# Three-point functions in Landau gauge from $N_f = 2$ lattice QCD

## — quark-photon and 3-gluon vertex —

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(with P.-H. Balduf and M. Leutnant)

Friedrich-Schiller-Universität Jena, Germany

WE-Heraeus-Seminar: *"From correlation functions to QCD phenomenology"*

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# Motivation

**Research in hadron physics / QCD thermodynamics**

- lattice QCD currently preferred tool to provide theoretical estimates
- full control over systematic error, hard/expensive in practise
- New: PDFs and PDAs also available via quasi-function/amplitudes (due to Ji (2013) and collaborators since then)
- Many new studies recently, requires much effort

**Lattice is not the only nonperturbative framework for QCD**

- Bound-state / Dyson-Schwinger equations
- Functional Renormalization group
- Pros/Cons different to lattice
- Input: nonperturbative n-point functions (in a gauge)
- **Problem**: truncation of infinite system of equations / of effective action

# Motivation

**Lattice can help to control systematic error of truncation**

- Lattice QCD + gauge-fixing: access to $n$-point functions
- Lattice results for propagators helped to settle qualitative features of momentum dependence in Landau gauge

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}, \qquad G(p) = \frac{J(p^2)}{p^2}, \qquad S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

**Lattice results are untruncated**

- but discretization and volumes effects should not be ignored
- violation of $O(4)$ symmetry cause deviations at large $p^2$: $F(p^2) \to F(p)$
- Wilson term changes momentum behavior $\propto O(a^2p^2)$
- In addition: finite momentum range and Gribov problem
- Challenge: continuum- and infinite-volume extrapolated results
- very large & fine lattices are required

**Good news:** Lattice methods currently keep up with required precision

# Three projects on 3-point functions in Landau gauge

**Most important** is quenched + unquenched lattice QCD data for

- **Quark-gluon vertex** (work in progress) *AS* et al.(2016) arXiv:1702.00612
  in collaboration with Kızılersü, Oliveira, Silva, Skullerud, Williams

- **Triple-gluon vertex** *AS* et al.(2016) arXiv:1702.00612
  BSc. project with P. Balduf (FSU Jena) (this talk)

$$\Gamma_{\mu\nu\lambda}(p,q) = \sum_{i=1}^{14} f_i(p,q)\, P^{(i)}_{\mu\nu\lambda}(p,q) \qquad \rightarrow \qquad \Gamma^{T}_{\mu\nu\rho}(p,q) = \sum_{i=1}^{4} F_i\, \tau^{\mu\nu\rho}_{i\perp}(p,q)$$

- **Quark-photon vertex** (this talk)
  MSc. project with M. Leutnant (FSU Jena)

$$\Gamma_\mu(k,Q) = \underbrace{i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3]}_{\Gamma^{\mathrm{BC}}_\mu(k,Q)} + \sum_{j=1}^{8} i t_j\, T^{(j)}_\mu(k,Q)$$

# Parameters of our gauge field ensembles ($N_f = 0, 2$)

**Lattice action**

- Wilson gauge action
- Wilson clover fermions
- Landau gauge
  (after thermalization)

**Can study:**

- quenched vs. unquenched
- quark mass dependence
- discret. + volume effects
- infrared behavior (3-gluon)

| $\beta$ | $\kappa$ | $L_s^3 \times L_t$ | $a$ [fm] | $m_\pi$ [MeV] |
|---|---|---|---|---|
| 5.20 | 0.13584 | $32^3 \times 64$ | 0.08 | 411 |
| 5.20 | 0.13596 | $32^3 \times 64$ | 0.08 | 280 |
| 5.29 | 0.13620 | $32^3 \times 64$ | 0.07 | 422 |
| 5.29 | 0.13632 | $32^3 \times 64$ | 0.07 | 295 |
| 5.29 | 0.13632 | $64^3 \times 64$ | 0.07 | 290 |
| 5.29 | 0.13640 | $64^3 \times 64$ | 0.07 | 150 |
| 5.40 | 0.13647 | $32^3 \times 64$ | 0.06 | 426 |
| 5.40 | 0.13660 | $48^3 \times 64$ | 0.06 | 260 |
| 6.16 | — | $32^3 \times 64$ | 0.07 | — |
| 5.70 | — | $48^3 \times 96$ | 0.17 | — |
| 5.60 | — | $72^3 \times 72$ | 0.22 | — |

**Acknowledgements**

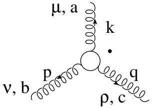
- $N_f = 2$ configurations provided by RQCD collaboration (Regensburg)
- Gauge-fixing and calculation of propagators at the HLRN, LRZ and FSU Jena

## Triple-gluon vertex in Landau gauge

(in collaboration with MSc. P. Balduf)

# Triple-gluon vertex in Landau gauge

$$\mathbf{\Gamma}_{\mu\nu\lambda}(\mathbf{p},\mathbf{q}) = \sum_{i=1,\ldots,14} f_i(p,q)\, P^{(i)}_{\mu\nu\lambda}(p,q)$$




- Perturbation theory: $f_i$ known up to three-loop order (Gracey)
- Some studies of nonperturbative structure last years (few results DSE/lattice)
- Needed for improved truncations of quark-DSE

**DSE results**

- Huber and von Smekal, JHEP 1304 (2013) 149 (new ansatz for vertex)
- Blum et al., PRD89(2014)061703 (improved truncation)
- Eichmann et al., PRD89(2014)105014 (full transverse form)
- Williams et al., PRD93(2016)034026 (unquenching effects)
- ...

