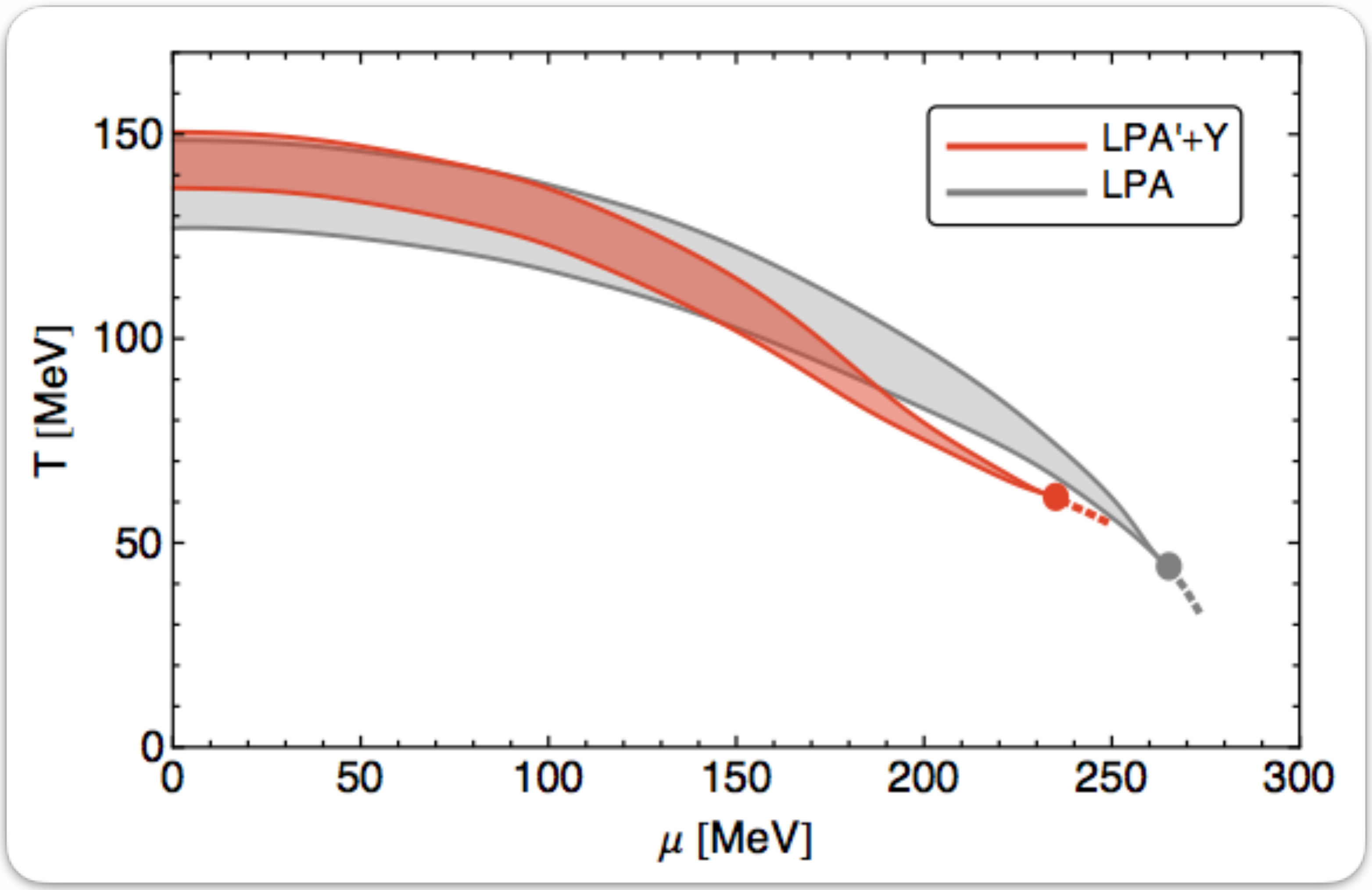
truncation	running couplings
LPA′+Y	$\bar{\tilde{U}}_k,\ \bar{h}_{l,k},\ \bar{h}_{s,k},\ Z_{l,k},\ Z_{s,k},\ Z_{\phi,k}$
LPA+Y	$\bar{\tilde{U}}_k,\ \bar{h}_{l,k},\ \bar{h}_{s,k}$
LPA	$\bar{\tilde{U}}_k$

Chiral Phase Diagram

[F. Rennecke, BJS 2017]

- Critical Endpoint for different truncations:

truncation	running couplings	$(T_{\mathrm{CEP}}, \mu_{\mathrm{CEP}})$ [MeV]
LPA′+Y	$\tilde{\bar{U}}_k$, $\bar{h}_{l,k}$, $\bar{h}_{s,k}$, $Z_{l,k}$, $Z_{s,k}$, $Z_{\phi,k}$	(61,235)
LPA+Y	$\tilde{\bar{U}}_k$, $\bar{h}_{l,k}$, $\bar{h}_{s,k}$	(46,255)
LPA	$\tilde{\bar{U}}_k$	(44,265)



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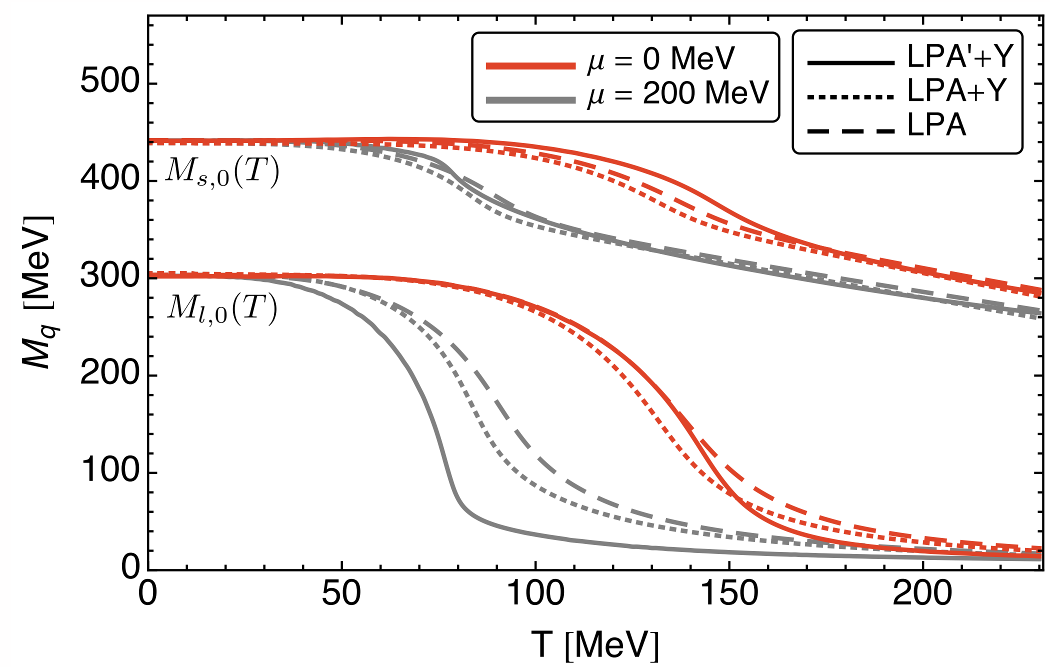
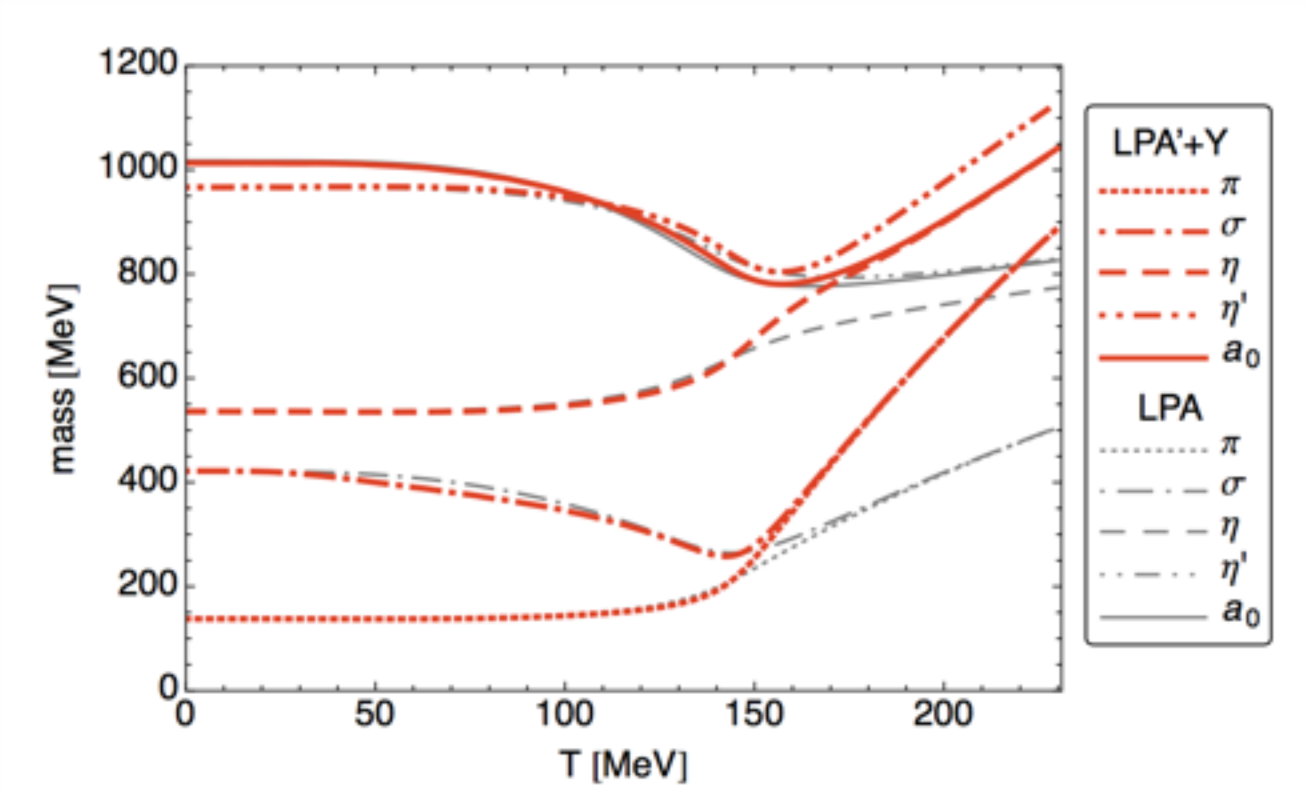
Masses

