PERTURBATIVE DYNAMICS of MASSIVE GLUONS

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OUTLOOK

- Motivation: the Curci-Ferrari (CF) model

- Perturbative (one-loop) analysis in the infrared safe renormalization scheme

- Application to QCD: spont. chiral sym. breaking (SχSB)

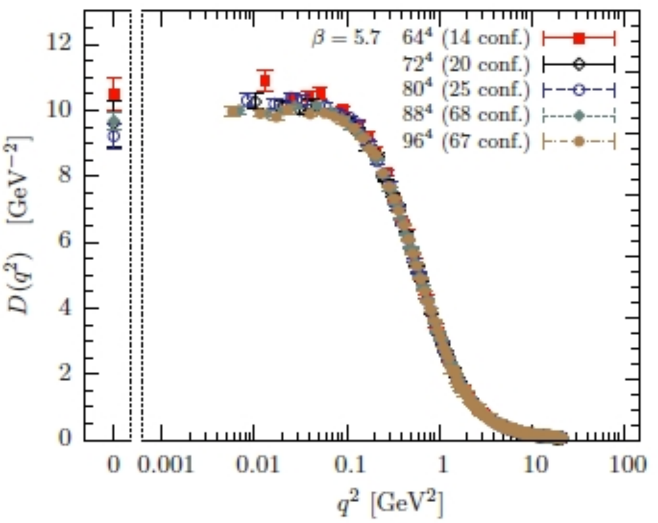
- Conclusions and perspectives

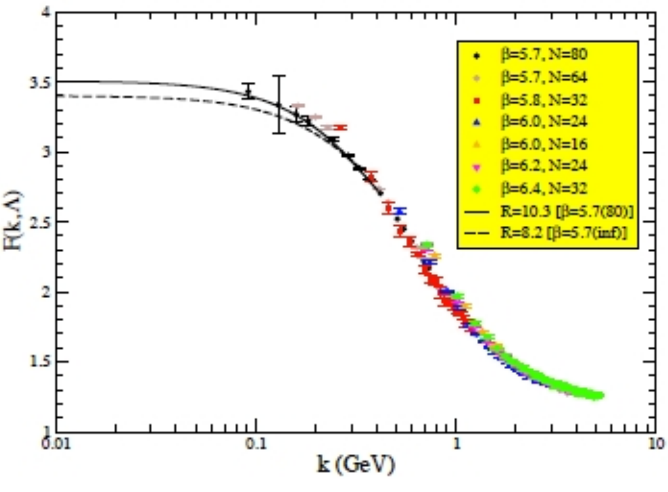
Motivations : general frame

Yang-Mills Green's function (gluon and ghost fields) are the basic ingredients of continuum approaches

- Bound states
- Nonzero T and μ
- etc.

Lattice results (Landau gauge $SU(3)$, $d=4$)





Massive gluon

Massless ghost

The massive FP (mFP) theory (I)

IR momenta are not accessible from perturbation theory based on the FP Lag. (Landau pole) ⇛ Nonperturbative continuum approaches

- DSE [..., Alkofer, Hauck, von Smekal ('97), ...]
- FRG [Ellwanger, Hirsch, Weber ('96) ...]
- HF [Feuchter, Reinhardt ('04) ...]

BUT Not exactly the FP Lagrangian

UV / IR regulator breaks the BRST symmetry which generates quadratic div.

⇛ accounted for by suitable subtraction / initial cond's.

Equivalent to a mass (counter) term

⇛ Range of parameters for which the mFP provides a viable realization of YM?

The mFP theory (II)

The FP quantization procedure is not valid / complete : <u>Gribov ambiguities</u>

- Minimal Landau gauge on the lattice (no continuum formulation known)
- (refined) Gribov-Zwanziger approach [Gribov ('78), Zwanziger ('89)...] restrict the path integral to the 1st Gribov region
- Average of copies [Serreau, Tissier ('12)]

⇒ **BRST is (softly) broken**

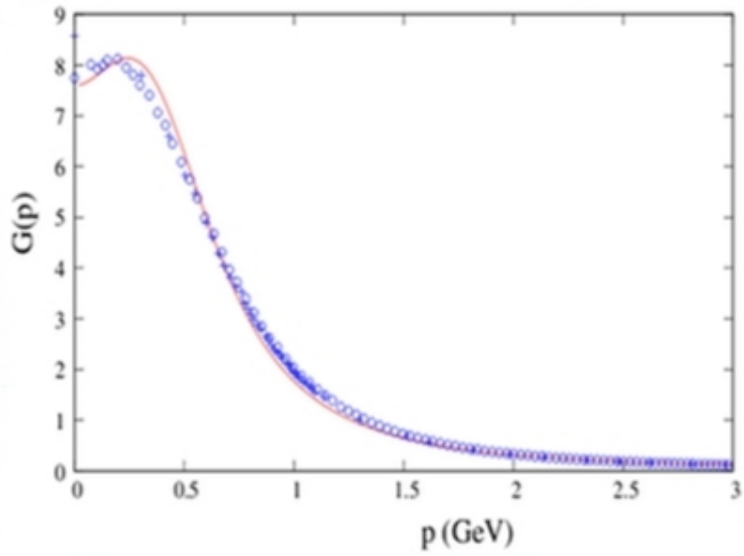
<u>Effective theory</u> : the minimal deformation of the FP theory with the correct UV behavior is <u>mFP</u>

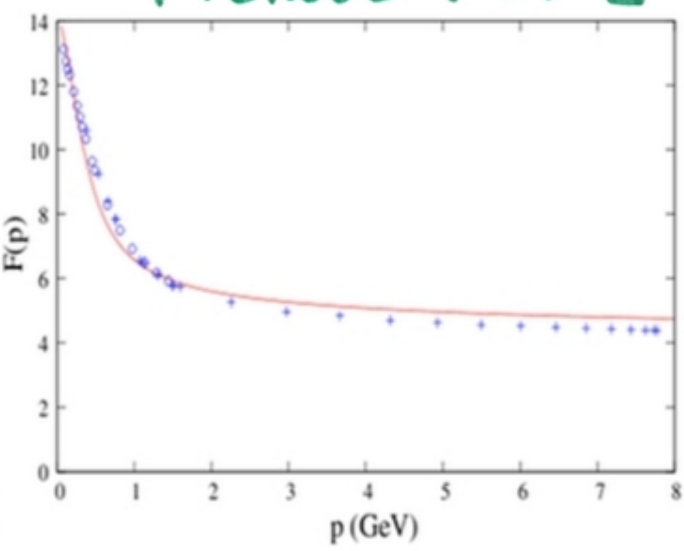
mFP : review

$$\mathcal{L} = \underbrace{\frac{1}{4} F^2 + \partial_\mu \bar{c} D_\mu c + i h \partial_\mu A_\mu}_{\text{FP (Landau gauge)}} + \underbrace{\frac{m^2}{2} A^2}_{\text{mFP}}$$

particular case of the Curci-Ferrari Lagrangian
[Curci, Ferrari ('76)]

- perturbatively renormalizable ($d \leq 4$)
- IR-safe RG trajectories (no Landau pole)
  [Tissier, Wschebor ('11)]
- Accurate description of vacuum YM correlators [Tissier, Wschebor ('10) + Peláez ('13)]





- Successful nonzero T, μ applications
  [Reinosa et al ('14) ...]

PART I : RG analysis in the IR-safe scheme

[Tissier-Wschebor ('11)]

$$G^{-1}(\mu) = \mu^2 + m^2 \quad , \quad F^{-1}(\mu) = 1$$

$$Z_{m^2} Z_A Z_c = Z_g \sqrt{Z_A}\, Z_c = 1$$

[Nonrenormalization theorems from modified Slavnov-Taylor id's (mSTIs)]

Defining $\lambda = \dfrac{g^2 N}{16\pi^2}$ for $SU(N)$

$$\beta_{m^2} = m^2(\gamma_A + \gamma_c) \quad ; \quad \beta_\lambda = \lambda(\gamma_A + 2\gamma_c)$$

with $\gamma_{A,c} = \dfrac{d \ln Z_{A,c}}{d \ln \mu}$

$$F(p) = \frac{m_0^2}{\lambda_0} \frac{\lambda(p)}{m^2(p)}$$

$$G(p) = \frac{\lambda_0}{m_0^4} \frac{m^4(p)}{\lambda(p)} \frac{1}{p^2 + m^2(p)}$$

$m_0^2 = m^2(\mu_0)$

$\lambda_0 = \lambda(\mu_0)$

The one-loop RG flow

$$d=4 \quad , \qquad t = \frac{p^2}{m^2(p)} \equiv \frac{1}{\hat{m}^2(p)}$$

$$\boxed{\begin{aligned} \gamma_c &= -\frac{\lambda}{2t^2}\Big[2t^2+2t-t^3\ln t \\ &\qquad + (t+1)^2(t-2)\ln(t+1)\Big] \\ \gamma_A &= \frac{\lambda}{6t^3}\Big[-17t^3+74t^2-12t+t^5\ln t \\ &\qquad + \dots\Big] \end{aligned}}$$

$\Rightarrow$ Correct (asymptotic freedom) behavior in the UV $\mu\to\infty$, where

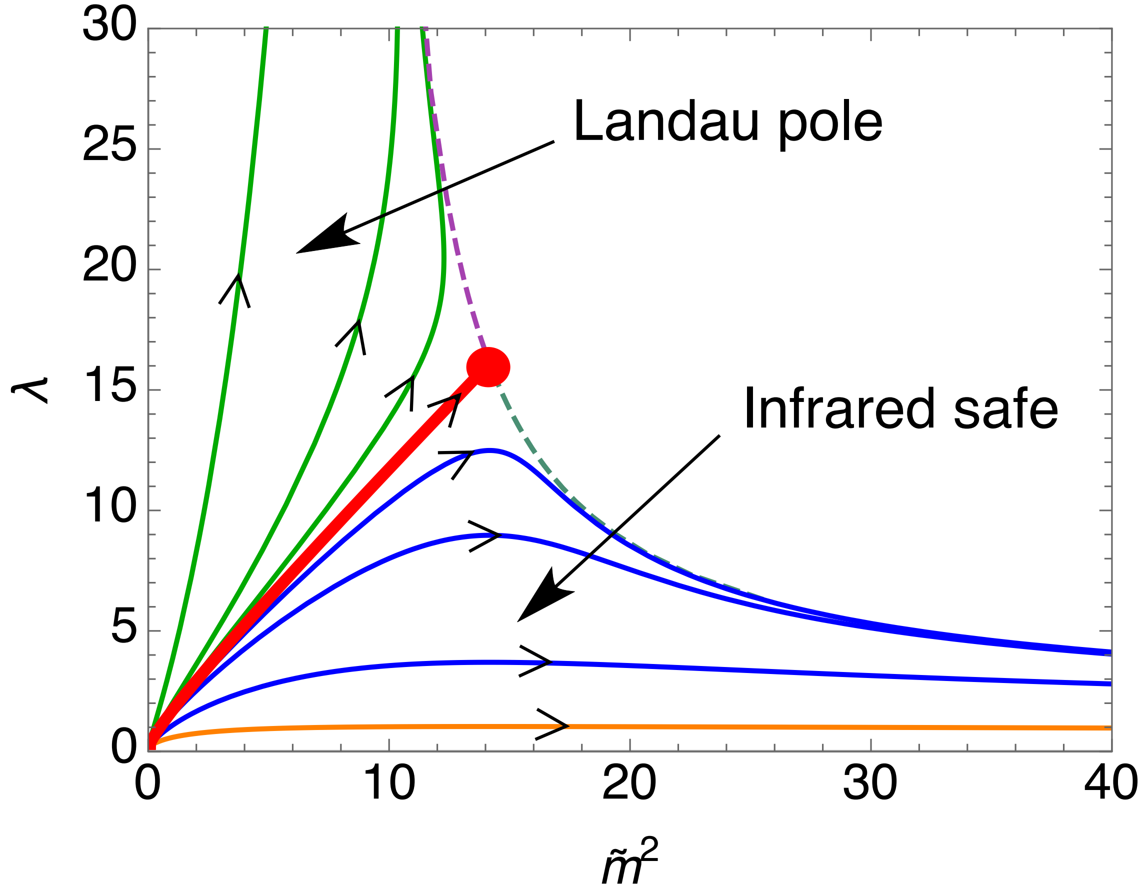
$$\frac{m^2(\mu)}{\mu^2} \longrightarrow 0$$

The relevant loop-expansion parameter

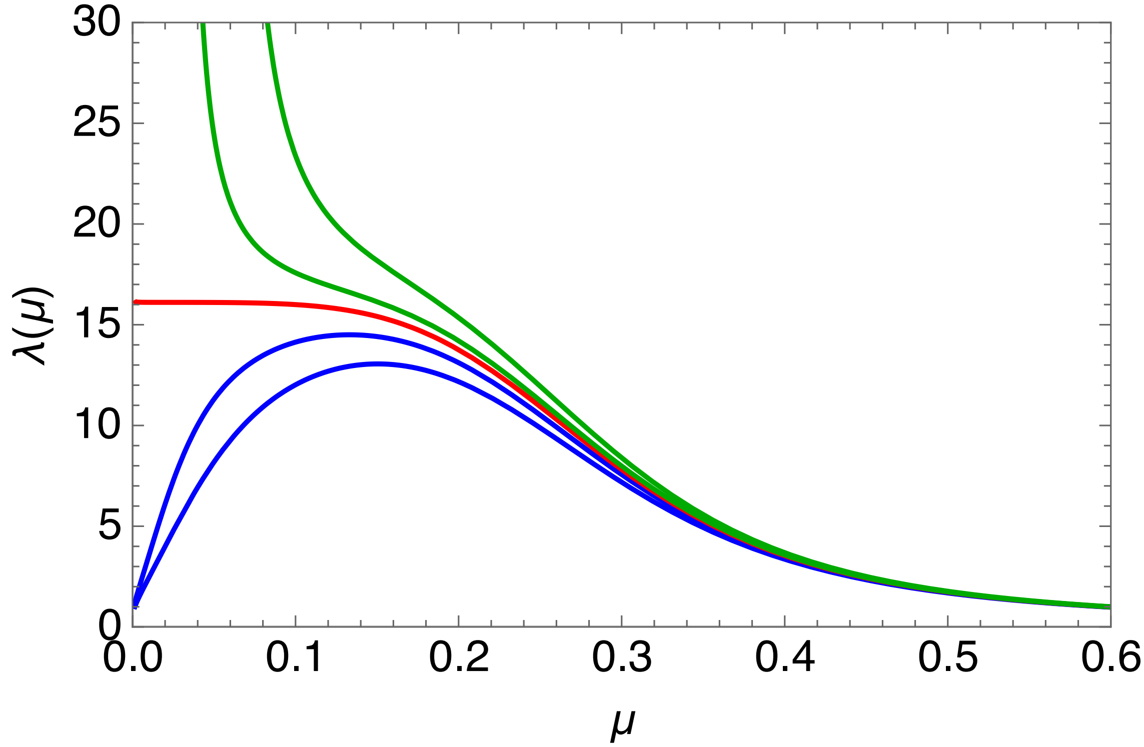
$$\boxed{\lambda_T(p) = \lambda_0 p^2 G(p) F^2(p) = \frac{\lambda(p)}{1+\hat{m}^2(p)}}$$

