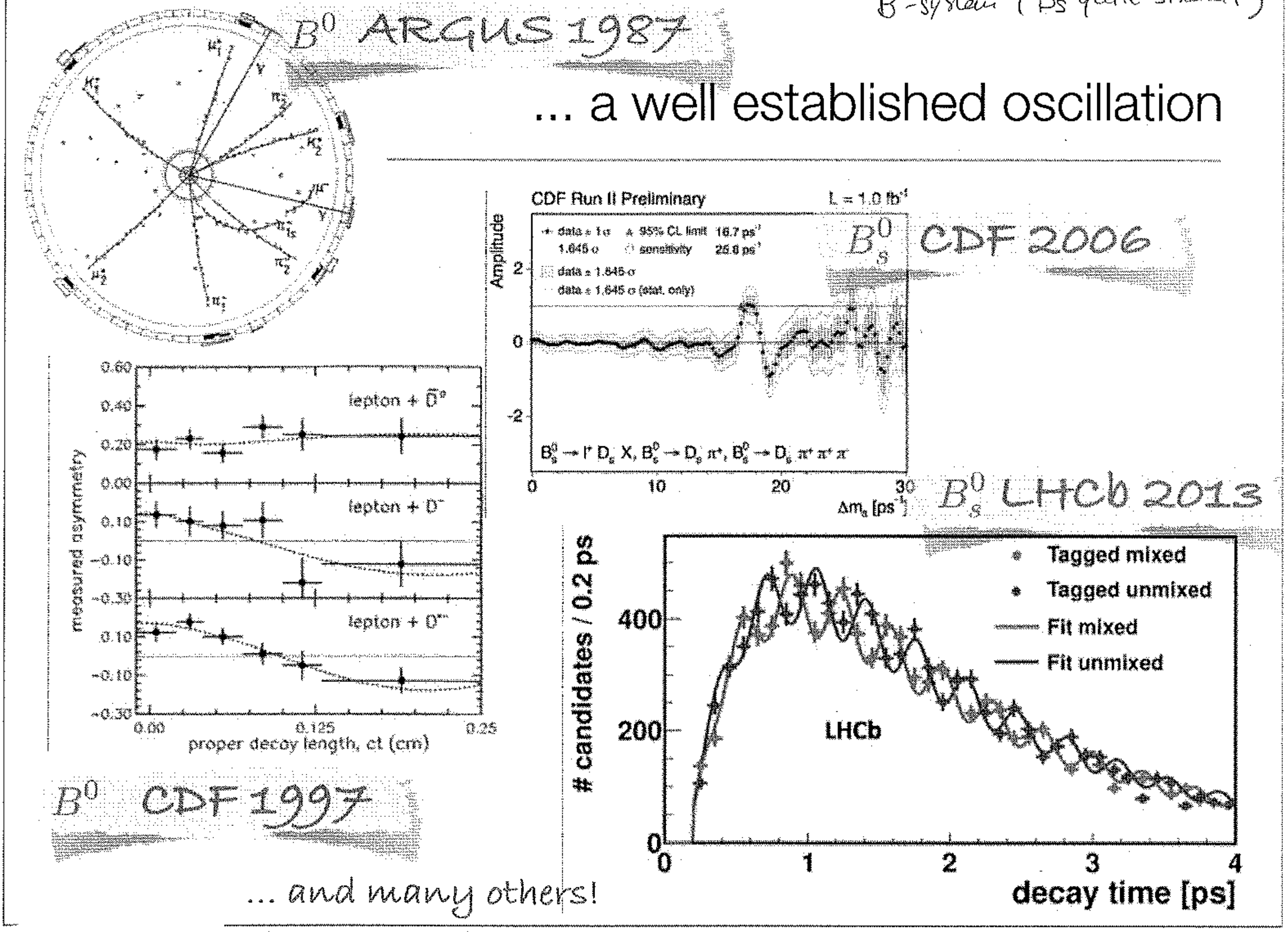
CP violation in neutral B^0 meson mixing

0.) Introduction.