[F. Rennecke, BJS 2017]

• quark masses



• meson masses



driven by the mesonic wave-function renormalization

mesons decouple more rapidly beyond LPA

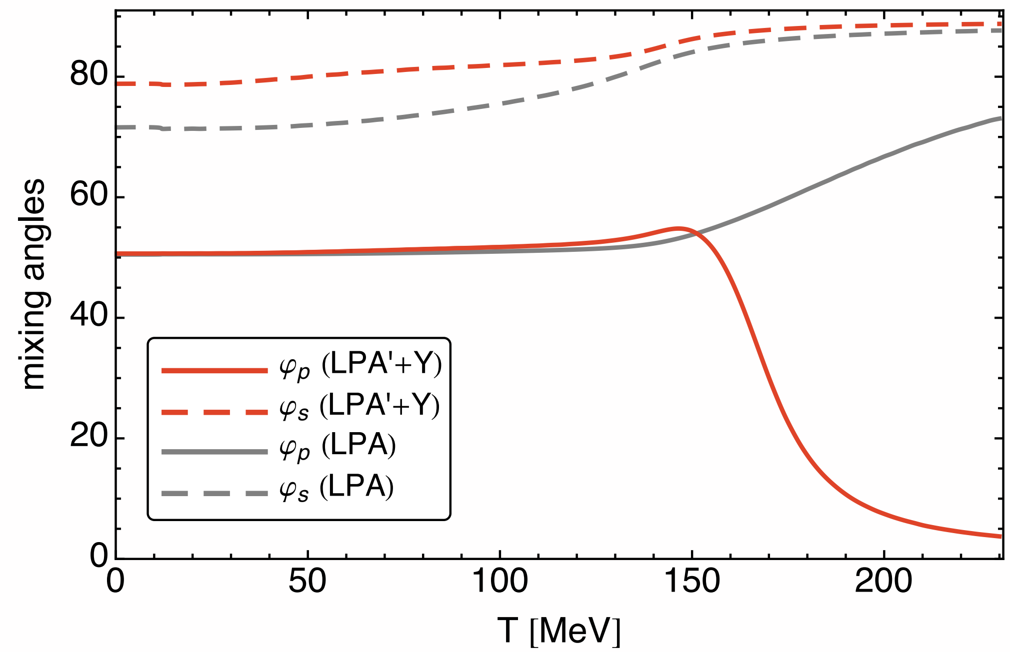
Mixing angles

[F. Rennecke, BJS 2017]

- mixing angles determine light and strange quark content of σ, f0, η, η' mesons

$$\begin{pmatrix} f_0 \\ \sigma \end{pmatrix} = \begin{pmatrix} \cos\varphi_s & -\sin\varphi_s \\ \sin\varphi_s & \cos\varphi_s \end{pmatrix} \begin{pmatrix} \sigma_l \\ \sigma_s \end{pmatrix}$$

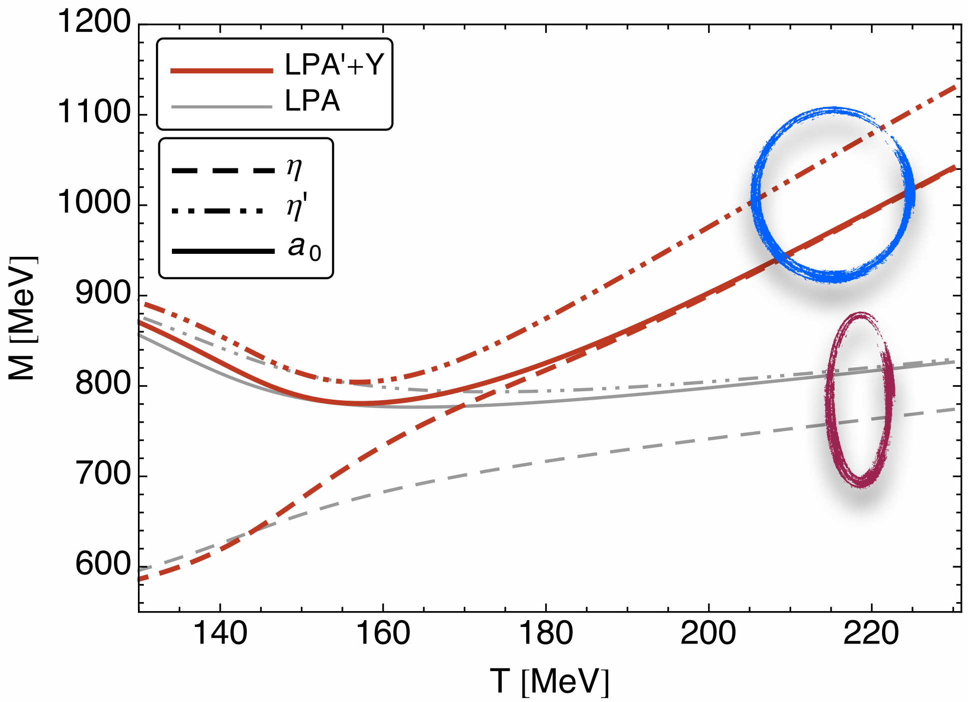
$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\varphi_p & -\sin\varphi_p \\ \sin\varphi_p & \cos\varphi_p \end{pmatrix} \begin{pmatrix} \eta_l \\ \eta_s \end{pmatrix}$$



significant effects on pseudoscalar mixing beyond LPA!

consequence: chiral partners of η and η' change!

mean-field/LPA	LPA' + Y
(η, f_0)	(η, a_0)
(η', a_0)	(η', f_0)



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Agenda

- Role of Fluctuations:

  from mean-field approximations to FRG

- **Columbia plot**

  with Simon Resch and Fabian Rennecke

Simon Resch
awarded Kaki-Prize 2016
(best theoretical master thesis)