(Taylor coupling)

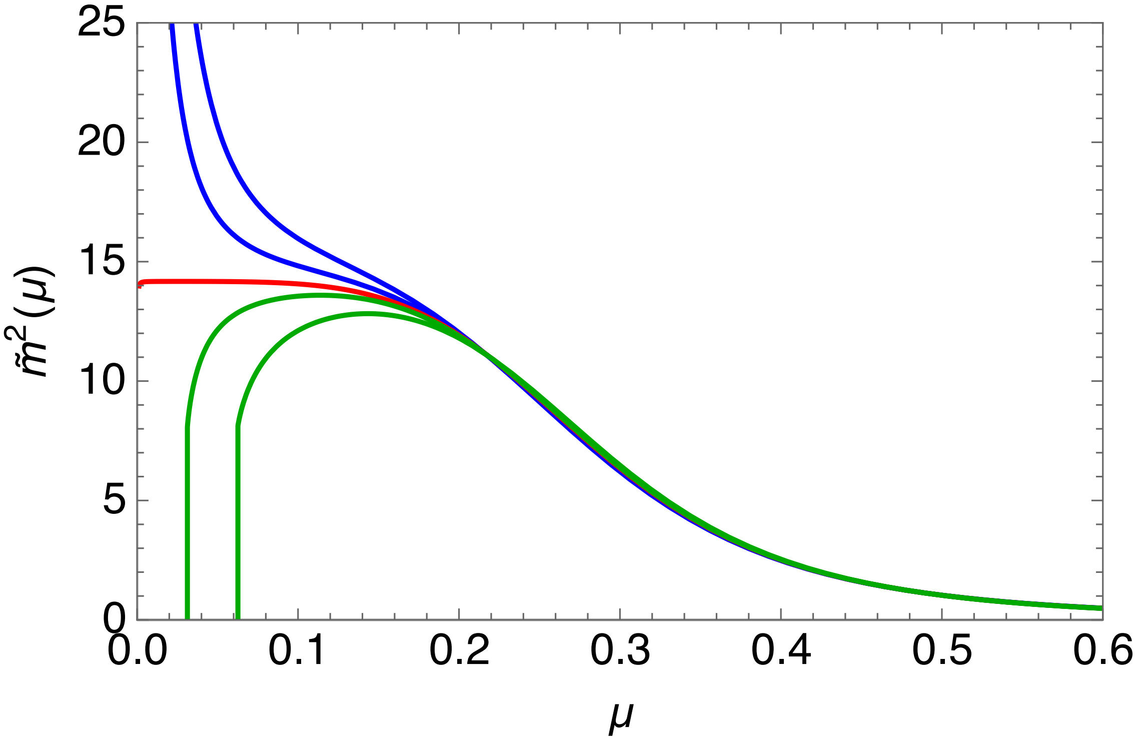
Phase diagram



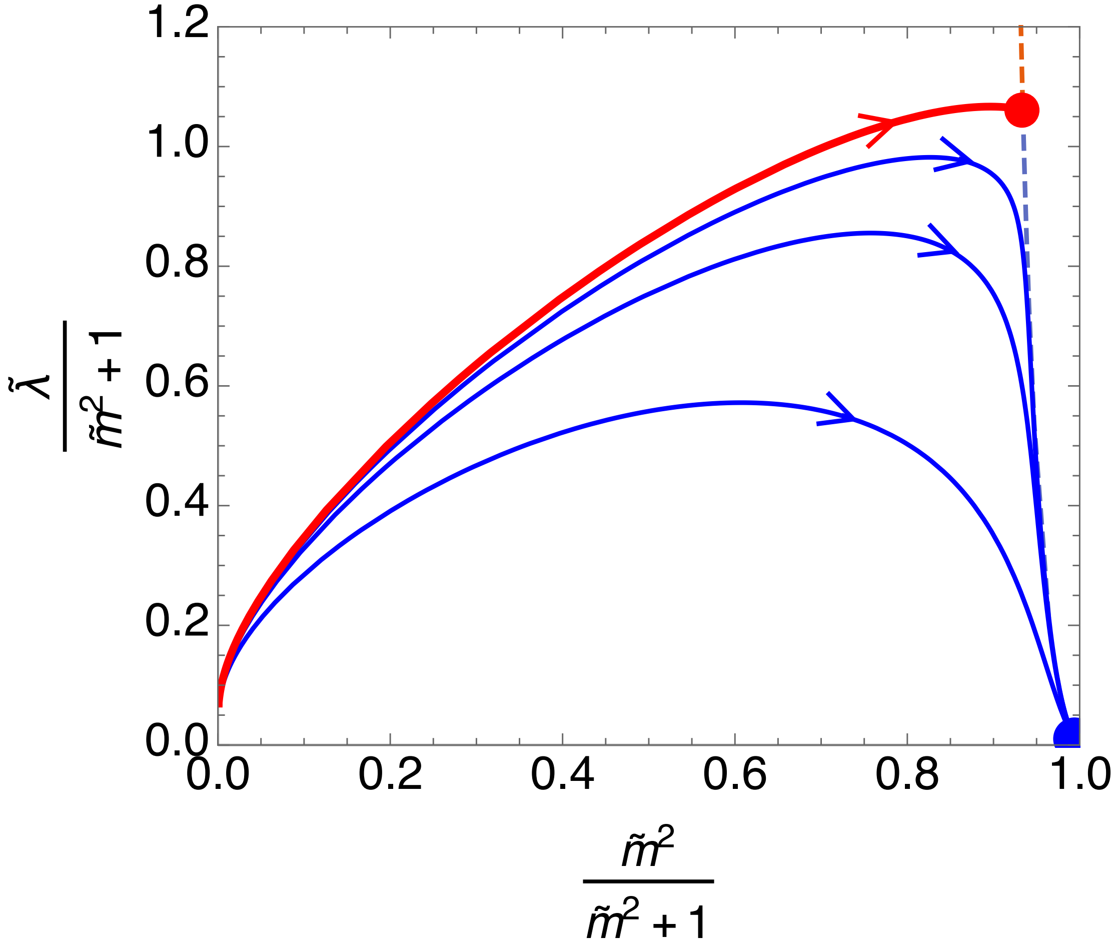
[Reinosa, Serreau, Tissier, Wschebor, PRD 96 (2017)]



30
25
20
15
10
5
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
λ(μ)
μ



25
20
15
10
5
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
m̃²(μ)
μ



1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
$\frac{\tilde{\lambda}}{\tilde{m}^2+1}$
$\frac{\tilde{m}^2}{\tilde{m}^2+1}$

The decoupling fixed point

The flow has an infrared attractive fixed point $\boxed{\tilde{m}_*^2 = \infty\ ,\ \lambda_* = 0 = \lambda_T^*}$

Perturbative !

massive regime
$(t \ll 1)$

$\gamma_A \simeq \frac{\lambda}{3}\ ,\ \gamma_c \simeq 0$

$$\Rightarrow \boxed{m^2(\mu) \underset{\mu \to 0}{\sim} \lambda(\mu) \sim \frac{3}{\ln \bar{\mu}/\mu}} \text{ (IR safe)}$$

AND

$$\Rightarrow \boxed{\left.\begin{array}{l} F(p \to 0) \to \text{const.} \\ G(p \to 0) \to \text{const.} \end{array}\right\} \text{“decoupling” behavior}}$$

N.B.: Here, $\boxed{F(0)G(0) = \frac{1}{m_0^2}}$

due to mSTIs.

The scaling fixed point

The flow has another f.p. at

$$0 < \tilde{m}_*^2 , \lambda_* < \infty .$$

Demanding $\frac{\beta_{\tilde{m}^2}}{\tilde{m}^2} = \frac{\beta_\lambda}{\lambda} = 0$

$\Rightarrow$ $\boxed{\gamma_A^* = 4 \quad ; \quad \gamma_c^* = -2}$

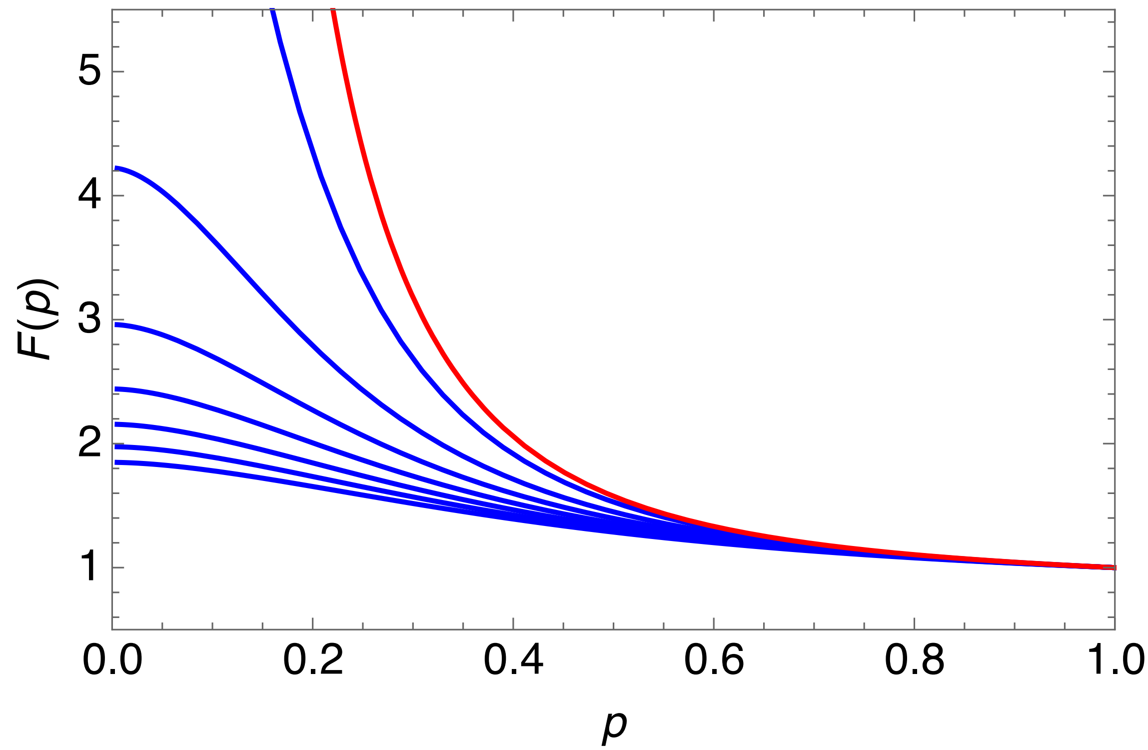
All-orders !

$\Rightarrow$ $\boxed{\begin{array}{l} F(p) \propto p^{-2} \\ G(p) \propto p^{2} \end{array}}$ } "scaling" behav. (Gribov)

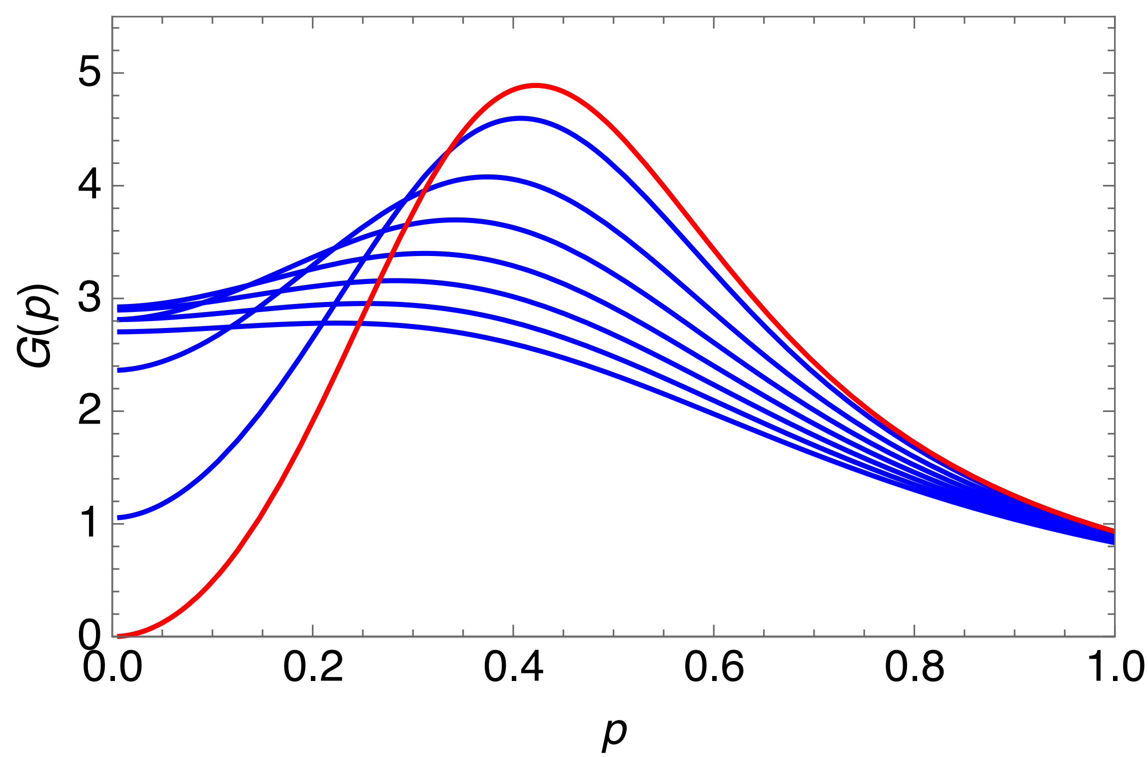
At one-loop order :

$$\tilde{m}_*^2 \simeq 14.18 \quad ; \quad \lambda_* \simeq 16.11$$

$\Rightarrow$ $\boxed{\lambda_T^* \simeq 1.06}$ Strongly coupled !