→ neutral mesons:

	quark content	$\tau = 1/\Gamma$	Δm
K-system	$(K^0 = d\bar{s},\ \bar{K}^0 = \bar{d}s)$	$0.26 \cdot 10^{-9}$ s	$5.29\ ns^{-1}$
D-system	$(D^0 = c\bar{u}\ \ \bar{D}^0 = \bar{c}u)$	$0.41 \cdot 10^{-12}$ s	$0.0024\ ps^{-1}$
B-system	$(B^0 = \bar{b}d\ \ \bar{B}^0 = b\bar{d})$	$1.53 \cdot 10^{-12}$ s	$0.507\ ps^{-1}$
B_s-system	$(B^0_s = \bar{b}s\ \ \bar{B}^0_s = b\bar{s})$	$1.47 \cdot 10^{-12}$ s	$17.77\ ps^{-1}$

we will focus on the B-system (B_s quite similar)



→ CP violation is usually classified in 3 types, the focus of this lecture is on two of these "types": CP violation in decay and CP violation in the interference between a decay with and without mixing.

→ Considering the latter type it is possible to get an idea about where the CKM phase enters the measured asymmetry.

Classification of the CP Violating Effects

1) CPV in decay: when the decay rate of a B to a final state f differs from the decay rate of the $\overline{B}$ to the CP-conjugated final state $\overline{f}$

$$\Gamma(B \to f) \neq \Gamma(\overline{B} \to \overline{f})$$

A example: $B^0 \to K^+\pi^-$

$$A_{CP} = \frac{\Gamma(B^0 \to K^+\pi^-) - \Gamma(\overline{B^0} \to K^-\pi^+)}{\Gamma(B^0 \to K^+\pi^-) + \Gamma(\overline{B^0} \to K^-\pi^+)}$$

Important note: observing complex phases.

Considering a single amplitude ; $A = |A| e^{i\varphi}$

$A^{\dagger}A = |A|^2 e^{i(\varphi-\varphi)} = |A|^2$ That shows how complex phases CANNOT be observed with a single amplitude. Several decay amplitudes can contribute to an amplitude $A = \sum_i A_i$, $A_i = |A_i| e^{i\varphi_i}$

Each phase consists of a phase ϕ_i, changing sign under CP tranformations (CP-odd) originating from the complex coupling constants, and a CP-even phase δ_i (originating from processes like gluon exchanges in the final state

$$\boxed{\begin{aligned} A_i &= A_i\, e^{i(\phi_i + \delta_i)} \\ \overline{A_i} &= A_i\, e^{i(-\phi_i + \delta_i)} \end{aligned}}$$

Given 2 amplitudes contributing to the total amplitude $A(B \to f)$ the magnitude of the total amplitude is:

$$|A|^2 = |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + |A_1 A_2|\, e^{i((\phi_1+\delta_1)-(\phi_2+\delta_2))} + e^{i((-\phi_1+\delta_1)+(\phi_2+\delta_2))}$$

$$= |A_1|^2 + |A_2|^2 + 2|A_1 A_2| \cos(\Delta\phi + \Delta\delta)$$

Considering the CP-conjugate $\bar{A}(\bar{B} \to \bar{f})$

$$|\bar{A}|^2 = |\bar{A}_1 + \bar{A}_2|^2 = |A_1|^2 + |A_2|^2 + |A_1 A_2| \left(e^{i((-\phi_1+\delta_1)-(-\phi_2+\delta_2))} + e^{i(-(-\phi_1+\delta_1)+(-\phi_2+\delta_2))} \right)$$

$$= |A_1|^2 + |A_2|^2 + 2|A_1 A_2| \cos(-\Delta\phi + \Delta\delta)$$

Take home message: the total CP-conjugated amplitude will have different magnitude wrt the total amplitude if there are 2 phases of which one flip signs under CP tranformation.