Columbia plot

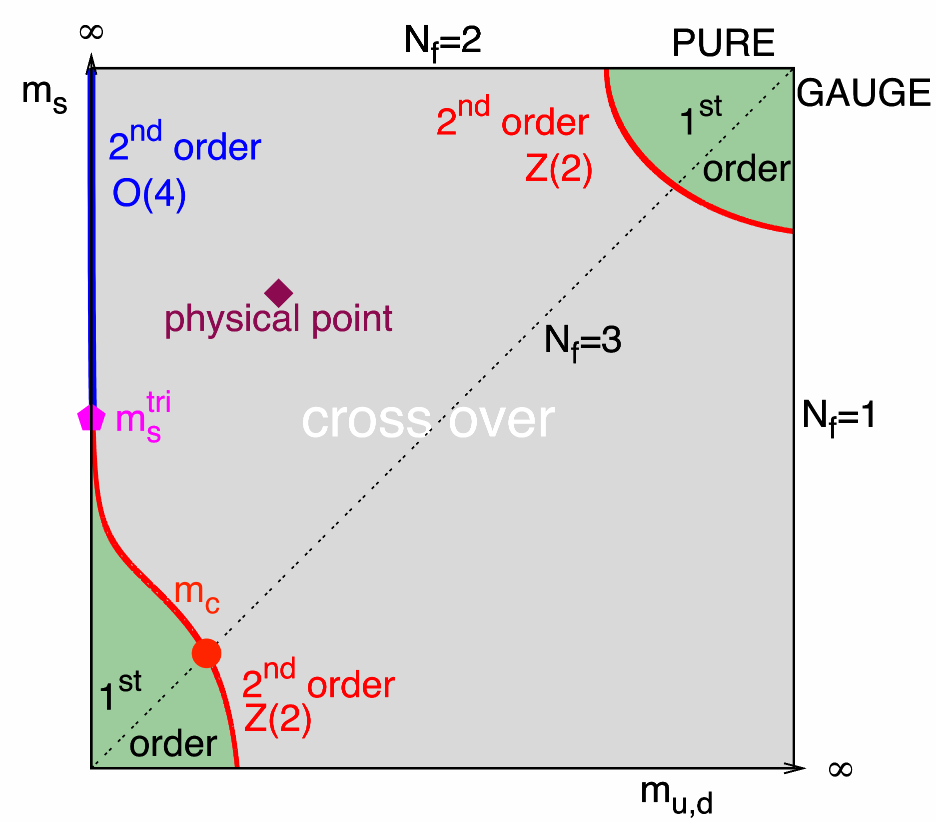
For physical quark masses: smooth phase transitions ➜ deconfinement: analytic change of d.o.f.

➜ associated global QCD symmetries only exact in two mass limits

1.infinite quark masses (center symmetry) — Order parameter: VEV of traced Polyakov loop(s)

2. massless quarks (chiral symmetry) — Order parameter: chiral condensate(s)

for finite quark masses:
both symmetries
explicitly broken



Columbia plot

For physical quark masses: smooth phase transitions ➜ deconfinement: analytic change of d.o.f.

➜ associated global QCD symmetries only exact in two mass limits

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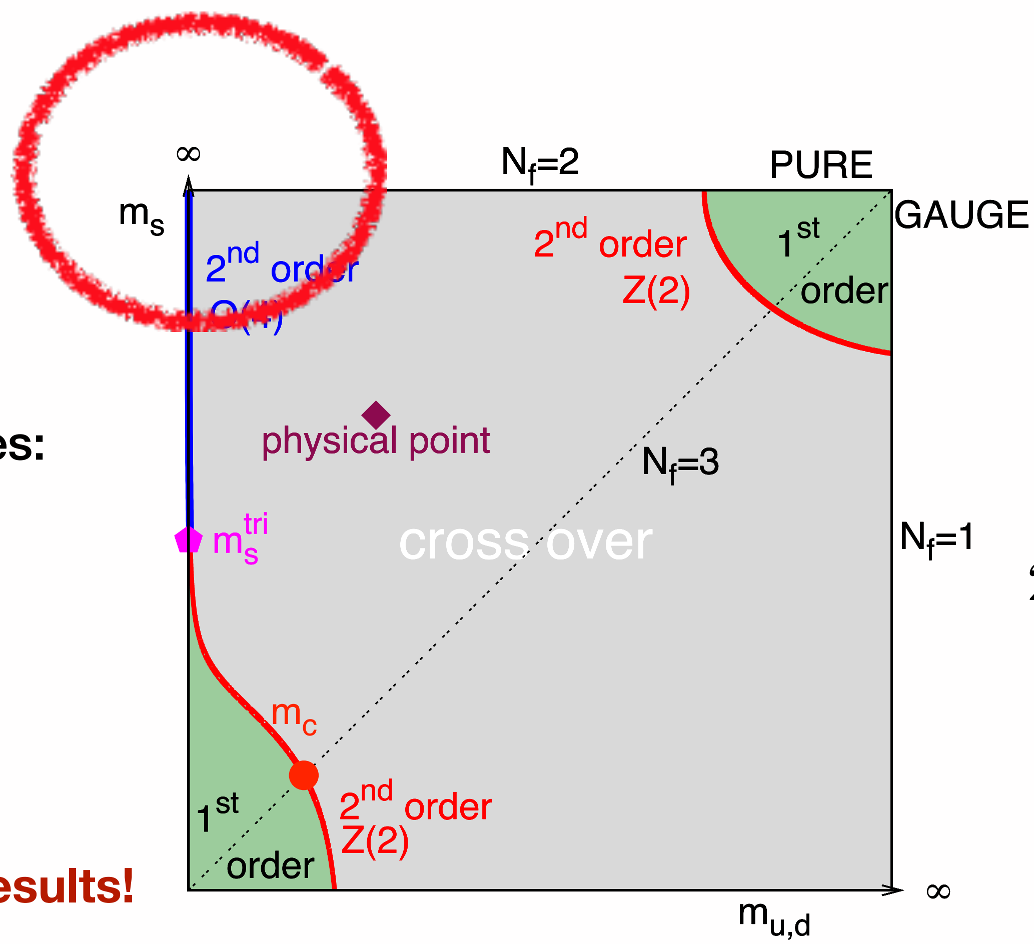
for finite quark masses: both symmetries explicitly broken

still conflicting lattice results!



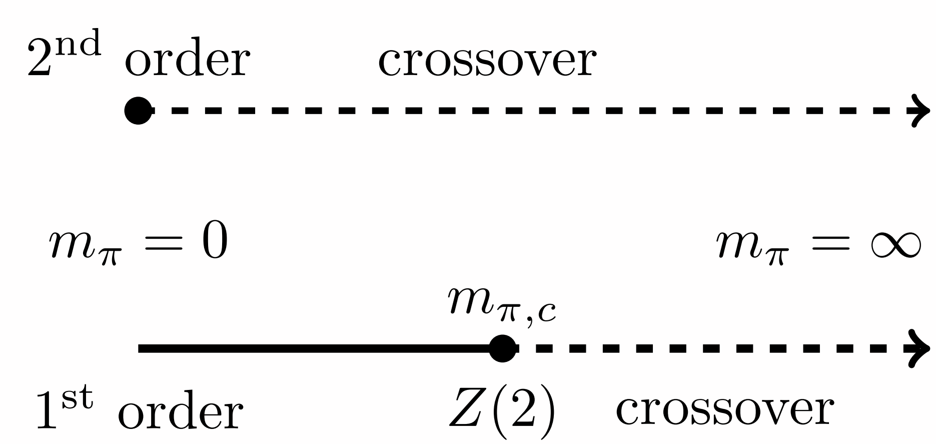
open issue: Nf=2: O(4)?

$U(2)_L \times U(2)_R / U(2)_V$?