F(p)
5
4
3
2
1
0.0
0.2
0.4
0.6
0.8
1.0
p



G(p)
5
4
3
2
1
0
0.0
0.2
0.4
0.6
0.8
1.0
p

Generalization to $2 \leq d \leq 4$

- dimensionless coupling $\tilde{\lambda} = \mu^{d-4} \lambda$
- scaling exp's $\boxed{p^2 G(p) \sim p^{2\alpha_G}\,, \quad F(p) \sim p^{2\alpha_F}}$

$\Rightarrow$ Decoupling fixed point (IR stable)

$$\lambda^{dec}_{T,*} = 1/\tilde{m}^{2\,dec}_{T,*} = 0$$

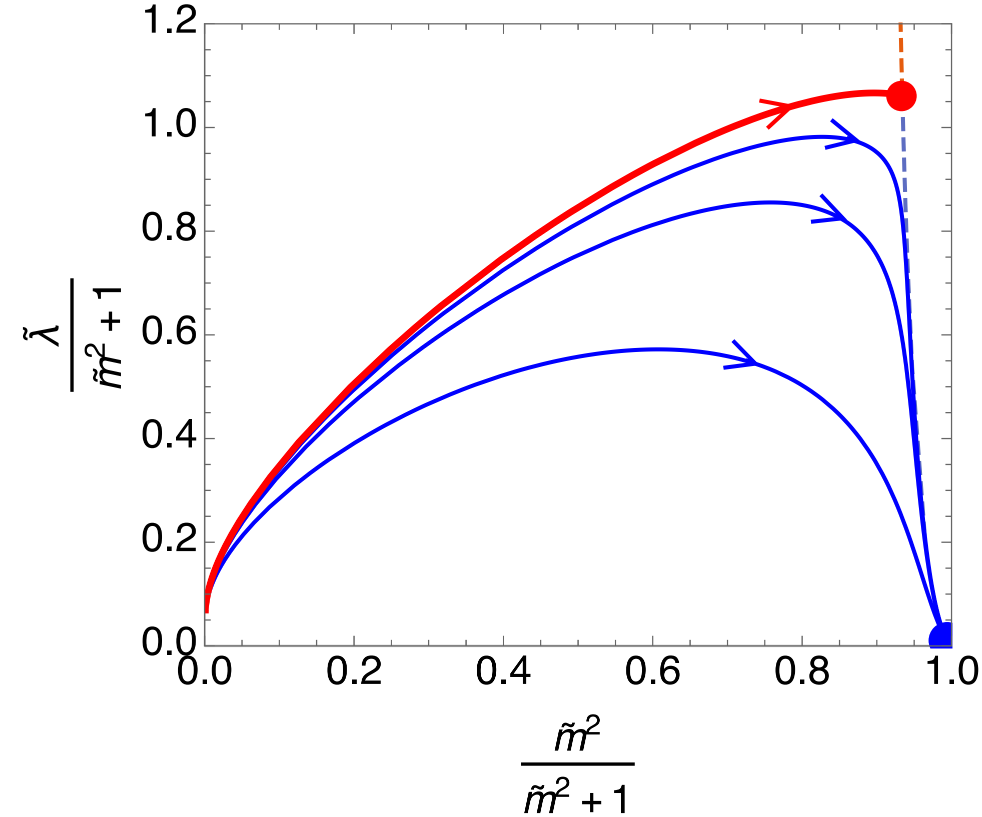
$$\boxed{\alpha_F = 0 \quad ; \quad \alpha_G = 1}$$

$\Rightarrow$ Scaling fixed point (IR unstable)

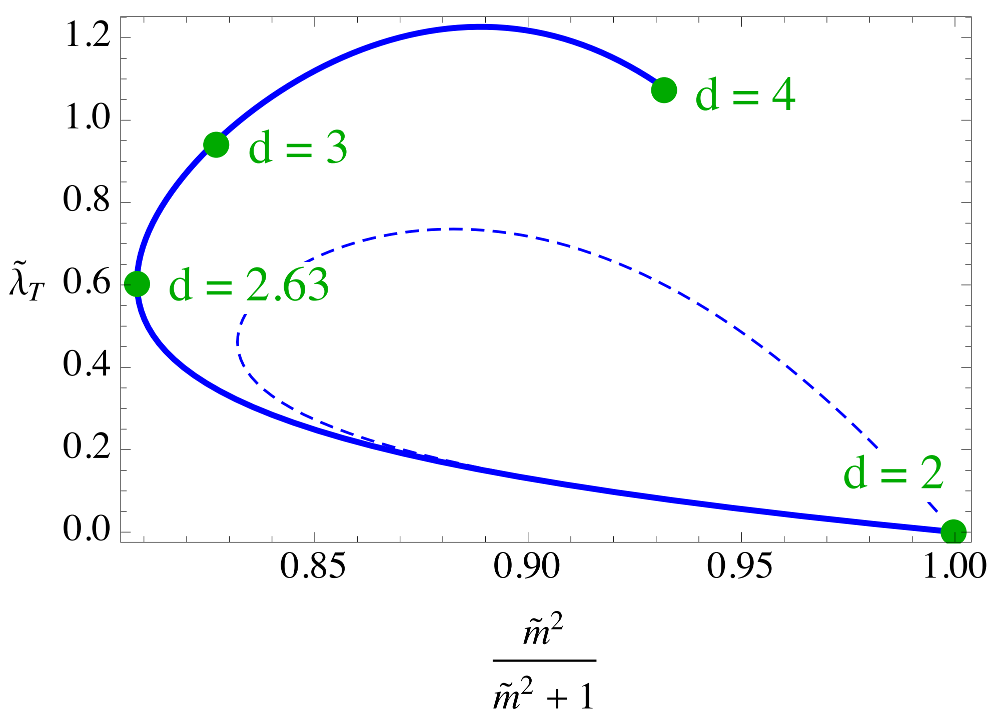
$$\frac{\beta_\lambda}{\lambda} = \frac{\beta_{\hat{m}^2}}{\hat{m}^2} = 0 \quad \Rightarrow \quad \gamma_A^* = d\,, \quad \gamma_c^* = 2-d$$

$$\boxed{\alpha_F = \frac{2-d}{2} \quad ; \quad \alpha_G = \frac{d}{2}}$$

(Gribov-type)



1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
$\frac{\tilde{\lambda}}{\tilde{m}^2+1}$
$\frac{\tilde{m}^2}{\tilde{m}^2+1}$



1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\tilde{\lambda}_T$
d = 4
d = 3
d = 2.63
d = 2
0.85
0.90
0.95
1.00
$\frac{\tilde{m}^2}{\tilde{m}^2+1}$

$$\boxed{G^{-1}(0) \text{ versus } m_0^2}$$

gluon screening mass squared

control mass parameter

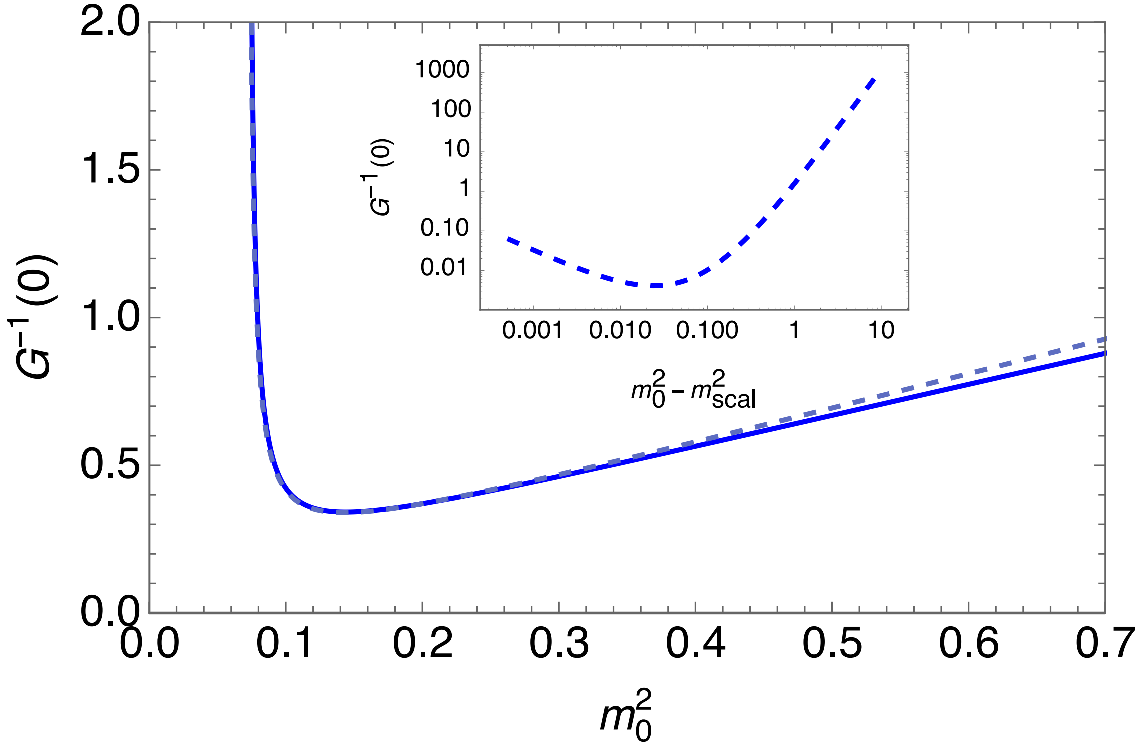
mSTIs $\Rightarrow$ $G_B(0)\,F_B(0) = m_B^{-2}$

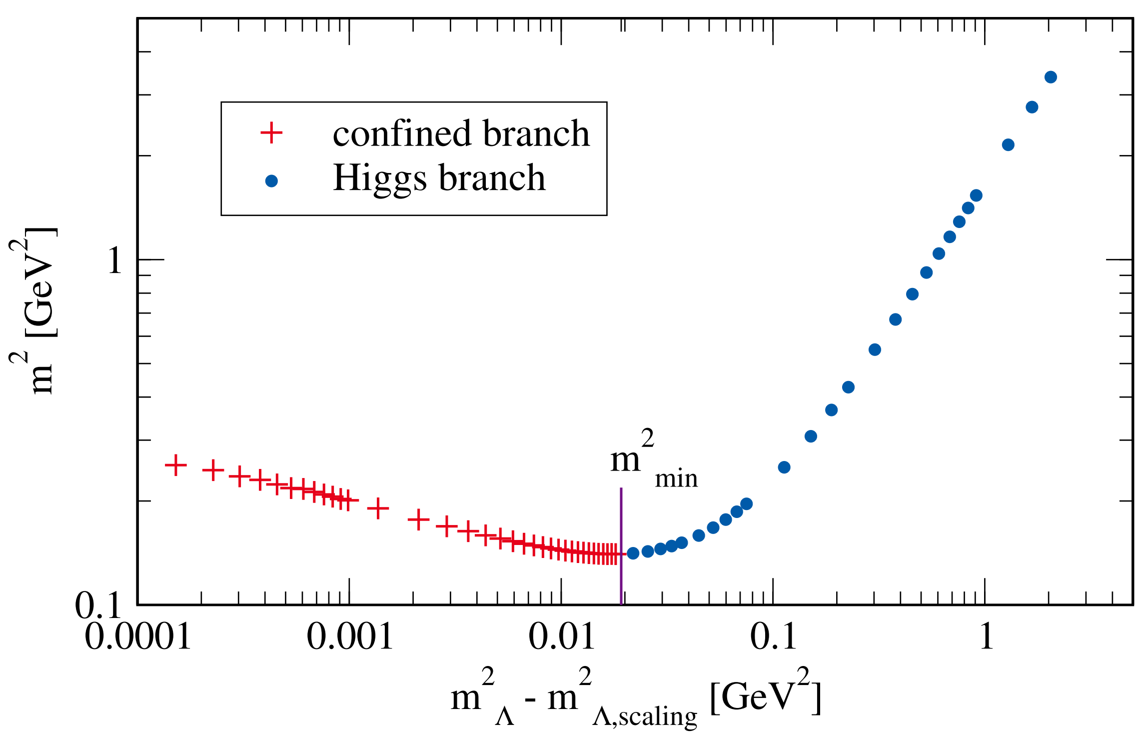
Present RG scheme $\Rightarrow$ $\boxed{G(0)\,F(0) = \frac{1}{m_0^2}}$

$$\Rightarrow \quad G^{-1}(0) = \frac{m_0^4}{c\,\lambda_0\,(m_0^2 - m_{scal}^2)}$$

minimum at $\boxed{m_{min}^2 = 2\,m_{scal}^2}$

N.B.: No sign of "confined" or "Higgs" phases as advocated in [Cyrol et al. ('16)]





[Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD94 (2016)]

