

Weak Gravitational Lensing with Euclid

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Outline

- What is Euclid?
- Basics of Lensing
- Weak Lensing

What is Euclid?

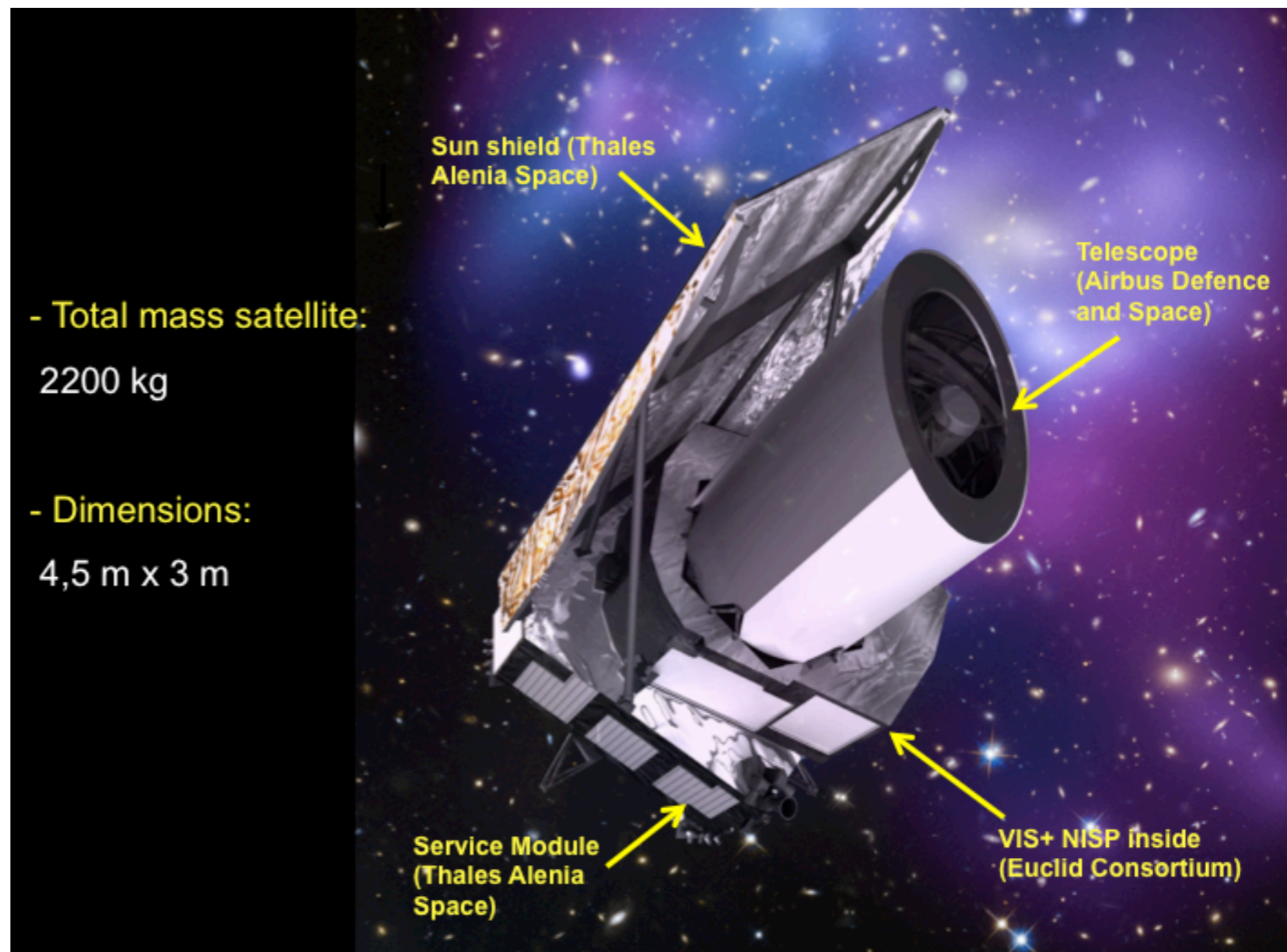
Euclid is an ESA medium class astronomy and astrophysics space mission

Aims: understanding accelerated expansion of the Universe and its cause (dark energy)

Two complementary cosmological probes:

- Weak gravitational Lensing ***This talk!***
- Galaxy Clustering (Baryonic Acoustic Oscillations and Redshift Space Distortion)

Renata's talk!



Basics of Lensing

Fermat's principle

How do we determine path of light from source to observer?

Fermat's principle:

“Light follows path along which travel time

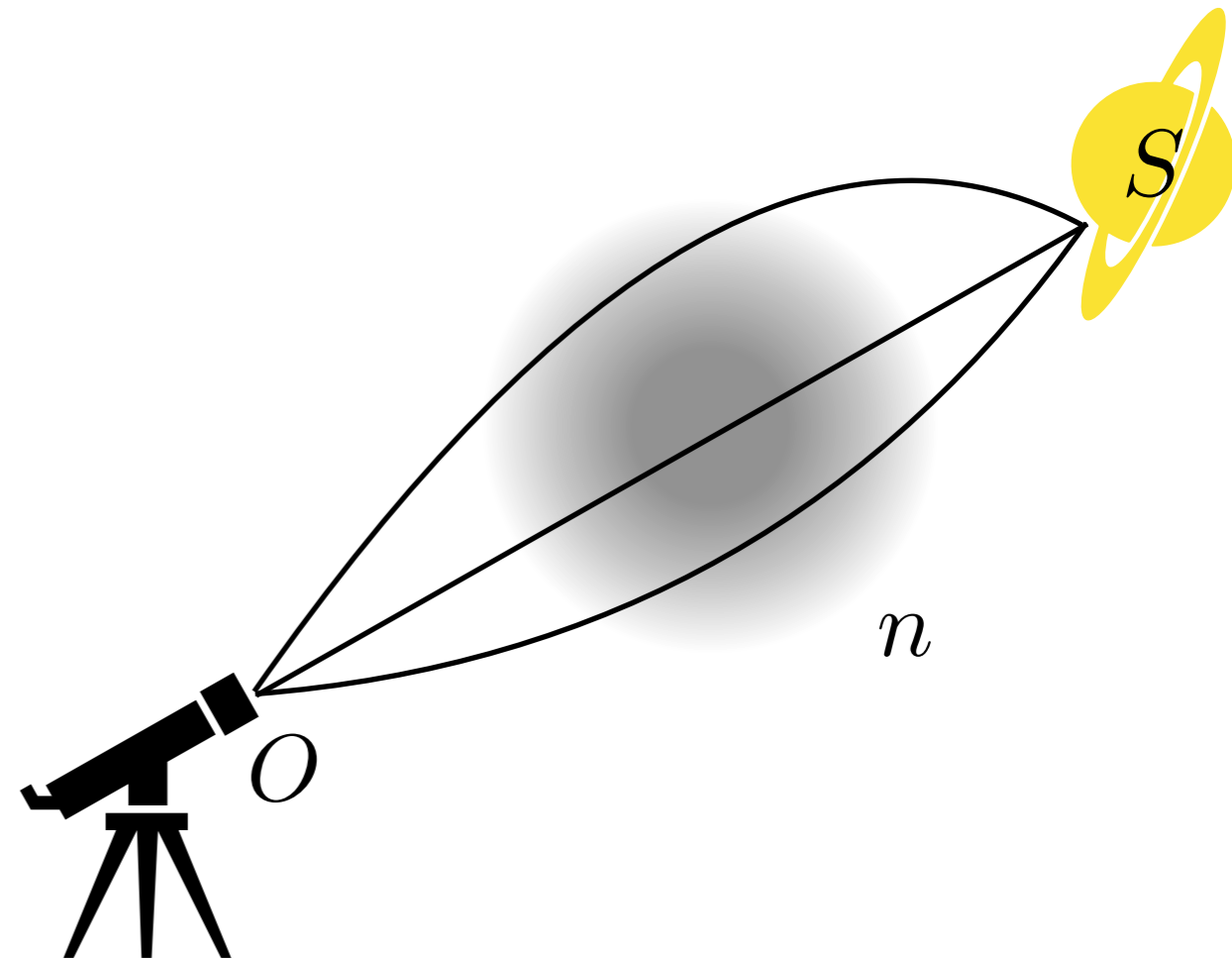
$$t = \int \frac{n(l)}{c} dl$$

is extremal.”

