

# Measuring $|V_{ub}|$ at LHCb

Svende Braun

RTG Students Lecture

January/February 2018



UNIVERSITÄT  
HEIDELBERG  
ZUKUNFT  
SEIT 1386



## ① Lecture Today

- CKM Mechanism
- How to measure CKM matrix elements in general?

## ② Lecture: How to measure CKM matrix elements in B-decays?

- $|V_{cb}|$
- $|V_{ub}|$

## ③ Lecture: Specific LHCb measurements

- $\Lambda_b \rightarrow p \mu \nu$
- $B_s^0 \rightarrow K^- \mu^+ \nu_\mu$

# 1. Part: CKM mechanism

# Mass generation of Quarks

- In the SM quarks and leptons acquire mass through the Yukawa terms in the Lagrangian:

$$\mathcal{L}_{Yukawa} = -\frac{v}{\sqrt{2}}(\bar{d}_L Y_d d_R + \bar{u}_L Y_u u_R) + h.c.$$

left-handed quarks are coupled to right-handed ones

- $q$  are the weak-eigenstates
- $Y_u, Y_d$  are 3x3 complex Yukawa matrices  
→ not diagonal in “generation space”

→ weak eigenstates  $\neq$  physical mass eigenstates

# Diagonalization

- to get mass eigenstates  $u'$ ,  $d'$  diagonalize Yukawa matrices by unitary transformations:

$$q'_A = V_{A,q} q_A$$

- with  $q = u, d$ ,  $A = L, R$  and  $V_{A,q} V_{A,q}^\dagger = 1$

- $V_{A,q}$  are determined by:

$$M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} = \text{diag}(m_u, m_c, m_t) = \frac{v}{\sqrt{2}} V_{L,u} Y_u V_{R,u}^\dagger,$$

$$M_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} = \text{diag}(m_d, m_s, m_b) = \frac{v}{\sqrt{2}} V_{L,d} Y_d V_{R,d}^\dagger$$

$$\Rightarrow \mathcal{L}_{\text{Yukawa}} = -\bar{d}'_L M_d d'_R + \bar{u}'_L M_u u'_R + h.c.$$

# CKM matrix

- these transformations leave all parts of SM Lagrangian unchanged, except for term describing the charged current weak interaction:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}(\bar{u}_L \gamma^\mu W_\mu^+ d_L + \bar{d}_L \gamma^\mu W_\mu^- u_L)$$

$$\rightarrow \mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}(\bar{u}'_L \gamma^\mu W_\mu^+ \underbrace{V_{L,u} V_{L,d}^\dagger}_{V_{CKM}} d'_L + \bar{d}'_L \gamma^\mu W_\mu^- \underbrace{V_{L,d} V_{L,u}^\dagger}_{V_{CKM}^\dagger} u'_L)$$

- ⇒ The charged-current interaction gets a flavor structure which is encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix
- ⇒ element  $(V_{CKM})_{ij}$  connects the LH u-type quark of the  $i$ th generation with the LH d-type quark of the  $j$ th generation

# CKM matrix

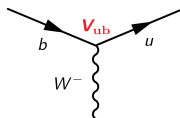
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- describes relation between weak eigenstates  $q$  and mass eigenstates  $q'$ :

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

→ weak eigenstates are a mixture of mass eigenstates

- weak interaction allows for transitions between different generations of quarks through charged current interactions



# Parameters of CKM matrix

in general:  $N \times N$  matrix

- $n^2$  complex elements =  $2n^2$  real parameters
- unitarity condition:  $V^\dagger V = \mathbb{1}$  implies  $n^2$  constraints
  - $n$  unitary conditions
  - $n^2 - n$  orthogonality relations
- removing quark phases:  $2n-1$  constraints