Spectral positivity violation

Non monotonous gluon propagator

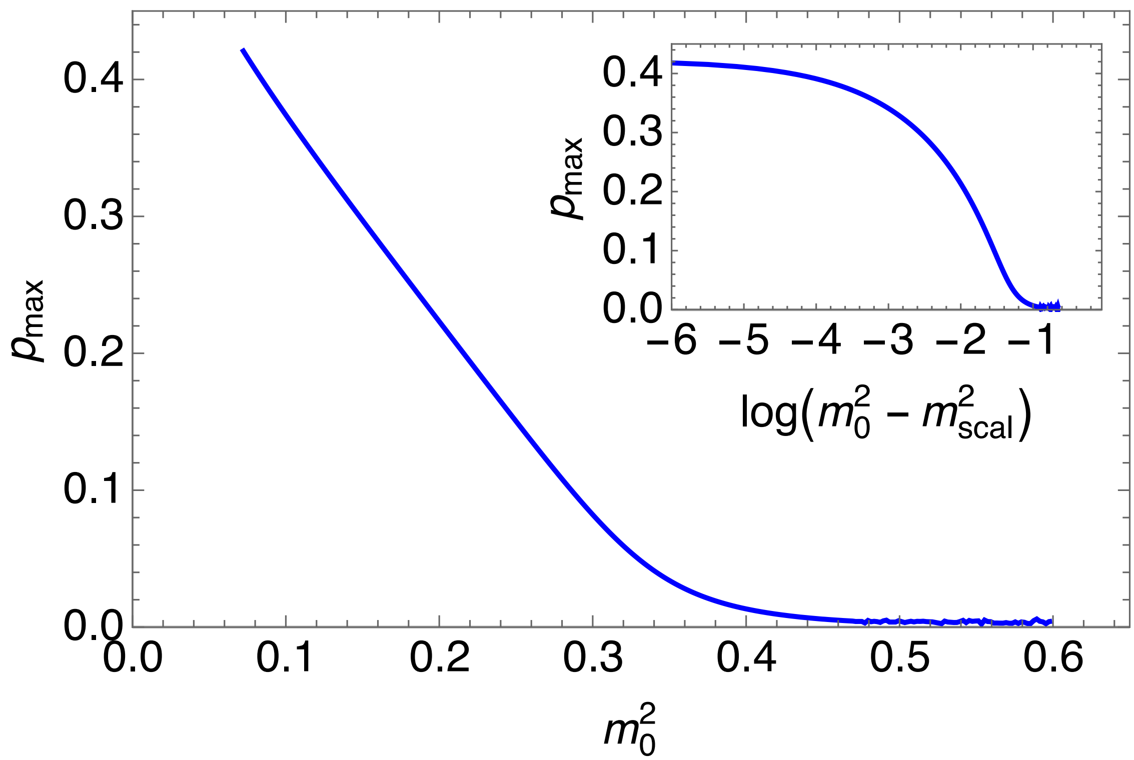
$\Rightarrow$ non positive-definite spectral weight

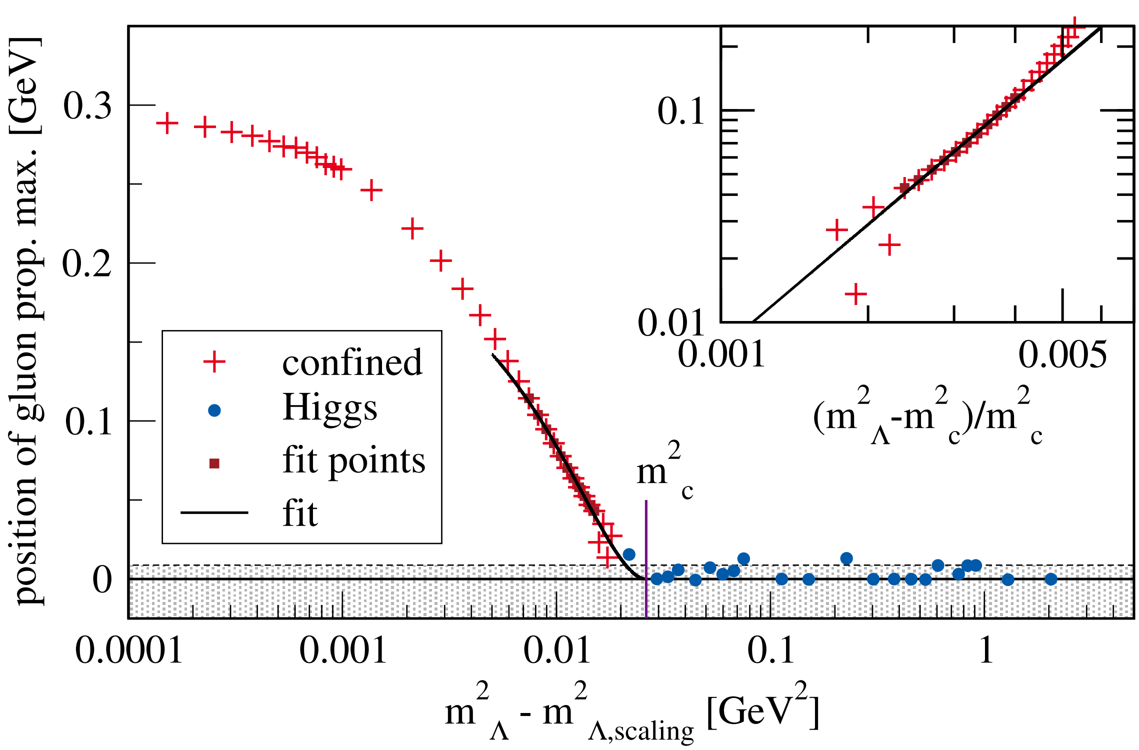
$$\frac{d \ln G(p)}{d \ln p} = - \frac{(2+\gamma_A)t + \gamma_c}{1+t}$$

$$\Rightarrow \boxed{\left.\frac{dG(p)}{dp^2}\right|_{\substack{p\to 0 \\ (d=4)}} = \frac{\lambda_0}{4 m_0^4} \ln\left(\frac{m^2(p)}{\lambda(p)}\right) > 0}$$

$G(p)$ is always non monotonous

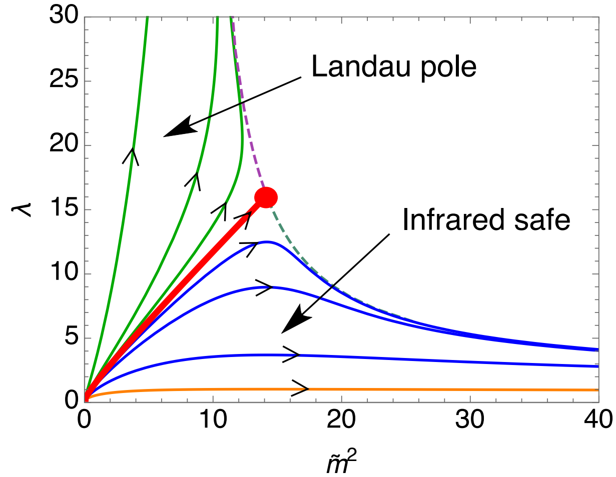
(No transition to a "Higgs" phase)

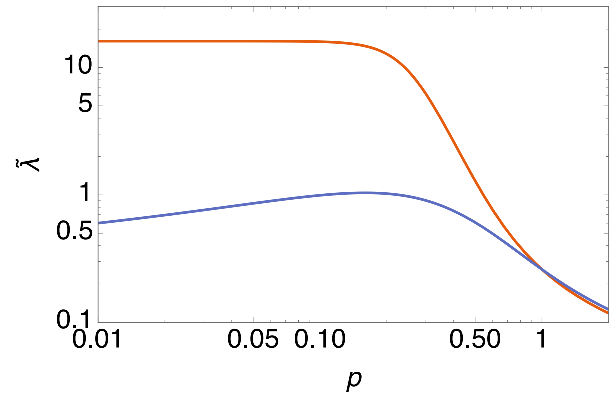


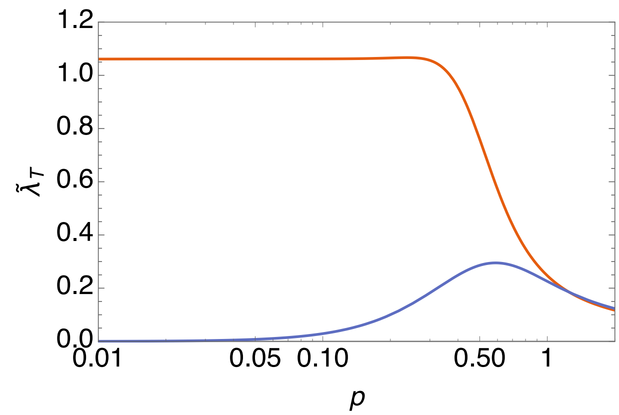


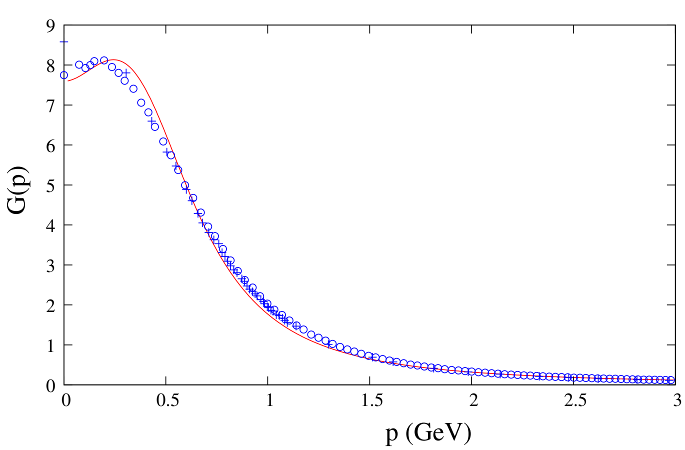
[Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD94 (2016)]

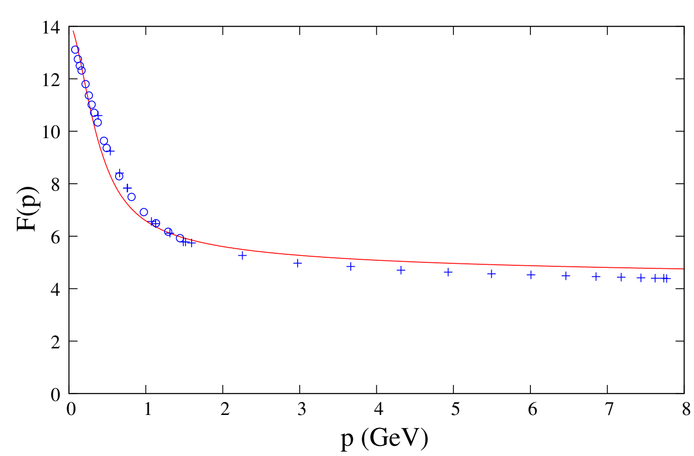
Lattice results: moderate coupling!









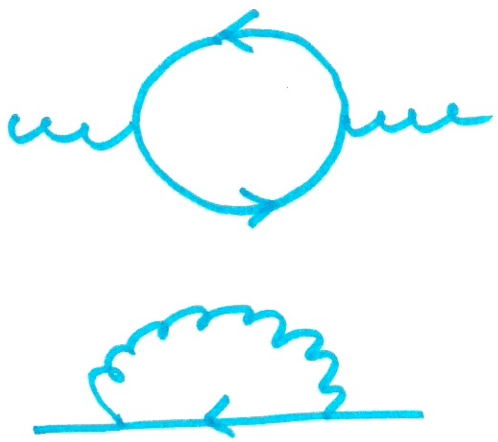


[Tissier, Wschebor PRD84 (2011) 045018]

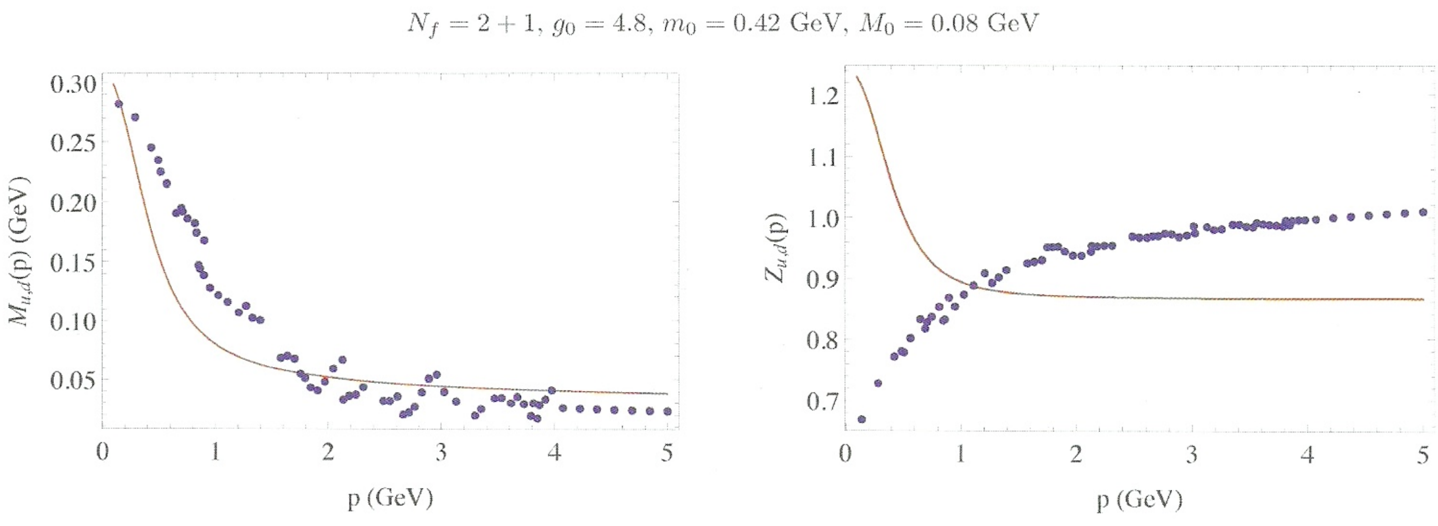
Data from [Bogolubsky *et al.*, PLB 676 (2009) 69 ; Dudal, Oliveira, Vandersickel, PRD 81 (2010) 074505]

PART II : QCD dynamics

$$\mathcal{L} = \frac{1}{4}F^2 + ih\partial A + \partial\bar{c}Dc + \underline{\underline{\frac{m^2}{2}A^2}}$$
$$+ \underline{\bar{\psi}(\not{D}+M)\psi}$$



