

2TG - etc - New, Dec 2017 Dominik Witze

1) Charming Way to probe

the Standard Model 1)

Introduction Basic concepts

I 1. theoretical prediction and experimental discovery

I 2. Mass generation and flavour transitions in the SM

Charm physics phenomenology

II 1. charm decays

II Neutral meson mixing

CP violation in charm

Introduction

Prelude

- Nowadays: the theory that describes fundamental particles and their interactions is the Standard Model.

Matter Content: 3 generations of fundamental particles with same charges under the SU gauge group, but very different masses

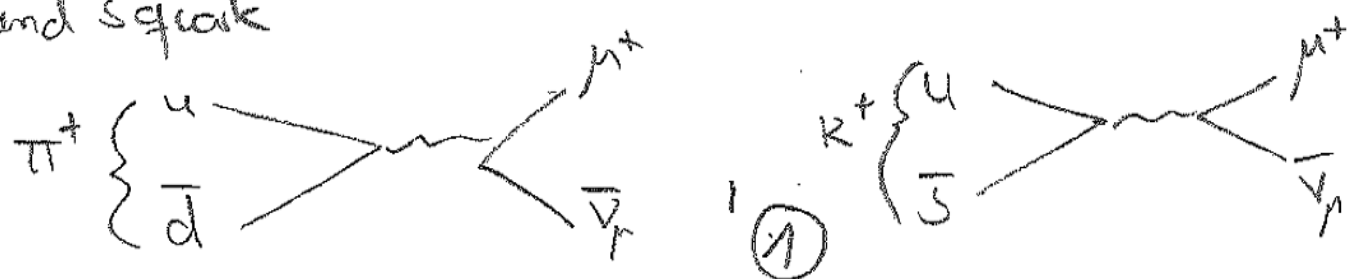
	I	II	III	Q [e]	
Quarks	u	c	t	+2/3	+ Antiparticles
	d	s	b	-1/3	
Leptons	ν_e	ν_μ	ν_τ	0	
	e	μ	τ	-1	

Flavour physics: Study difference between the generations such as masses, flavour transitions and CP-violation. Charm physics is flavour physics involving production and decay of hadrons containing c-quarks:

Examples = $J/\psi = |c\bar{c}\rangle$ 'charmonium', 'hidden charm'
 $D^0 = |c\bar{u}\rangle$, $D^+ = |c\bar{d}\rangle$, $D_s^+ = |c, \bar{s}\rangle$ 'open charm'
 $\Lambda_c = |cud\rangle$ 'charmed baryons'

I.1. Charm prediction and experimental discovery

- Let's move back to 1960. Known (quark) matter content: u, d, s
 observation of pion and kaon decays showed that the u-quark couples to both the d and s quark



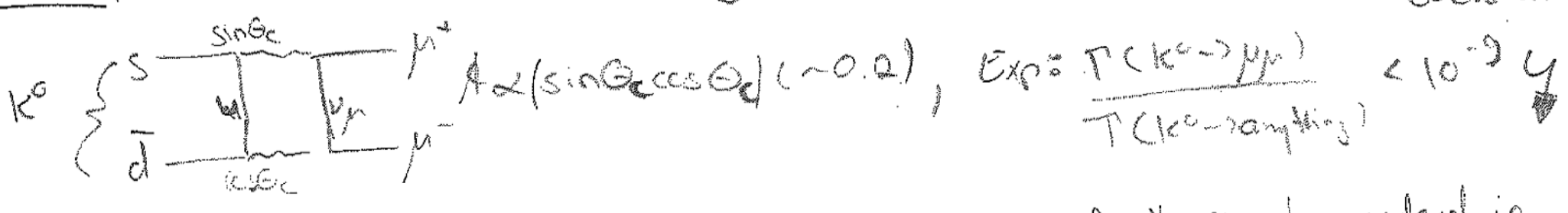
measured ratio: $\frac{\Gamma(K^+ \rightarrow \mu^+ \nu)}{\Gamma(K^+ \rightarrow \pi^+ \nu)} \sim 1.3$ contradictory to 'naive' theoretical prediction of ~ 18 (assuming universal coupling of the weak interaction)

Glashow, Iliopoulos, Maiani 1970: weak eigenstates are not pure flavour states: Introduce the doublet

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d \cos \theta_c + s \sin \theta_c \end{pmatrix}, \text{ with } \theta_c \text{ Cabibbo angle. } \begin{matrix} u \\ \swarrow \\ d \end{matrix} \propto \cos \theta_c, \begin{matrix} u \\ \swarrow \\ \bar{s} \end{matrix} \propto \sin \theta_c$$

$$\frac{\Gamma(K \rightarrow \mu \nu)}{\Gamma(K \rightarrow \pi \nu)} \propto \tan^2 \theta_c \Rightarrow \theta_c \sim 13^\circ. \text{ Cabibbo's idea solved that problem.}$$

however, this would lead to the following transitions, again contradictory to experimental observations



Glashow, Iliopoulos, Maiani 1970: predict existence of a new fourth quark, ordered in 2 generations:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \text{ with } \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos \theta_c |d\rangle + \sin \theta_c |s\rangle \\ -\sin \theta_c |d\rangle + \cos \theta_c |s\rangle \end{pmatrix}$$

this leads to a second process with negative sign, explaining the large suppression:

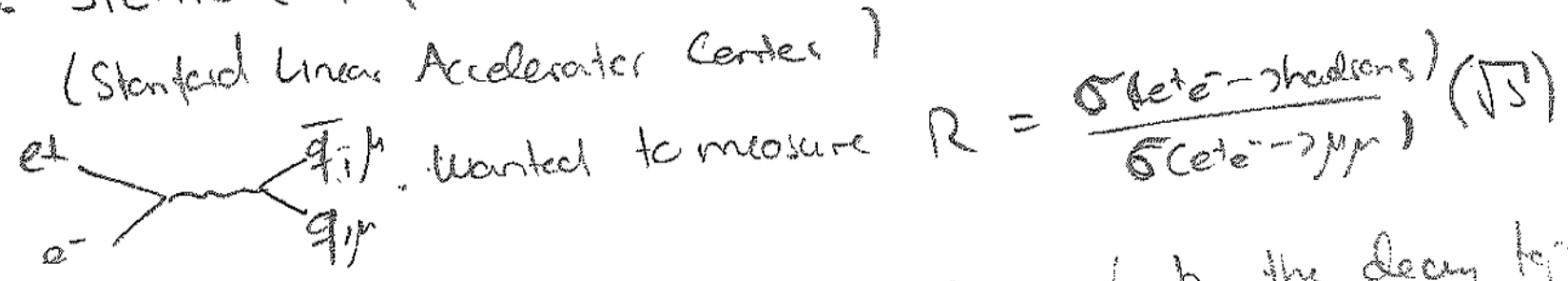
$$\begin{matrix} s \\ \swarrow \\ u \end{matrix} \propto \sin \theta_c \cos \theta_c + \begin{matrix} s \\ \swarrow \\ c \end{matrix} \propto -\sin \theta_c \cos \theta_c = 0 \text{ [if } m_c = m_u \text{]}$$

this is usually called the "GIM mechanism". The charm prediction and completion of the 2nd family was an important step towards nowadays SM.

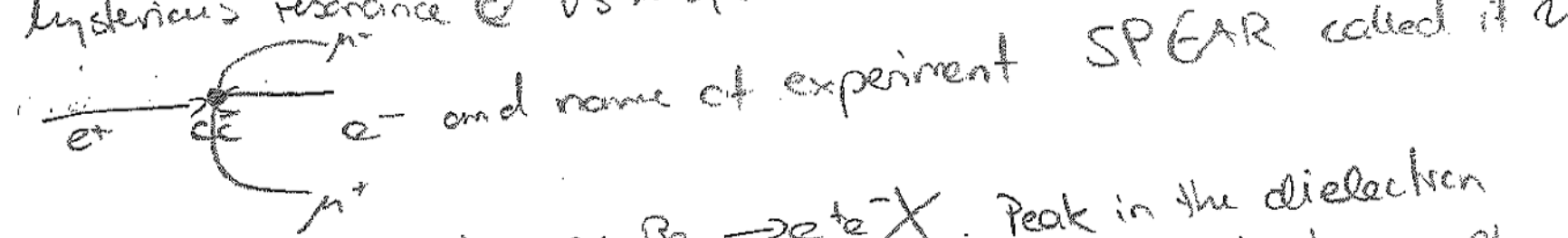
experimental discovery

1974 two experiments reported observation of a new long lived particle (independent)

Dichter et al: SPEAR (Stanford Positron Electron Asymmetric Ring) at SLAC (Stanford Linear Accelerator Center)



mysterious resonance @ $\sqrt{s} \approx 3.1 \text{ GeV}$. Inspired by the decay topology



et al = Brookhaven-Experiment: $p + Be \rightarrow e^+ e^- X$. Peak in the dielectron spectrum at $m_{ee} = 3.1 \text{ GeV}$. Family name Ting ~ Symbol 'J' looks like a "J" Nobel price in 1976, combined name J/psi ($c\bar{c}$ -resonance)

why: Why was the new resonance interpreted as $c\bar{c}$ state? The high mass and long lifetime could not be explained by the existence of 3 quarks (u, d, s). The final confirmation of the charm discovery was given by the measurement of 'open charm' (D^0, D^+) also in 1976. Today's picture was completed by the discovery of the b (1977) and top quark (1995). Theoretically, the third generation was already predicted in 1973 to be able to explain CP violation observed in the kaon system - Let's move back to 2017

2. Mass generation and flavour transitions in the SM

the SM, each generation consists of 3 (quark) fermion multiplets. (we ignore lepton sector in the following)

$$\mathcal{Q}_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

Left handed $SU(2)$ doublets

$$\mathcal{R}_i = (u_R, c_R, t_R)$$

right handed $SU(2)$ singlets

$$\mathcal{L}_i = (d_R, s_R, b_R)$$

Let's look at the Yukawa couplings that give mass to the fermions, as ordinary terms such as $\sim m \bar{\psi} \psi$ are not respecting the gauge symmetry.

$$\mathcal{L}_{Yukawa} = Y_{ij}^d \bar{\mathcal{Q}}_{Li} \phi d_{Rj} + Y_{ij}^u \bar{\mathcal{Q}}_{Li} \hat{\phi} u_{Rj} + h.c.$$

\uparrow Yukawa matrices complex, non-diagonal
 \uparrow Higgs
 $\hat{\phi} = i\sigma_2 \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ after symmetry breaking

$$= \frac{v}{\sqrt{2}} \left(\bar{d}_{Li} Y_{ij}^d d_{Rj} + \bar{u}_{Li} Y_{ij}^u u_{Rj} \right) + h.c.$$

$$= \frac{v}{\sqrt{2}} \left(\bar{d}_{Li} V_{Ld}^\dagger V_{Ld} Y_{ij}^d V_{Rd}^\dagger V_{Rd} d_{Rj} + \bar{u}_{Li} V_{Lu}^\dagger V_{Lu} Y_{ij}^u V_{Ru}^\dagger V_{Ru} u_{Rj} \right) + h.c.$$

$$= \bar{d}_{Li} (M_d)_{ij} d_{Rj} + \bar{u}_{Li} (M_u)_{ij} u_{Rj} + h.c.$$

what happened? We inserted a "V", but the implications are important:

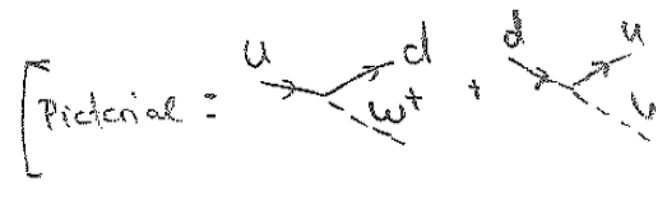
$$\text{chose } V_{Lq} \text{ such that: } M_d = \frac{v}{\sqrt{2}} V_{Ld}^\dagger Y^d V_{Rd}^\dagger = \text{diag}(m_d, m_s, m_b)$$

$$M_u = \frac{v}{\sqrt{2}} V_{Lu}^\dagger Y^u V_{Ru}^\dagger = \text{diag}(m_u, m_c, m_t)$$

transform the fields $\hat{u}_L = V_{Ll} u_L$ $\hat{d}_L = V_{Lld} d_L$
 $\hat{u}_R = V_{Rl} u_R$ $\hat{d}_R = V_{Rd} d_R$

this also has crucial impact on the weak interaction:

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} (\bar{u}_{Li} \delta^{ij} W_{\mu}^{+} d_{Lj} + \bar{d}_{Li} \delta^{ij} W_{\mu}^{-} u_{Lj})$$



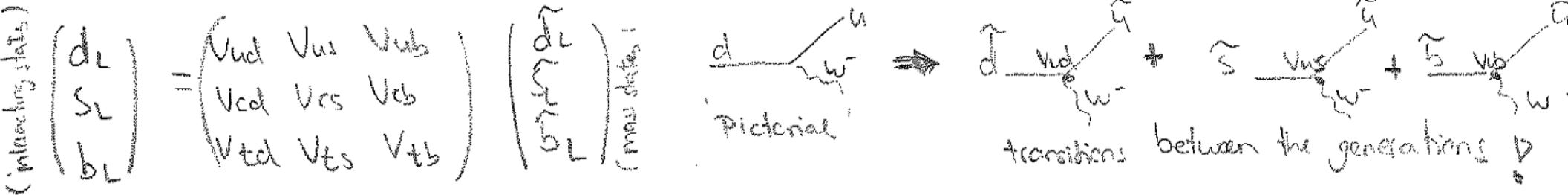
$$= -\frac{g}{\sqrt{2}} (\bar{u}_{Li} \delta^{ij} W_{\mu}^{+} \underbrace{(V_{Lij} V_{Lld}^{\dagger})}_{=V_{CKM}} \hat{d}_{Lj} + \bar{d}_{Li} \delta^{ij} W_{\mu}^{-} \underbrace{(V_{Lid} V_{Lij}^{\dagger})}_{=V_{CKM}^{\dagger}} \hat{u}_{Lj})$$

insert transformed fields

$$\begin{bmatrix} d_L = V_{Lld} \hat{d}_L \\ \bar{u}_L = \bar{\hat{u}}_L V_{Ll} \end{bmatrix}$$

Note: The neutral current $\mathcal{L}_A \sim \sum_{\mu} \bar{u} (C_V - C_A \gamma^5) \delta^{\mu} u$ is not affected (no FCNC @ tree level)

Now, let's have a closer look at $d_{Li} = (V_{CKM})_{ij} \hat{d}_{Lj}$: weak states are mixture of mass states!

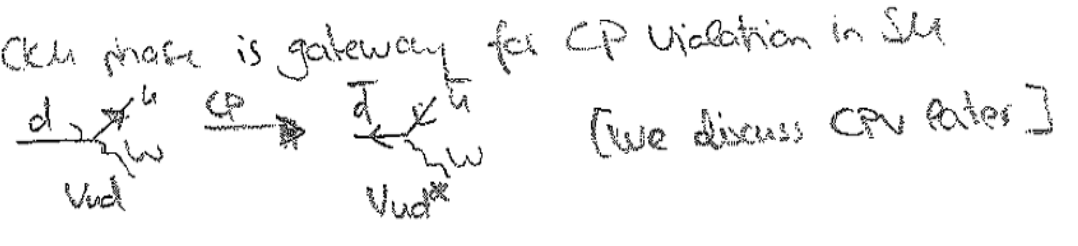


Properties of V_{CKM} : - unitary (as it is a product of unitary matrices)

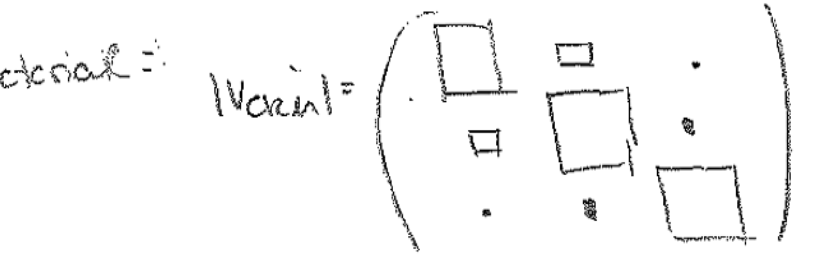
- 3 real parameters and 1 phase.
- in general: for N generations $\frac{N(N-1)}{2}$ real parameters and $\frac{N(N-1)(N-2)}{2}$ phases

Remark: starting from 18 (9 real + 9 phases)
 unitary: 9 - (3 real + 6 phases) = 5 phases are absent
 quark phases: 4 (3 real + 1 phase)

Very important: for $N < 3$ no phases left;



Hierarchy of CKM matrix



a priori, there is no fundamental reason for the observed magnitude of the CKM elements. The picture is purely based on measurements. But the hierarchy allows to use the Wolfenstein parametrization in $\lambda = |V_{us}| \sim 0.22$

$$V_{CKM} \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 (\beta - i\gamma) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3 (1 - \beta i\gamma) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$A, \beta, \gamma \in \mathcal{O}(1)$, for exact definition see for example PDG review.