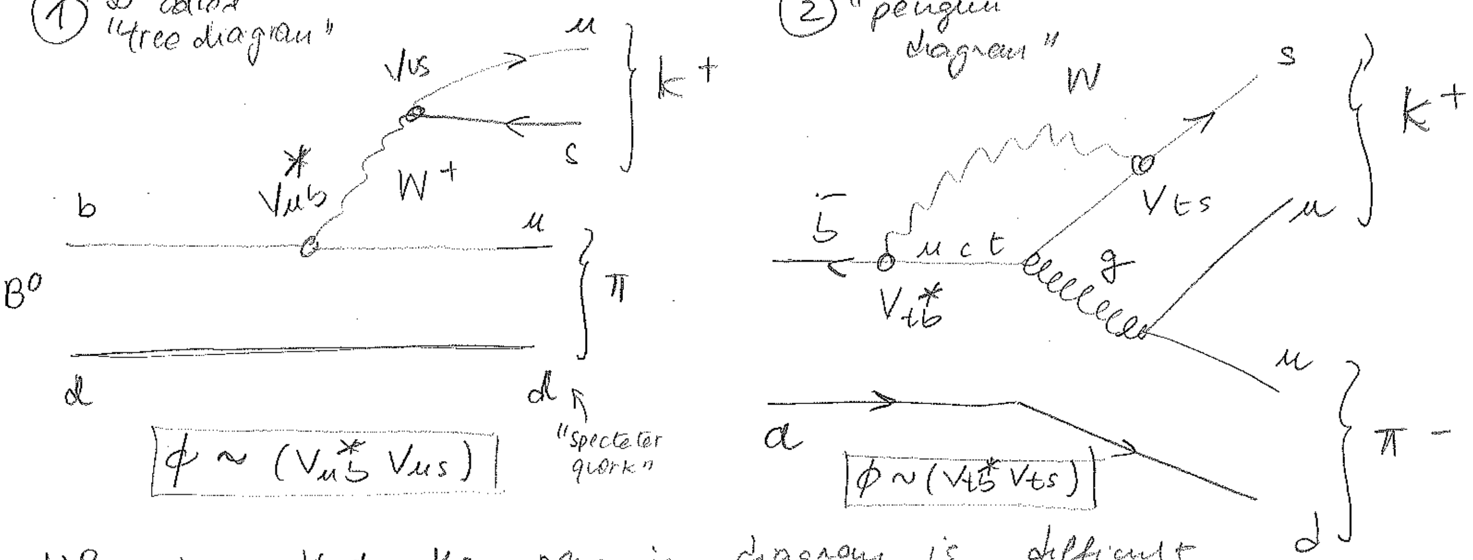
Schematically:

The example: $B^0 \to K^+\pi^-$

We need 2 amplitudes:

① so called "tree diagram"

② "penguin diagram"



$\boxed{\phi \sim (V^*_{ub} V_{us})}$

$\boxed{\phi \sim (V^*_{tb} V_{ts})}$

NB: given that the penguin diagram is difficult to calculate, it is difficult to interpret this result in terms of CKM angles. The weak phase (ϕ) difference can be seen from the weak interaction vertices

Discovery of direct CP violation in $B^0 \to K^+\pi^-$ in 2004.
Here the measurement from LHCb of the same quantity, together with the first observation of CP violation in $B_s^0 \to K^+\pi^-$ decays
(LHCb, $1 fb^{-1}$)