One-loop contributions

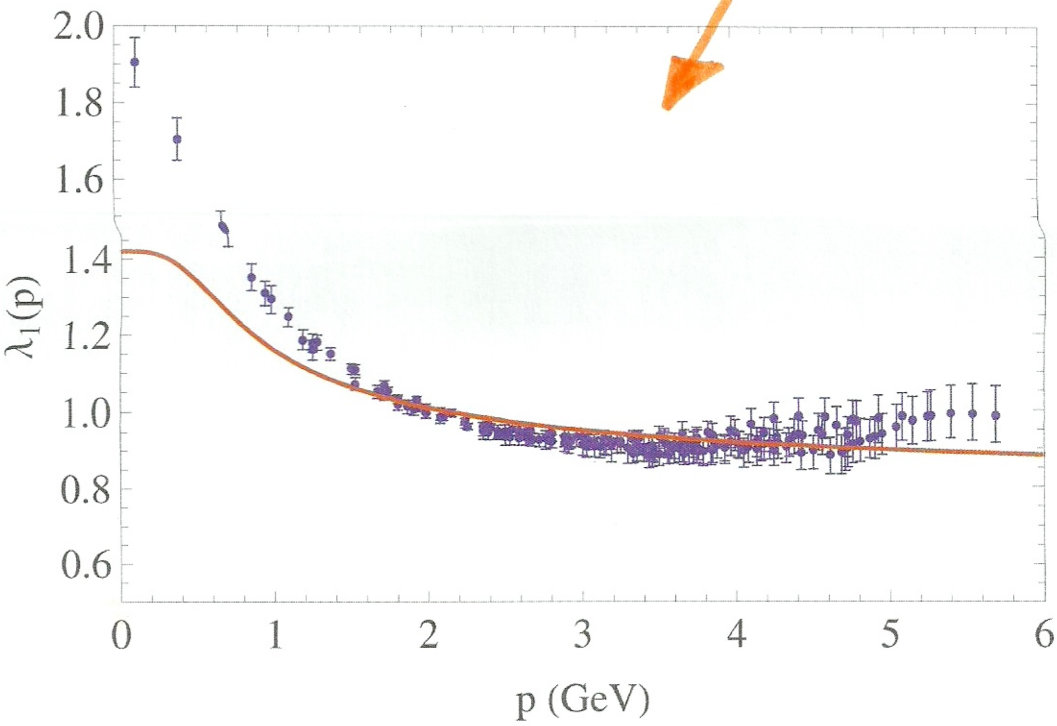


[Peláez, Tissier, Wschebor PRD 90 (2014) 065031]
Data from [Bowman *et al.* PRD 71 (2005) 54507]

The quark-gluon coupling

$$= -i\, g_q(p)\, \gamma_\mu + \dots$$

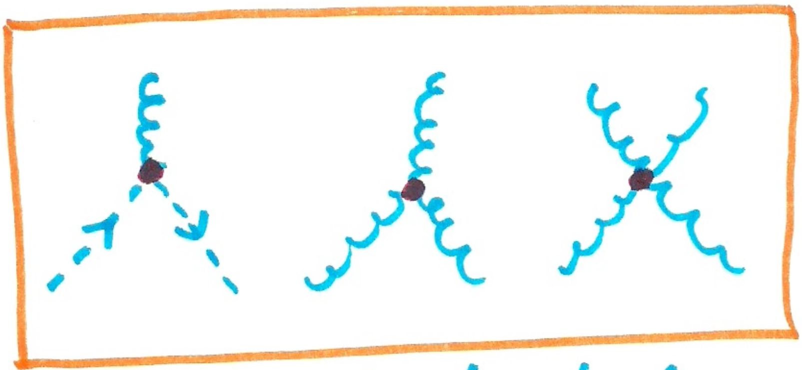
$$\boxed{g_q(p) = \underbrace{g_g(p)}_{\text{ghost-gluon}}\, \lambda_1(p)}$$



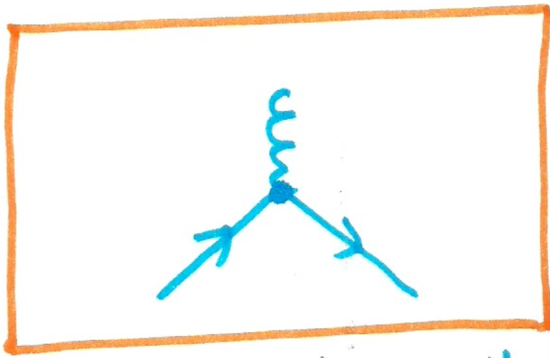
[Peláez, Tissier, Wschebor PRD 92 (2015) 045012]
Data from [Skullerud *et al.* JHEP 0304 (2003) 047]

Small parameters in infrared QCD

[Peláez, Reinosa, Serreau, Tissier, Wschebor ('17)]



g_g : can be treated perturbatively



g_q : not small !!

① Expand in g_g, not in g_q

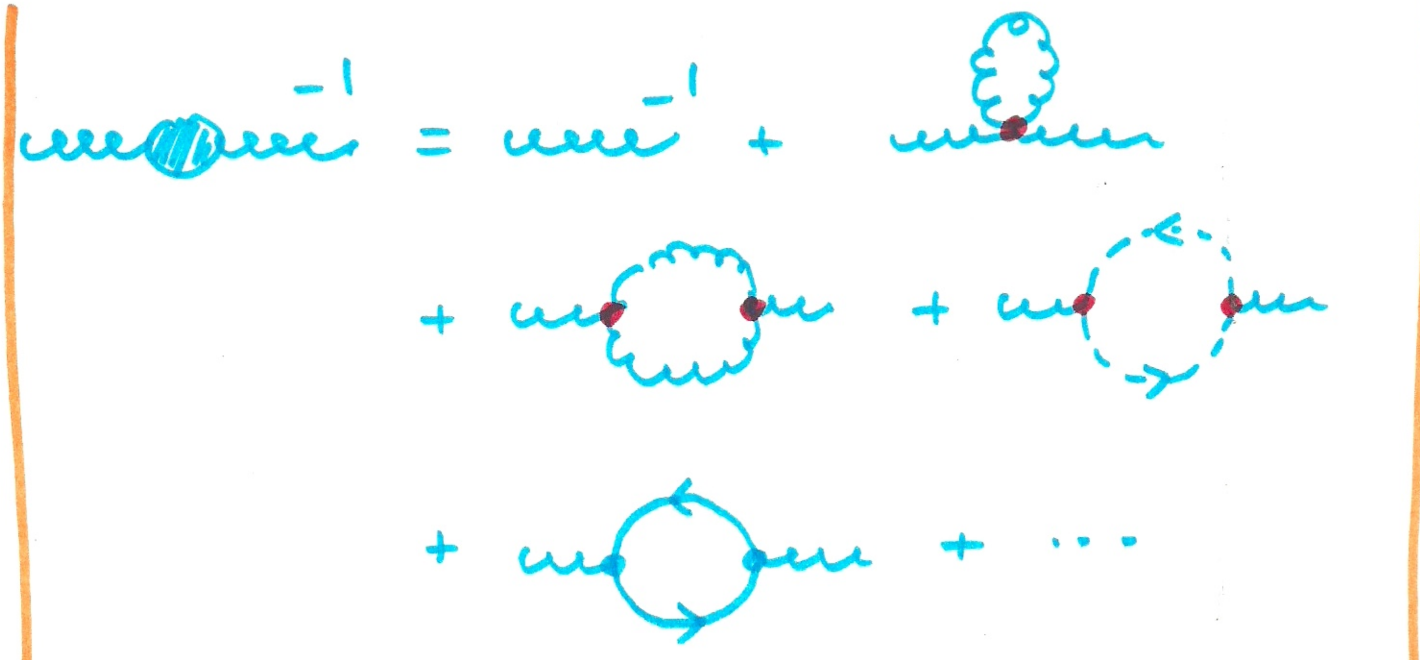
e.g. : only QED-like diagrams survive at L.O. in the quark sector

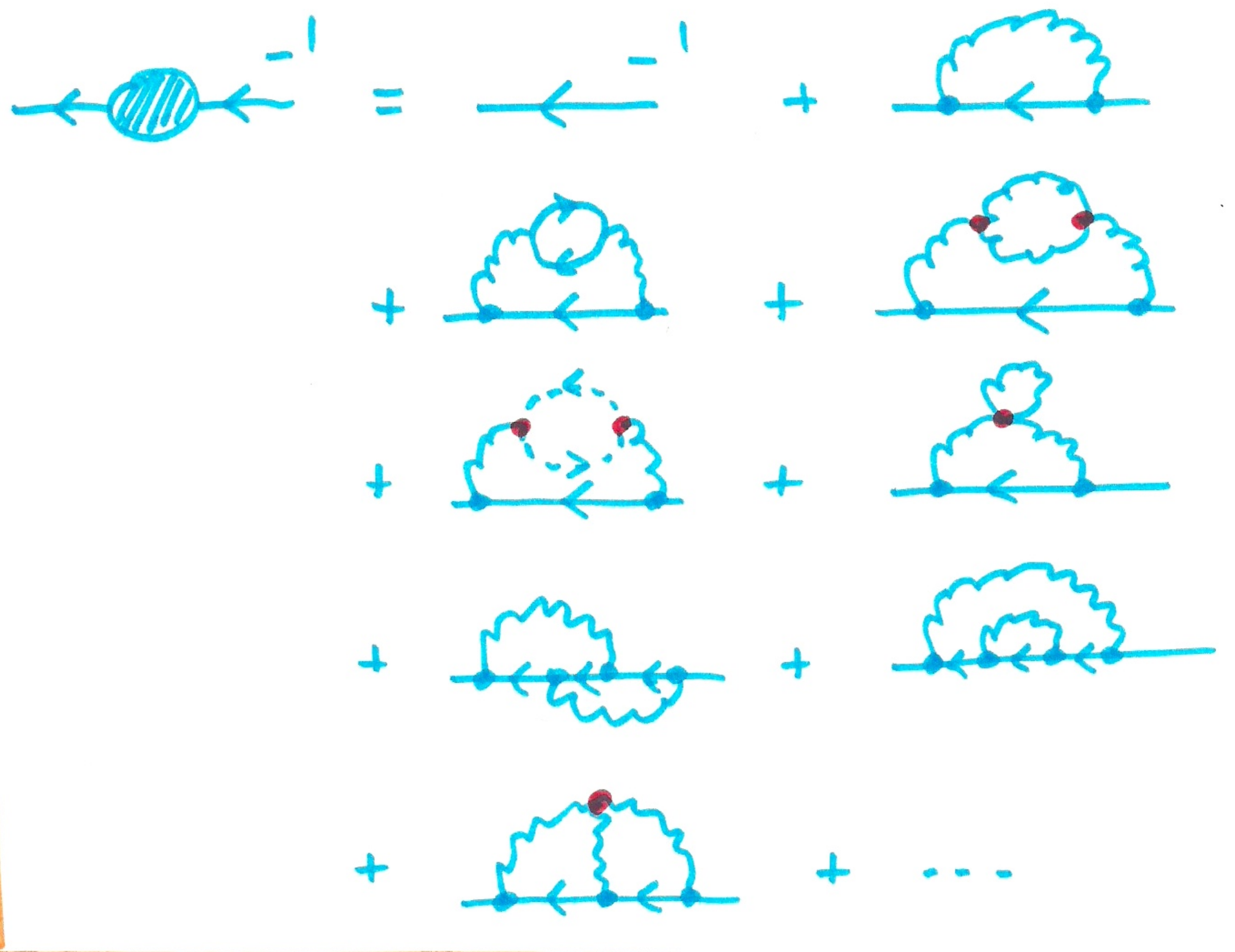
② Expand in $1/N_c$ with $g_q^2 N_c$ fixed

['t Hooft ('74) ...]

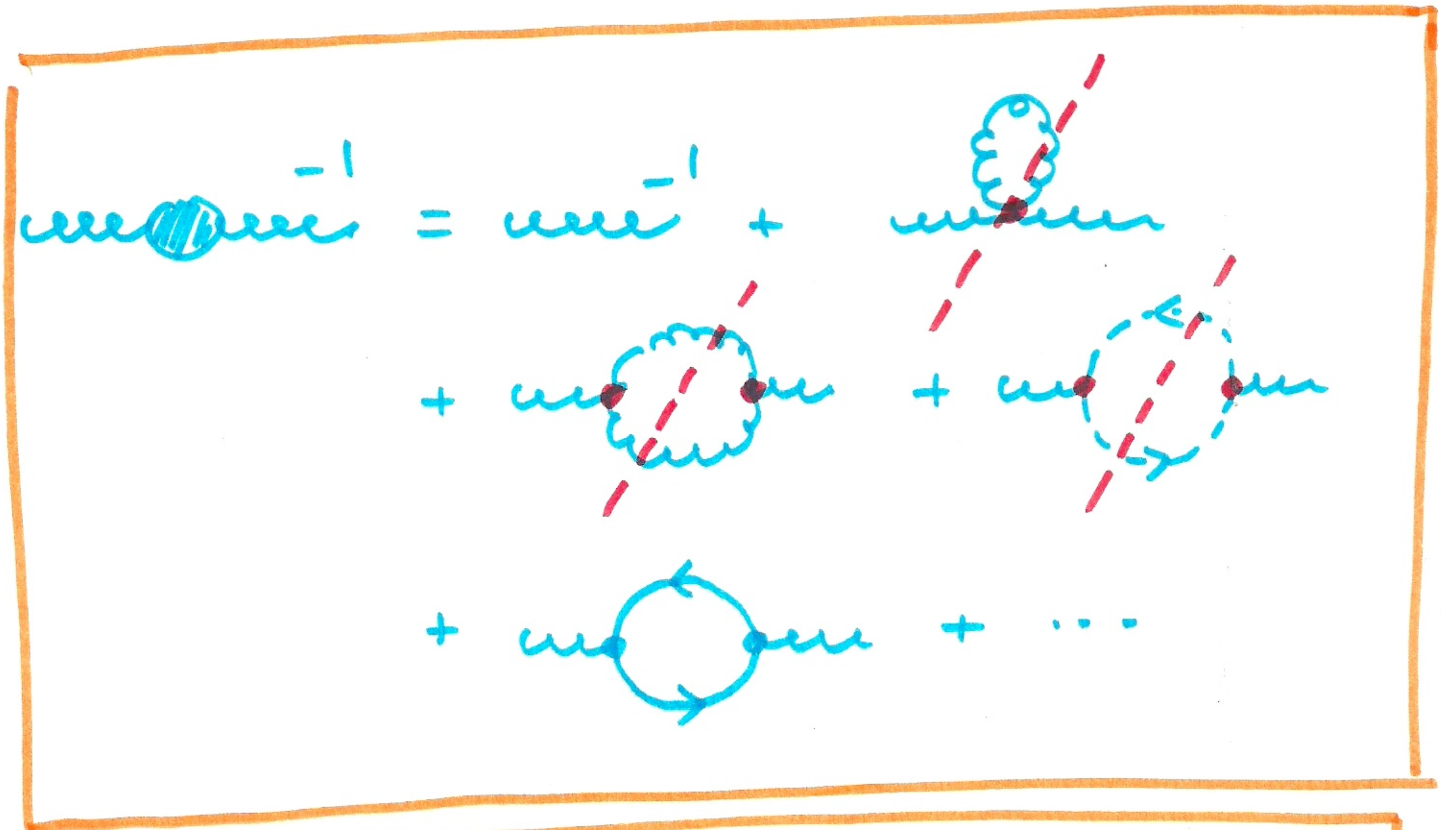
→ Captures essential aspects of QCD dynamics