Remark: unitarity gives rise to 9 equations such as $V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$
 \rightarrow 6 'triangles' in complex plane

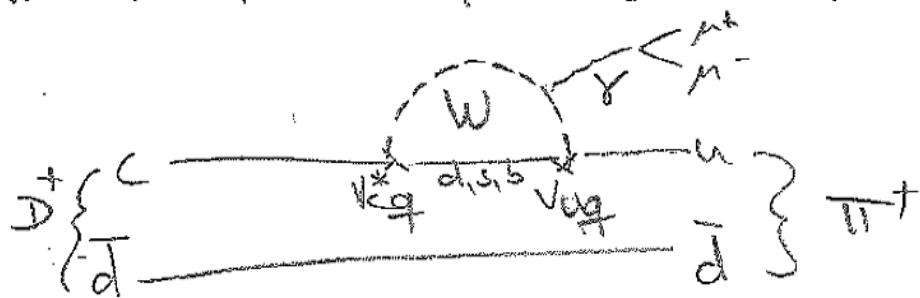
take home message: (1) The charm quark played a major role in the development of the SM
 (2) In the SM, mass generation and flavour transitions have a common origin. Transitions are governed by unitary 3×3 CKM matrix. (4)

Charm physics phenomenology

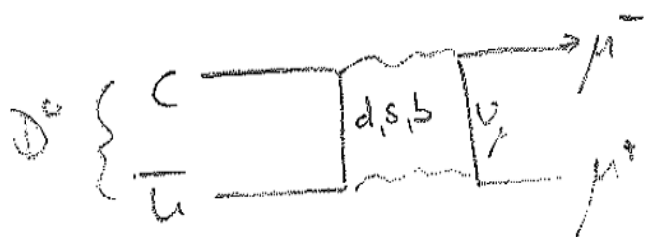
I.1 Rare charm decays

last week we saw that FCNC are not possible at tree level. However, higher order transitions are possible which allow to search for NP contributions beyond the mass scale accessible in direct search

examples:



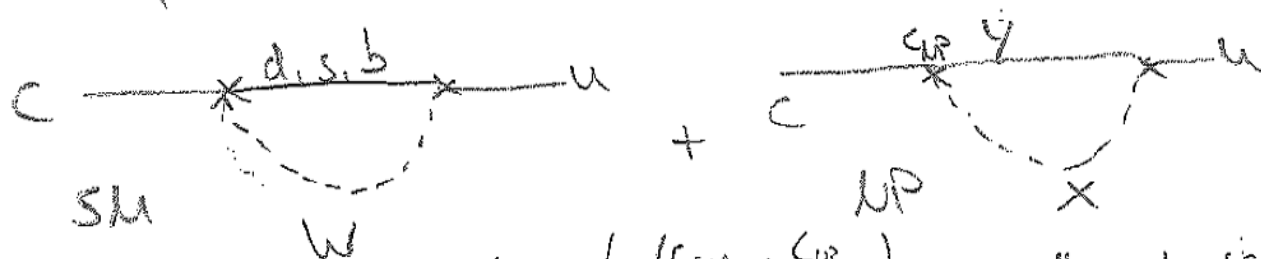
$D^+ \rightarrow \pi^+ \mu^+ \mu^-$ via "Penguin diagram"
Quark flavour changes in internal W-loop



$D^0 \rightarrow \mu^+ \mu^-$ via "box diagram"

they all have the FCN $c \rightarrow u$ transition in common. As the internal particles are produced 'virtually', potential very heavy and yet unobserved particles can enter the loop and change the rate. Even particles too heavy for direct production can contribute (indirect approach)

Schematic:



$$A_{tot} = A_0 \left(\frac{C_{SM}}{m_W^2} + \frac{C_{NP}}{\Lambda^2} \right)$$

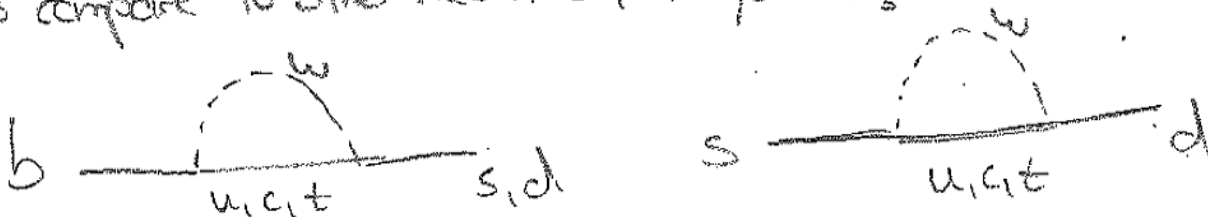
This allows to set limits on $\frac{C_{NP}}{\Lambda^2}$

SM contributions:

$$\begin{aligned} \mathcal{A} &\propto V_{cd}^* V_{ud} f\left(\frac{m_d^2}{m_W^2}\right) + V_{cs}^* V_{us} f\left(\frac{m_s^2}{m_W^2}\right) + V_{cb}^* V_{ub} f\left(\frac{m_b^2}{m_W^2}\right) \\ &\stackrel{\text{unitarity}}{=} V_{cs}^* V_{us} \left(f\left(\frac{m_s^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right) + V_{cb}^* V_{ub} \left(f\left(\frac{m_b^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right) \approx 10^{-8} \end{aligned}$$

~ 0 , as $m_s^2 \sim m_d^2$ 'GIM suppressed'
 $\sim \lambda^5 (0.010^{-4})$ 'CKM suppressed'
 $f\left(\frac{m_q^2}{m_W^2}\right) \sim \frac{1}{16\pi^2} \frac{m_q^2}{m_W^2}$

- Remark: Just a simple estimation. QCD effects enhance the rate and are hard to estimate
Theory: $BR(C \rightarrow u \mu^+ \mu^-) \leq 10^{-9}$ [1101.1053], $BR(D^+ \rightarrow \pi^+ \mu^+ \mu^-) \leq 3.7 \cdot 10^{-12}$ [1510.003117]
- How does this compare to other meson (b, s) systems?
 $BR(D^0 \rightarrow \mu^+ \mu^-) \leq 6 \cdot 10^{-11}$ [011235v2]



