# Axions

First lecture 2.7.21

Why are axions interesting? (Outline of lecture series)

1. New particle that solves a fundamental problem in particle physics (strong CP problem)
2. They are an attractive dark matter candidate.
3. They are experimentally within reach. Large parts of interesting parameter space will be explored in the coming years.

## 1. Strong CP problem

Suppose we want to construct a Lagrangian for a gauge potential $A_\mu$ (as in electrodynamics).

We know: field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\,[A_\mu, A_\nu]$$

Gauge and Lorentz invariant Yang-Mills Lagrangian

$$\mathcal{L}_{YM} = -\frac{1}{2}\,\mathrm{Tr}[F^{\mu\nu}F_{\mu\nu}]$$

$$\overset{U(1)}{=} -\frac{1}{2}\,(\vec{B}^2 - \vec{E}^2)$$

Equations of motion

$$D_\mu F^{\mu\nu} = 0 \xrightarrow{U(1)} \underbrace{\partial_\mu F^{\mu\nu} = 0}_{\text{2 out of 4 Maxwell eq.}}$$

This is a Lagrangian formulation of a $U(1)$ or $SU(N)$ gauge theory.

E.g.

Electromagnetism: $\mathcal{L} = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu}$

Electroweak theory: $\mathcal{L} = -\frac{1}{2}\mathrm{Tr}[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$

QCD: $\mathcal{L} = -\frac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$

**But** The most general Lagrangian contains all terms allowed by symmetries

We have to add this term: $F^{\mu\nu}\tilde{F}_{\mu\nu}$

$$\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$$

In electrodynamics $\tilde{F}$ is $F$ with $\vec{B} \to -\vec{E}$ and $\vec{E} \to \vec{B}$

$\tilde{F}$ is called dual field strength tensor.

$\Rightarrow$ Most general Lagrangian

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}[F^{\mu\nu}F_{\mu\nu}] - \frac{g^2\theta}{16\pi^2}\,\mathrm{Tr}[F_{\mu\nu}\tilde{F}^{\mu\nu}]$$

$$\overset{U(1)}{=} -\frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \qquad + \frac{e^2\theta}{8\pi^2}\,\vec{B}\cdot\vec{E}$$

Let's talk about this new term:

- It is CP violating

$$\overset{+}{|}\xleftarrow{\vec{E}}\overset{-}{|} \quad \xrightarrow{P} \quad \overset{-}{|}\xrightarrow{\vec{E}}\overset{+}{|}$$

$$\xrightarrow{CP} \quad \overset{+}{|}\xleftarrow{\vec{E}}\overset{-}{|}$$

$$\vec{B} \propto \vec{r}\times\vec{J} \quad \xrightarrow{P} \quad (-\vec{r})\times(-\vec{J}) \propto \vec{B}$$

$$\xrightarrow{CP} \quad (-\vec{r})\times\vec{J} \quad \propto -\vec{B}$$

$$\Rightarrow \quad F^{\mu\nu}\tilde{F}_{\mu\nu} \propto \vec{E}\cdot\vec{B} \quad \xrightarrow{P} \quad -F^{\mu\nu}\tilde{F}_{\mu\nu}$$

$$\xrightarrow{CP} \quad -F^{\mu\nu}\tilde{F}_{\mu\nu}$$

CP and P violation. Both are violated in the SM. So this is not problematic.

- It is a total derivative!

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = \partial_\mu V^\mu$$

with $V_\mu \propto \varepsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}[A^\nu\partial^\rho A^\sigma - \frac{2}{3}g A^\nu A^\rho A^\sigma]$

$\Rightarrow$ classically: no contributions to E.o.M.

in QFT: no additional Feynman rules

It is still observable due to instantons.

Instantons are field configurations which solve the classical (euclidean) equations of motion and have a finite action. They exist in SU(2) and SU(3) but not in U(1) gauge theories.

# BPST Instanton

$$A_\mu^a = 2\,\frac{\eta^a{}_{\mu\nu}\,(x-x_0)^\nu}{(x-x_0)^2+\rho^2} \sim \frac{1}{r}$$

$\rho$: size
$x_0$: position
$r \equiv |x-x_0|$

Instantons are localized in space and time
$\Rightarrow$ They are processes <u>not</u> particles

Contribution to the action:

$$S = \int d^4x\, \mathcal{L}_\theta \;\hat{=}\; \int d^4x\, \partial_\mu V^\mu \;\hat{=}\; \int d\Omega\, r^3 \underbrace{(A^3 + A\partial A)}_{\sim \frac{1}{r^3}} = \text{finite}$$

We can show
a) For one (anti-) instanton $\frac{g^2}{16\pi^2}\int F\tilde{F}\,d^4x = \pm 1$
b) Superposition of Instantons at large distances is also a solution (Dilute instanton gas)

$\Rightarrow$ $\frac{g^2}{16\pi^2}\int d^4x\, F\tilde{F} \in \mathbb{Z}$

$\Rightarrow$ $\theta$ is an angle: $\quad e^{iS} = e^{i\theta \underbrace{\frac{g^2}{16\pi^2}\int F\tilde{F}d^4x}_{\in \mathbb{Z}}}$

$\theta \to \theta + 2\pi$ does not change any observable.