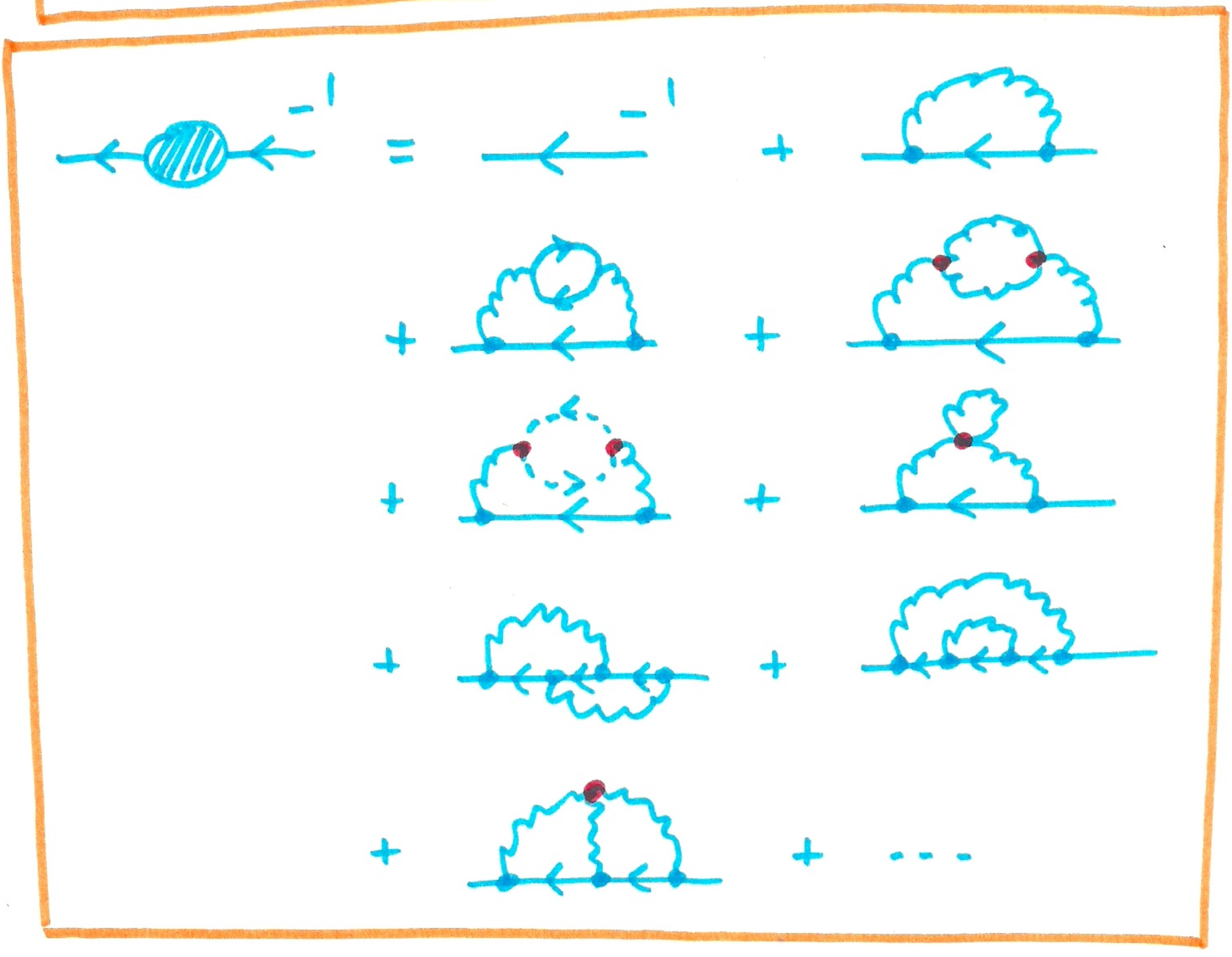
"Rainbow-improved" loop expansion



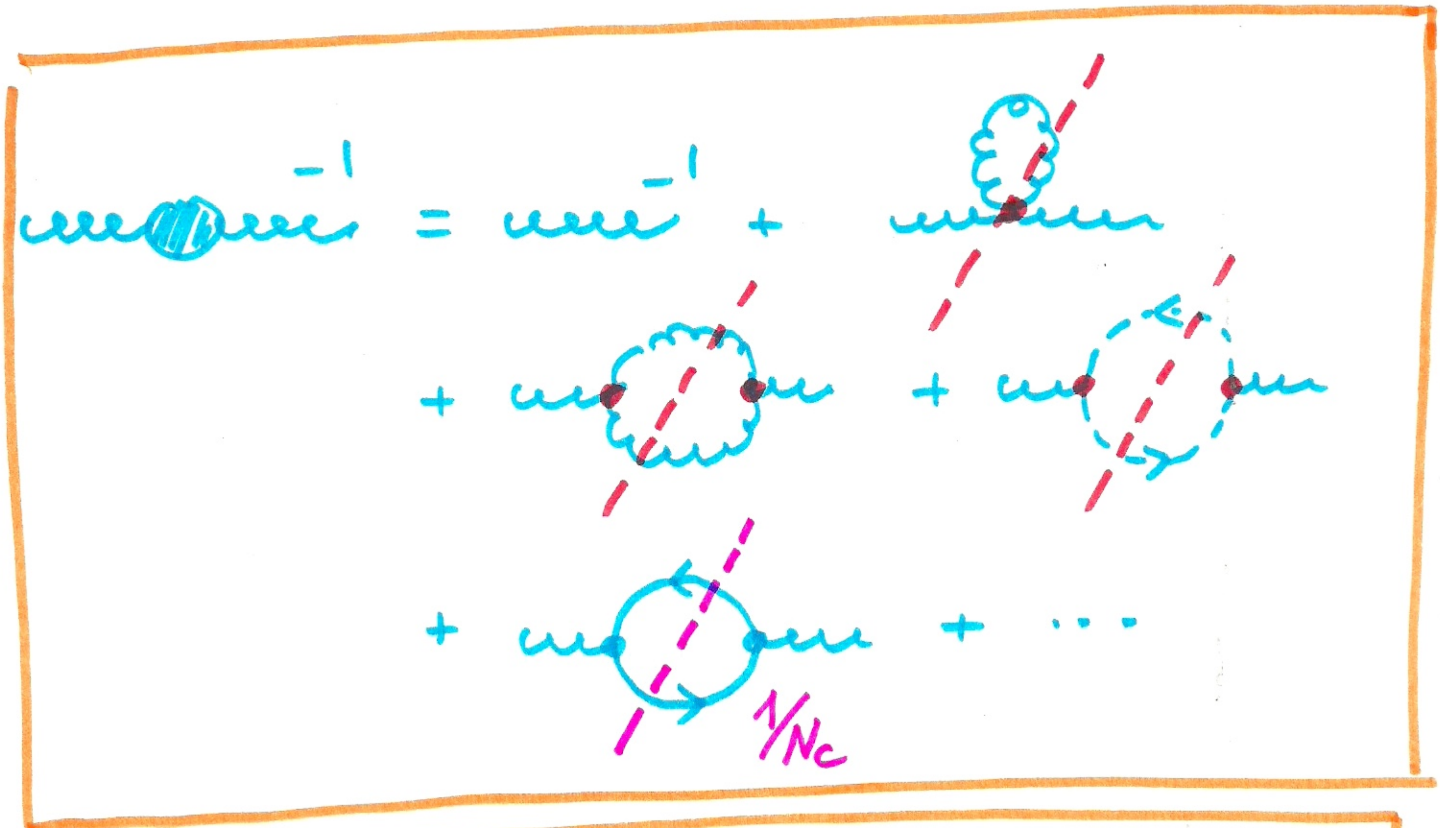


"Rainbow-improved" loop expansion

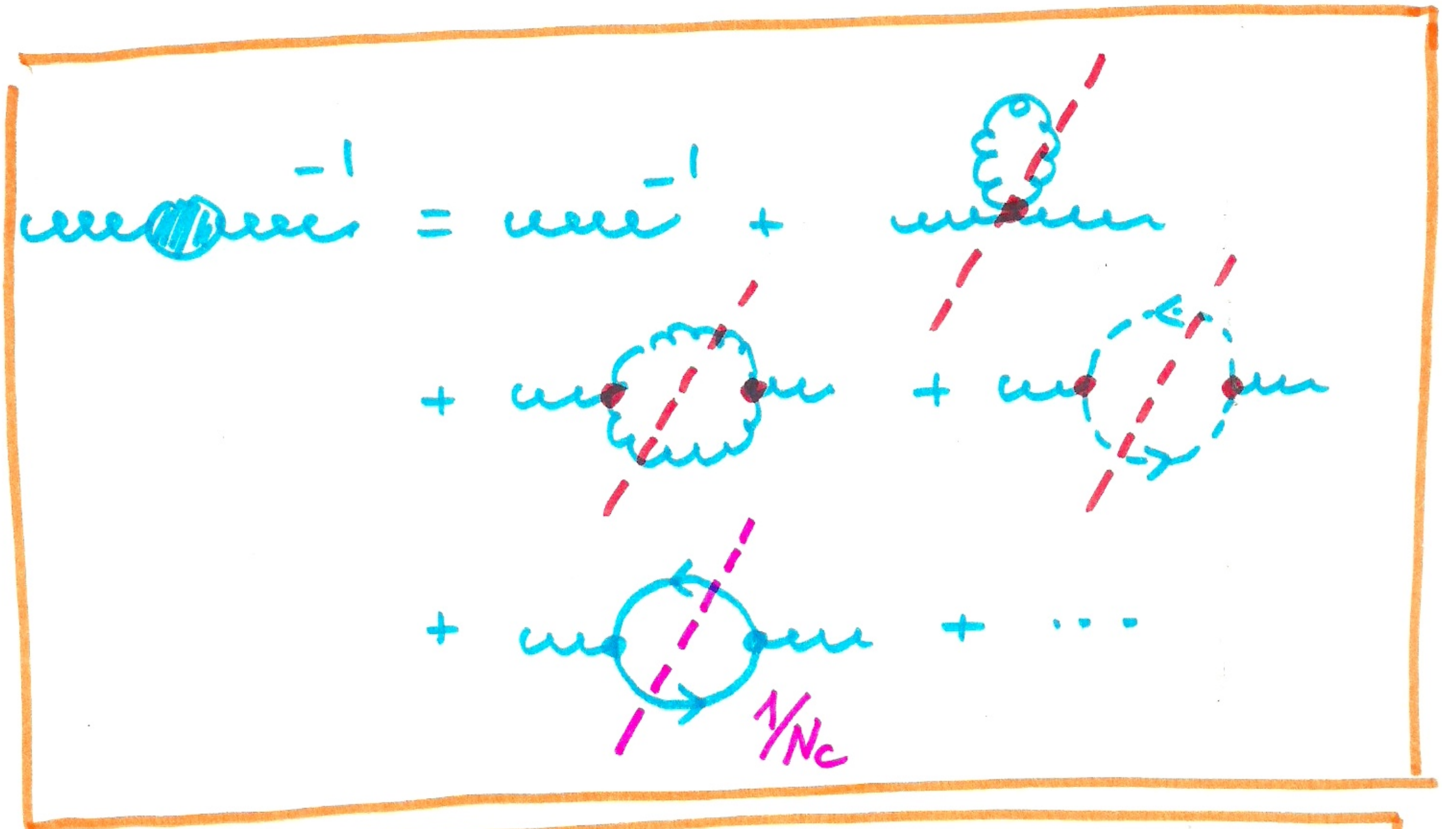




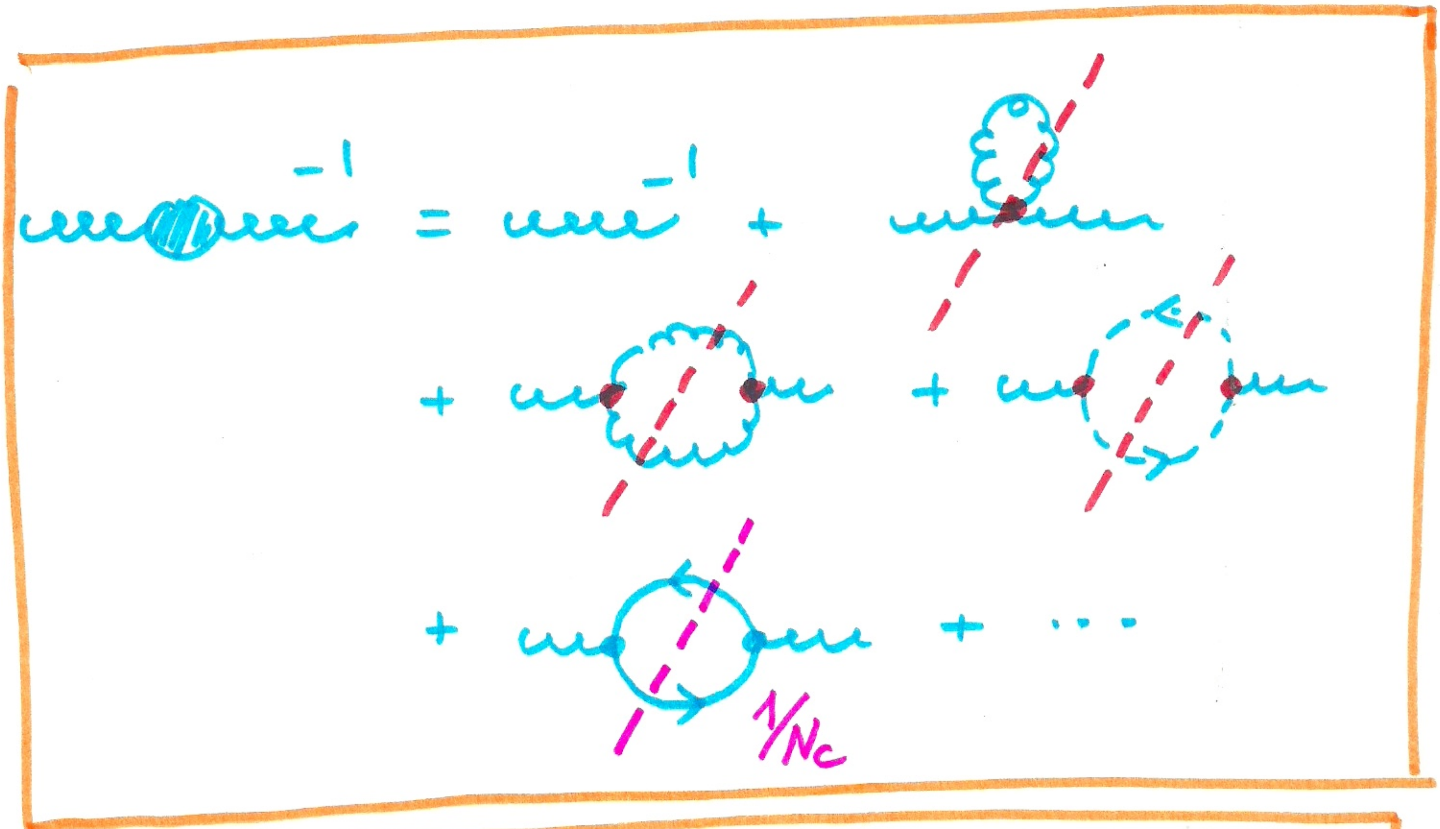
"Rainbow-improved" loop expansion



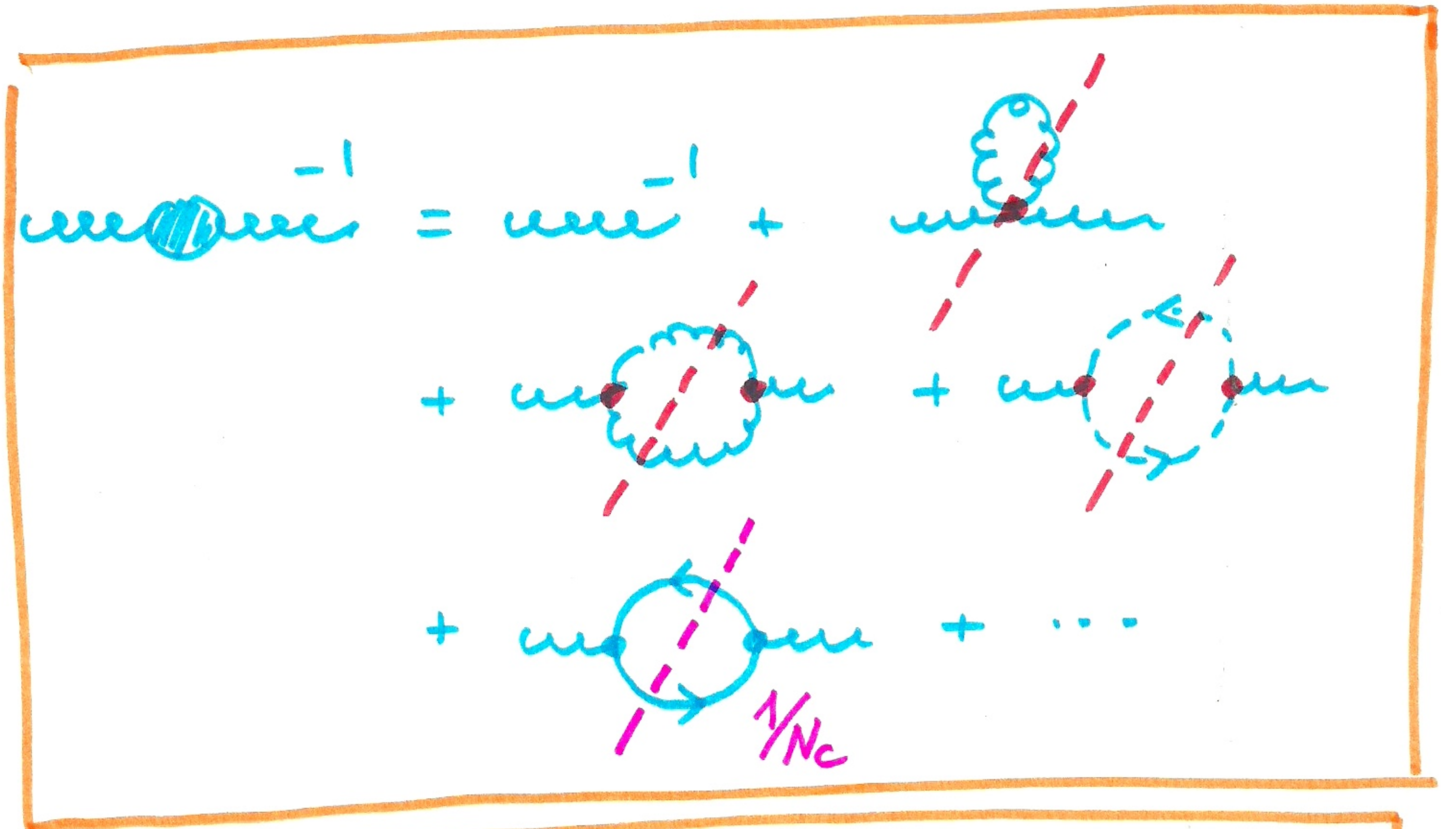
"Rainbow-improved" loop expansion



"Rainbow-improved" loop expansion



"Rainbow-improved" loop expansion



Rainbow-improved : Leading Order (RiLO)

L.O. : only rainbow diagrams survive in the quark propagator

$$S(p) = \left(iA(p)\not{p} + B(p)\right)^{-1} = -i\tilde{A}(p)\not{p} + \tilde{B}(p)$$

$$\frac{\tilde{A}}{A} = \frac{\tilde{B}}{B} = \frac{1}{A^2p^2 + B^2}$$

$$\tilde{A}(p) = 1 - g_q^2 C_F \int_q \frac{f(q,p)\,\tilde{A}(q)}{(p+q)^2 + \underline{m}^2}$$

$$\tilde{B}(p) = M + 3g_q^2 C_F \int_q \frac{\tilde{B}(q)}{(p+q)^2 + \underline{m}^2}$$

CF mass

$$f(p,q) = \frac{2p^2q^2 + 3(p^2+q^2)(p.q) + 4(p.q)^2}{p^2(p+q)^2}$$

RILO : Renormalization

① $A \to \sqrt{Z_A}\, A \quad ; \quad \psi \to \sqrt{Z_\psi}\, \psi$

$m^2 \to Z_{m^2} m^2 \; ; \; M \to Z_M M \; ; \; g_q \to Z_g g_q$

② $S^{-1}(p=\mu_0) = -i\not{\mu}_0 + M(\mu_0)$

③ $Z_A = Z_{m^2} = Z_g Z_\psi \sqrt{Z_A} = 1$ at L.O.

$$\tilde{A}(p) = \underline{Z_\psi} - \underline{g_q}^2 C_F \int_q \frac{f(q,p)\,\tilde{A}(q)}{(p+q)^2 + \underline{m}^2}$$