→ leaves  $(n - 1)^2$  free parameters

=  $\frac{1}{2}n(n - 1)$  rotation angles +  $\frac{1}{2}(n - 1)(n - 2)$  phases

⇒ for  $3 \times 3$  matrix: **3 rotation angles + 1 CP violating phase**



# Parametrization of CKM matrix

PDG parametrization: 3 Euler angles  $\theta_{23}, \theta_{13}, \theta_{12}$  and 1 Phase  $\delta$

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \\ \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

where  $c_{ij} = \cos \theta_{ij}$  and  $s_{ij} = \sin \theta_{ij}$

# Wolfenstein Parametrization

Reflects the hierarchical structure of the CKM matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} \text{u} & \color{red}{\blacksquare} & \color{red}{\blacksquare} & \color{red}{\blacksquare} \\ \text{c} & \color{red}{\blacksquare} & \color{red}{\blacksquare} & \color{red}{\blacksquare} \\ \text{t} & \color{red}{\blacksquare} & \color{red}{\blacksquare} & \color{red}{\blacksquare} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$\lambda, A, \rho, \eta$  with  $\lambda = 0.22$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$|V_{ub}| \times e^{-i\gamma}$

$|V_{td}| \times e^{-i\beta}$

with  $\lambda = \sin \theta_{12}$ ,  $A\lambda^2 = \sin \theta_{23}$  and  $\sin \theta_{23} e^{i\delta} = A\lambda^3(\rho + i\eta)$

# Unitarity Condition

$$V^\dagger V = VV^\dagger = \mathbb{1}$$

$$\Leftrightarrow \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

leads to following three unitary relations:

$$V_{ud} V_{ud}^* + V_{us} V_{us}^* + V_{ub} V_{ub}^* = 1$$

$$V_{cd} V_{cd}^* + V_{cs} V_{cs}^* + V_{cb} V_{cb}^* = 1$$

$$V_{td} V_{td}^* + V_{ts} V_{ts}^* + V_{tb} V_{tb}^* = 1$$

- *weak universality*: squared sum of coupling strengths of u-quark to d, s and b-quarks is equal to overall charged coupling of c and t-quark
- adding up to 1, no probability remaining to couple to 4th down-type quark

# Unitarity Triangles

remaining orthogonality conditions can be described by an triangle relations in the complex plane:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad (\text{db})$$

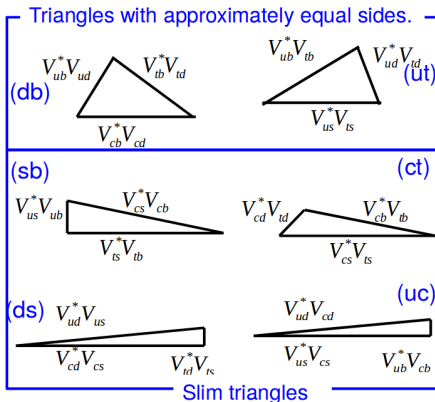
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \quad (\text{sb})$$

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \quad (\text{ds})$$

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0 \quad (\text{ut})$$

$$V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0 \quad (\text{ct})$$

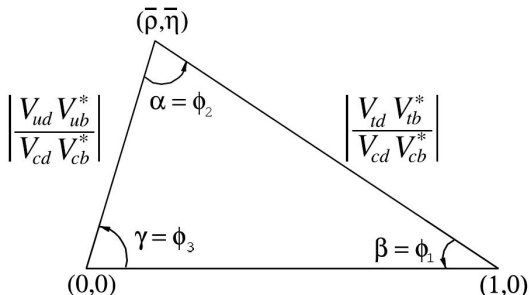
$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0 \quad (\text{uc})$$



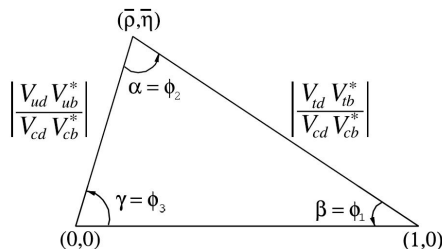
→ all 6 triangles have same area  $J_{CP}/2$ , called the Jarlskog invariant, measure of CP violation of the SM