# Triple-gluon vertex in Landau gauge

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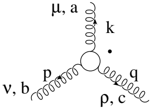


- Perturbation theory: $f_i$ known up to three-loop order (Gracey)
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**Lattice results** (“zero-crossing of deviation from tree-level)

- SU(2) YM: Cucchieri et al., [PRD77(2008)094510]
  - “zero-crossing” at small momenta (2d, 3d)
- SU(3) YM: “zero-crossing” at small momenta (4d)
  - Athenodorou et al. [1607.01278]: for $p^2 < 170\,\text{MeV}$ for symmetric momenta
  - Duarte et al. [1607.03831]: for $p^2 \sim 220\,\text{MeV}$ for $p = -q$
  - Boucaud et al. [1701.07390]: for $p \sim 100 - 200\,\text{MeV}$ for symmetric momenta

# Lattice calculation

## Monte Carlo averages

$$G_{\mu\nu\rho}(p,q) = \langle A_\mu(p)\, A_\nu(q)\, A_\rho(-p-q)\rangle_U \;, \quad D_{\mu\nu}(p) = \langle A_\mu(p) A_\nu(-p)\rangle_U$$

- Consider all pairs of nearly diagonal $|p| = |q|$
- Angles: $\angle(p,q) = \phi = 60^\circ, 90^\circ, 120^\circ$ and $180^\circ$
- Average data for $a|p| = a|q|$ and nearby momenta (vary binsize)

## Projection onto tree-level

$$G_1(p,q) = \frac{\Gamma^{(0)}_{\mu\nu\rho}}{\Gamma^{(0)}_{\mu\nu\rho}} \frac{G_{\mu\nu\rho}(p,q,p-q)}{D_{\mu\lambda}(p) D_{\nu\sigma}(q) D_{\rho\omega}(p-q)\, \Gamma^{(0)}_{\lambda\sigma\omega}}$$

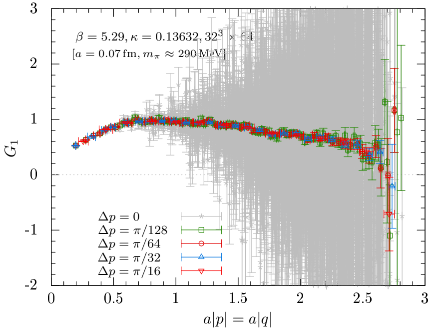
## Lattice tree-level expression

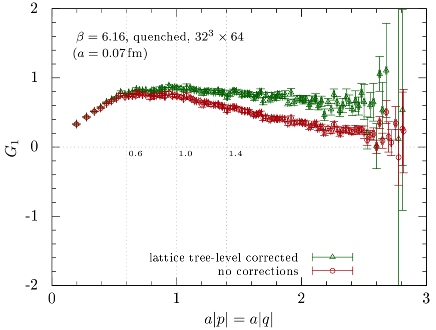
$\mathrm{s}(k_\mu) \equiv \frac{2}{a}\sin(ak_\mu/2), \quad \mathrm{c}(k_\mu) \equiv \frac{1}{a}\cos(ak_\mu/2)$

$$\Gamma^{(0)}_{\mu\nu\rho} = (2\pi)^4 i g_0 f^{abc} \Big[\mathrm{s}(k'_\lambda - k_\lambda) c(k_\mu + k'_\mu)\,\delta_{\mu\nu} + \mathrm{s}(2k_\nu + k'_\nu) c(k'_\lambda)\,\delta_{\mu\lambda} - \mathrm{s}(2k'_\mu + k_\mu) c(k_\nu)\,\delta_{\nu\lambda}\Big]$$

$$\xrightarrow{a\to 0} (2\pi)^4 i g_0 f^{abc} \Big[(k'_\lambda - k_\lambda)\,\delta_{\mu\nu} + (2k_\nu + k'_\nu)\,\delta_{\mu\lambda} - (2k'_\mu + k_\mu)\,\delta_{\nu\lambda}\Big]$$

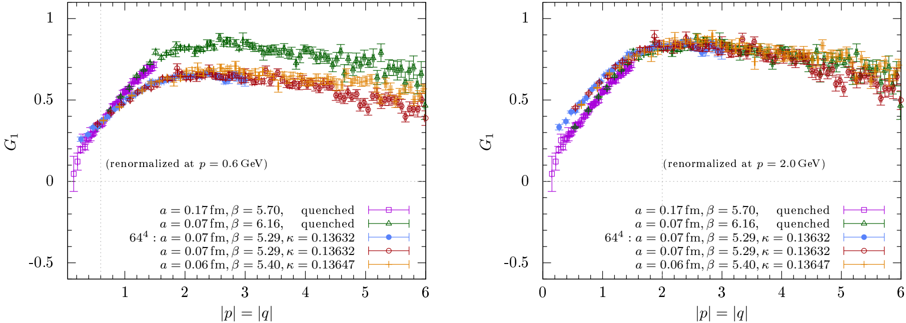
# Effect of binning and tree-level improvement




- Binning reduces statistical noise significantly
- Lattice tree-level improvement relevant for larger $ap_\mu$ (expected)

# Comparison: quenched versus unquenched ($N_f = 2$)




- Where to renormalize ?
- Different slope at small momenta ($N_f = 0$ vs. $N_f = 2$)
- Consistent with findings from Williams et al., PRD93(2016)034026
- Size of unquenching effect changes with renormalization point