$$\tilde{B}(p) = \underline{M} + 3\underline{g_q}^2 C_F \int_q \left\{ \frac{\tilde{B}(q)}{(p+q)^2 + \underline{m}^2} - (p \leftrightarrow \mu_0) \right\}$$

RiLO : RG improvement

① $$\left(\frac{\partial}{\partial \ln\mu} - \gamma_\psi + \beta_x \frac{\partial}{\partial x}\right) S^{-1}(p) = 0$$

$$x = m, M, g_q$$

$$\gamma_\psi = \frac{\partial \ln Z_\psi}{\partial \ln \mu} \; ; \; \beta_x = \frac{\partial x}{\partial \ln\mu}$$

② $$A(p,\mu_0) = \frac{Z_\psi(\mu_0)}{Z_\psi(p)}$$

$$B(p,\mu_0) = A(p,\mu_0)\, M(p)$$

$$\boxed{\begin{aligned} Z_\psi(p) &= 1 + \frac{g_q^2(p) C_F}{Z_\psi(p)} \int_q \frac{Z_\psi(q)}{q^2 + M^2(q)} \frac{f(qp)}{(p+q)^2 + m^2(p)} \\ p M'(p) &= \gamma_\psi(p) M(p) - 3\, \frac{g_q^2(p) C_F}{Z_\psi(p)} \\ &\quad \times \int_q \frac{Z_\psi(q) M(q)}{q^2 + M^2(q)} \frac{2p^2 + 2p.q}{(p+q)^2 + m^2(p)} \end{aligned}}$$

RiLO : UV limit

The large p regime is dominated by $\boxed{m \lesssim q \lesssim p}$

$$Z_\psi(p) \simeq 1$$

$$p\,M'(p) \simeq -\frac{3 g_q^2(p) C_F}{8\pi^2 p^2} \int_{m^2}^{p^2} dq^2\, M(q)$$

$\oplus$ $g_q^2(p) = \dfrac{1}{\beta_0 \ln p^2/\mu_0^2}$; $\beta_0 = \dfrac{\frac{11}{3}N_c - \frac{2}{3}N_f}{16\pi^2}$

Massive solution

$$M(p) \sim \left(1 + \beta_0 g_0^2 \ln p^2/\mu_0^2\right)^{-\alpha}$$

SχSB solution

$$M(p) \sim p^{-2} (\ln p)^{\alpha-1}$$

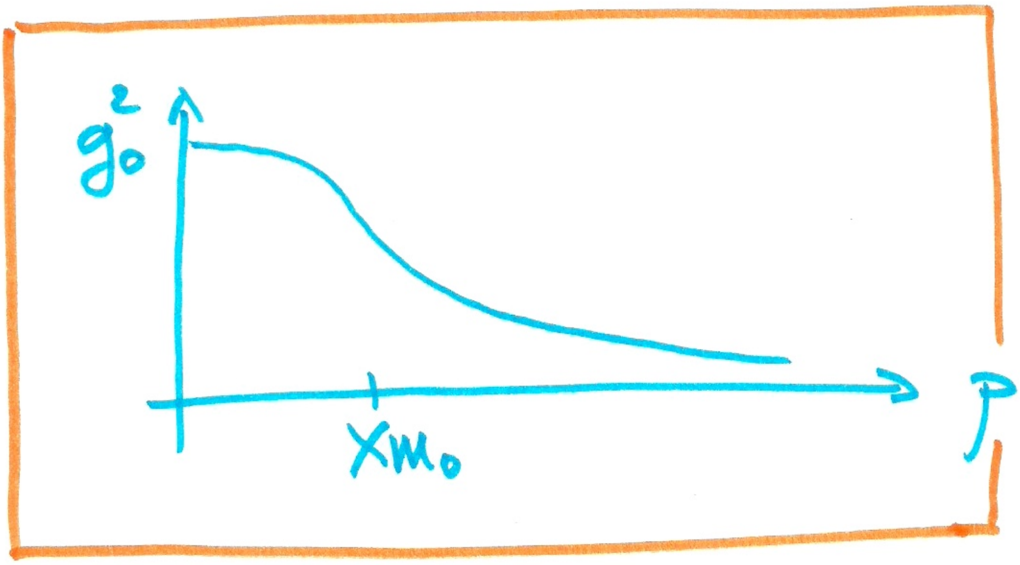
$$\alpha = \frac{3C_F}{16\pi^2 \beta_0}$$

[Miransky ('85); Atkinson et al. ('88); Fischer et al. ('03)...]