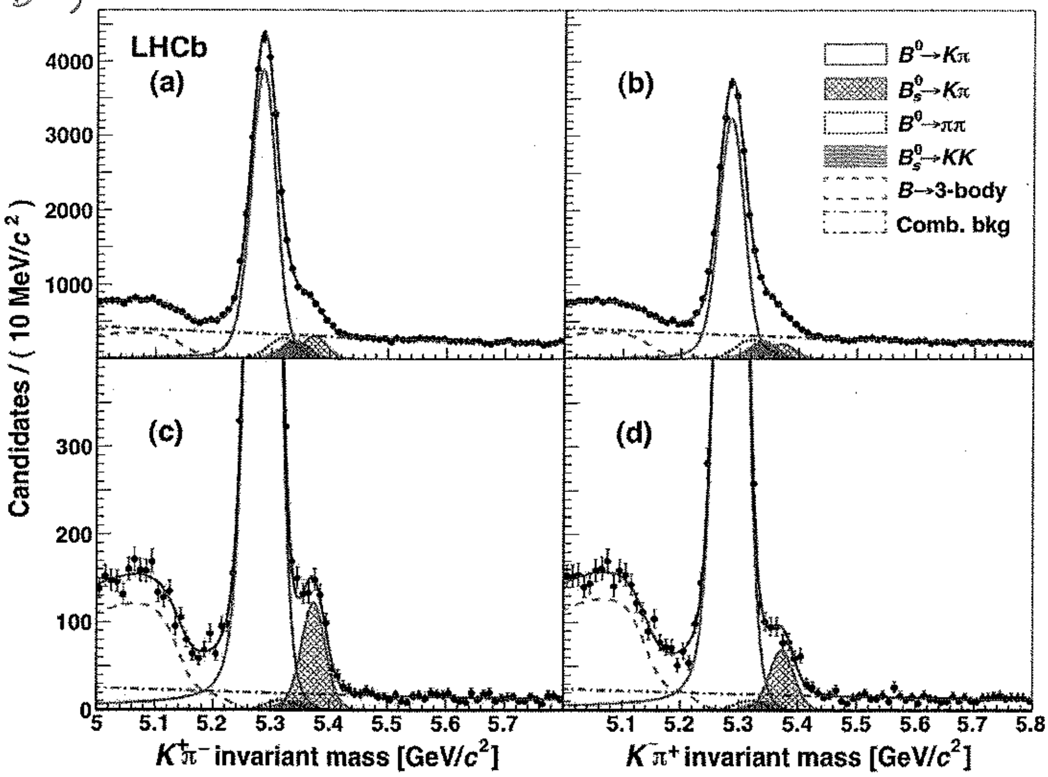


FIG. 1. Invariant mass spectra obtained using the event selection adopted for the best sensitivity on (a, b) $A_{CP}(B^0 \to K^+\pi^-)$ and (c, d) $A_{CP}(B_s^0 \to K^-\pi^+)$. Panels (a) and (c) represent the $K^+\pi^-$ invariant mass, whereas panels (b) and (d) represent the $K^-\pi^+$ invariant mass. The results of the unbinned maximum likelihood fits are overlaid. The main components contributing to the fit model are also shown.

arXiv:1304.6173v2

Recall: LHCb forward spectrometer @ LHC collider
Important for these analysis: the Particle Identification!
"Raw" asymmetry: extracted from the signal yields (hight of the peaks in the plots)
The raw asymmetry needs to be corrected for two effects: Detection asymmetry and Production asymmetry (diluted by the $B^0_{(s)}$ oscillations). More about these topics on the last lecture

$$A_{CP}(B_s^0 \to K^-\pi^+) = 0.27 \pm 0.04\,(stat) \pm 0.01\,(syst)$$
$$A_{CP}(B^0 \to K^+\pi^-) = -0.080 \pm 0.007\,(stat) \pm 0.003\,(syst)$$

2) CP violation in mixing

$$\text{Probability}(B \rightarrow \overline{B}) \neq \text{Probability}(\overline{B} \rightarrow B)$$

We have already seen in the first lecture, that occurs when

$$\left|\frac{q}{p}\right| \neq 1$$

remember:

$$|B_H\rangle = p|B^0\rangle + q|\overline{B^0}\rangle$$
$$|B_L\rangle = p|B^0\rangle - q|\overline{B^0}\rangle$$