Find $\vec{x}(l)$ s.t. :

$$\delta t = \delta \int_S^O n(\vec{x}(l)) dl = 0$$

For WL: $n(\vec{x}) \approx 1 - \frac{2\Phi(\vec{x})}{c^2}$

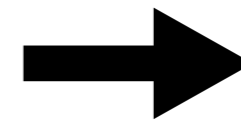


Deflection angle

- From solving the variational problem one finds:

$$\hat{\vec{\alpha}}(b) = \frac{2}{c^2} \int_S^O \vec{\nabla}_{\perp} \Phi dz$$

- For single mass point $\Phi = -\frac{GM}{r}$



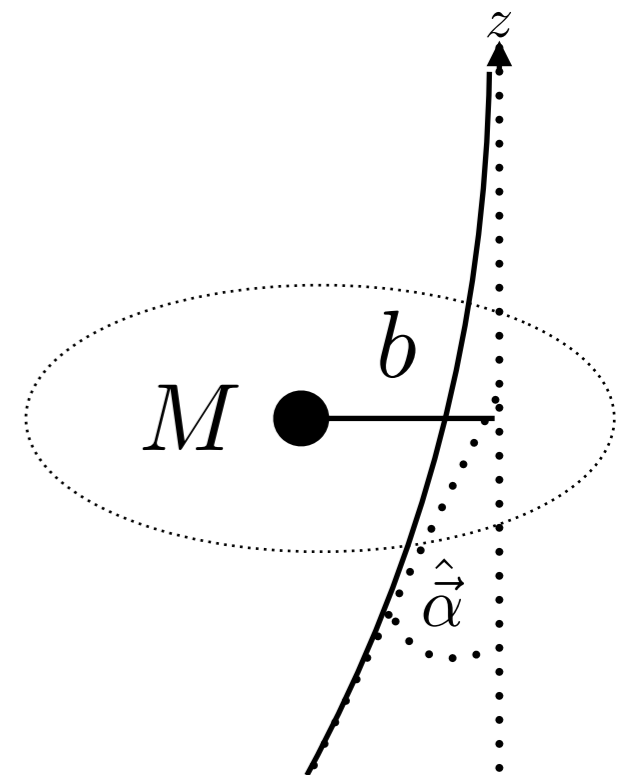
$$|\hat{\vec{\alpha}}| = 4 \frac{GM}{c^2 b}$$

- Thin screen approximation for source distr:

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z) dz$$

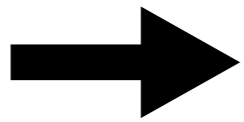
- Total deflection angle:

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}') \Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} d^2 \xi'$$



Lens equation

For small $\theta, \beta, \hat{\alpha}$: $\hat{\alpha} D_{LS} - \vec{\theta} D_S = \vec{\beta} D_S$

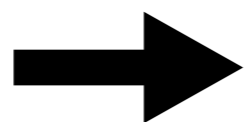


$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

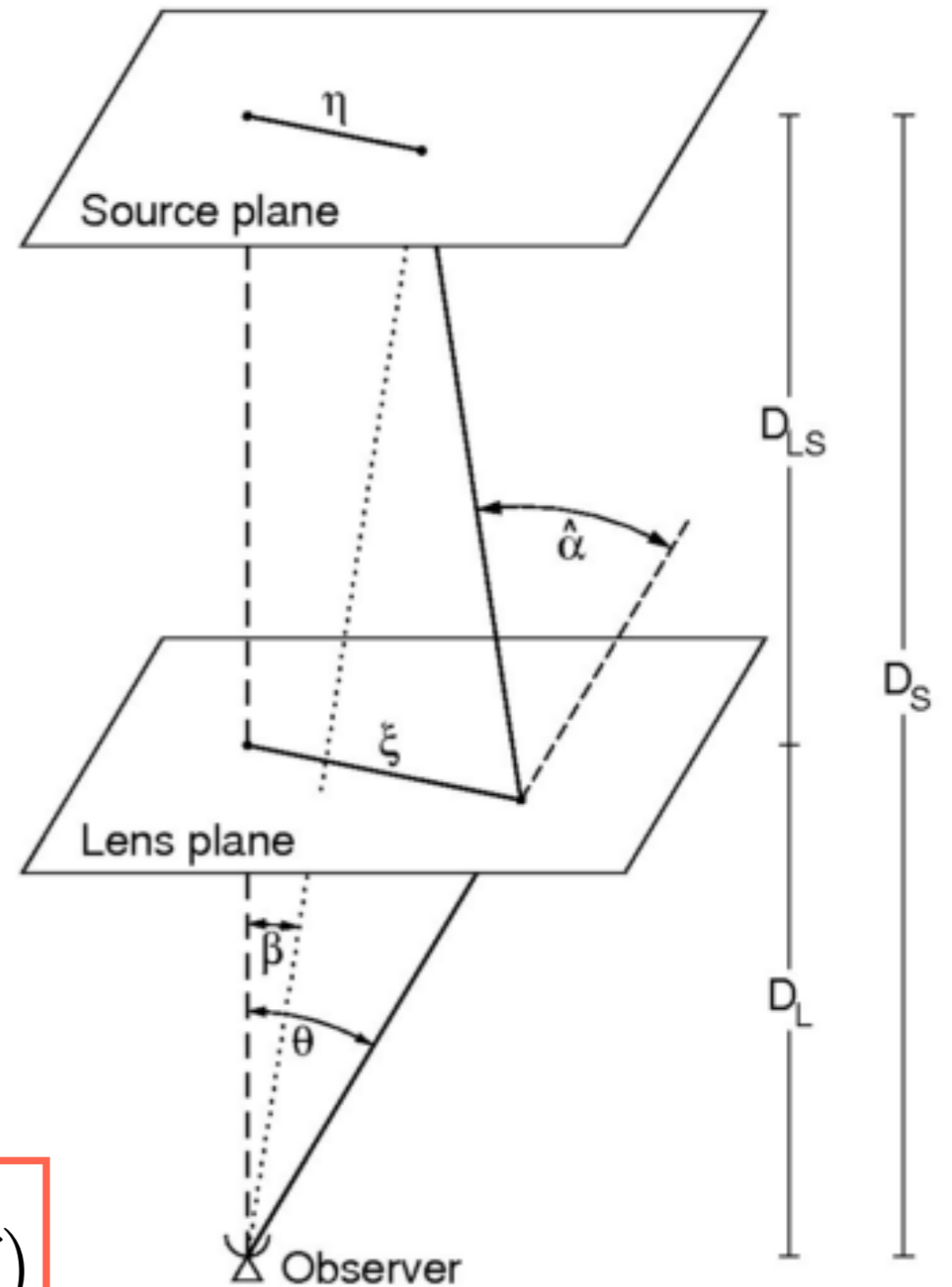
with reduced $\vec{\alpha}(\vec{\theta}) \equiv \frac{D_{LS}}{D_S} \hat{\alpha}$

Introducing dimensionless coordinates:

$$\vec{x} = \frac{\vec{\xi}}{\xi_0}; \quad \vec{y} = \frac{\vec{\eta}}{\eta_0}$$



$$\vec{y} = \vec{x} - \vec{\alpha}(\vec{x})$$



$$\vec{\eta} = \vec{\beta} D_S; \quad \vec{\xi} = \vec{\theta} D_L$$

Lensing potential

- Introduce 2D planar Newtonian potential

$$\psi(\theta) = \frac{D_{LS}}{\xi_0 D_S} \frac{2}{c^2} \int \Phi(D_L \vec{\theta}, z) dz$$

- Properties: 1) $\vec{\nabla}_x \psi(\vec{x}) = \vec{\alpha}(\vec{x}) \quad \longrightarrow \quad \text{differential deflection!}$

2) $\Delta_x \psi(\vec{x}) = 2\kappa(\vec{x}) \quad \longrightarrow \quad \text{Poisson equation}$
($\Delta\Phi = 4\pi G\rho$)

- Convergence density $\kappa(\vec{x}) \equiv \frac{\Sigma(\vec{x})}{\Sigma_{cr}}$ is dimensionless surface