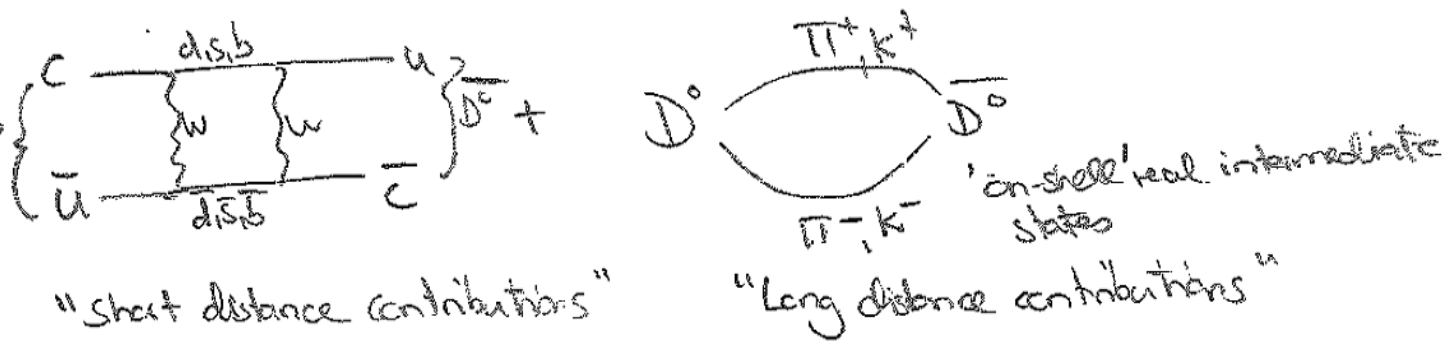
$BR(B \rightarrow X_s \ell \ell) = 7 \cdot 10^{-4}$ [1312.5364]
 $BR(B^+ \rightarrow \pi^+ \mu^+ \mu^-) = 2 \cdot 10^{-9}$ [1509.00414]
 $BR(B_s \rightarrow \mu^+ \mu^-) = 3 \cdot 10^{-9}$ [1703.05747]

processes are dominated by the t-quark: $f\left(\frac{m_t^2}{m_W^2}\right) \sim \mathcal{O}(1)$

- Charm system: only system with d-type quarks in the loop (important if NP acts differently)
Suppression higher ("effectively uncoupled from third generation") Extremely low SM rate that competes with NP processes.

7.2 Neutral charm mixing

$D^0 (c\bar{u})$ and $\bar{D}^0 (\bar{c}u)$ are accessible through common intermediate states:



the propagating mass states are therefore a superposition of the flavour states. Describe the system by the following 'effective' Schrödinger equation

$$i \frac{d}{dt} \begin{pmatrix} |D^0(t)\rangle \\ |\bar{D}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} \mu - i\Gamma \end{pmatrix} \begin{pmatrix} |D^0(t)\rangle \\ |\bar{D}^0(t)\rangle \end{pmatrix} \quad \mu, \Gamma \text{ are } 2 \times 2 \text{ matrices}$$

with eigenstates: $|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle, p^2 + q^2 = 1$

and time evolution $D_{1,2}(t) = e^{-im_{1,2}t} e^{\frac{1}{2}\Gamma_{1,2}t} |D_{1,2}(0)\rangle$ with different masses $m_{1,2}$ and widths $\Gamma_{1,2}$

usually one defines: $x = \frac{m_1 - m_2}{\Gamma}, y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma}, \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$

An initially ($t=0$) produced D^0 (\bar{D}^0) meson can oscillate in its antiparticle! (and vice versa)

$$P(D^0 \rightarrow \bar{D}^0 | t) = |\langle \bar{D}^0(t) | D^0(0) \rangle|^2 \frac{|q|^2}{|p|^2} \frac{e^{-\Gamma t}}{2} [\cosh(y\Gamma t) - \cos(x\Gamma t)]$$

$$P(\bar{D}^0 \rightarrow D^0 | t) = |\langle D^0(t) | \bar{D}^0(0) \rangle|^2 \frac{|p|^2}{|q|^2} \frac{e^{-\Gamma t}}{2} [\cosh(y\Gamma t) - \cos(x\Gamma t)]$$

(different, if $\frac{|q|^2}{|p|^2} \neq \frac{|p|^2}{|q|^2}$)

= mixing, if either x or y (or both) $\neq 0$!

SM predictions:

In general, predictions are very hard due to the non-perturbatively calculable long distance contributions. However, due to the small FCNC processes $|x, y| \leq 10^{-3}$.

How does this compare to b and K meson physics?

	x	y
K^0	0.477	1
B_d^0	0.78	0.0015*
B_s^0	26.1	0.06*
D^0	0.003	0.007

The observable quantities x and y really depend on the system (i.e. involved quarks and couplings)

Again, in charm the SM mixing rate (and ΔT) are really tiny, making charm mixing sensitive for NP contributions

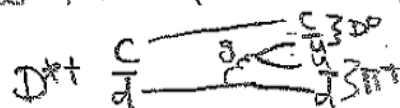
* theoretical values
† fun fact: K meson mixing dominated by charm quark in the loop

Experimental measurement:

to measure mixing, one needs to know the flavour at production and moment of decay

(1) at $t=0$: We select a flavour specific decay $D^{*+} \rightarrow D^0 \pi^+$ i.e. once we know $D^{*+} \rightarrow \bar{D}^0 \pi^+$

the charge of the π^{+-} , we know the flavour of the D^0 (strong decay)

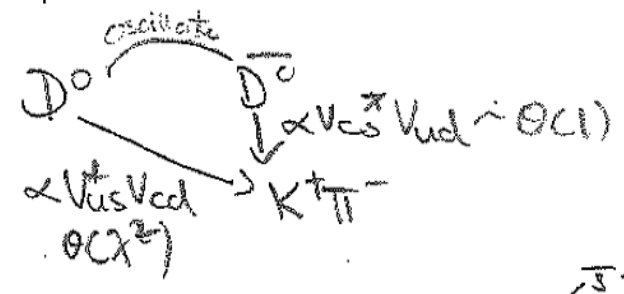


flavour conserved!