In B^0 and $\overline{B^0}$ the CP violation in mixing is predicted to be very small, but it is the dominant effect in the kaon system. The LHCb measurement of CP violation in B^0-$\overline{B^0}$ mixing will be presented during the last lecture

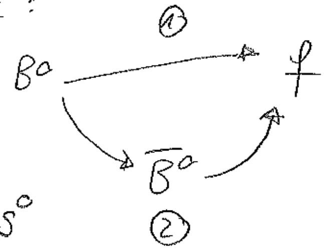
3) CP violation in interference between a decay with and without mixing

This form of CP violation is measured in decays to a final state accessible for both the $B^0_{(s)}$ and $\overline{B}^0_{(s)}$ mesons.
CP is violated iff

$$\Gamma(B^0 \rightarrow \overline{B^0} \rightarrow f)(t) \neq \Gamma(\overline{B^0} \rightarrow B^0 \rightarrow f)(t)$$

In this case, due to $\overline{f} = f$, the two amplitudes contributing to the final state f will be:

① $A(B^0 \rightarrow f)$ ② $A(B^0 \rightarrow \overline{B^0} \rightarrow f)$

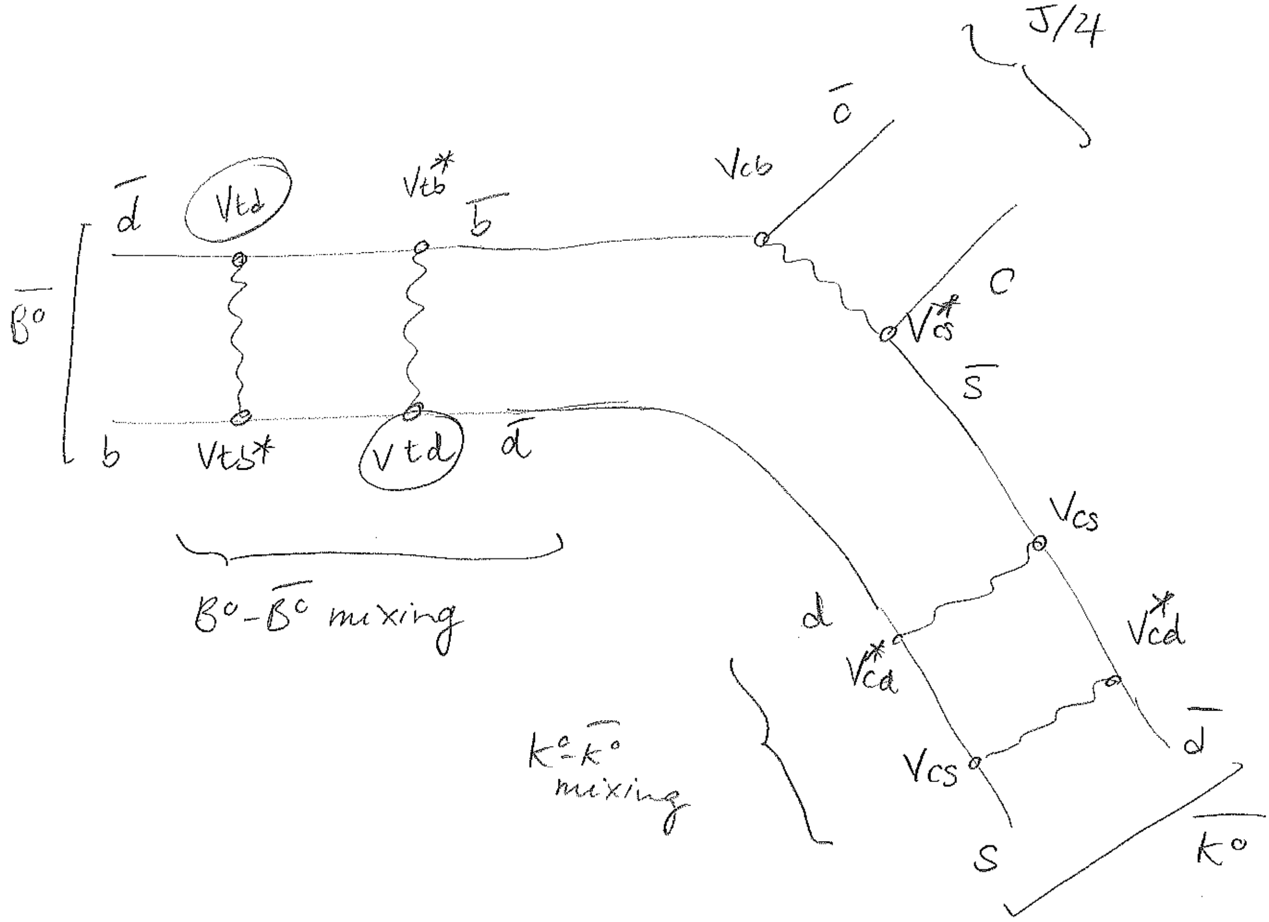


Example of a measurement : $B^0 \rightarrow J/\psi\, K_S^0$

$-$ $B^0 \rightarrow J/\psi\, K^0$
$\overline{B^0} \rightarrow J/\psi\, \overline{K^0}$ } to obtain the same final state we need to consider

$$|K_S^0\rangle = p|K^0\rangle + q|\overline{K^0}\rangle$$

diagrams of interest:



A usually used quantity to classify CPV is λ_f

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} = \frac{q}{p}\frac{A(\bar{B}^0\to f)}{A(B^0\to f)} \qquad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}$$

in this case we have:

$$\lambda_{J/\psi K_S} = \left(\frac{q}{p}\right)_{B^0}\left(\eta_{J/\psi K^0_S}\,\frac{\bar{A}_{J/\psi K^0_S}}{A_{J/\psi K^0_S}}\right)\left(\frac{p}{q}\right)_{K^0}$$