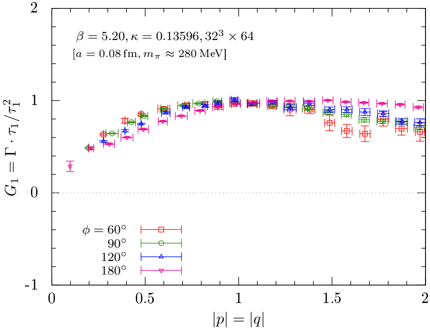
# Angular dependence

**Consider**

- Consider $|p| = |q|$ and $\phi = \angle(p, q)$
- $\phi = 60^\circ, 90^\circ, 120^\circ$ and $180^\circ$

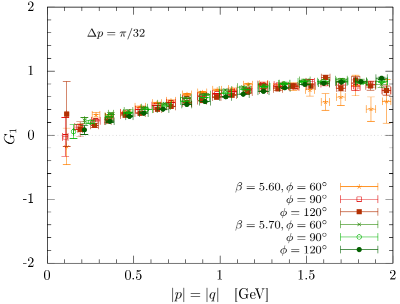
**Find**

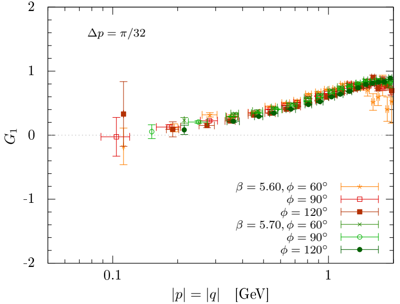
- small angular dependence
- small $|p|$: $\phi = 180^\circ$ data increase less strong than $\phi = 60^\circ$ data
- large $|p|$: $\phi = 180^\circ$ data falls less strong than $\phi = 60^\circ$ data
- seen for <u>all</u> lattice data



# “Zero crossing” for dressing function?

considered here for $N_f = 0$ and different bin sizes




- Reach momenta below 100 MeV
- Data points touch zero, no clear signal for zero crossing (smaller momenta?)
- Either scenario consistent with data

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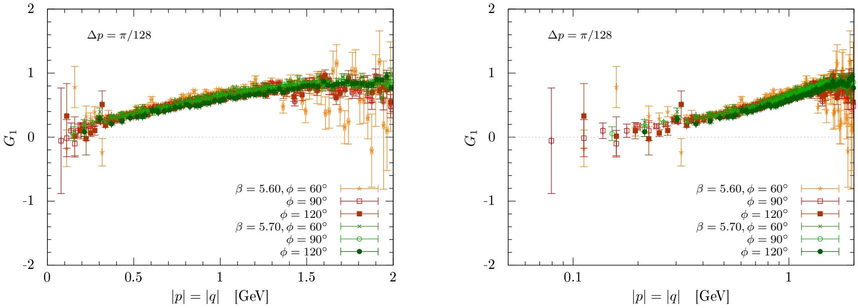


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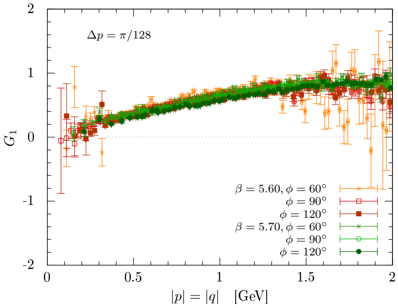
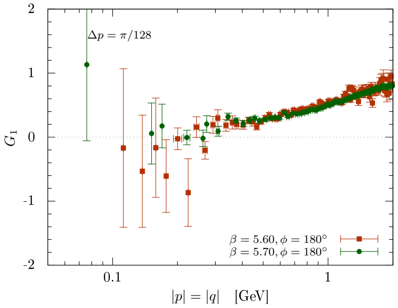



- Reach momenta below 100 MeV
- Data points touch zero, no clear signal for zero crossing (smaller momenta?)
- Either scenario consistent with data

# Bose-symmetric transverse tensor structure

Eichmann et al., PRD89(2014)105014

**Tensor structure** of 3-gluon vertex

$$\Gamma_{\mu\nu\lambda}(p,q) = \sum_{i=1}^{14} f_i(p,q) P^{(i)}_{\mu\nu\lambda}(p,q)$$

**Tensor structure** of transversely-projected vertex

$$\Gamma^{T}_{\mu\nu\rho}(p,q) = \sum_{i=1}^{4} F_i(\mathcal{S}_0,\mathcal{S}_1,\mathcal{S}_2)\, \tau^{\mu\nu\rho}_{i\perp}(p,q)$$

with the Lorentz invariants $[p,q,r=-(p+q)]$

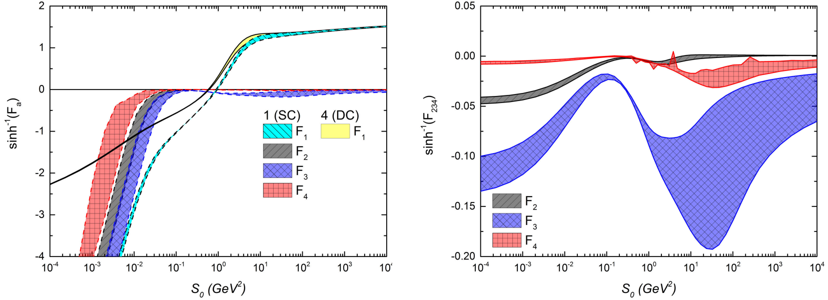
$$\mathcal{S}_0 \equiv \mathcal{S}_0(p,q,r) = \frac{1}{6}\left(p^2+q^2+r^2\right)$$

$$\mathcal{S}_1 \equiv \mathcal{S}_1(p,q,r) = a^2+s^2 \qquad \in [0,1] \qquad \text{with} \quad a \equiv \frac{\sqrt{3}}{6\mathcal{S}_0}\left(q^2-p^2\right)$$

$$\mathcal{S}_2 \equiv \mathcal{S}_2(p,q,r) = 3sa^2 - s^3 \qquad \in [-1,1] \qquad \text{and} \quad s \equiv \frac{1}{6\mathcal{S}_0}\left(p^2+q^2-2r^2\right)$$