➜ crit. exp. similar

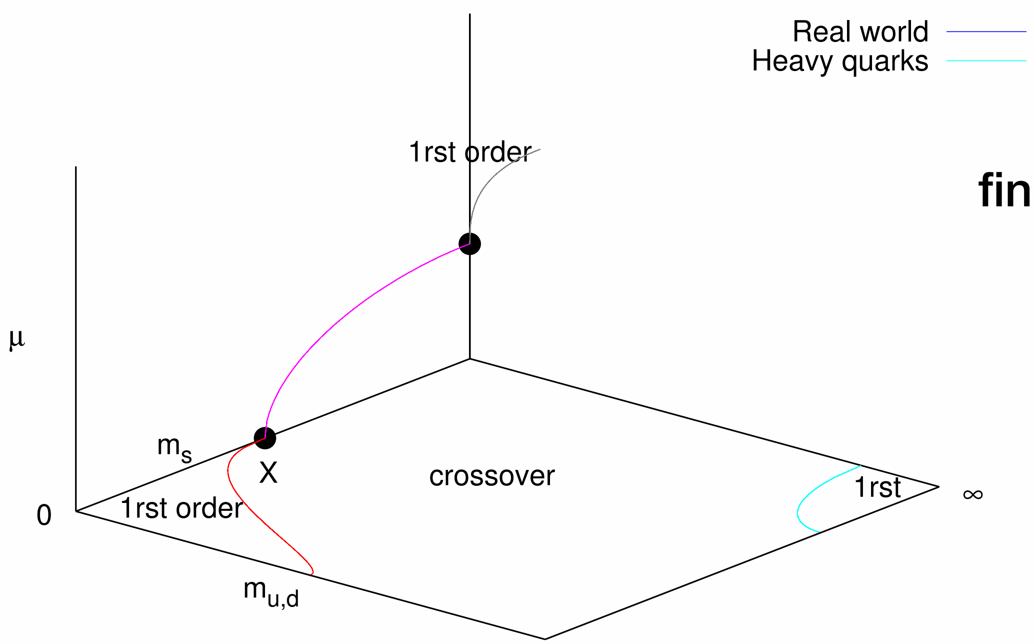
or even 1st order?

dep. on strength of axial anomaly!

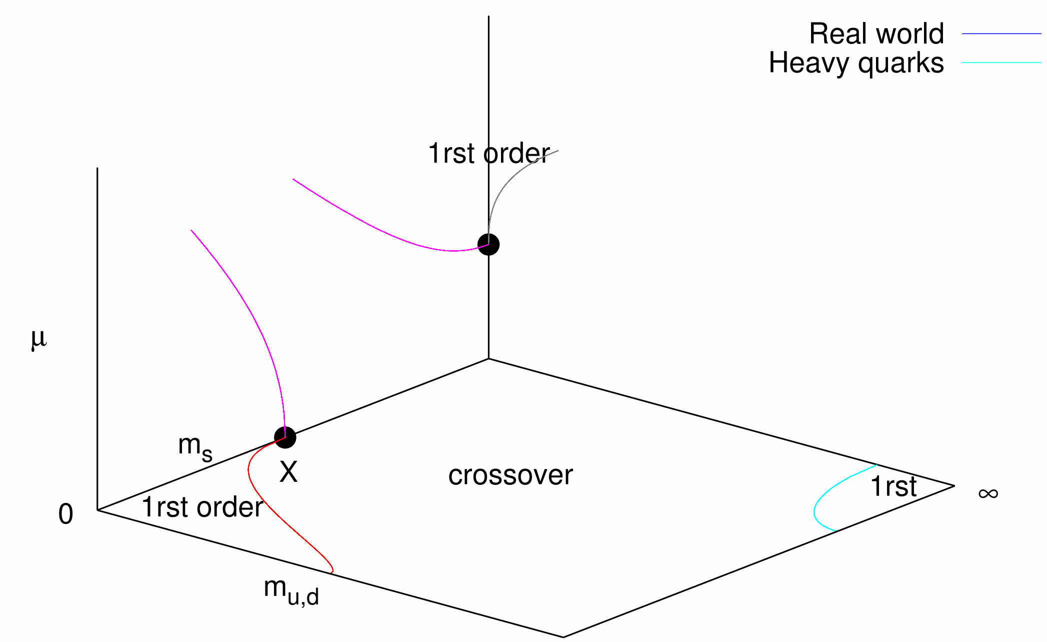


Columbia plot (MFA)

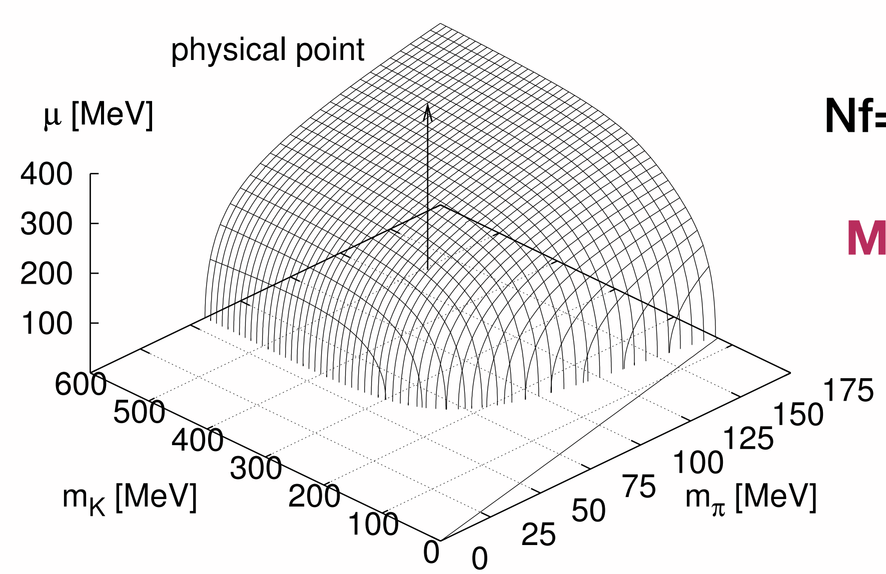
chiral critical surface
standard scenario



chiral critical surface
non-standard scenario



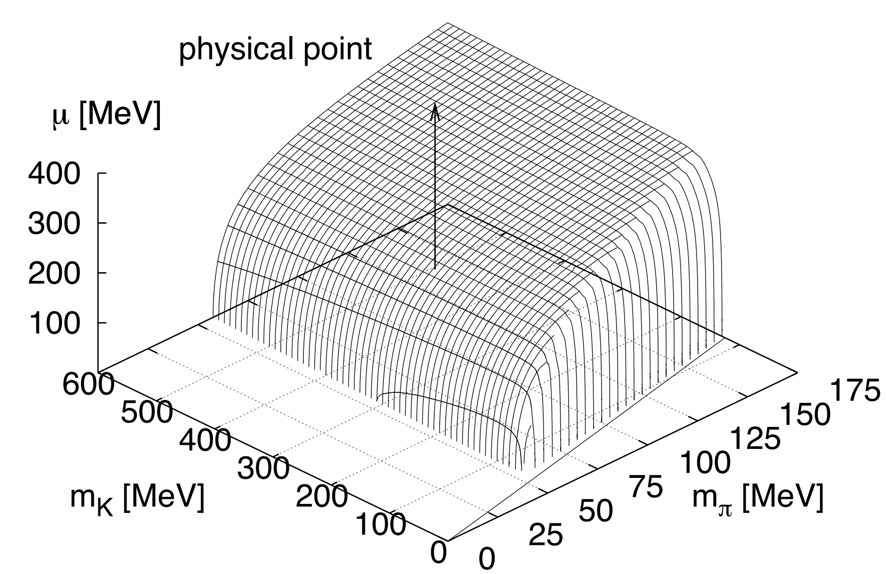
finite chemical potential



with $U_A(1)$-anomaly

Nf=2+1 quark-meson model

Mean-field approximation



without $U_A(1)$-anomaly

[BJS, M. Wagner 2009]

Columbia plot (FRG)