# Unitarity Triangle

- (db) and (ut) have both sides with similar length, terms of order  $\lambda^3$ , better to visualize  
→ (db) triangle used as 'the unitarity triangle' for historic reasons (b-factories Belle & Babar):  
$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$
- divide each side by best-known one  $V_{cd} V_{cb}^*$  gives:



# The Unitarity Triangle



- angles of unitarity triangle:

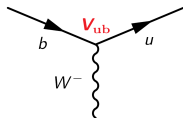
$$\alpha \equiv \arg \left[ -\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right], \quad \beta \equiv \arg \left[ -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right], \quad \gamma \equiv \arg \left[ -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right]$$

- CKM matrix elements are fundamental parameters of the SM  
 → no theory predictions!  
 ⇒ precise determination of parameters very important
- **Goal of flavour physics:** overconstrain unitarity triangle by complementary measurements & search for New Physics

## 2.Part: How to measure CKM matrix elements in general?

# How to measure CKM matrix elements in general?

- as shown before CKM matrix elements included in charged current weak interaction Lagrangian
- magnitude can be measured from rates of respective flavour changing transitions



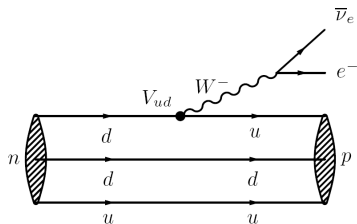
→ emission of W boson

- rate of that is proportional to the coupling strength  $|V_{ij}|^2$ :

$$\frac{d\Gamma}{dq^2} \propto G_F^2 |V_{ij}|^2 |f^+(q^2)|$$

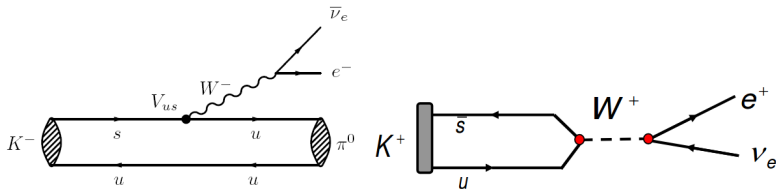
→ needs as input non-perturbative Form factor ( $f^+(q^2)$ ) calculations from LQCD or QCD sum rules





Experimental measurements:

- measured from nuclear  $\beta$  decay:  $n \rightarrow pe^- \bar{\nu}_e$   
 $|V_{ud}| = 0.97417 \pm 0.00021$
- error dominated by theoretical uncertainties due to binding energy corrections in nuclei
- also from neutron lifetime
- measure  $\pi^+ \rightarrow \pi^0 e^+ \nu_e$  branching ratio (PIBETA experiment)



- from **semileptonic Kaon decays** (many different experiments):

$K_L^0 \rightarrow \pi \mu \nu$ ,  $K^\pm \rightarrow \pi^0 e^\pm \nu$ ,  $K^\pm \rightarrow \pi^0 \mu^\pm \nu$  and  $K_S^0 \rightarrow \pi e \nu$  can extract  $|V_{us}| f_+(0)$   
 $\rightarrow$  need to know form factor (FF)  $f_+(0)$  from Lattice calculations (LQCD)

- also from **fully leptonic decays** (KLOE experiment):

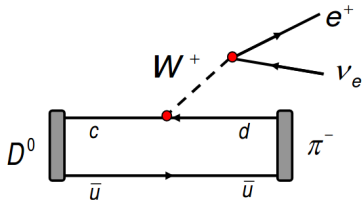
$|V_{us}/V_{ud}|$  from  $K \rightarrow \mu \nu(\gamma)$  and  $\pi \rightarrow \mu \nu(\gamma)$  together with ratio of decay constants  $f_K/f_\pi$  from LQCD

average of both gives:

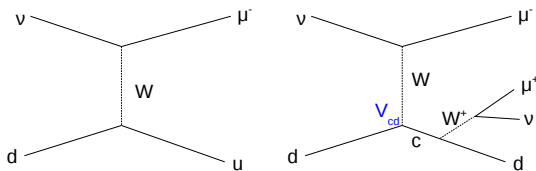
$$|V_{us}| = 0.2248 \pm 0.0006$$

- also from Hyperon decays and hadronic tau decays  $\tau \rightarrow h \nu$  ( $h = \pi, K$ )

- from **semileptonic charm decays** (BaBar, Belle, BESIII, CLEO-c):  
 $D \rightarrow \pi l \nu$ ,  $D \rightarrow K l \nu$  together with FFs  $f_+^{D\pi}(0)$  from LQCD
- from **leptonic charm decays** (BESIII, CLEO):  
 $D^+ \rightarrow \mu^+ \nu$  and decay constant  $f_D$  from LQCD

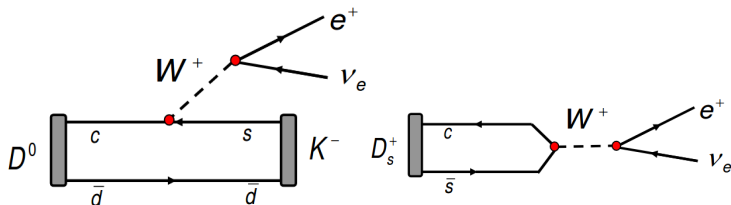


- from **neutrino scattering data** (CDHS, CCFR, CHARM II):  
The difference of the ratio of double-muon to single-muon production by neutrino and antineutrino beams is proportional to the charm cross section off valence d quarks, and therefore to  $|V_{cd}|^2$



gives total average of all 3 methods:

$$|V_{cd}| = 0.220 \pm 0.005$$



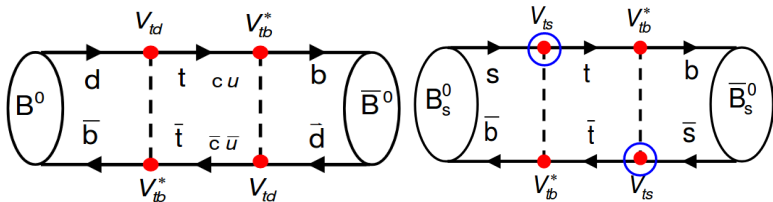
- from **semileptonic D decays** (Belle, BABAR, CLEO-c, BESIII):  
 $D \rightarrow K l \nu$  together with FFs  $f_+^{DK}(0)$  from LQCD
- from **leptonic  $D_s$  decays** (Belle, BABAR, CLEO-c):  
 $D_s \rightarrow \mu \nu, \tau \nu$  using the PDG values for mass and lifetime of the  $D_s$ , the masses of the leptons and  $f_{D_s}$  from LQCD

averaging both gives:

$$|V_{cs}| = 0.995 \pm 0.016$$

- also from tagged on-shell  $W^+ \rightarrow c \bar{s}$  decays at LEP II

- in general measurable in tree level top decays, but not precise enough
- can be measured via virtual effects:
  - 1 from  $B - \bar{B}$ ,  $B_s - \bar{B}_s$  oscillations mediated by box diagrams with top quarks and LQCD input for  $f_{B_d}$  and  $f_{B_s}$



$$\Delta m_d = (0.5064 \pm 0.0019) ps^{-1} \sim (V_{td}^* V_{tb})^2, \quad \Delta m_s = (17.757 \pm 0.021) ps^{-1} \sim (V_{ts}^* V_{tb})^2$$

$$|V_{td}| = (8.2 \pm 0.6) \times 10^{-3}, \quad |V_{ts}| = (40.0 \pm 2.7) \times 10^{-3}$$

can be measured via virtual effects:

- 1 from  $B - \bar{B}$  oscillations mediated by box diagrams with top quarks
  - 2 loop-mediated rare K and B decays
    - $B \rightarrow X_s \gamma$  penguin process  $\sim |V_{ts}^* V_{tb}|^2$
    - $B_s \rightarrow \mu\mu$  box diagram
- theoretical uncertainties in hadronic effects limit accuracy of current determinations, reduced when measuring  $|V_{td}/V_{ts}|$  instead:

$$|V_{td}/V_{ts}| = 0.215 \pm 0.001 \pm 0.011$$

- theoretically clean determination of  $|V_{ts}^* V_{td}|$  from  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , so far only 7 events observed from E949 & E787 Collaboration (*arXiv:0903.0030*), NA62 expected to improve that

- top decays at hadron colliders CDF, D0, ATLAS, CMS:

$$R = \mathcal{B}(t \rightarrow Wb) / \mathcal{B}(t \rightarrow Wq) = |V_{tb}|^2 / (\sum_q |V_{tq}|^2) \text{ with } q=b,s,d$$
$$\Rightarrow R = |V_{tb}|^2 \text{ assuming unitarity}$$

- direct determination from single top-quark production cross section through  $Wtb$  vertex

combination of both:

$$|V_{tb}| = 1.009 \pm 0.031$$

dominated by experimental systematic uncertainties



# Putting it all together

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
$$= \begin{pmatrix} 0.97417 \pm 0.00021 & 0.2248 \pm 0.0006 & V_{ub} \\ 0.220 \pm 0.005 & 0.995 \pm 0.016 & V_{cb} \\ 0.0082 \pm 0.0006 & 0.04 \pm 0.0027 & 1.009 \pm 0.031 \end{pmatrix}$$

clear hierarchy visible as described in Wolfenstein parametrization:

$$|V_{CKM}| \sim \begin{pmatrix} 1 & \lambda & V_{ub} \\ \lambda & 1 & V_{cb} \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

with  $\lambda \sim 0.22$

# Take Home Message

- CKM matrix encodes the flavour structure of charged-current interaction
- weak interaction allows for transition between quark generations
- CKM matrix is unitary, complex 3x3 matrix  
→ 4 free parameters: 3 rotation angles & 1 CP violating phase
- CKM matrix elements are fundamental parameters of the SM → need to be extracted from several different measurements
- from unitarity condition get the unitarity triangle
- **Goal of Flavour Physics** to over constrain this by complementary measurements

→ next week discussion on  $V_{cb}$  and  $V_{ub}$

- Physics at the Terrascale, Ian Brock, Thomas Schörner-Sadenius, 2011, Chapter Quark Flavour Physics
- Review of Particle Physics, C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update, <http://pdg.lbl.gov/2017/reviews/rpp2017-rev-ckm-matrix.pdf>, <http://pdg.lbl.gov/2017/reviews/rpp2017-rev-vud-vus.pdf>
- Quark and Lepton Flavor Physics Lectures by Ulrich Uwer <https://www.physi.uni-heidelberg.de/uwer/lectures/Flavor/notes.html>
- Standard Model Lecture by Ulrich Uwer <https://www.physi.uni-heidelberg.de/uwer/lectures/StandardModel/notes.html>
- The Standard Model of Particle Physics Lectures by Andre Schöning and [https://www.mpi-hd.mpg.de/manitop/StandardModel/lectures/Lecture\\_SM\\_22.pdf](https://www.mpi-hd.mpg.de/manitop/StandardModel/lectures/Lecture_SM_22.pdf)
- Lectures on CP violation, Particle Physics II January 2015, P. Kooijman & N. Tuning, <https://www.nikhef.nl/h71/Lectures/2015/ppII-cpviolation-29012015.pdf>