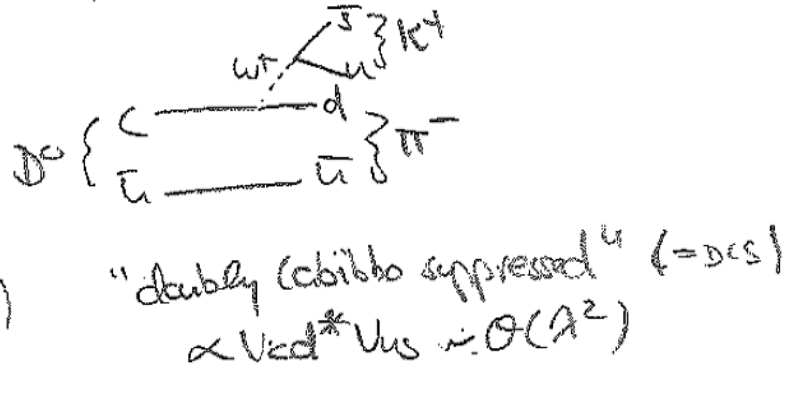
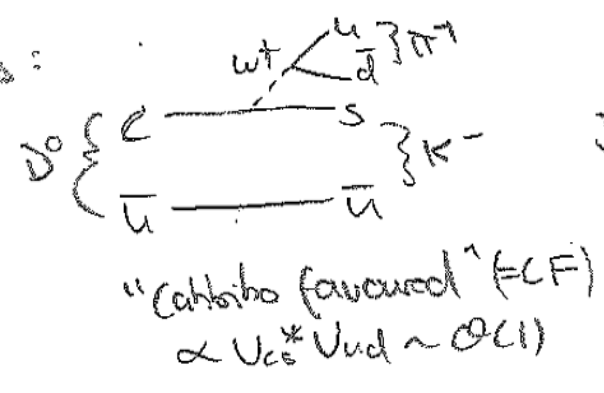
⑥

at the moment of decay: It would be very convenient to take also a flavour specific decay such as $D^0 \rightarrow K^- \mu^+ \nu$. This is experimentally very challenging

• Easier: Take $D^0 \rightarrow K^- \pi^+$, $\bar{D}^0 \rightarrow K^+ \pi^-$. However, the \bar{D}^0 can also directly decay to $K^+ \pi^-$ (very suppressed)



Feynman graphs:



• Measuring the rate. $D^0 \rightarrow K^+ \pi^-$ ("wrong sign decay") includes both mixing and direct (very) suppressed decay. Comparing with the rate $D^0 \rightarrow K^- \pi^+$ ("right side") gives:

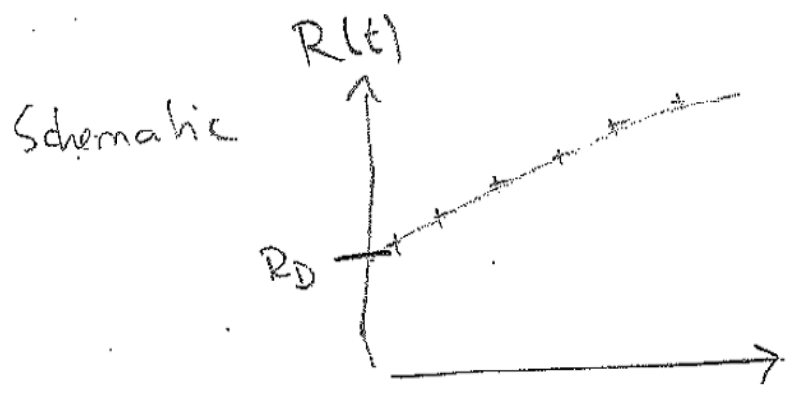
$$R(t) = \frac{N_{ws}}{N_{rs}}(t) = R_D + \sqrt{R_D} \cdot y' \cdot \sqrt{t} + \frac{x'^2 + y'^2}{4} (t)^2$$

\uparrow Ratio Rates $\propto t$ $\propto t^2$
 DCS / CF decay

$$x' = x \sin \delta + y \cos \delta$$

$$y' = -x \cos \delta + y \sin \delta$$

δ = "strong phase difference DCS and CF decay paths"



- non trivial dependence on decay time indicates mixing!
- The US $D \rightarrow K \pi$ mixing analysis was the first one showing 5 σ significance from a single experiment (arxiv 1211.1230, nowadays more measurements exist. (2012))

III.1 CP Violation in charm



• What is it and why do we care at all?

CP transformation is the combined charge (C) and parity (P) transformation of a system. CP is violated if the system is not invariant under the transformation

The main reason to look for CPV: As pointed out by Sakharov, CP violation is a necessary condition for the observed cosmological baryon asymmetry. The CPV in the SM is not sufficient to explain the observed asymmetry

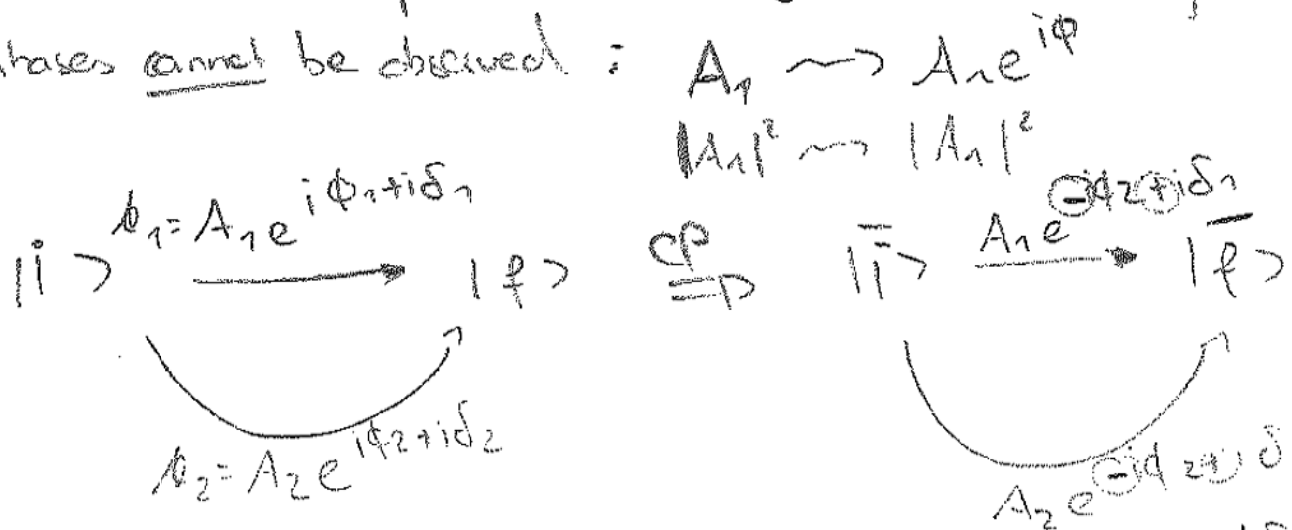
More practical: In the SM, the complex phase of the CKM matrix is the only source of CPV in the SM. Any SM extension (enlarging the particle content) or new interactions with