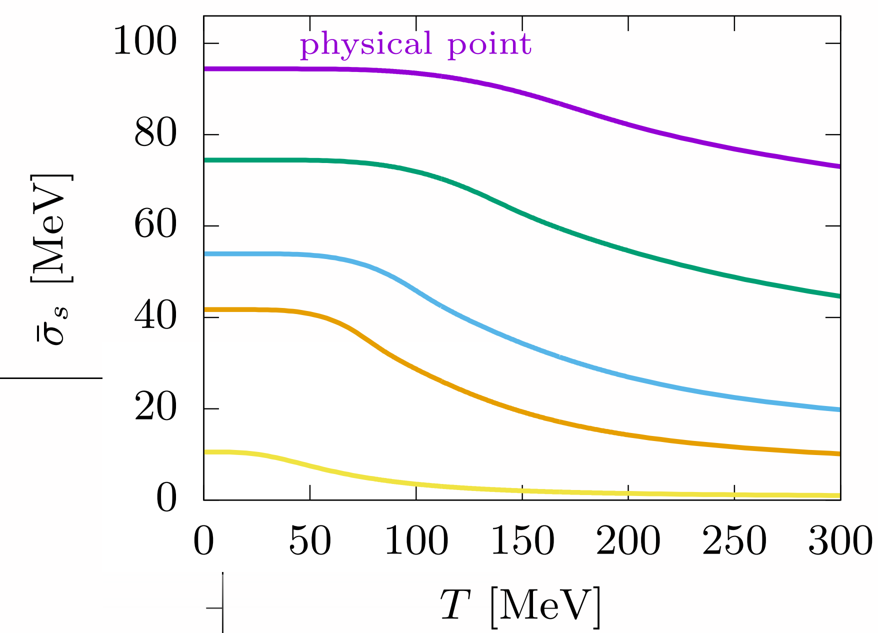
Nf=2+1 quark-meson truncation with FRG

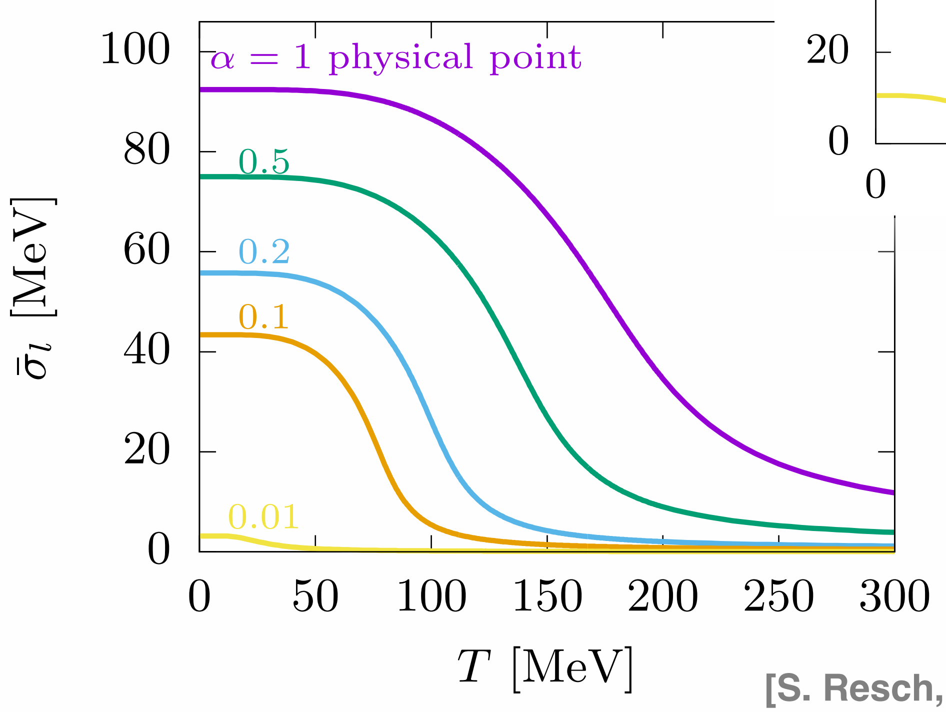
Fixing initial action in the UV: 4 coupling parameters, 2 explicit symmetry breaking, 1 ‘t Hooft determinant

axial $U_A(1)$ symmetry: consider two extrema on or off

How to fix initial action in the UV away from the physical mass point?

1.) **fixed UV -scenario**:
vary only explicit SB
remaining parameter
adjusted @ physical
point

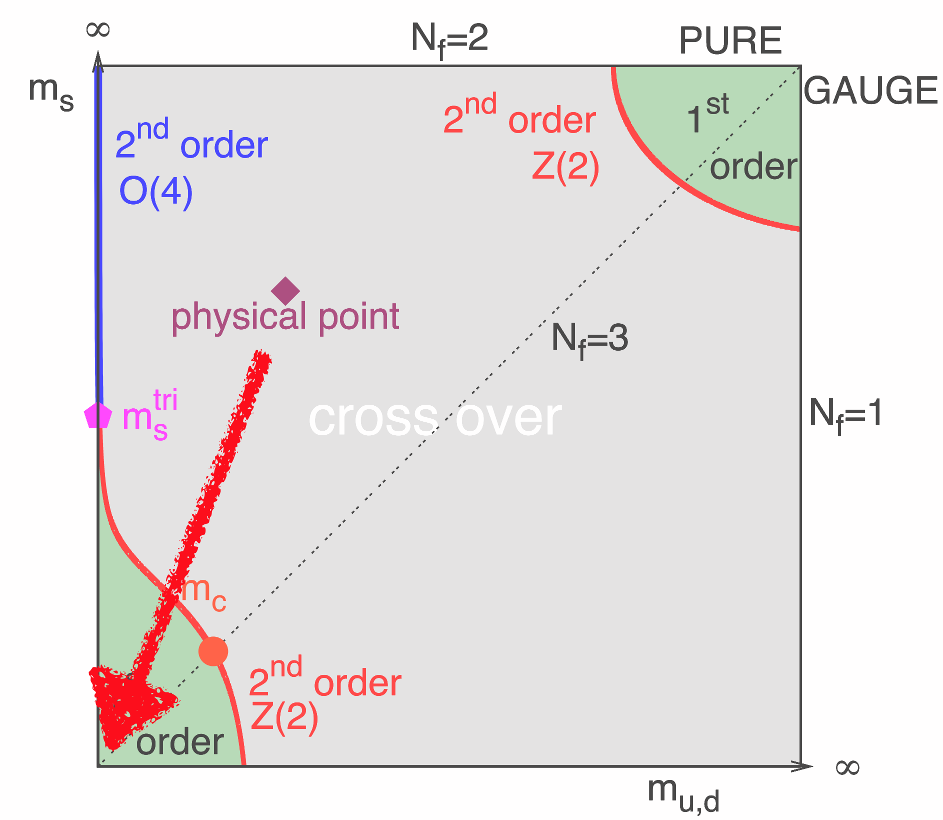




No symmetry breaking in the chiral limit!

α=1 physical mass point

α=0 chiral limit



[S. Resch, F. Rennecke, BJS arXiv:1712.07961]

Columbia plot (FRG)