**Thanks for your attention!**

# Backup Slides

$$|V_{ud}|$$

superallowed ( $\Delta I = 0$ ) nuclear, neutron, and pion decays

→ most precise determination from **superallowed nuclear beta-decays** ( $0^+ \rightarrow 0^+$  transitions):

- with half-lives,  $t$ , and  $Q$  values (total energy released in a given nuclear decay) that give the decay rate factor,  $f$ :

$$ft = \frac{K}{2G_V^2(1 + \Delta)}, \quad |V_{ud}| = G_V/G_F, \quad \Rightarrow |V_{ud}|^2 = \frac{2984.48(5)\text{sec}}{ft(1 + \Delta)}$$

- $\Delta$  denotes the electroweak radiative corrections (RC), nuclear structure, and isospin violating nuclear effects → nucleus dependent
- most recent analysis of 14 precisely measured superallowed transitions gives a weighted average:

$$|V_{ud}| = 0.97420(10)_{\text{exp.,nucl.}}(18)_{\text{RC}}(\text{superallowed})$$

$$|V_{ud}|$$

## Neutron lifetime:

- together the ratio of axial-vector/vector couplings,  $g_A \equiv G_A/G_V$  via neutron decay asymmetries gives:

$$|V_{ud}|^2 = \frac{4908.7(1.9)\text{sec}}{\tau_n(1 + 3g_A^2)}$$

- error from uncertainties in the electroweak radiative corrections due to hadronic loop effects
- Using world averages for  $\tau_n^{ave} = 879.3(9)\text{sec}$  and  $g_A^{ave} = 1.2723(23)$  gives:

$$|V_{ud}| = 0.9763(5)_{\tau_n}(15)_{g_A}(2)_{RC}$$

- error dominated by  $g_A$  uncertainties
- value of  $|V_{ud}|$  is high by 1.3 sigma wrt. superallowed nuclear beta-decay

$$|V_{ud}|$$

## pion decay:

- PIBETA experiment at PSI measured the very small ( $\mathcal{O}(10^{-8})$ ) branching ratio for  $\pi^+ \rightarrow \pi^0 e^+ \nu_e$  with about  $\pm 0.5\%$  precision, gives:

$$|V_{ud}| = 0.9749(26) \left[ \frac{BR(\pi^+ \rightarrow e^+ \nu_e(\gamma))}{1.2352 \times 10^{-4}} \right]^{1/2}$$

- Theoretical uncertainties in the pion  $\beta$ -decay determination are very small
- much higher statistics required to make this approach competitive with others



$$|V_{US}|$$

from **kaon decays**, hyperon decays, and tau decays

$$\Gamma_{Kl3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1 + \delta_K^l + \delta_{SU2}) C^2 |V_{us}|^2 f_+^2(0) I_K^l$$

$M_K$  kaon mass,  $S_{EW}$  short-distance radiative correction,  $\delta_K^l$  mode-dependent long-distance radiative correction,  $f_+^2(0)$  calculated form factor at zero momentum transfer for the  $l\nu$  system,  $I_K^l$  phase-space integral,  $\delta_{SU2}$  for charged kaon decays deviation from one of the ratio of  $f_+(0)$  for the charged to neutral kaon decay,  $C^2$  is 1 (1/2) for neutral (charged) kaon decays

- Many measurements during last decade have resulted in a significant shift in  $|V_{us}|$  due to difference in  $K \rightarrow \pi e^+ \nu$  branching fraction as a result of inadequate treatment of radiation in older experiments