more couplings "comes with new sources of CPV. This makes CPV very attractive place to look for NP.

the SM:  \xrightarrow{CP}  , phases of the CKM matrix elements change sign

How to measure CPV?

As all observable quantities are in general obtained by "squaring" the amplitudes, overall phases cannot be observed: $A_1 \rightarrow A_1 e^{i\phi}$



ϕ changes sign under CP
 δ is 'strong phase' not changing sign. under CP.

total amplitude $A = A_1 + A_2 = C (1 + R e^{i\Delta\phi + i\Delta\delta})$; $R = \frac{A_2}{A_1}$
 $\bar{A} = \bar{A}_1 + \bar{A}_2 = C (1 + R e^{-i\Delta\phi + i\Delta\delta})$

define the asymmetry $A_{CP} = \frac{\Gamma(Ci \rightarrow f) - \Gamma(Ci \rightarrow \bar{f})}{\Gamma(Ci \rightarrow f) + \Gamma(Ci \rightarrow \bar{f})} = 2 \sin(\Delta\phi) \sin(\Delta\delta)$
 $\Gamma \propto |A|^2$
 suppressed if one amplitude dominates!

Important remark: If there's no strong phase difference, $A_{CP} = 0!$

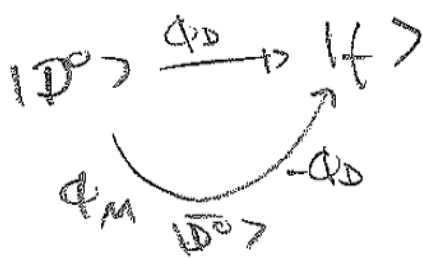
Why? $|A|^2 \sim \cos(\Delta\phi)$
 $|\bar{A}|^2 \sim \cos(-\Delta\phi)$
 $|A|^2 - |\bar{A}|^2 = 0$
 $[\cos(-\Delta\phi) = \cos(\Delta\phi)]$

Take home message: CPV is only possible if several amplitudes with different strong and weak phases contribute.

Classification of CPV

CPV in the decay: $\frac{|\bar{A}_f|}{|A_f|} \neq 1$ (i.e. $\Gamma(D^0 \rightarrow f) \neq \Gamma(D^{\bar{0}} \rightarrow \bar{f})$)

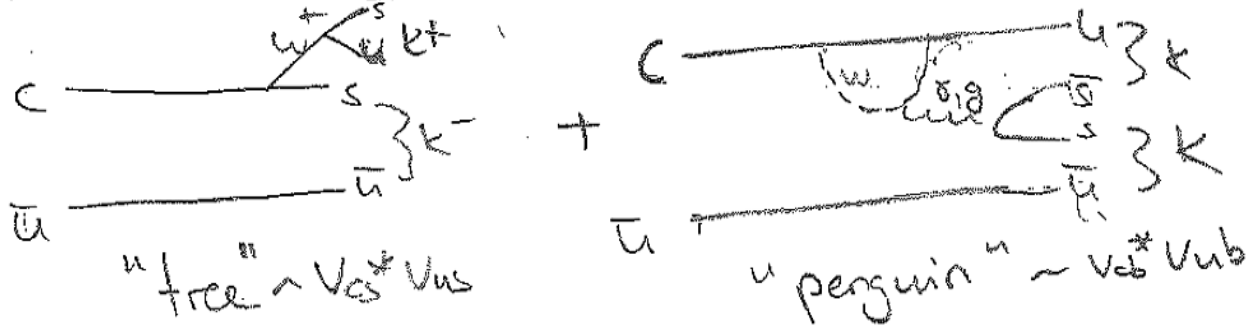
CPV in mixing: $|\frac{q}{p}| \neq 1$ (i.e. $\Gamma(D^0 \rightarrow D^0) \neq \Gamma(D^0 \rightarrow \bar{D}^0)$)

CPV in interference between mixing and decay (same final state for D^0 and \bar{D}^0): $\text{Im} \left(\frac{q}{p} \frac{\bar{A}_f}{A_f} \right) \neq 0$
 [i.e. $\Gamma(D^0 \rightarrow D^0 \rightarrow f) \neq \Gamma(D^0 \rightarrow \bar{D}^0 \rightarrow f)$]

SM predictions for charm decays

- As we need two competing amplitudes, usually one is very dominant (tree) over a higher order process (often penguin, box...): $A_{CP} \sim R \cdot \sin(\Delta\phi) \sin(\Delta\delta)$

Let's look at $D^0 \rightarrow \bar{K} K^+$



Remark: due to the " $\sin \frac{\pi}{4}$ ", only $D \rightarrow KK$ and $D \rightarrow \pi\pi$ can receive penguin contributions

- We already saw that the $c \rightarrow u$ transitions are tiny. In charm, $R \sim \frac{|V_{cb}^* V_{ub}|}{|V_{cs}^* V_{us}|} \sim \frac{\lambda^5}{\lambda} \approx \lambda^4$

- For indirect CP, the calculations go beyond the scope of this lecture.