$\sqrt{\frac{M_{12}^*}{M_{12}}}$ i.e. neglecting Γ_{12}

$= -1$ because $J/\psi K^0_S$ is a CP-odd final state (J/ψ spin 1 is CP-even, K_S spin-0 is (ALMOST) CP-even

$$\left(\frac{q}{p}\right)_{B^0} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \qquad \frac{\bar{A}}{A} = \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \qquad \left(\frac{p}{q}\right)_K = \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}$$

$$\lambda_{J/\psi K^0_S} = -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}$$

Taking its imaginary part

$$\Im\,\lambda_{J/\psi K_S} = -\sin\left\{\arg\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\,\frac{V_{cb} V_{cd}}{V_{cb}^* V_{cd}}\right)\right\}$$

$$= -\sin\left\{2\arg\left(\underbrace{\frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*}}_{\beta}\right)\right\} \equiv \sin 2\beta$$

Experimentally let's define $A_{CP}(t) = \dfrac{\Gamma(B^0(t)\to f) - \Gamma(\bar{B}^0(t)\to f)}{\Gamma(B^0(t)\to f) + \Gamma(\bar{B}^0(t)\to f)}$

Writing the expressions for the decay rates, assuming $|q/p| = 1$, and $|A_f| = |\bar{A}_f|$, and $\Delta\Gamma = 0$,

$$A_{CP}(t) = -\Im\lambda_f \sin(\Delta m t)$$

$$= -\sin(2\beta)\sin(\Delta m t)$$

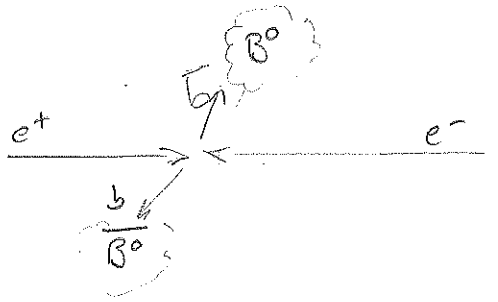
V_{td} is the only CKM element with non-vanishing imaginary part. The phase difference between $B^0 \to J/\psi K_S$ and $B^0 \to \bar{B}^0 \to J/\psi K_S$ originates from V_{td} (box-diagram)