## <u>$\theta$ in the SM</u>

Where could we observe $\theta$?
CP violating observable in QCD: neutron electric dipole moment

we measure: $|d_n| < 3.0 \cdot 10^{-13}\, e\,\text{fm}$
we calculate: $|d_n| = (2.4 \pm 1.0)\,|\theta|\, 10^{-3}\, e\,\text{fm}$

$$\Rightarrow \quad |\theta| < 1.3 \cdot 10^{-10}$$

So $\theta$ could have any value between $-\pi$ and $\pi$ and miraculously chooses to be very (!!!) close to 0.
$\Rightarrow$ strong CP problem

## 2. Axion solution

Replace constant $\theta$ by a scalar field.

$$a \equiv \theta \cdot f_a \qquad \text{Axion field}$$

angle ↗ ↑ some energy scale

$$\Rightarrow \mathcal{L}_a = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{a}{f_a}\frac{g^2}{16\pi^2}\,\mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}] + \text{shift symm. terms}$$

The axion obtains a potential from instanton contributions to the path integral.

Vafa & Witten: Minimum of the potential is always at CP conserving value. Vacuum of the scalar field at $\theta = 0$.

$\Rightarrow$ The problem solves itself, when $\theta$ is dynamical i.e. an axion exists.

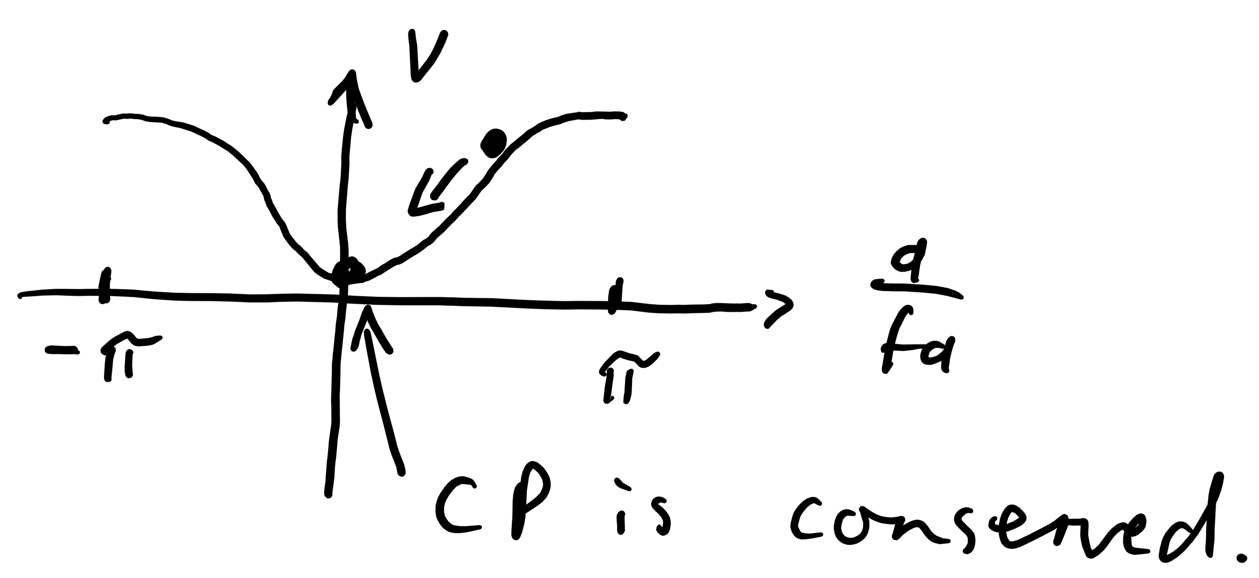
"Proof" of Vafa Witten theorem:

CP violating terms are the only imaginary contributions to the euclidean action:

$$\begin{aligned}
\exp(-V_{eff}(a)) &= \left| \int DG^\mu \exp\left(-S[\phi, G^\mu] - i\int d^4x \frac{g^2}{16\pi^2}\frac{a}{f_a} G\tilde{G}\right)\right| \\
&\leq \int DG^\mu \left| \exp\left(-S[\phi, G^\mu] - i\int d^4x \frac{g^2}{16\pi^2}\frac{a}{f_a} G\tilde{G}\right)\right| \\
&\leq \int DG^\mu \exp(-S[\phi, G^\mu]) \\
&\leq \exp(-V_{eff}(0))
\end{aligned}$$

$$\Rightarrow V_{eff}(0) \leq V_{eff}(a)$$

Minimum at $a = 0$.



