Lattice Ghost Propagator in Linear Covariant Gauges

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- In a Quantum Field Theory, knowledge of all Green's functions allows a complete description of the theory
- In QCD, propagators of fundamental fields (e.g. quark, gluon and ghost propagators) encode information about non-perturbative phenomena
- In particular, gluon and ghost propagators encode information about confinement/deconfinement
- Since gluon/ghost propagators are gauge dependent quantities, we need to choose a gauge
  - Landau gauge $\partial_\mu A_\mu = 0$
  - Linear covariant gauge $\partial_\mu A_\mu = \Lambda$

Landau gauge fixing on the lattice

- gauge fixing functional

$$F^{Landau}(U^g) = -\mathrm{Re}\,\mathrm{tr}\sum_{x,\mu}\left[\,g(x)\,U_\mu(x)\,g^\dagger(x+e_\mu)\right]$$

- First variation: Landau gauge condition $\partial_\mu A^a_\mu = 0$
- Second variation: defines a symmetric matrix

$$\begin{aligned} M^{ab}_{x,y} &= \sum_\mu \mathrm{Re}\,\mathrm{tr}\left[\left\{t^a, t^b\right\}(U_\mu(x) + U_\mu(x-\hat{\mu}))\right]\delta_{xy} \\ &- 2\,\mathrm{Re}\,\mathrm{tr}\left[t^b t^a U_\mu(x)\right]\delta_{x+\hat{\mu},y} - 2\,\mathrm{Re}\,\mathrm{tr}\left[t^a t^b U_\mu(x-\hat{\mu})\right]\delta_{x-\hat{\mu},y} \end{aligned}$$

  - continuum limit $-\frac{1}{2}\left(\partial_\mu D^{ab}_\mu + D^{ab}_\mu \partial_\mu\right)$
  - in Landau gauge: $= -\partial_\mu D^{ab}_\mu$

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Linear covariant gauges (LCG) on the lattice

- gauge fixing functional

$$F^{LCG}(U^g; g) = F^{Landau}(U^g) + \mathrm{Re\,tr} \sum_x [ig(x)\Lambda(x)]$$

- First variation: LCG condition $\partial_\mu A^a_\mu = \Lambda^a(x)$
- Second variation defines the same symmetric matrix as in Landau gauge

$$\begin{aligned} M^{ab}_{x,y} &= \sum_\mu \mathrm{Re\,tr}\left[\left\{t^a, t^b\right\}(U_\mu(x) + U_\mu(x-\hat{\mu}))\right]\delta_{xy} \\ &- 2\,\mathrm{Re\,tr}\left[t^b t^a U_\mu(x)\right]\delta_{x+\hat{\mu},y} - 2\,\mathrm{Re\,tr}\left[t^a t^b U_\mu(x-\hat{\mu})\right]\delta_{x-\hat{\mu},y} \end{aligned}$$

- continuum limit $-\frac{1}{2}\left(\partial_\mu D^{ab}_\mu + D^{ab}_\mu \partial_\mu\right)$
- not the correct one...

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Linear covariant gauges on the lattice

- A simple solution: correct $M^{ab}(x,y)$

$$[\Delta M]^{ab}_{xy} = \mathrm{Re}\,\mathrm{tr} \sum_\mu \left[\left[t^a, t^b \right] \left(U_\mu(x) - U_\mu(x-\hat{\mu}) \right) \right] \delta_{xy}$$

$$\begin{aligned} \left[M^+ \right]^{ab}_{xy} &= M^{ab}_{xy} + [\Delta M]^{ab}_{xy} \to -[\partial_\mu D_\mu]^{ab}_{xy} \\ \left[M^- \right]^{ab}_{xy} &= M^{ab}_{xy} - [\Delta M]^{ab}_{xy} \to -[D_\mu \partial_\mu]^{ab}_{xy} \end{aligned}$$

- M, M^+, M^- can not be distinguished as quadratic forms:

$$\omega^a(x)\,[\Delta M]^{ab}_{xy}\,\omega^b(y) = \omega^a(x) f_{abc}\,\mathrm{Re}\,\mathrm{tr}\left[i t^c \ldots \right] \omega^b(y) = 0$$

- the minimizing condition can only define a symmetric quadratic form

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Practical issues

- M^+ is a real non-symmetric matrix; can not be inverted using Conjugate Gradient (as one does in Landau gauge)
  - Generalized Conjugate Residual