→ Let's represent the CKM matrix as:

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

NB: CPV requires $V_{ij} \neq V_{ij}^*$ ⟹ β, β_s, γ need to be different from zero (more details: see ADDITIONAL MATERIAL 1 and references)

The value of $\sin 2\beta$ has been measured very accurately by the B-factories. They exploit the process: $e^+e^- \rightarrow \Upsilon \rightarrow B^0\bar{B}^0$



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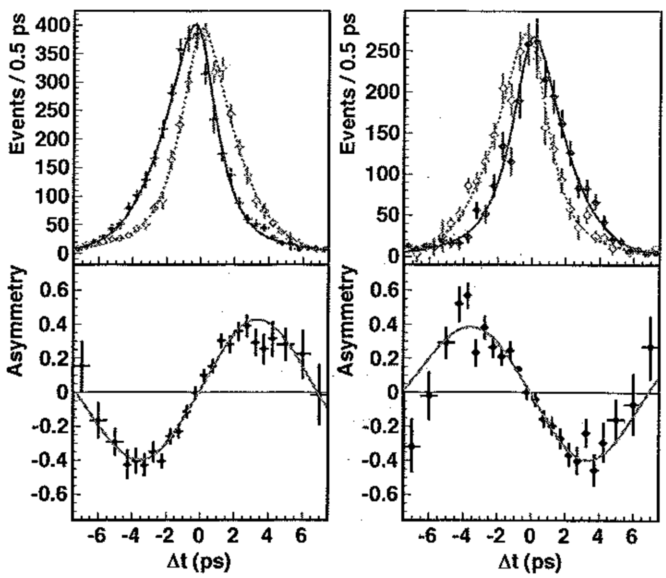


FIG. 2: (color online) The background-subtracted Δt distribution (top) for $q = +1$ (red) and $q = -1$ (blue) events and asymmetry (bottom) for good tag quality ($r > 0.5$) events for all CP-odd modes combined (left) and the CP-even mode (right).

$q = \eta_{J/\psi K^0}$ CP eigenvalue

arXiv:1201.4643v2

- $B^0\bar{B}^0$ pair coherently produced $\Rightarrow$ the lifetime of the B^0 meson is expressed as time difference between the B^0 and the $\bar{B}^0$ decays (Δt)

- The number of B^0 decays is determined by requiring the other B has decayed as $\bar{B}^0$ ("Flavor tagging")

$$A_{CP}(\Delta t) = \frac{\Gamma(B^0(\Delta t) \rightarrow f) - \Gamma(\bar{B}^0(\Delta t) \rightarrow f)}{\Gamma(B^0(\Delta t) \rightarrow f) + \Gamma(\bar{B}^0(\Delta t) \rightarrow f)}$$

$$= \eta_f \sin(2\beta)\sin(\Delta m \Delta t)$$

This paper from Belle reports:

$$\sin 2\beta = 0.667 \pm 0.023(\text{stat}) \pm 0.012\ (\text{syst})$$

PDG average: 0.675 ± 0.020

References; in addition to quoted papers (Lecture 1 and 2)

- P. Kooijman and N. Tuning "CP Violation" Lectures (Nikhef)
- U. Nierste "Three Lectures on Meson Mixing and CKM phenomenology" arXiv:0904.1869.v1
- H. Perkins: "Introduction to High Energy Physics" 4th Edition
- U. Uwer "Flavor Physics" lectures
- S. Braibant, G. Giacomelli, M. Spurio "Particles and Fundamental Interactions"
- Particle Data Group