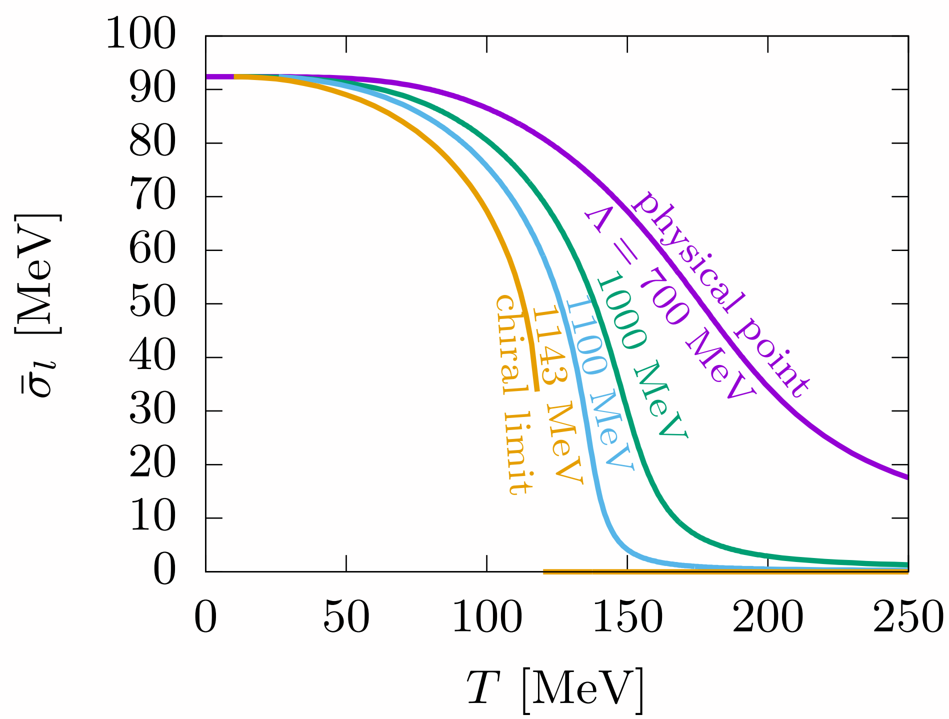
Nf=2+1 quark-meson truncation with FRG

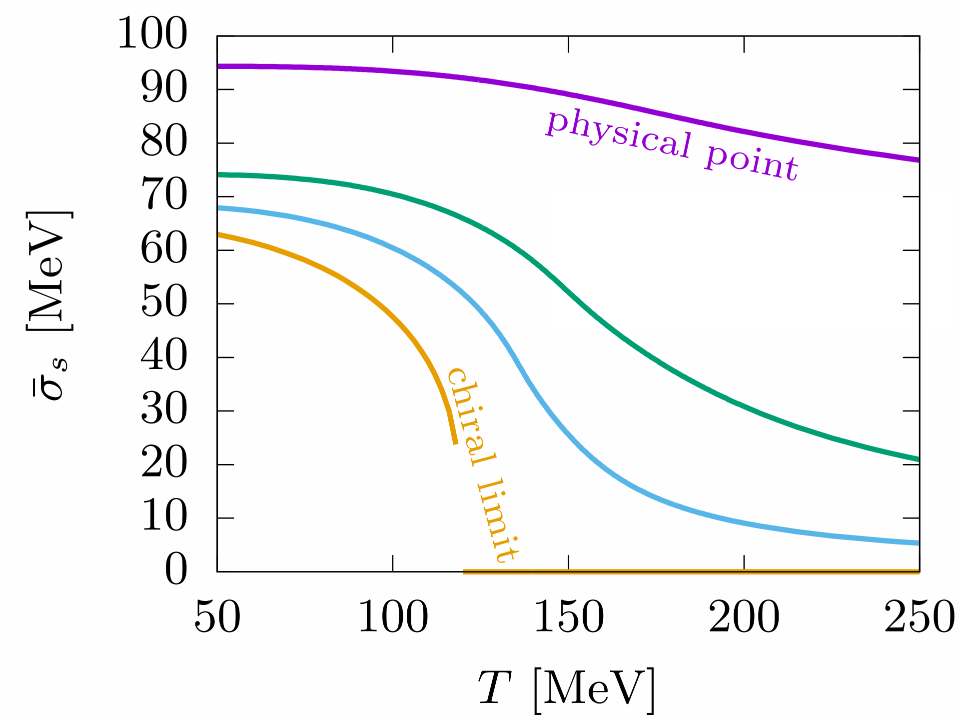
Fixing initial action in the UV: 4 coupling parameters, 2 explicit symmetry breaking, 1 ‘t Hooft determinant

axial $U_A(1)$ symmetry: consider two extrema on or off

How to fix initial action in the UV away from the physical mass point?

2.) **fixed f_π-scenario**: (motivated by chiral perturbation theory)
fix f_π in the infrared when explicit SB is varied
trick: vary only UV cutoff while initial action is fixed

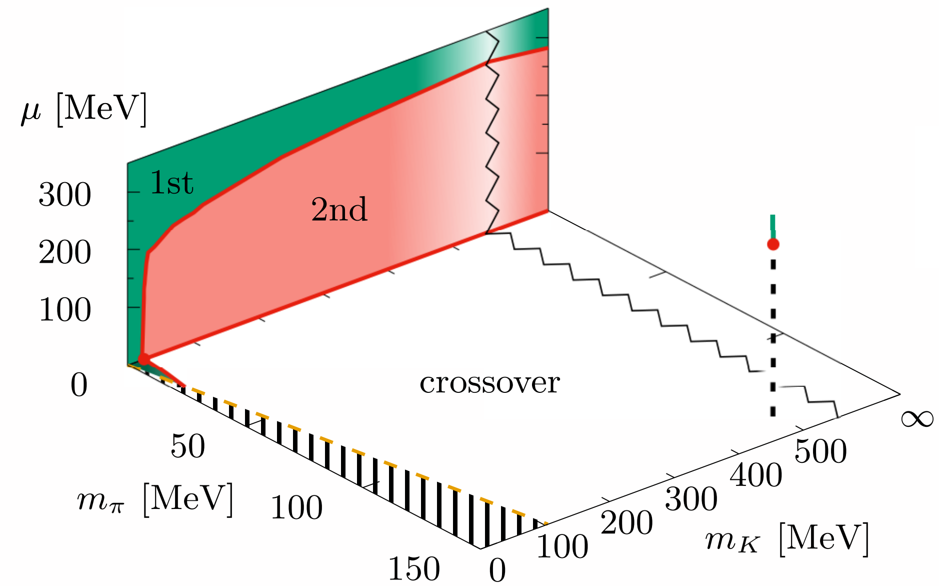




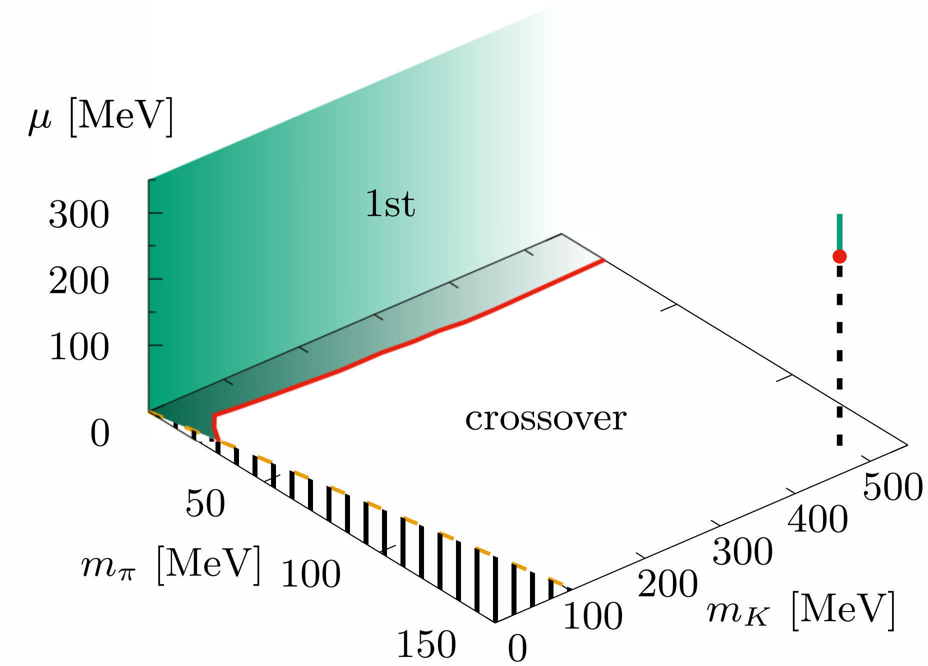
[S. Resch, F. Rennecke, BJS arXiv:1712.07961]