Y. Saad, Iterative Methods for Sparse Linear Systems

- What about zero modes?
  - one should work in the subspace orthogonal to the null space of M^+
  - constant vectors are *not* zero modes of M^+

$$M^+M^+x = M^+b$$

Suman, Schilling, PLB373(1996)314

  - can be solved in two steps

$$\begin{aligned} M^+Y &= M^+b \\ M^+X &= Y \end{aligned}$$

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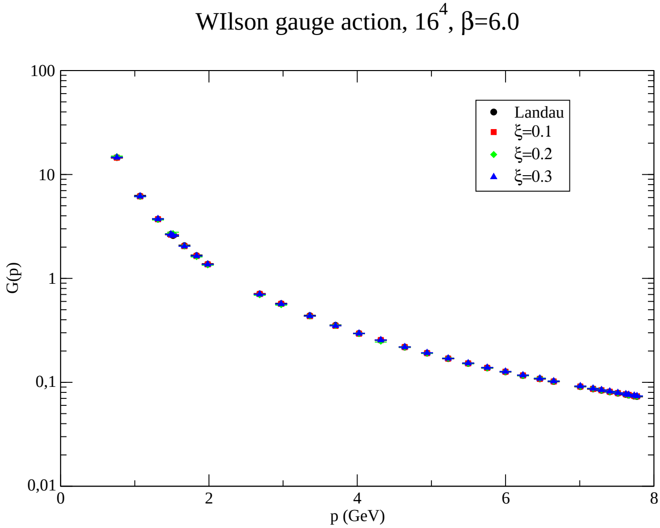
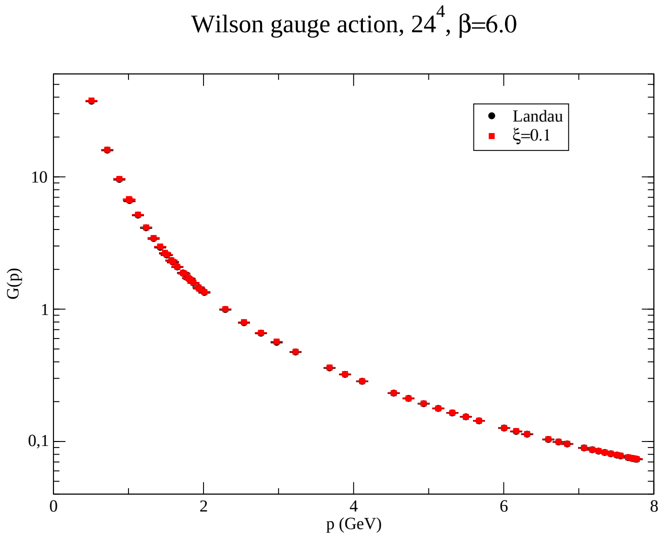
Results

- 100 U's, 20 Λ's



- 100 U's, 20 Λ's



Outlook

- FP sector of the gauge fixed Lagrangian

$$\mathcal{L}_{\text{ghost}} = \overline{c}^a \partial_\mu D^{ab}_\mu c^b$$

- correct Hermiticity assignment for the ghost fields is

$$c^\dagger(x) = c(x), \qquad \overline{c}^\dagger(x) = -\overline{c}(x)$$

Alkofer, Von Smekal, Phys. Rep. 353 (2001) 281

- can be achieved by choosing two independent real Grassman ghost fields

$$c(x) \to u(x), \qquad \overline{c}(x) = iv(x)$$

- demanding $\mathcal{L}_{\text{ghost}} = \mathcal{L}^\dagger_{\text{ghost}}$

$$\mathcal{L}_{\text{ghost}} = \frac{1}{2} \begin{pmatrix} u & v \end{pmatrix} \begin{pmatrix} 0 & -iD\partial \\ i\partial D & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} := \frac{1}{2} \bar{\phi} \mathfrak{D} \phi$$

- operator $\mathfrak{D}$ is Hermitian
- seems a suitable candidate to use on the lattice

Conclusions

- brief discussion of how to compute ghost propagator in LCG
- Very preliminary results for small lattice volumes
  - no differences with Landau gauge results
- Outlook
  - comparison with the inversion of $\mathfrak{D}$
  - larger volumes to access infrared region

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