# DSE results for transverse form factors

Eichmann et al., PRD89(2014)105014, Fig.9




Scaling solution:

- $F_i$ diverge for $q \to 0$

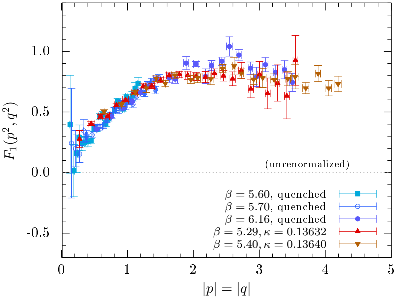
Decoupling solution

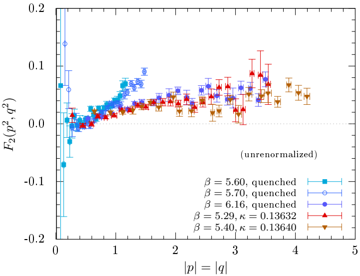
- Only $F_1$ significantly different from 0
- $F_{2,3,4}$ almost flat $\approx 0$

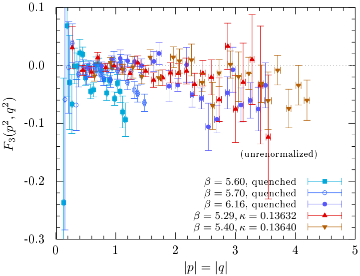
No lattice results available for a cross-check . . . until today.

# First lattice results for transverse tensor structure

**Lattice results** for $F_1$, $F_2$ and $F_3$




**Lattice results** confirm

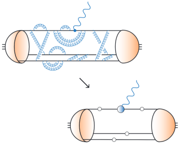
- Leading form factor is $F_1$
- $F_{i=2,3,4}$ close to zero $\forall$ momenta

# Quark-Photon vertex in Landau gauge

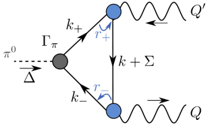
(in collaboration with M. Leutnant)

**Input for hadron phenomenology based on DSE + bound-state equations:**

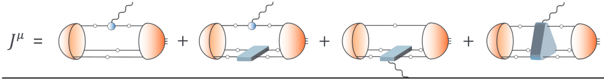
N transition form factors
(G. Eichmann 1602.03462)



Meson transition form factor $\pi^0 \to \gamma^*\gamma^*$
(E. Weil et al. (2017))




Nucleon electromagnetic current
(G. Eichmann 1602.03462)

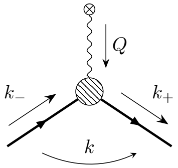



No lattice data available up to now

# Quark-Photon vertex

## in approximation of external photon

**Common parametrization** (3+8 form factors)




$$\Gamma_\mu(k,Q) = \underbrace{i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3]}_{\Gamma^{\mathrm{BC}}_\mu(k,Q)} + \sum_{j=1}^{8} i\tau_j\, T^{(j)}_\mu(k,Q)$$

- Nonperturbative form factors dominated by QCD corrections ($\rightarrow$ external photon)
- Momenta: $k_\pm = k \pm \frac{Q}{2} \quad \Longleftrightarrow \quad Q = k_+ - k_-$ and $k = \frac{1}{2}(k_+ + k_-)$
- $\Gamma^{\mathrm{BC}}_\mu$ solves vector WTI $\rightarrow \lambda_i$ given by quark dressing functions

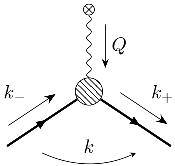
$$\lambda_1 = \frac{A(k_+^2) + A(k_-^2)}{2}, \qquad \lambda_2 = \frac{A(k_+^2) - A(k_-^2)}{k_+^2 - k_-^2}, \qquad \lambda_3 = \frac{B(k_+^2) - B(k_-^2)}{k_+^2 - k_-^2}$$

- Inverse quark propagator

$$S^{-1}(k_\pm) = i\gamma_\mu A(k_\pm^2) + B(k_\pm^2) \qquad \text{with} \quad k_\pm^2 = k^2 + \frac{Q^2}{4} \pm k\cdot Q$$

# Quark-Photon vertex

in approximation of external photon




**Common parametrization** (3+8 form factors)

$$\Gamma_\mu(k,Q) = \underbrace{i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3]}_{\Gamma^{BC}_\mu(k,Q)} + \sum_{j=1}^{8} i\tau_j\, T^{(j)}_\mu(k,Q)$$

- Nonperturbative form factors dominated by QCD corrections ($\to$ external photon)
- Momenta: $k_\pm = k \pm \frac{Q}{2} \iff Q = k_+ - k_-$ and $k = \frac{1}{2}(k_+ + k_-)$
- $\Gamma^{BC}_\mu$ solves vector WTI $\to \lambda_i$ given by quark dressing functions
- Transverse part not fixed by WTI, vanishes for $Q \to 0$ [Eichmann et al.]