Vacuum fluctuations

FRG

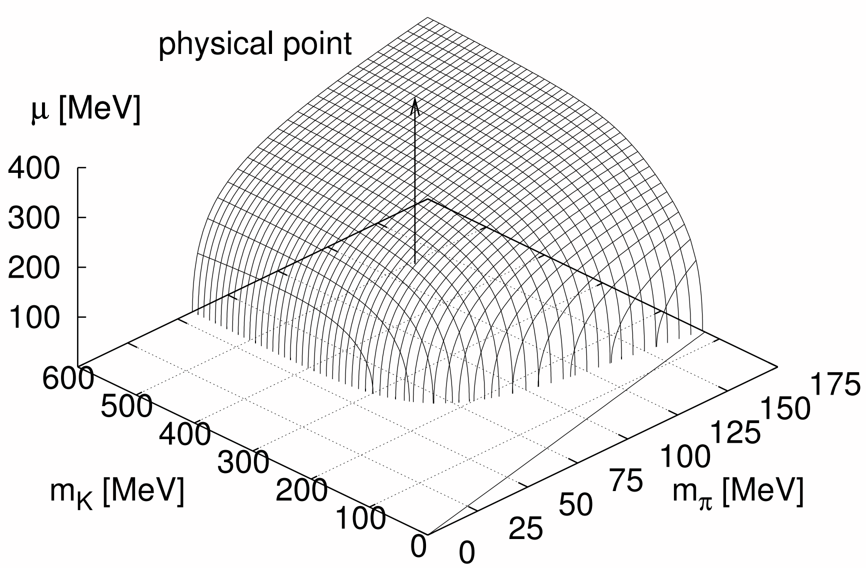


all fluctuations
quarks & mesons

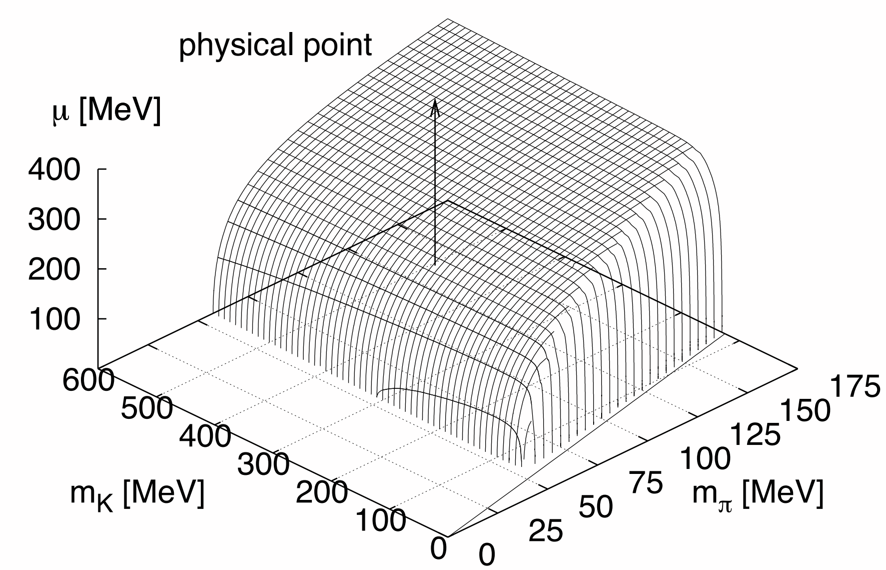


with $U_A(1)$-anomaly

without $U_A(1)$-anomaly



Mean-field analysis

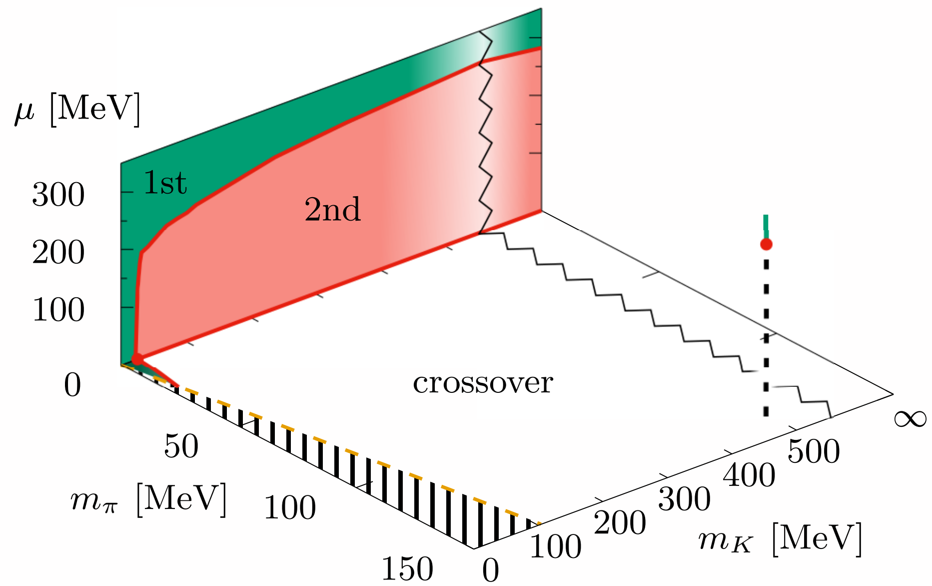


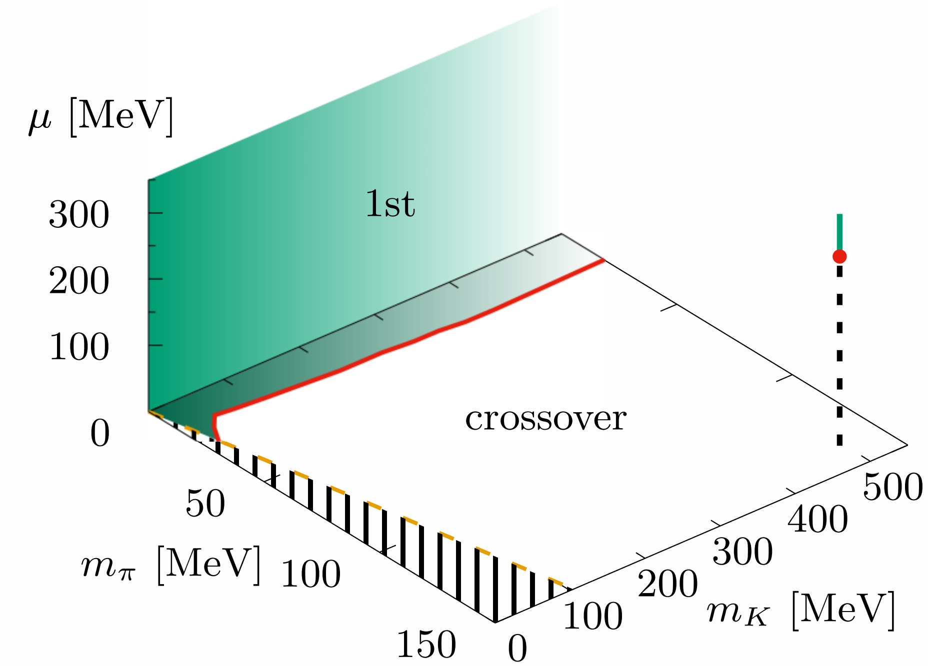
[S. Resch, F. Rennecke, BJS arXiv:1712.07961]