- experiments: BNL E865, KTeV, KLOE, NA48, ISTRA+
- new measurements of branching fractions, of lifetimes and form factors resulted in improved precision for all of the experimental inputs, gives:

$$f_+(0)|V_{us}| = 0.2165(4)$$

- using recent FLAG averages:

$$\begin{aligned} |V_{us}| &= 0.2238(4)_{exp+RC}(6)_{lattice} \quad (N_f = 2 + 1, K_{l3} \text{ decays}) \\ &= 0.2231(4)_{exp+RC}(7)_{lattice} \quad (N_f = 2 + 1 + 1, K_{l3} \text{ decays}) \end{aligned}$$

$$|V_{US}|$$

from a comparison of the radiative inclusive decay rates for  $K \rightarrow \mu\nu(\gamma)$  and  $\pi \rightarrow \mu\nu(\gamma)$  together with lattice calculation of  $f_{K^+}/f_{\pi^+}$  gives:

$$\frac{|V_{US}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = 0.23871(20) \left[ \frac{\Gamma(K \rightarrow \mu\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} \right]^{1/2}$$

small error coming from electroweak radiative corrections and isospin breaking effects, using experimental measurements for  $\Gamma$  together with FLAG averages gives:

$$\begin{aligned} |V_{US}| &= 0.2256(10) \quad (N_f = 2 + 1, K_{\mu 2} \text{ decays}) \\ &= 0.2253(7) \quad (N_f = 2 + 1 + 1, K_{\mu 2} \text{ decays}) \end{aligned}$$

give similar results as from  $Kl3$  decays

$$|V_{us}|$$

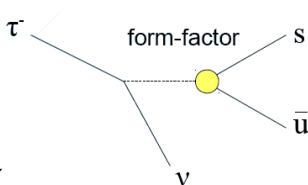
from **hyperon beta decays**:

$$\Lambda \rightarrow pe^{-}\bar{\nu}, \Sigma^{-} \rightarrow ne^{-}\bar{\nu}, \Xi^{-} \rightarrow \Lambda e^{-}\bar{\nu}, \Xi^{0} \rightarrow \Sigma^{+}e^{-}\bar{\nu}$$

gives:

$$|V_{us}| = 0.2250(27)$$

which neglects SU(3) breaking effects



from **tau decays**:  $\tau \rightarrow \nu KX$

averaging both inclusive and exclusive measurements, gives:

$$|V_{us}| = 0.2202(15)$$

differs by about 3 sigma from the kaon determination, mainly from inclusive determination

# CKM Unitarity Constraints

Unitarity Requirement should be fulfilled:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

using PDG averages of  $|V_{us}|$  (using only kaon decays )

$$\begin{aligned} |V_{us}| &= 0.2244(6) \quad (N_f = 2 + 1) \\ &= 0.2243(5) \quad (N_f = 2 + 1 + 1) \end{aligned}$$

and  $|V_{ud}|$  (using only superallowed nuclear beta decays )

$$|V_{ud}| = 0.97420(10)_{exp.,nucl.}(18)_{RC}$$

( $|V_{ub}|^2 \simeq 1.7 \times 10^{-5}$  is negligibly small, ignored here) gives:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(5)$$

$\Rightarrow$  strong confirmation of SM, better than  $50 \sigma$ , implies constraints on NP

Measurements of on-shell  $W^\pm$  decays sensitive to  $|V_{CS}|$  were made by LEP-2

- W branching ratio to each lepton flavour is:

$$1/\mathcal{B}(W \rightarrow l\bar{\nu}_l) = 3 \left[ 1 + \sum_{u,c,d,s,b} |V_{ij}|^2 (1 + \alpha_s(m_W)/\pi) + \dots \right]$$

- Assuming lepton universality, the measurement  $\mathcal{B}(W \rightarrow l\bar{\nu}_l) = (10.83 \pm 0.07 \pm 0.07)\%$  implies  $\sum_{u,c,d,s,b} |V_{ij}|^2 = 2.002 \pm 0.027$
- only flavour-tagged W-decays determine  $|V_{CS}|$  directly, DELPHI's tagged  $W^+ \rightarrow c\bar{s}$  analysis, yielding

$$|V_{CS}| = 0.94_{-0.26}^{+0.32} \pm 0.13$$