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

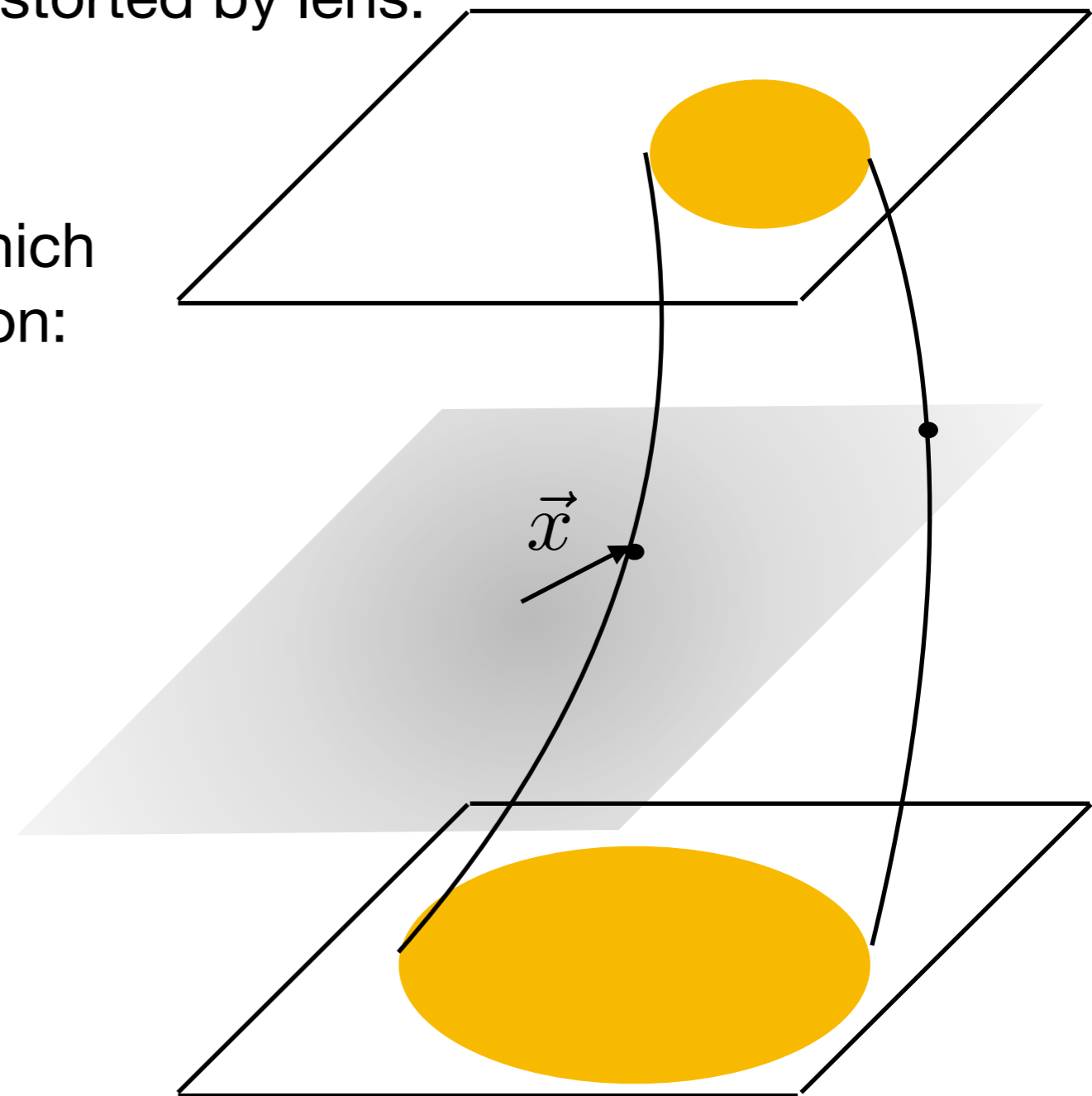
Distortion

Light bundles are deflected differentially!

Image of background galaxy can be distorted by lens:

If source much smaller than scale on which lens changes can linearize image position:

$$A \equiv \frac{\partial \vec{y}}{\partial \vec{x}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{x})}{\partial x_j} \right)$$
$$= \left(\delta_{ij} - \frac{\partial^2 \psi(\vec{x})}{\partial x_i \partial x_j} \right)$$



Distortion

- The Jacobian can be found:

$$\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22})$$

$$\gamma_2 = \psi_{12}$$

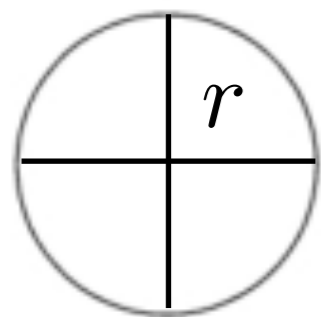
$$A = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_2 \end{pmatrix}$$

$$= (1 - \kappa) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}$$

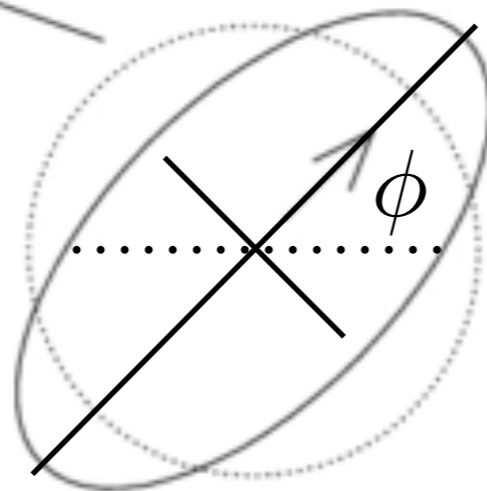
convergence

shear

Convergence alone



Source



Convergence + Shear

- A circular source gets mapped into an ellipse

$$a = \frac{r}{1 - \kappa - \gamma}$$

$$b = \frac{r}{1 - \kappa + \gamma}$$

Weak Lensing

Ellipticity

Ellipticity for small $\kappa, \gamma \lesssim 1$

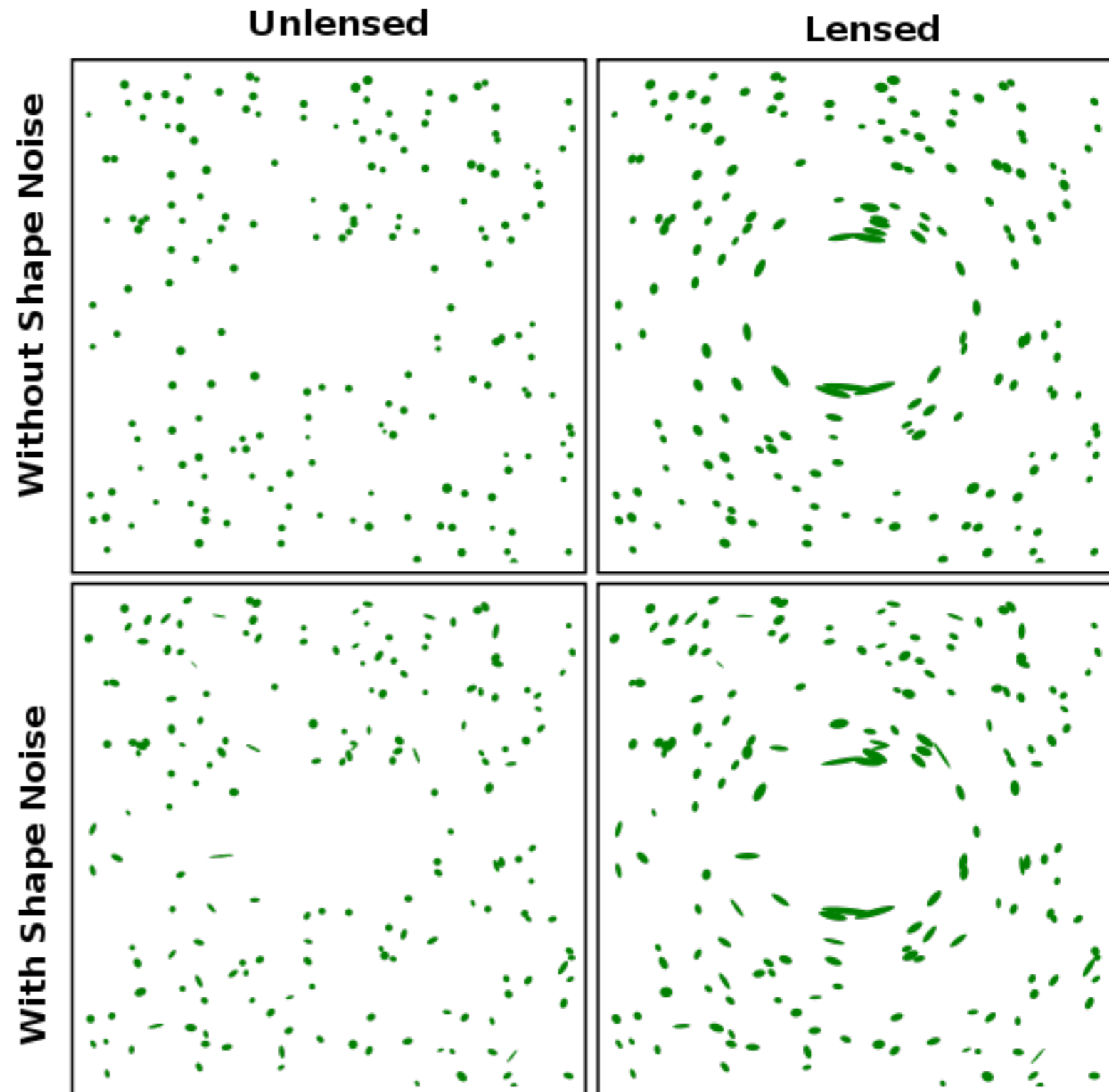
$$\epsilon = \frac{a - b}{a + b} = \frac{\gamma}{1 - \kappa} \approx \gamma$$