$$T^{(1)}_\mu = \mathrm{t}^{QQ}_{\mu\nu}\gamma_\nu, \quad T^{(2)}_\mu = \frac{ik\cdot Q}{2}\mathrm{t}^{QQ}_{\mu\nu}[\gamma_\nu,\not{k}], \quad T^{(3)}_\mu = \frac{i}{2}[\gamma_\mu,\not{Q}], \quad T^{(4)}_\mu = \frac{1}{6}[\gamma_\mu,\not{k},\not{Q}],$$

$$T^{(5)}_\mu = \mathrm{t}^{QQ}_{\mu\nu}ik_\nu, \quad T^{(6)}_\mu = \mathrm{t}^{QQ}_{\mu\nu}k_\nu\not{k}, \quad T^{(7)}_\mu = (k\cdot Q)\,\mathrm{t}^{kQ}_{\mu\nu}\gamma_\nu, \quad T^{(8)}_\mu = \frac{i}{2}\mathrm{t}^{kQ}_{\mu\nu}[\gamma_\nu,\not{k}]$$

with $\mathrm{t}^{kQ}_{\mu\nu} \equiv (k\cdot Q)\,\delta_{\mu\nu} - k_\mu Q_\nu$ and $[a,b,c] \equiv [a,b]c + [b,c]a + [c,a]b.$

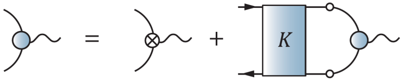
# Quark-Photon vertex

**Common parametrization**

$$\Gamma_\mu(k,Q) = \Gamma_\mu^{\mathrm{BC}}(k,Q) \;+\; \sum_{j=1}^{8} i\tau_j \; T_\mu^{(j)}(k,Q)$$

**Numerical solution** (continuum)

- from inhomogeneous BS equation in Rainbow-ladder truncation

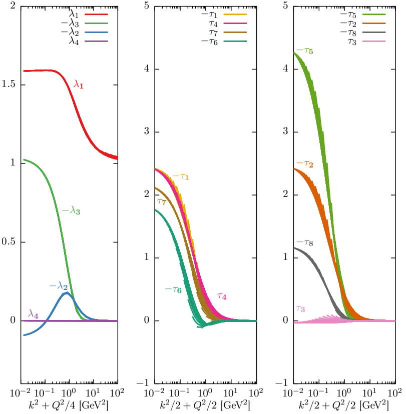



- Used since Maris & Tandy (1999)
- Ball-Chiu part given by quark DSE

Is this sufficient?

**Rainbow-ladder results**

(Sanchis-Alepuz & Williams, 1710.04903v2)




(similar Eichmann, 2014)

# Quark-Photon vertex from lattice QCD

**Monte Carlo averages:** quark bilinear and propagator in Landau gauge
(Inverse of $D_U$ via momentum sources, numerically demanding)

$$G_\mu^{\alpha\beta}(k,Q) = \sum_{x,y,z} e^{ik_+(x-z)} e^{ik_-(z-y)} \left\langle [D_U^{-1}]_{xz}^{\alpha\gamma} \gamma_\mu^{\gamma\delta} [D_U^{-1}]_{zx}^{\delta\beta} \right\rangle_U$$

$$S^{ab}(k_\pm) = \sum_{x,y} e^{ik_\pm(x-y)} \left\langle [D_U^{-1}]_{xy}^{ab} \right\rangle_U \qquad \text{with} \quad k_\pm = k \pm \frac{Q}{2}$$

**Vertex** from amputated 3-point function

$$\Gamma_\mu(k,Q) = S^{-1}\left(k - \frac{Q}{2}\right) G_\mu(k,Q)\, S^{-1}\left(k + \frac{Q}{2}\right)$$

**Form factors** from solving

$$\Gamma_\mu(k,Q) = i\gamma_\mu \lambda_1 + 2k^\mu [i\not{k}\,\lambda_2 + \lambda_3] + \sum_{j=1}^{8} i\tau_j\, T_\mu^{(j)}(k,Q)$$

# Quark-Photon vertex from lattice QCD

## Momenta

- Twisted boundary conditions

$$(ak_{\pm})_\mu = \frac{2\pi}{L_\mu}\left(n^{\pm}_\mu + \frac{\tau_\mu}{2}\right) \qquad \text{twist-angle: } \tau_\mu$$

- **Symmetric** momenta

$$\begin{aligned} n^- &= n\,(1,1,0,0) + (\tau,\tau,0,0) \qquad Q^2 = k_-^2 = k_+^2, \qquad k\cdot Q = 0 \\ n^+ &= n\,(0,1,1,0) + (0,\tau,\tau,0) \end{aligned}$$

- **Asymmetric** momenta

$$\begin{aligned} n^- &= n\,(2,1,0,0) + (\tau,\tau,0,0) \quad Q^2 = k_-^2 > k_+^2 = 2k_-k_+, \quad k\cdot Q = \frac{4\pi^2}{L_s^2}(3n^2 + 2n\tau) \\ n^+ &= n\,(0,1,1,0) + (0,\tau,\tau,0) \end{aligned}$$

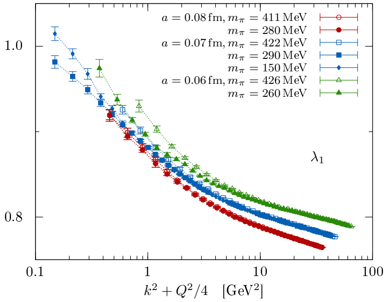
- Use: $n = 1, 2, \ldots, L_s/4$ and $\tau = 0,\ 0.4, 0.8, 1.2$ and $1.6$

# Ball-Chiu form factors

## symmetric momenta

**Bare lattice data**

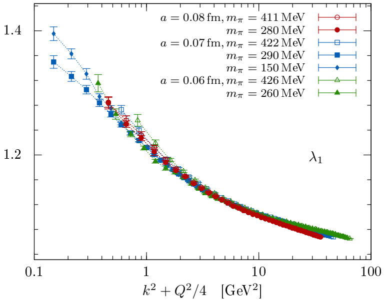
(multiplied by $Z_V(\beta)$ from RQCD coll.)