ADDITIONAL MATERIAL 1

digression about CKM parametrization : CPV parameters

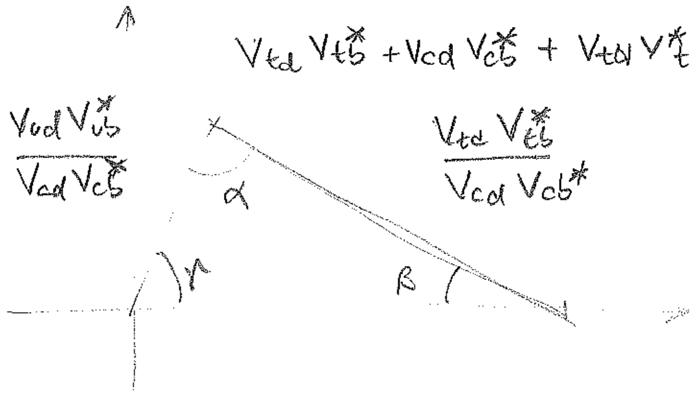
$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \delta V$$

$\boxed{\text{Wolfenstein parametrization}}$

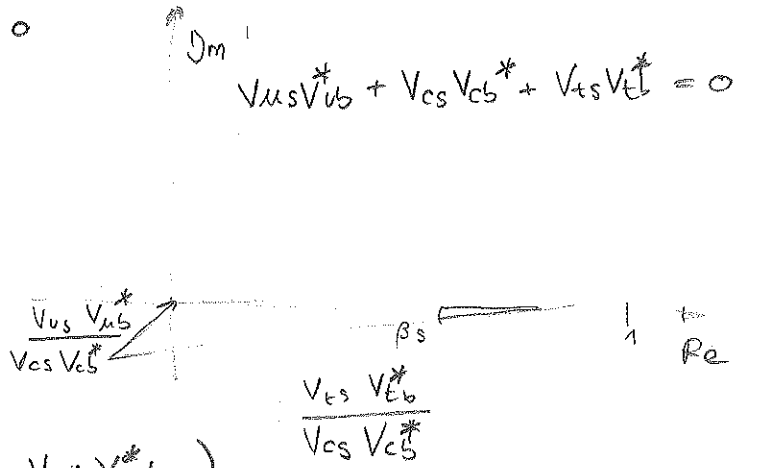
$$\delta V = \begin{pmatrix} -\frac{1}{8}\lambda^4 & 0 & 0 \\ \frac{1}{2}A^2\lambda^5(1-2(\rho+i\eta)) & -\frac{1}{8}\lambda^4(1+4A^2) & 0 \\ \frac{1}{2}A\lambda^5(\rho+i\eta) & \frac{1}{2}A\lambda^4(1-2(\rho+i\eta)) & -\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|\,e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|\,e^{-i\beta} & -|V_{ts}|\,e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

$V_{td}V_{tb}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$



$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$



$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \qquad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \qquad \beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

CPV requires $V_{ij} \neq V_{ij}^*$ $\Longleftrightarrow$ the triangle(s) have finite surface (*)

If one of the mixing angles is zero, the area is zero,

the CKM matrix would reduce to a 2×2 matrix $\Rightarrow$ no CPV

would be possible

* Footnote: CKM representation using Euler angles θ_{ij}

$c_{ij} = \cos\theta_{ij} \quad s_{ij} = \sin\theta_{ij}$

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

Jarlskog invariant $\quad J = \Im(V_{11} V_{22} V_{12}^{*} V_{21}^{*}) = \Im(V_{22} V_{33} V_{23}^{*} V_{32}^{*})$

$= \ldots$

$= 2 \times$ area of the triangle(s)

$= c_{12} c_{13}^{2} c_{23} s_{12} s_{13} s_{23} \sin\delta_{13}$

It is more clear from this notation that the area of the triangles occurs in all CP violating effects.

$\Rightarrow$ to describe CPV in weak interaction we measure

β, γ, β_s