Consider intrinsic ellipticity:

$$\epsilon_i = \epsilon_i^{(s)} + \gamma_i$$

Averaging over **many** sources

$$\langle \epsilon \rangle = \langle \gamma \rangle$$



Mass distribution map

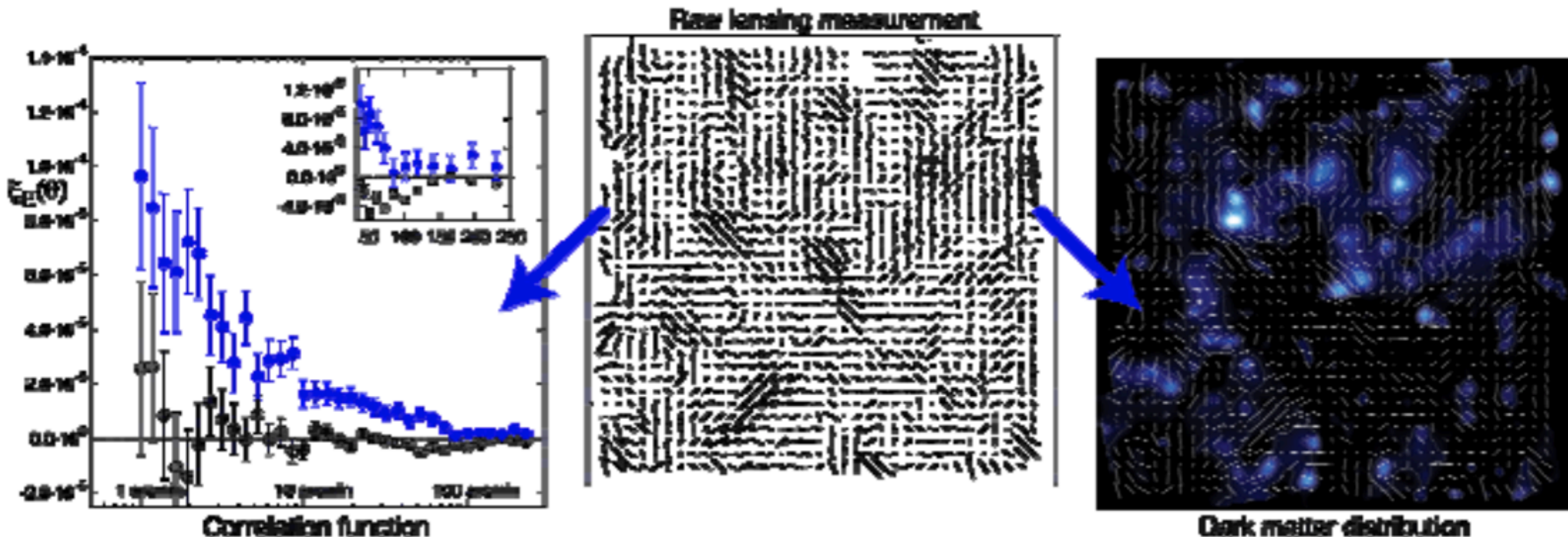
One finds via Fourier transforming lensing potential:

$$\kappa(\vec{\theta}) = \frac{1}{\pi} \int d^2\theta' [D_1(\vec{\theta} - \vec{\theta}')\gamma_1 + D_2(\vec{\theta} - \vec{\theta}')\gamma_2]$$

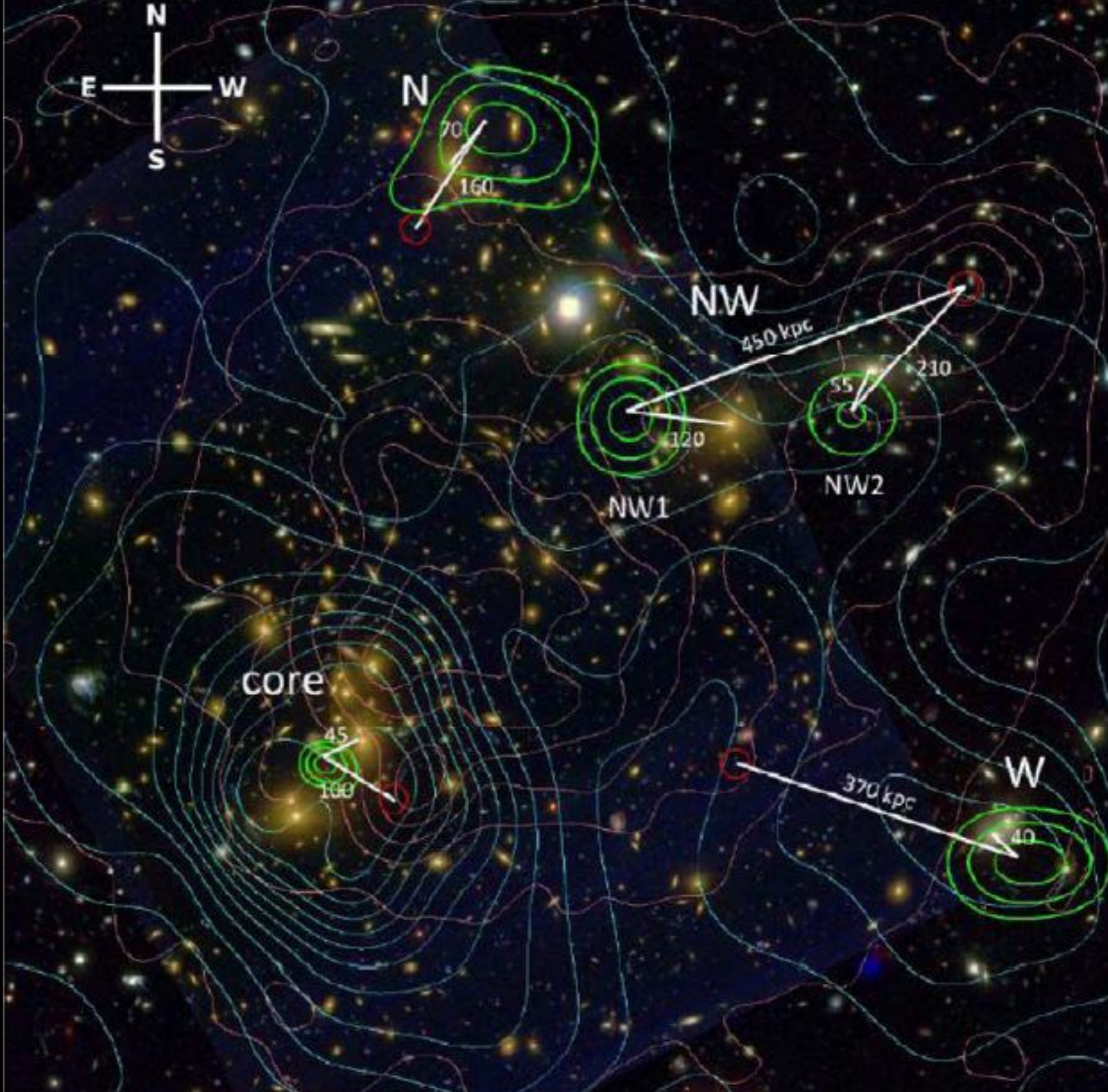
$$D_1(\vec{\theta} - \vec{\theta}') = \frac{\theta_1^2 - \theta_2^2}{\theta^4}$$