# 3. A glimpse at axion models

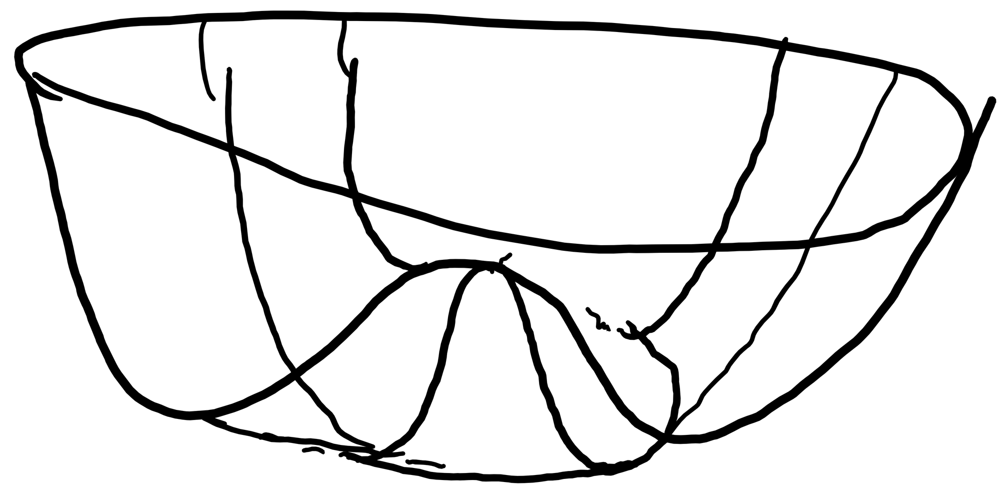
$\frac{a}{f_a} G\tilde{G}$ is a dimension 5 operator. $\Rightarrow$ Not UV complete

(Almost) All QCD axion models introduce a complex scalar field $\phi$. with a SSB potential

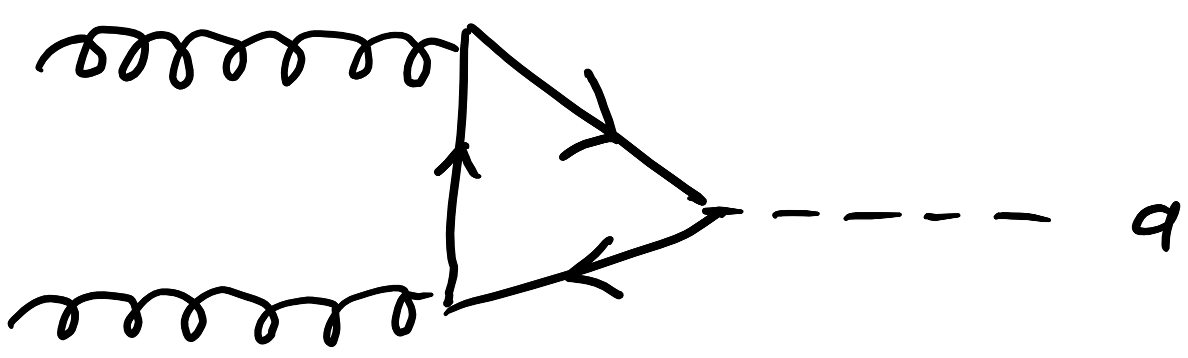
$$\mathcal{L}_\phi = -\partial_\mu\phi\,\partial^\mu\phi - \mu^2\phi^2 - \lambda\phi^4$$

axion

when $\phi$ obtains a vev : $\phi = (\langle\phi\rangle + \rho)\, e^{i\frac{a}{f_a}}$



axion needs to interact with SM or other fields charged under SU(3). Then we get the effective interaction




$\sim \frac{a}{f_a} G\tilde{G}$

All QCD axion models have this $aG\tilde{G}$ interaction. They differ in their other couplings to the SM

e.g. $c_f \frac{\partial_\mu a}{f_a} \bar{f}\gamma^\mu\gamma^5 f$ $\qquad$ $c_{FF} \frac{a}{f_a} F\tilde{F}$