RiLO : Results

Toy model for the flow of g_q^2 :

$$g_q^2(p) = \frac{g_0^2}{1 + \beta_0 g_0^2 \ln\left(\frac{p^2 + x^2 m_0^2}{x^2 m_0^2}\right)}$$

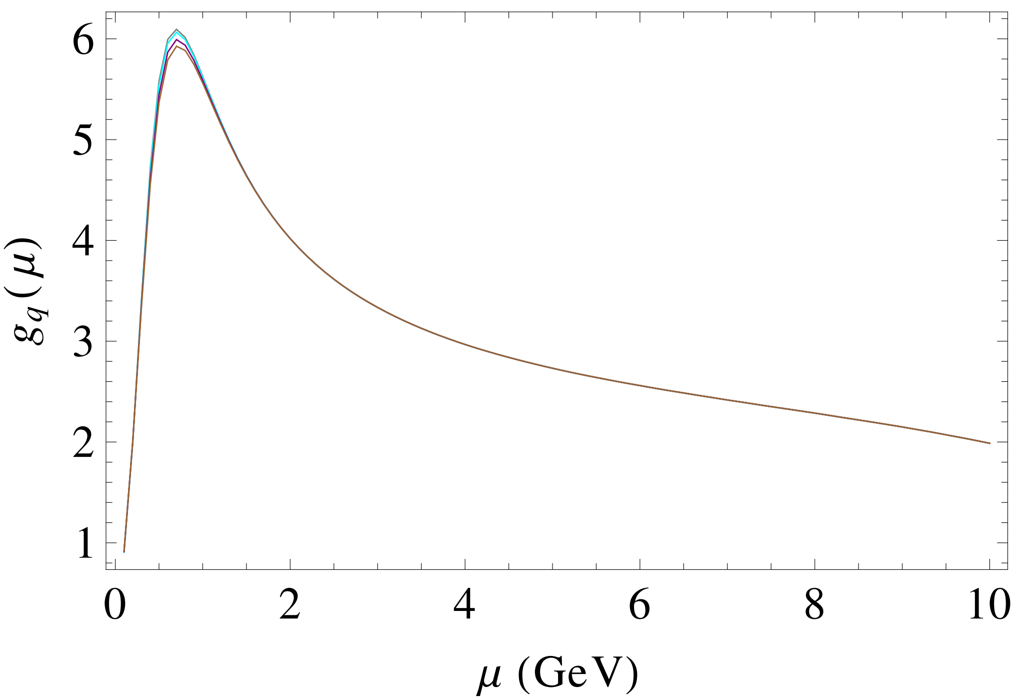


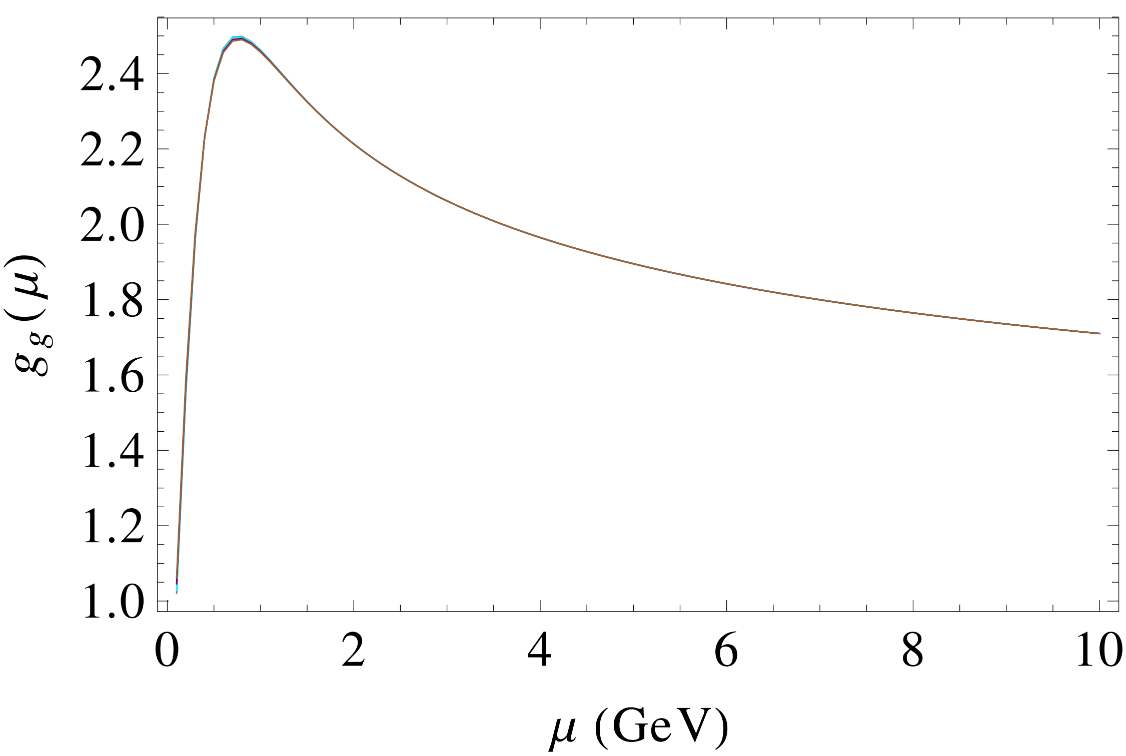
- $m_0 = 0.4$ GeV (from fits of YM propags)
- vary $M(10\,\text{GeV})$, g_0^2, x

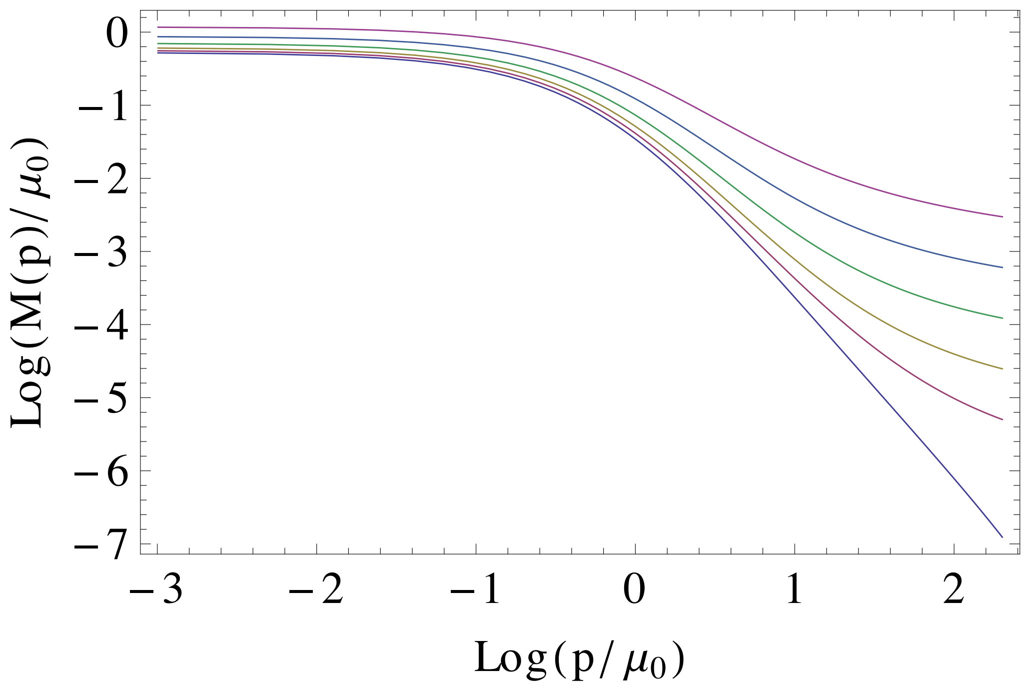
⚠ Same quality with the fully consistent flow computed at RiLO (no other parameters than m_0, g_0)

[in preparation]

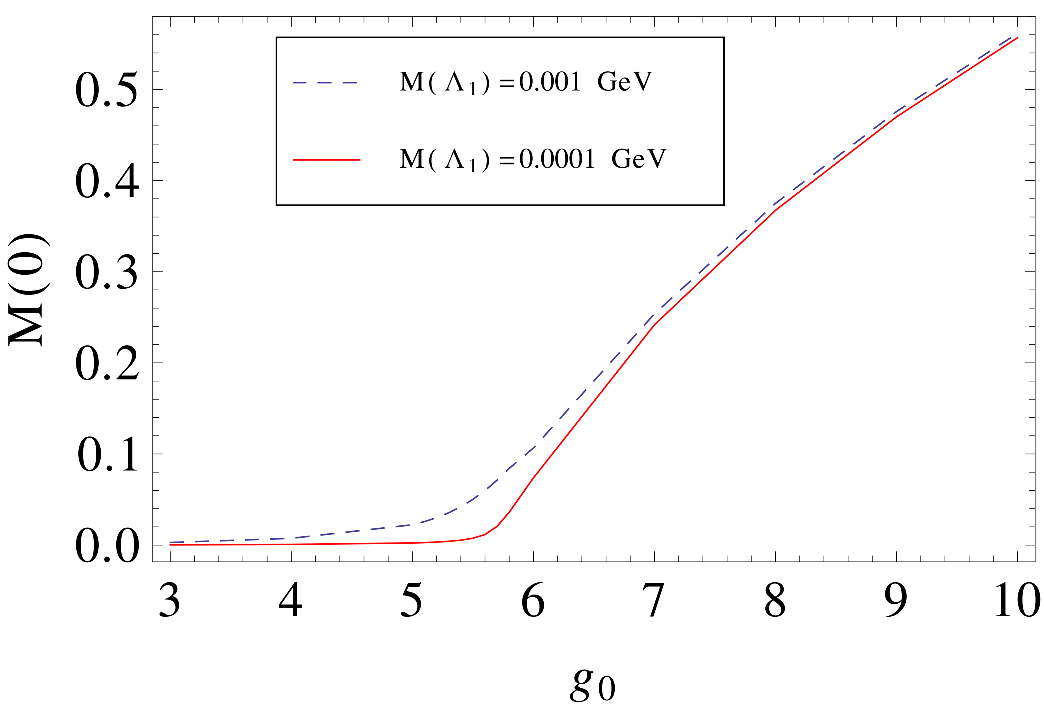
RILO: running couplings



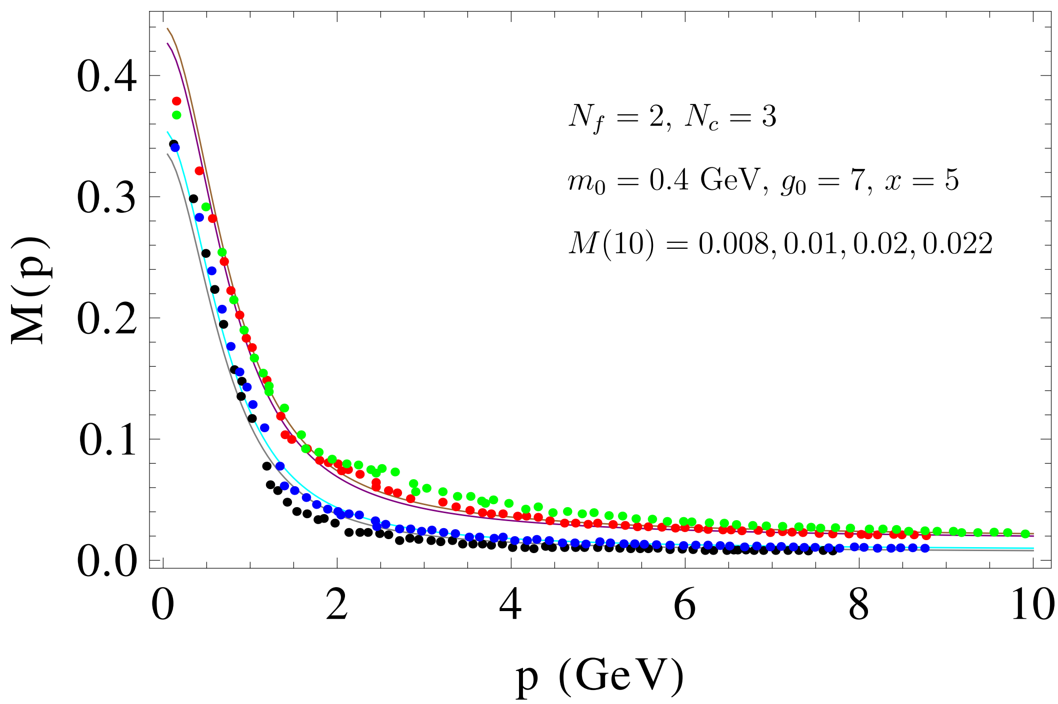




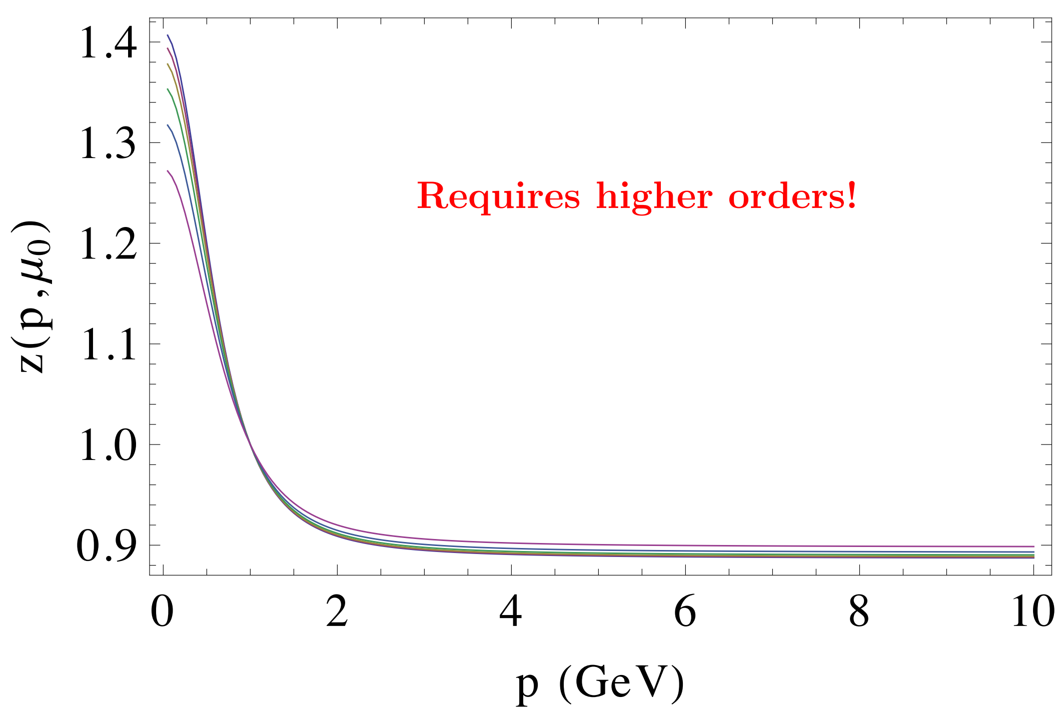
0
−1
−2
−3
−4
−5
−6
−7
Log(M(p)/μ_0)
−3
−2
−1
0
1
2
Log(p/μ_0)



M(Λ_1) = 0.001 GeV
M(Λ_1) = 0.0001 GeV
0.5
0.4
0.3
0.2
0.1
0.0
M(0)
3
4
5
6
7
8
9
10
g_0



Data from [Oliveira et al. arXiv:1605.09632]



CONCLUSIONS

- The CF model provides a satisfying description of various aspects of the dynamics of nonabelian gauge fields in perturbation theory.

- The dynamics of (light) quarks and, in particular, $S\chi SB$ is well described by a systematic expansion in g_g, $1/N_c$

  ⇒ rainbow resummation of fermion lines

PERSPECTIVES

- Higher (two) loop calculations in Yang-Mills theories and in QCD

- Full RiLO:
  - running couplings m, g_g, g_q
  - gluon/ghost/quark propagators
  - quark-gluon vertex

- Meson properties in RiLO
  e.g.: m_π, f_π as functions of m_0, g_0

- Finite T, μ

...