However, it turns out that $A_{CP}^{\text{mixing}} \propto x$, $A_{CP}^{\text{interfere}} \propto y$. So, as $x, y \lesssim 10^{-3}$, also indirect CPV is expected to be below 10^{-3} . [These are all heuristic arguments, theoretical predictions are really hard]

- Can we do better? It is attractive to look at decays, where R is enhanced, i.e. where the dominant amplitude is also very suppressed, such as $D^0 \rightarrow K_S K_S$ (no tree level possible)

- Experimentally: Charm is the only neutral meson system, where CPV has not been measured! (Limits are indeed $\lesssim \mathcal{O}(10^{-3})$)

Take home message: Everything is small in charm!

References

- Lecture notes 'Quark and Lepton Flavour physics' SS 2015 by Ulrich Lauer
- Lecture notes 'Standard Model of Particle physics' SS 2013 by Werner Rodejans and Andre Schöningh
- Lecture notes 'CP violation' by P. Koopman & D. Tuning April 2011 (white)
- Talk 'Quark Flavour Physics' by Uli Lauer, Neuchâten UCLB workshop 2017
- Dissertation by Sascha Stöckel 2014: "Measurement of CP violation in muon tagged $D^0 \rightarrow \pi K$ and $D^0 \rightarrow \pi\pi$ decays at LHCb"
- Lecture notes 'Teilchenphysik II-III - Ableitung IX' WS 2001 by André Rübli (ETH Zürich)

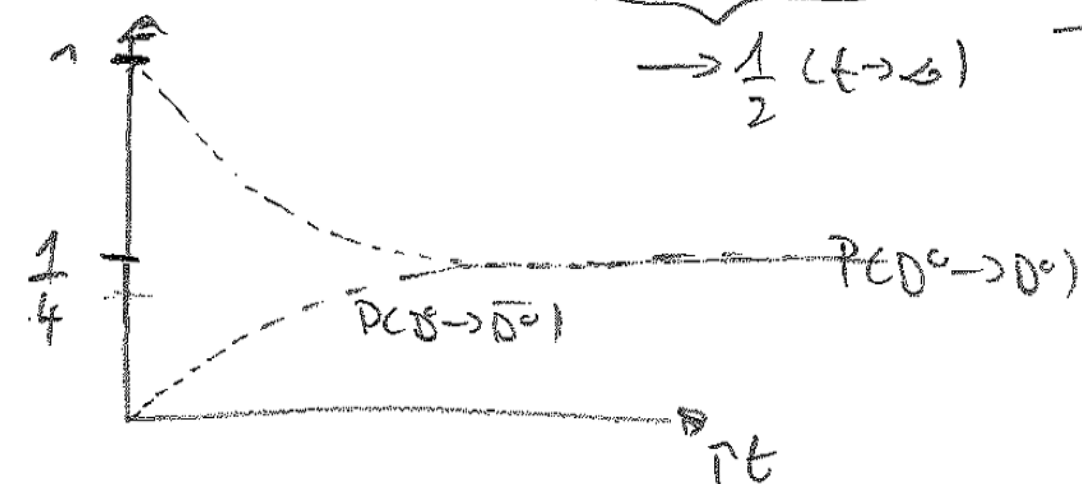
case 1.) $x=0$ $y=1$ (one is stable)

$$\left| \frac{q}{p} \right| = 1$$

$$P(D^0 \rightarrow D^0) = \frac{1}{2} e^{-\pi t} (\cosh(\pi t) - 1)$$

$$P(D^0 \rightarrow \bar{D}^0) = \frac{1}{2} e^{-\pi t} (\cosh(\pi t) + 1)$$

$$\rightarrow \frac{1}{2} (t \rightarrow \infty) \quad \rightarrow 0 (t \rightarrow \infty)$$

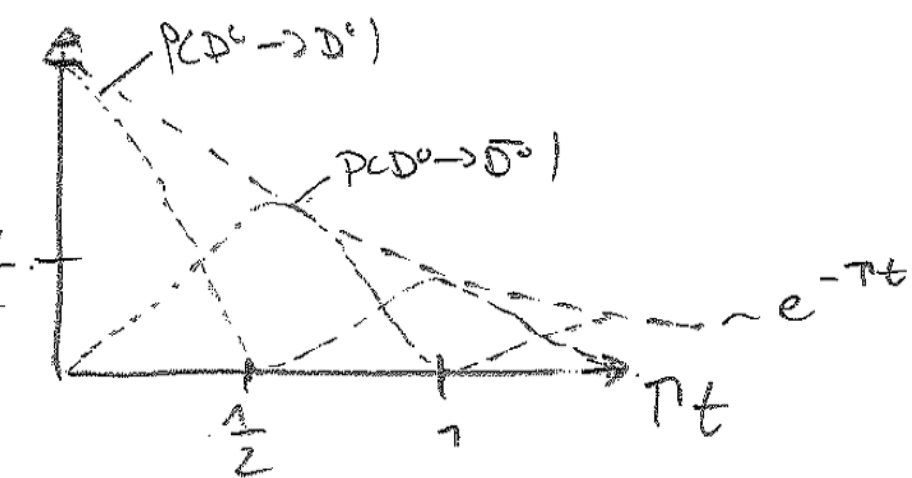


case 2.) $y=0$ $x=2\pi$ $\left| \frac{q}{p} \right| = 1$

$$P(D^0 \rightarrow D^0) = \frac{1}{2} e^{-\pi t} (1 + \cos(2\pi t))$$

$$P(D^0 \rightarrow \bar{D}^0) = \frac{1}{2} e^{-\pi t} (1 - \cos(2\pi t))$$

(periodic!)



luxury koen: $|U_{ts} U_{td}|^2 \sim \lambda^{10}$
 $|U_{cs} U_{cd}|^2 \sim \lambda^2$

[Extra]