- Quark mass effect not conclusive
- Discretization effects clearly visible

**Renormalized** at 5 GeV$^2$

(relative to $\beta = 5.29$ data and RL-curves)

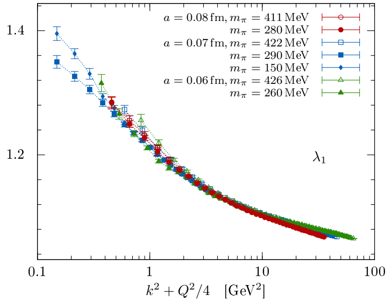



- $Z_2(5.20) = 1.397$, $Z_2(5.29) = 1.375$
- $Z_2(5.40) = 1.350$

# Ball-Chiu form factors

**Renormalized** at 5 GeV$^2$
(relative to $\beta = 5.29$ data and RL-curves)

(symmetric momenta)

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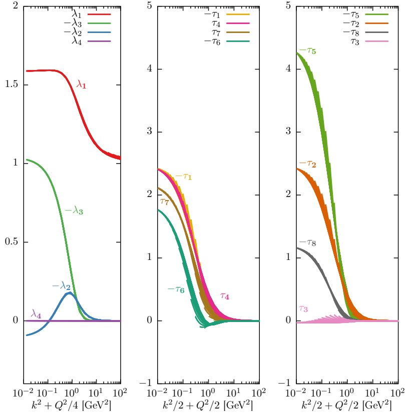
**Rainbow-ladder results**
(Sanchis-Alepuz & Williams, 1710.04903v2)

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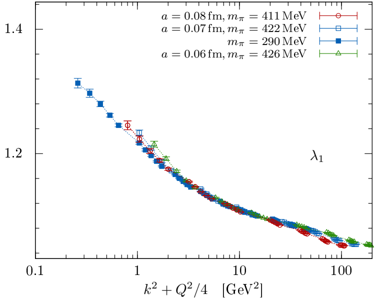


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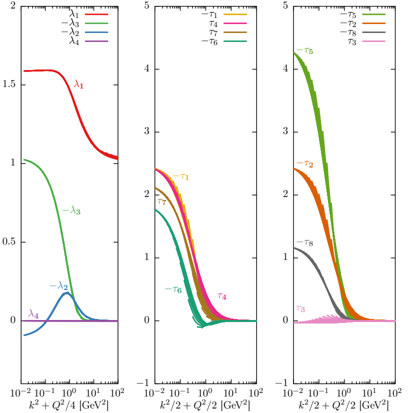


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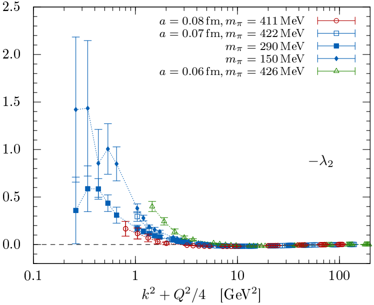


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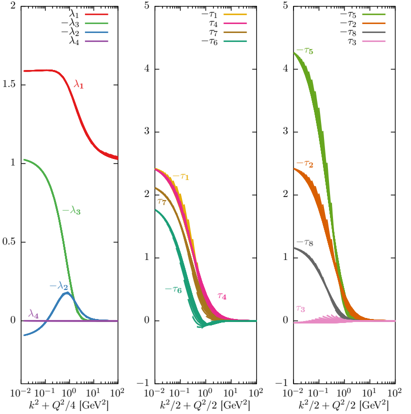


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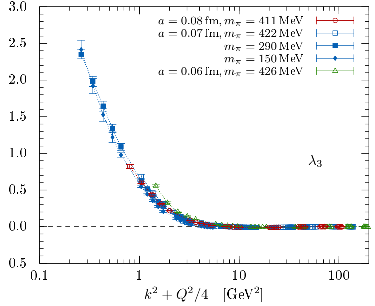


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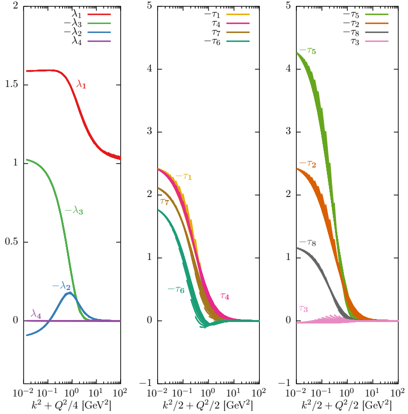


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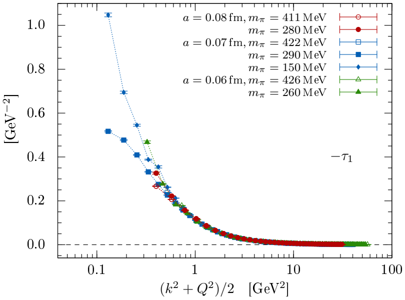


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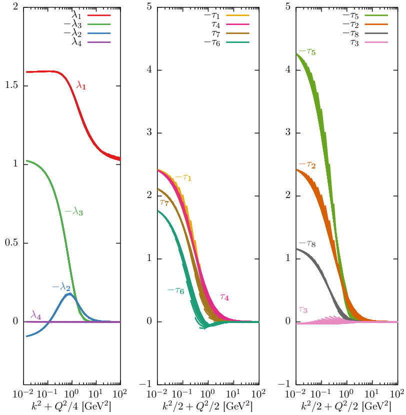