$$D_2(\vec{\theta} - \vec{\theta}') = \frac{2\theta_1\theta_2}{\theta^4}$$

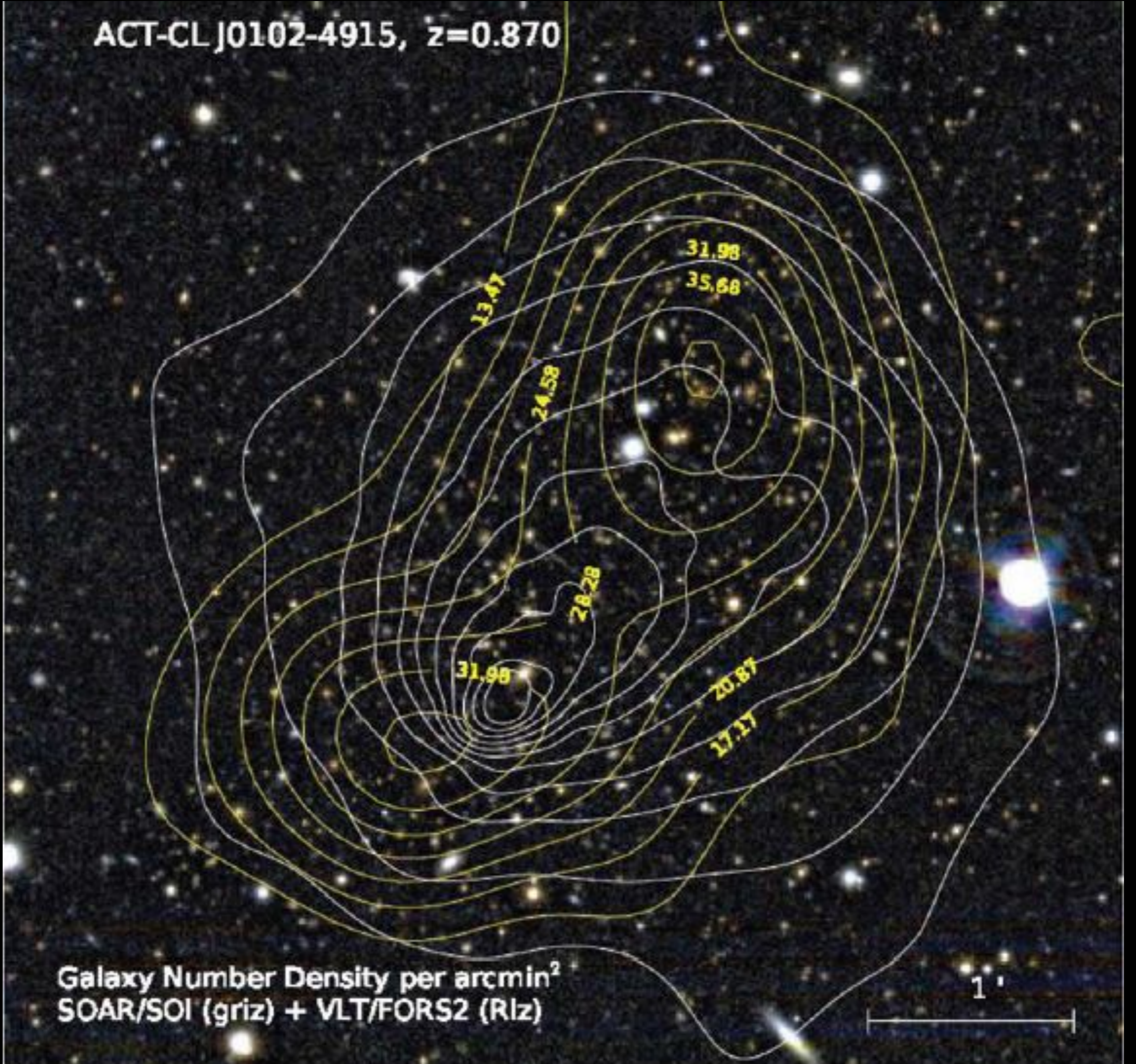
Can construct mass map from measuring shear γ at many positions $\vec{\theta}$







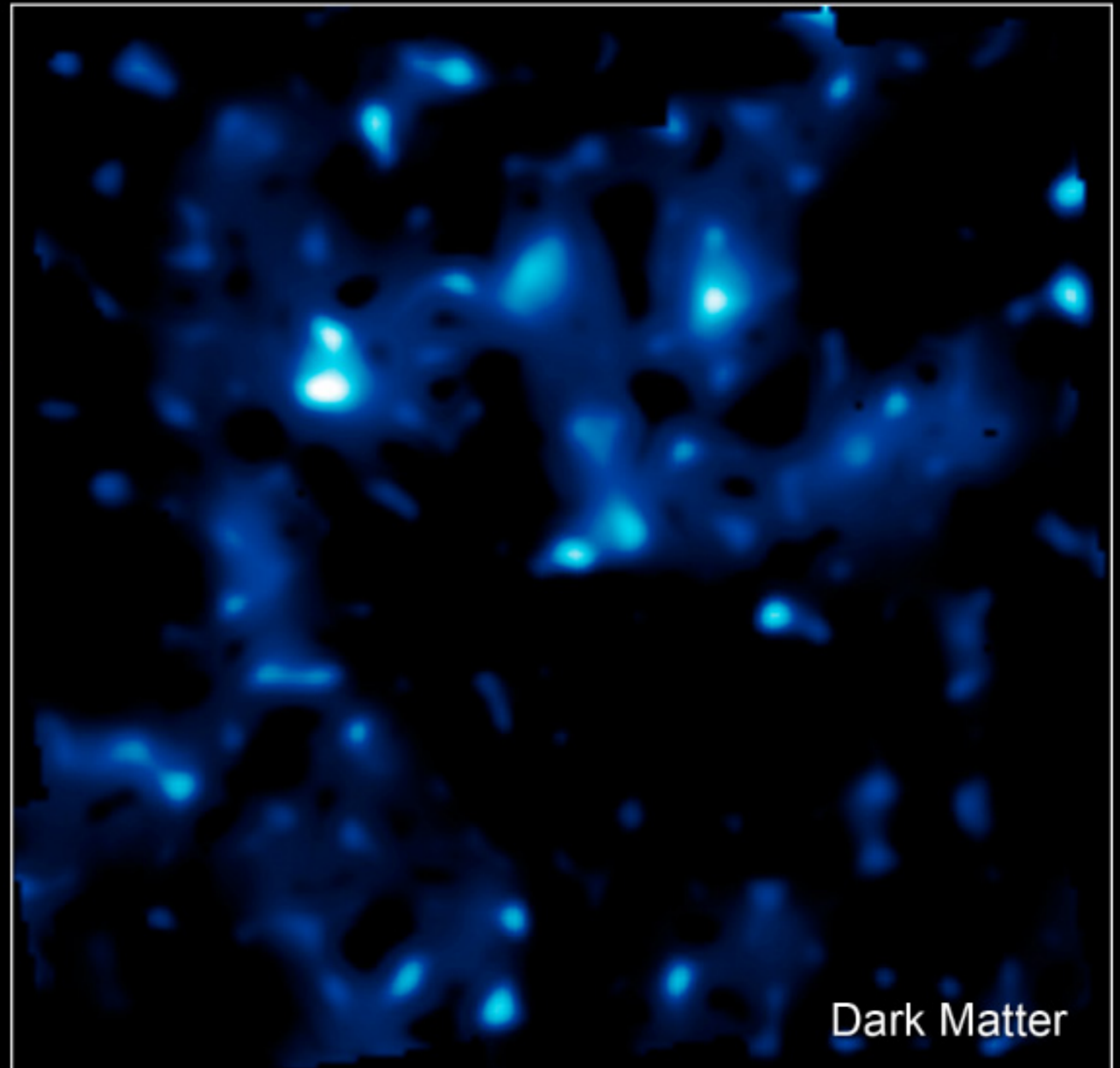
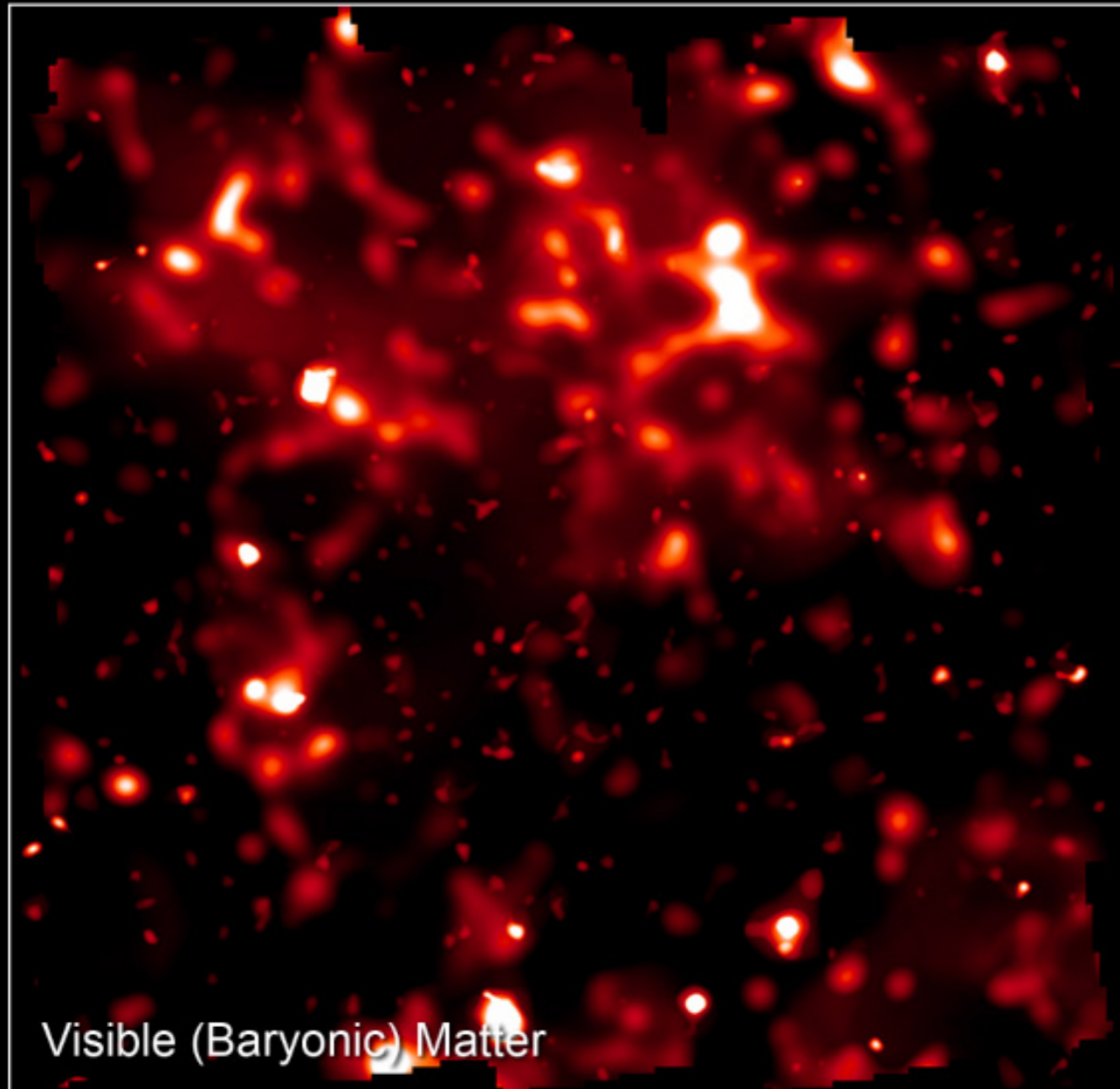
ACT-CL J0102-4915, $z=0.870$