Vacuum fluctuations

FRG

all fluctuations
quarks & mesons



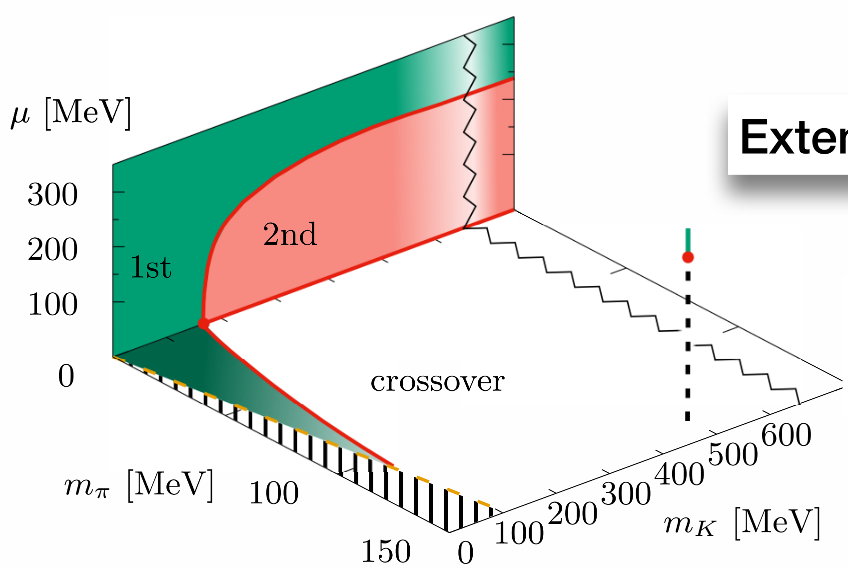


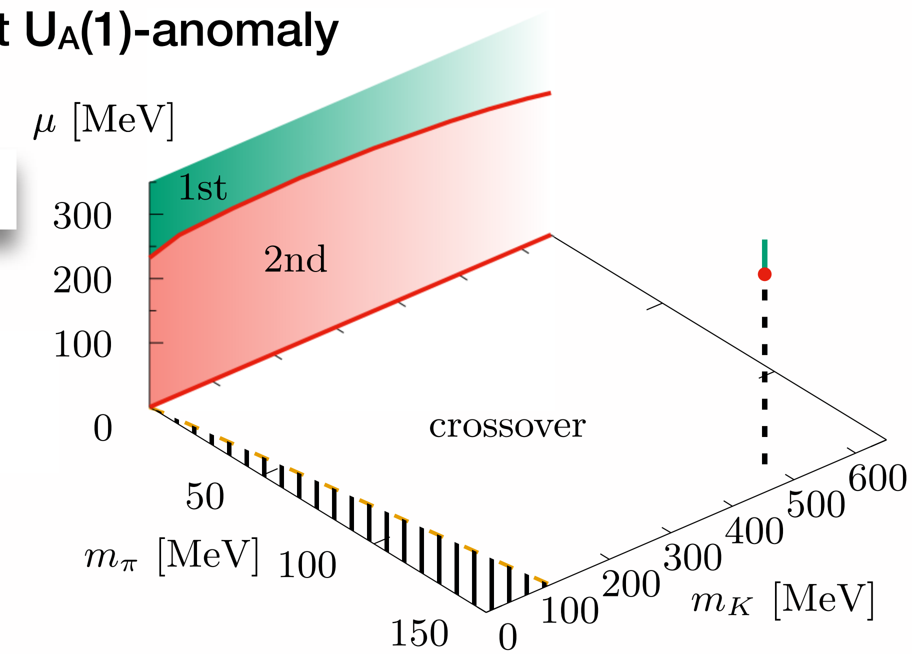
with $U_A(1)$-anomaly

without $U_A(1)$-anomaly

Extended Mean-field analysis

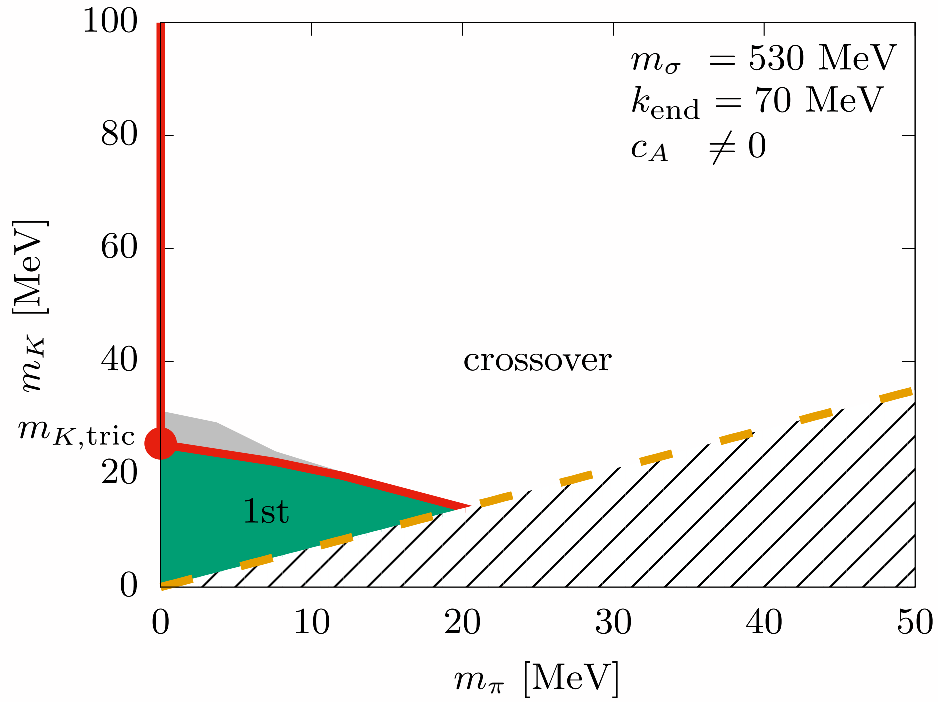
influence of vacuum
fluctuations of quarks





[S. Resch, F. Rennecke, BJS arXiv:1712.07961]

Columbia plot with the FRG



findings:

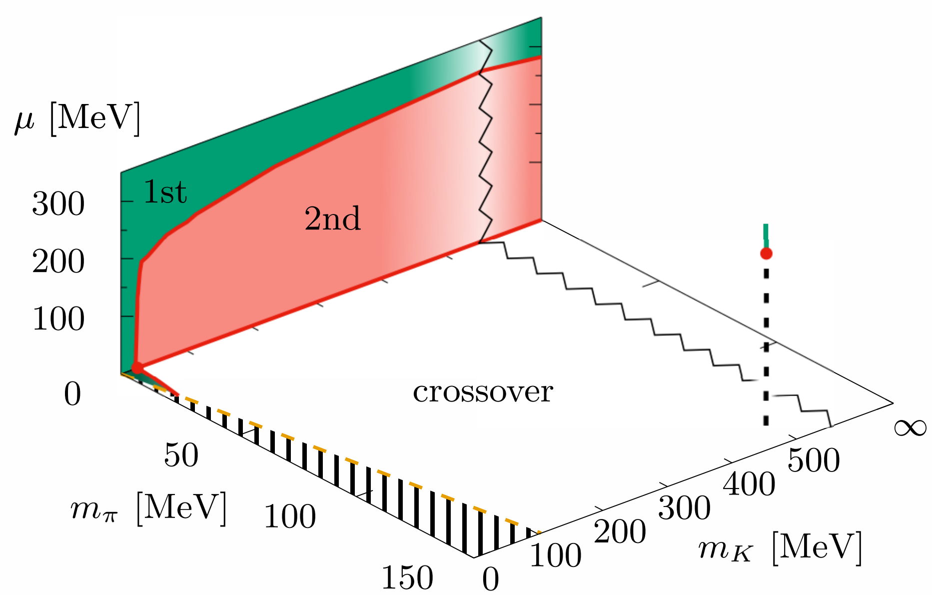
conventional bending of chiral critical surface
➜ critical endpoint @ physical mass point

tricritical strange quark mass far away from light chiral limit

First-order region around chiral limit very small

Nf=2+1 ➜ Nf=2 analytically connected two-flavor chiral limit

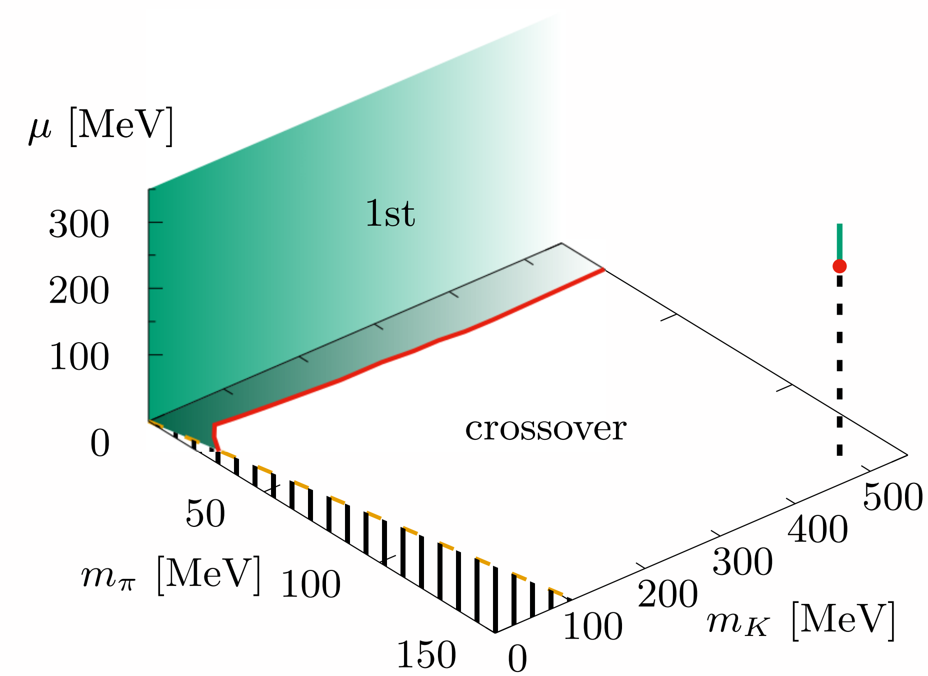
influence of axial anomaly on chiral critical line



with $U_A(1)$-anomaly

FRG

all fluctuations
quarks & mesons



without $U_A(1)$-anomaly

[S. Resch, F. Rennecke, BJS arXiv:1712.07961]

Summary & Conclusions

- effects of quantum and thermal fluctuations on QCD phase diagram

  FRG investigation with different truncations LPA, LPA’, LPA’+Y

  ➜ fluctuations are important

  ➜ mass sensitivity of the chiral phase structure (Columbia plot)