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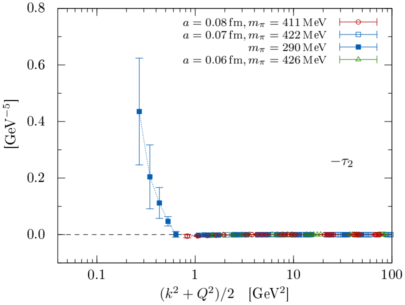


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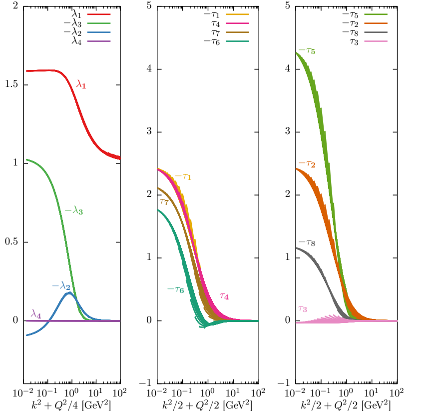


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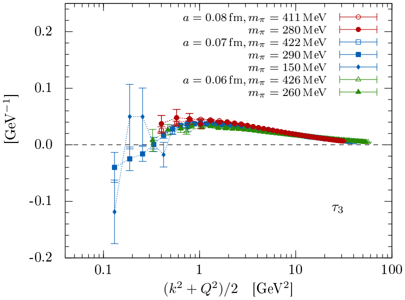


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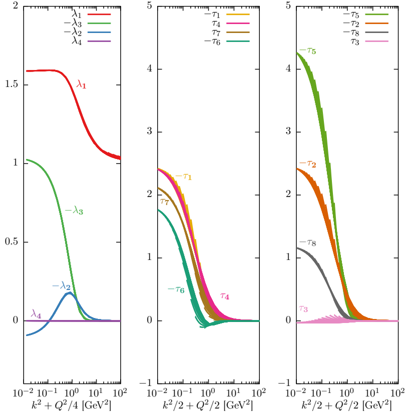


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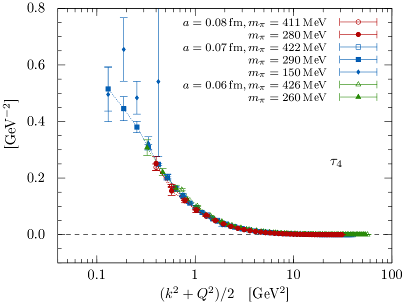


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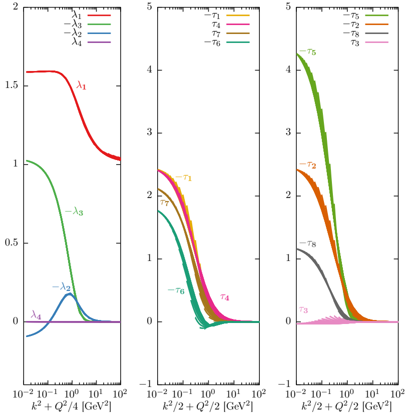


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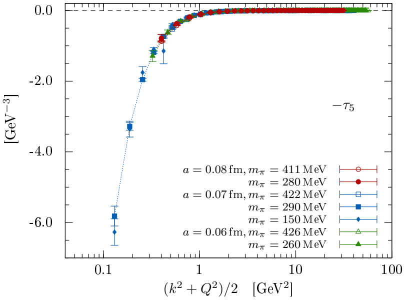


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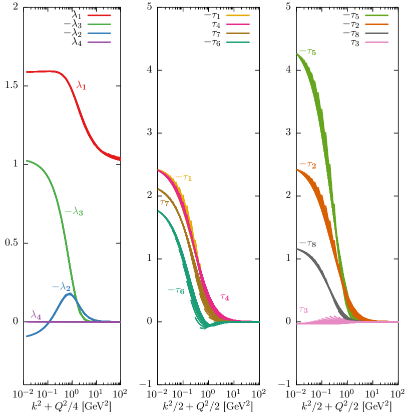


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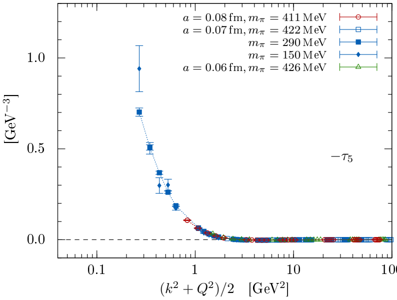


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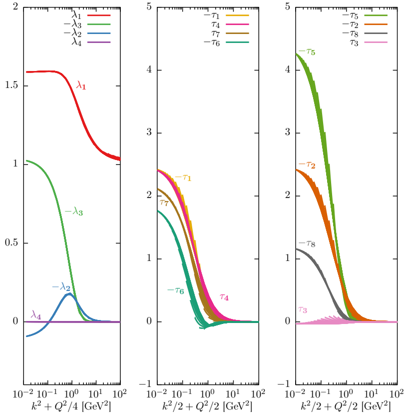


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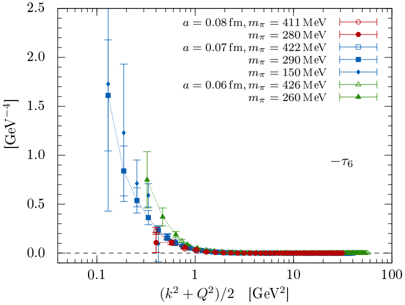