Overlay mass and EM



Construct DM map from difference



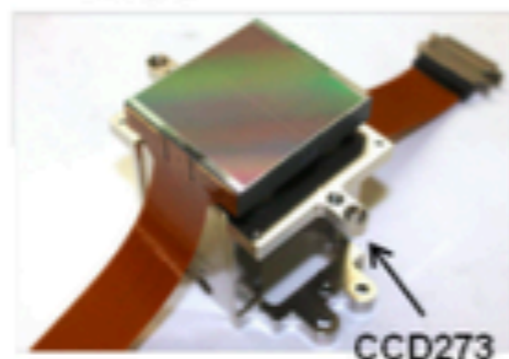
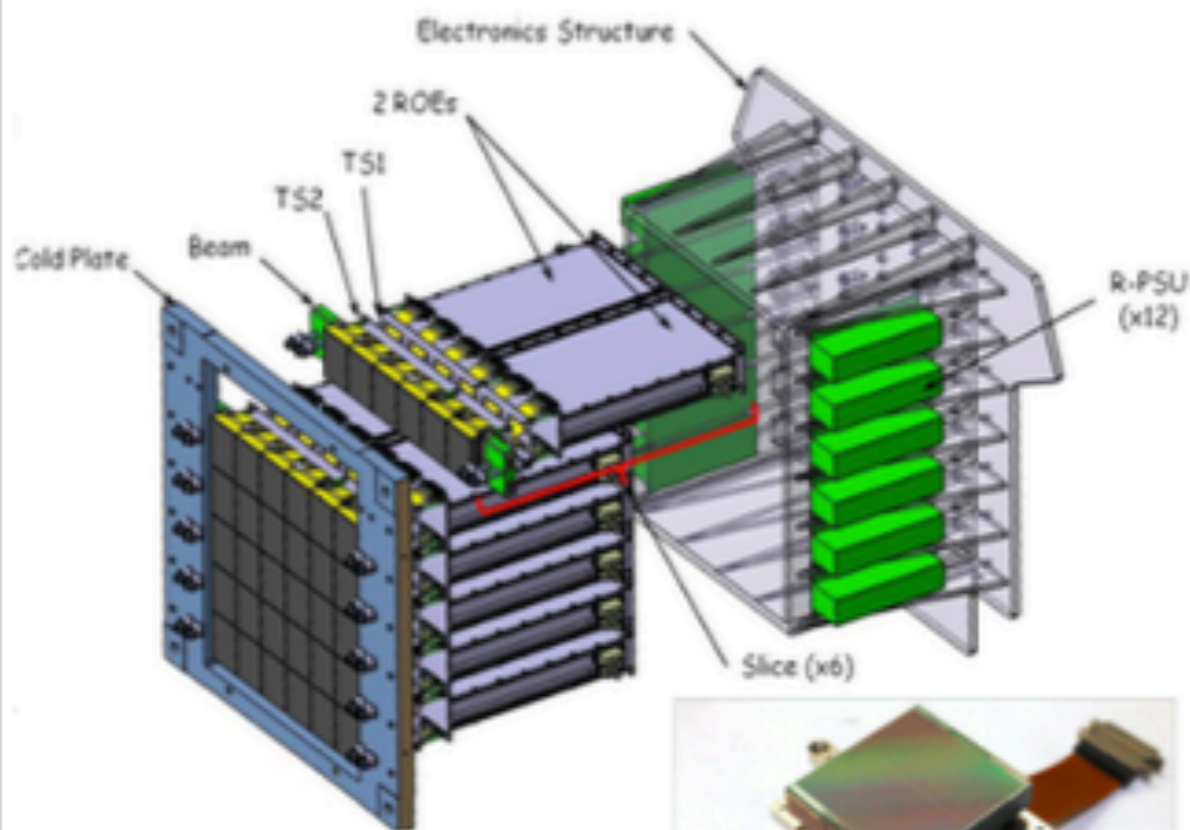
Distribution of Visible and Dark Matter • Cosmic Evolution Survey
Hubble Space Telescope • Advanced Camera for Surveys

Euclid

VIS

Courtesy: S. Pottinger, M. Cropper and the VIS team

- FoV: 0.54deg²
- Mass : 133 kg
- Telemetry: < 520 Gbt/day
- 36 4kx4K E2V CCDs, 12 micron pixels
- 0.1 arcsec pixel on sky
- **Limiting mag, wide survey AB : 24.5 (10 σ)**
- **1 Filter:** Y(R+I+Y): band pass 550-900nm

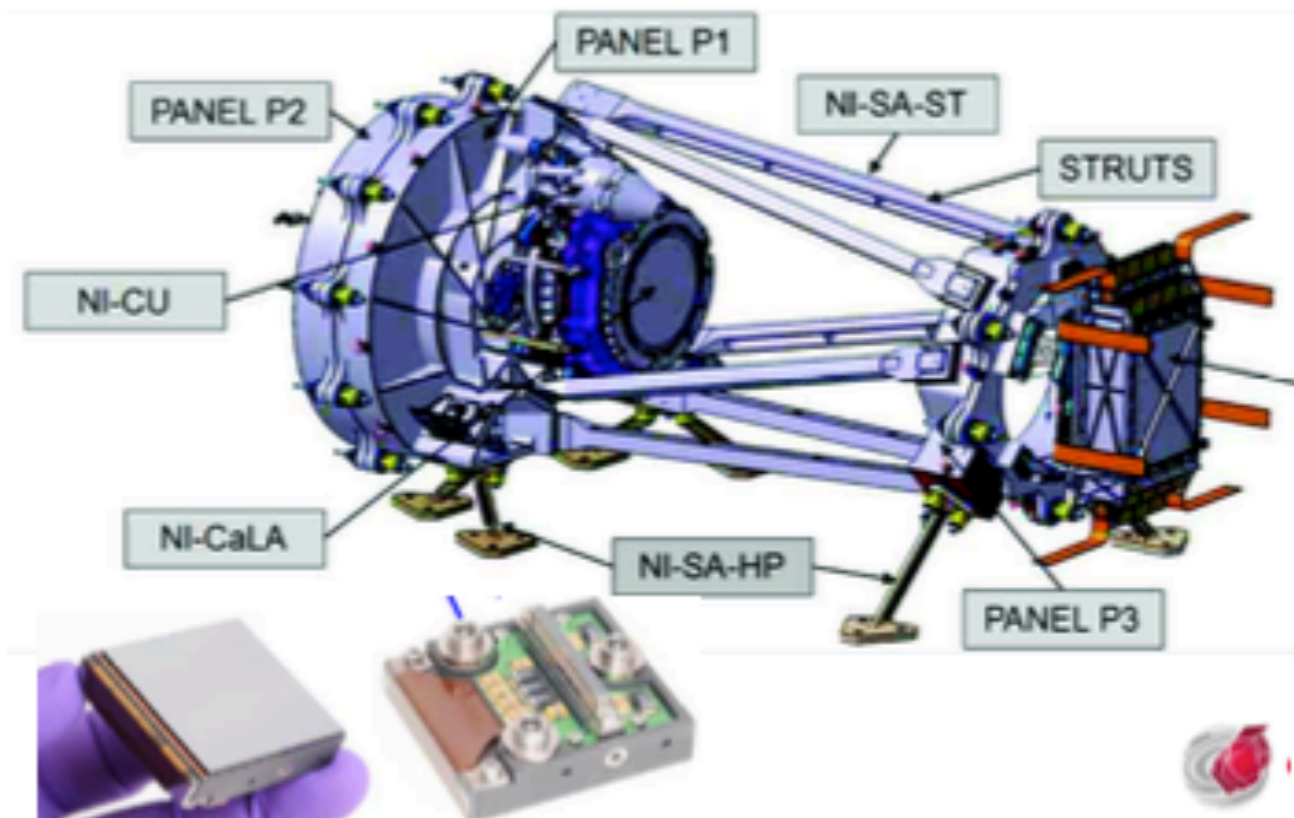


and

NISP

Courtesy: T. Maciaszek and the NISP team

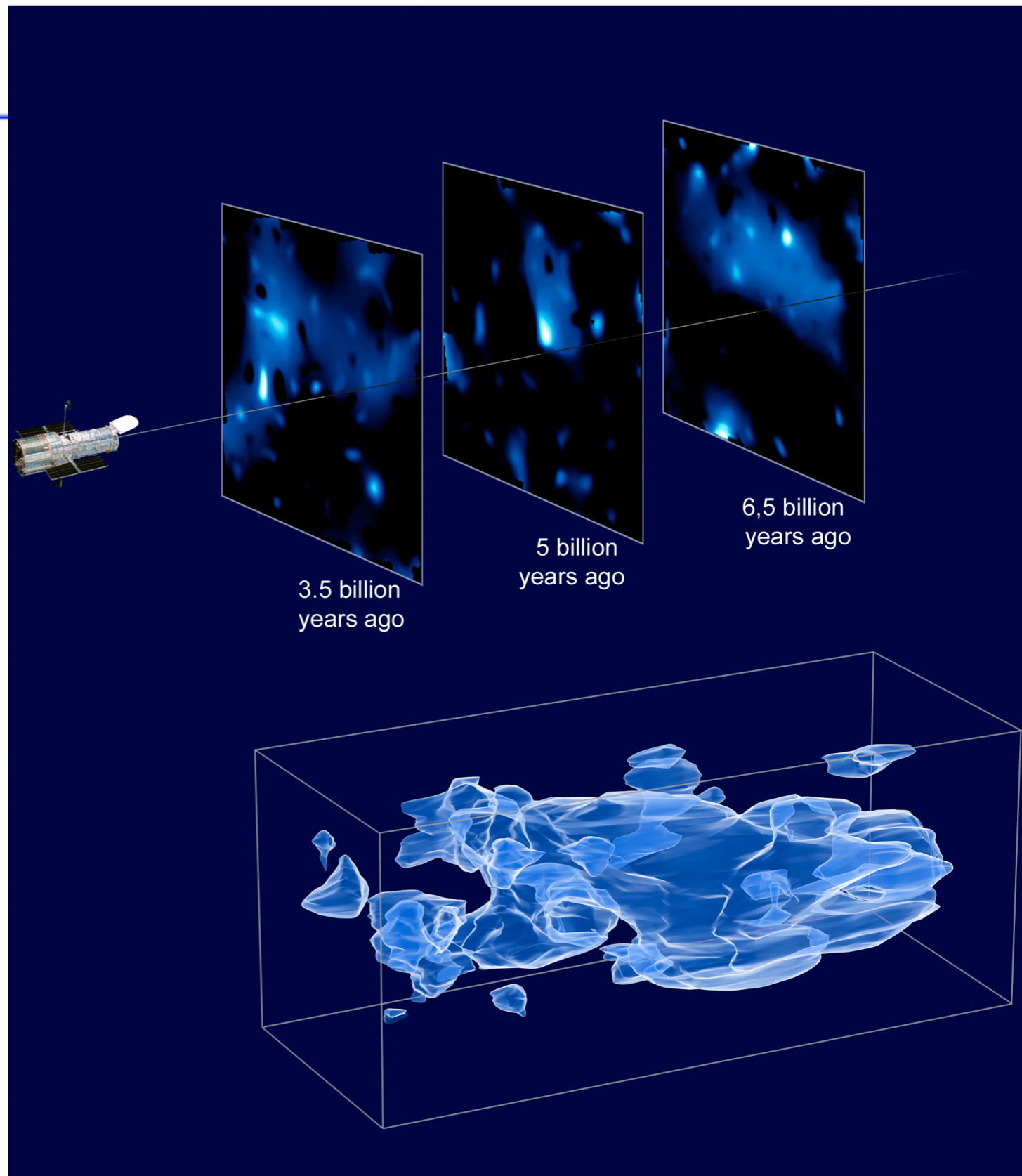
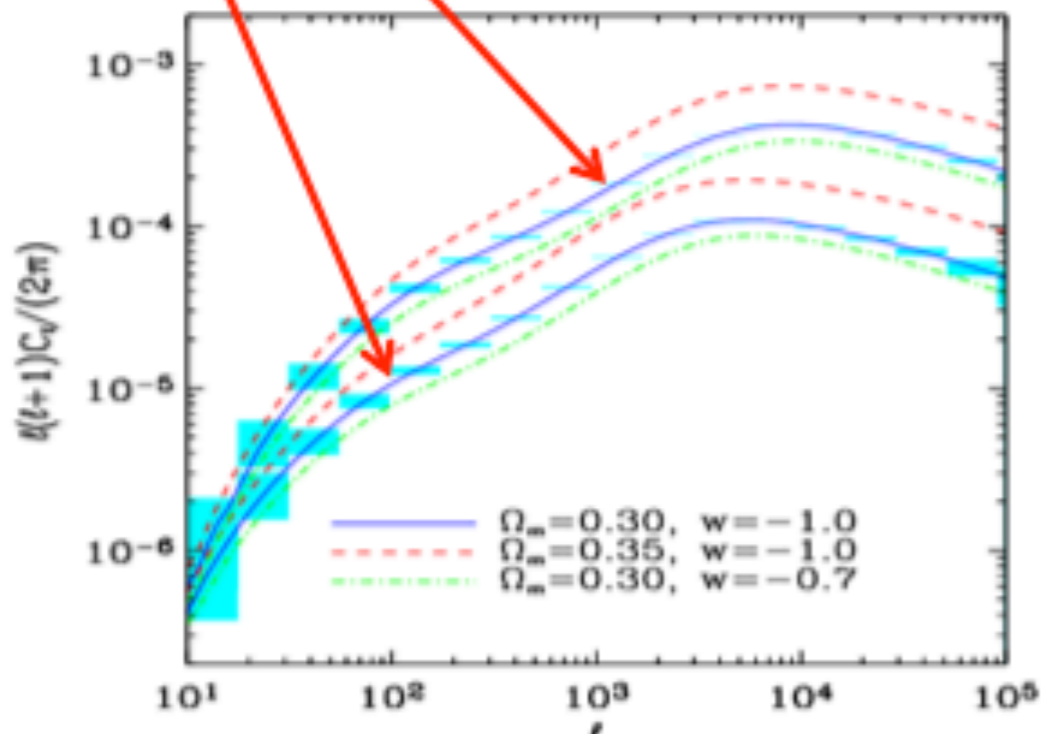
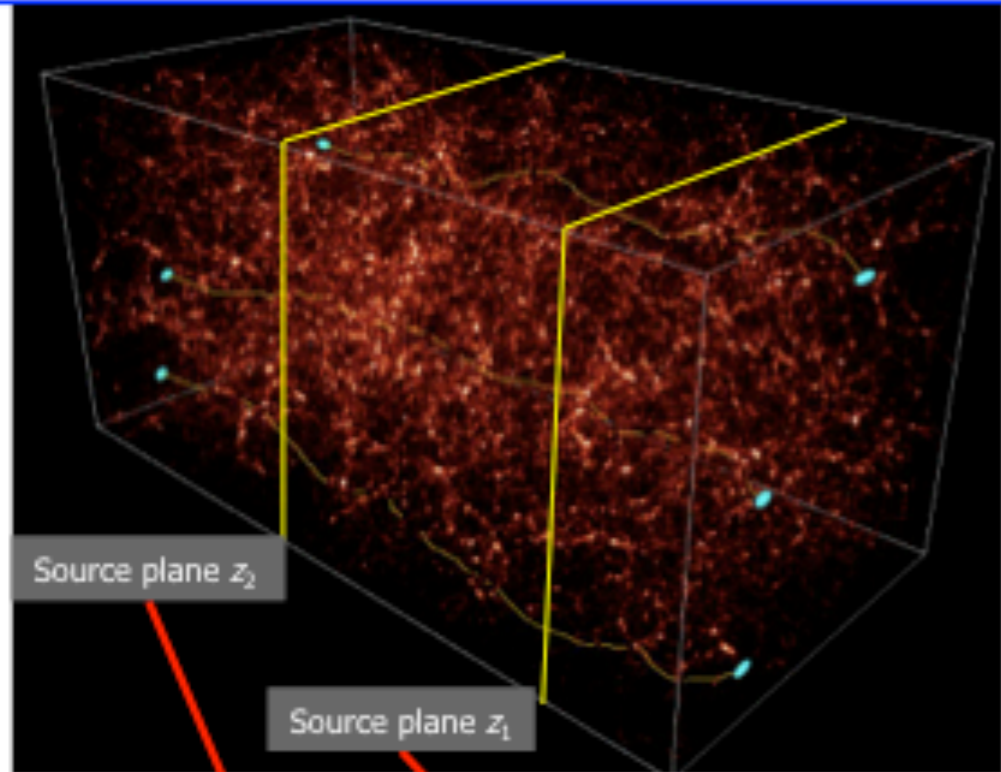
- FoV: 0.55 deg²
- Mass : 159 kg
- Telemetry: < 290 Gbt/day
- Size: 1m x 0.5 m x 0.5 m
- 16 2kx2K H2GR detectors
- 0.3 arcsec pixel on sky
- Limiting mag, wide survey AB : 24 (5 σ)
- **3 Filters:** Y, J, H
- **4 grisms:** 1B (920 – 1250) ,3R (1250 – 1850)