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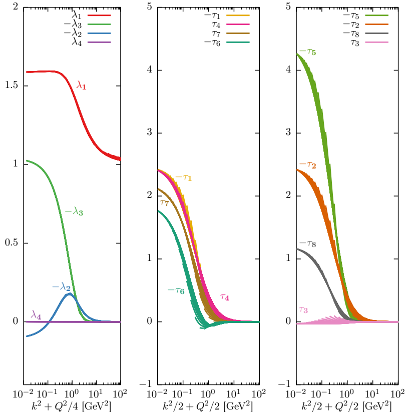


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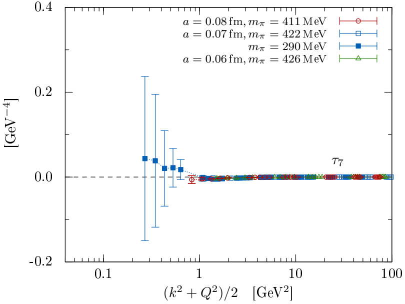


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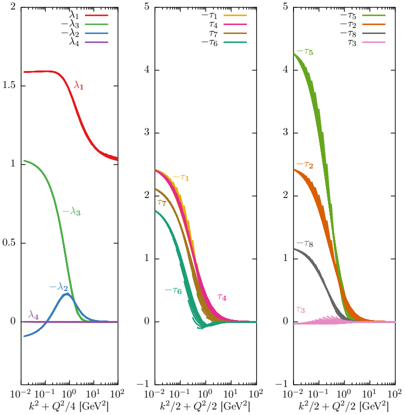


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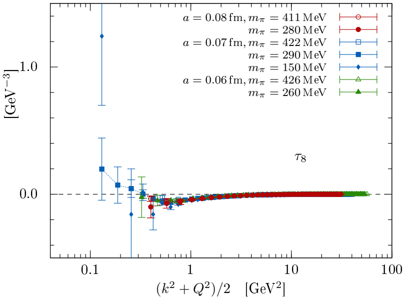


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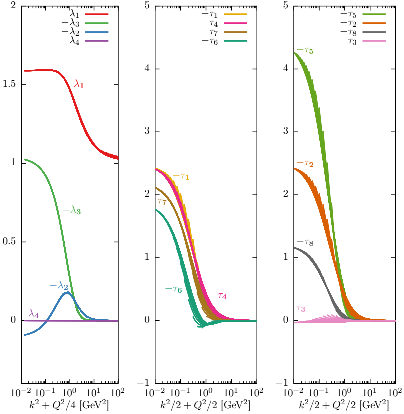


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# Summary

**New lattice data**

- 3-gluon vertex and QCD contributions to quark-photon vertex
- Quenched and unquenched QCD lattice simulations

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- existence of zero-crossing cannot be decided yet

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# Remark: Vector WT identity and the lattice

**Continuum:** Vector Ward-Takahashi identity (WTI)

$$Q_\mu G_\mu(k,Q) = S(k_-) - S(k_+) \qquad \text{with} \quad G_\mu = S(k_-)\,\Gamma_\mu(k,Q)\,S(k_+)$$

**Lattice:**

- three-point function:

$$G_\mu(k,Q) = \sum_{x,y,z} e^{ik_-(x-z)} e^{-ik_+(y-z)} \left\langle \psi_x V_\mu(z)\,\bar{\psi}_y \right\rangle$$

- Local vector current:

$$V_\mu^f(z) = \bar{\psi}_z \gamma_\mu \frac{\lambda^f}{2} \psi_z$$

- violates vector WTI with $S =$ Wilson quark propagator

$$Q_\mu G_\mu(k,Q) \neq S_W(k_-) - S_W(k_+)$$

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- Point-split vector current satisfies WTI [Karsten/Smit (1981)]

$$\tilde{V}^f_\mu(z) = \frac{1}{2}\left( \bar{\psi}_z[\gamma_\mu - 1] U_{x\mu} \frac{\lambda^f}{2} \psi_{z+a\hat{\mu}} + \bar{\psi}_{z+a\hat{\mu}}[\gamma_\mu + 1] U^\dagger_{x\mu} \frac{\lambda^f}{2} \psi_z \right)$$

- In practise: momentum dependence of correction ignored

$$\tilde{V}^f_\mu(z) = V^f_\mu(z) + aK_\mu(z) \quad \equiv \quad Z_V(g^2) V^f_\mu(z) + O(a)$$

# Remark: Vector WT identity and the lattice

**Lattice correction**

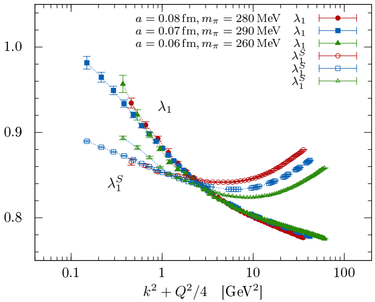
- Applied $Z_V(\beta) \sim 0.7x$ to QPV data (values from RQCD collaboration)
- Brings data in the right ballpark

**Form factor $\lambda_1$**

$$\lambda_1 = \frac{1}{12}\,\mathrm{Tr}(\gamma_\mu \Gamma_L) - (k\cdot Q)\, l_2$$

$$\lambda_1^S = \frac{A(k_+^2) + A(k_-^2)}{2}$$

**Comparison: $\lambda_1$ vs. $\lambda_1^S$**




- curves should approach each other for $a \to 0$
- Surprised about splitting at small momenta (momentum dependent correction to $Z_V$?)

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