WL probe: Cosmic shear over $0 < z < 2$:

1.5 billion galaxies shapes, gravitational shear and photometric redshifts (u,g,r,i,z,Y,J,H) with 0.05 $(1+z)$ accuracy over 15,000 deg^2

Figure 10.14.10



Thank You!

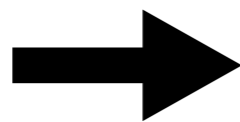
Sources:

- M. Meneghetti, “Introduction to Gravitational Lensing”, Lecture scripts
- P. Schneider, “Weak gravitational lensing”, arXiv:astro-ph/0509252
- H. Hoekstra, “Weak gravitational lensing”, arXiv:1312.5981 [astro-ph.CO]
- R. Laureijs, “Euclid Assessment Study Report for the ESA Cosmic Visions”, arXiv:0912.0914

Refractive index

Need to determine “refractive index” of lens in gravity.

- Assume that lens is weak and small compared to dimensions of optical system

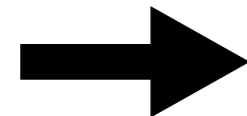


$$\frac{\Phi}{c^2} \ll 1$$

- Weak lens perturbs metric:

$$\eta_{\mu\nu} \rightarrow g_{\mu\nu} = \text{diag} \left(1 + \frac{2\Phi}{c^2}, -\left(1 - \frac{2\Phi}{c^2}\right), -\left(1 - \frac{2\Phi}{c^2}\right), -\left(1 - \frac{2\Phi}{c^2}\right) \right)$$

- Light follows geodesics $ds = 0$:


$$\left(1 + \frac{2\Phi}{c^2}\right)c^2 dt^2 = \left(1 - \frac{2\Phi}{c^2}\right)(d\vec{x})^2$$

Refractive index

- Speed of light in gravitational medium:

$$c' = \frac{|d\vec{x}|}{dt} \approx c \left(1 + \frac{2\Phi}{c^2}\right)$$

- The refractive index of the gravitational medium is then:

$$n = \frac{c}{c'} \approx 1 - \frac{2\Phi}{c^2}$$

- Can now solve variational problem and obtain Euler equation's of motion

$$\delta t = \delta \int_S^O n(\vec{x}(l)) dl = 0$$

$$n \dot{\vec{e}} = \vec{\nabla} n - \vec{e}(\vec{\nabla} n \cdot \vec{e})$$

where \vec{e} is the unit tangent vector along the light path.

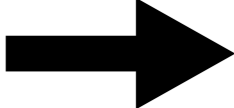
Distortion

- Can split off isotropic part:

$$S = \left(A - \frac{1}{2} \text{tr} A \cdot I_2 \right) = \begin{pmatrix} -\frac{1}{2}(\psi_{11} - \psi_{22}) & -\psi_{12} \\ -\psi_{12} & \frac{1}{2}(\psi_{11} - \psi_{22}) \end{pmatrix}$$

- Define pseudo-vector $\vec{\gamma} = (\gamma_1, \gamma_2)$

$$\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22}) \quad \gamma_2 = \psi_{12}$$

 $S = \begin{pmatrix} -\gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 \end{pmatrix} = |\vec{\gamma}| \begin{pmatrix} -\cos 2\phi & \sin 2\phi \\ \sin 2\phi & \cos 2\phi \end{pmatrix}$

- Remainder:

$$\frac{1}{2} \text{tr} A = (1 - \kappa) \delta_{ij}$$

Mass distribution map

Relate κ and γ to lensing potential ψ

FT

$$\kappa = \frac{1}{2}(\psi_{11} + \psi_{22}) \rightarrow \hat{\kappa} = -\frac{1}{2}(k_1^2 + k_2^2)\hat{\psi}$$

$$\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22}) \rightarrow \hat{\gamma}_1 = -\frac{1}{2}(k_1^2 - k_2^2)\hat{\psi}$$

$$\gamma_2 = \psi_{12} \rightarrow \hat{\gamma}_2 = -k_1 k_2 \hat{\psi}$$

Eliminate ψ :

$$\hat{\kappa} = k^{-2}[(k_1^2 - k_2^2)\hat{\gamma}_1 + 2k_1 k_2 \hat{\gamma}_2]$$

Dark Matter Map

- Construct dark matter map via *weak gravitational lensing*
- Perform measurement of *ellipticity* of galaxies

