Higgs Effective Field Theory

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11th – 25th January 2016

Abstract

This is a series of three short lectures for the Heidelberg RTG "Particle Physics beyond the Standard Model". After introducing the basic idea of effective field theories in general, they focus on the dimension-6 operators of linear Higgs effective field theory and their role at the LHC experiments.

Contents

1 Overview

Effective field theory (EFT) is a very general tool that plays a role in many, if not all, areas of physics. Whenever phenomena are spread out over different energy or length scales, an effective description can be valuable, either to simplify calculations, or to actually allow model-independent statements that would be impossible without such a framework.

This introduction is divided into two parts. In Sec. 2 we will go through the basic ideas behind effective field theories in rather general terms. We will mostly follow the classic example of Fermi theory throughout this section, but we will also explain the colour of the sky with EFT techniques. Most of the material in the first half is strongly influenced by Refs. [1, 2].

In Sec. 3 we will apply these general ideas to electroweak and Higgs physics at the TeV scale, currently probed by the ATLAS and CMS experiments. The effective model of interest is called **Higgs effective field theory** and has received a lot of attention lately. We will go through its building blocks, look at its phenomenology, and link it to a few example scenarios of physics beyond the Standard Model (SM). Finally we will discuss current constraints on dimension-6 operators and discuss if using this language makes sense at the LHC.

Throughout the lectures, we will focus on words and pictures rather than mathematical precision and technical details. This unfortunately means that some things will appear from nowhere, and some details will be entirely omitted. Several important examples of EFTs will never be mentioned, including χPT or dark matter EFTs. For a well-written, slightly more rigorous, and much more extensive introduction to EFTs, see for instance Refs. [1, 2].

2 Effective field theory essentials

2.1 Different physics at different scales

A hierarchy of scales

Our world behaves very differently depending on which energy and length scales we look at. At extremely high energies (or short distances), Nature might be described by a quantum theory of gravity. At energies of a few hundred GeV, the Standard Model of particle physics is (disappointingly) in agreement with everything. Going to lower energies (or larger distances), we do not have to worry about Higgs or W bosons anymore: there are electromagnetic interactions described by QED, weak interactions described by Fermi theory, and the strong physics of QCD. Below a GeV, quarks and gluons are replaced by pions and nucleons as the relevant degrees of freedom. Then by nuclei, atoms, molecules. At this point most physicists give up and let chemists (and ultimately biologists and sociologists) analyse the systems.

The important thing here is that the observables at one scale are not directly sensitive to the physics at significantly different scales. This is nothing new: for molecules to stick together, the details of the Higgs sector are not relevant; and we can calculate how an apple falls from a tree without knowing about quantum gravity. To do physics at one scale, we do not have to (and often cannot) take into account the physics from all other scales. Instead, we isolate only those features that play a role at the scale of interest.

An effective field theory is a physics model that includes all effects relevant at a given scale, but not those that only play a role at significantly different scales. In particular, EFTs ignore spatial substructures much smaller than the lengths of interest, or effects at much higher energies than the energy scale of interest.

We will mostly work on examples with one full or underlying theory and one effective theory. For simplicity, we pretend that the full theory describes physics correctly at all scales. The EFT is a simpler model than the full theory and neglects some phenomena (such as heavy particles) at energy scale Λ. However, it correctly describes the physics as long as the observables probe energy scales

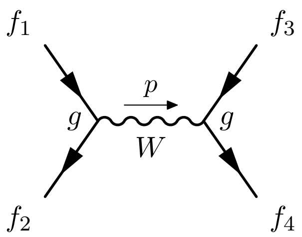
$$E \ll \Lambda \,, \tag{1}$$

within some finite precision. This **scale hierarchy** between the energy of interest and the scale of high-energy physics not included in the EFT is the basic requirement for the EFT idea. A validity range (1) is a fundamental property of each EFT.

Fermi theory

The textbook example for an EFT is Fermi theory. It describes the charged current interactions between quarks (or hadrons), leptons and neutrinos at low energies (we will write down the theory in detail in the next section). The underlying model here is the SM, in which this weak interaction is mediated by the exchange of virtual W bosons with mass m_W and coupling constant g:

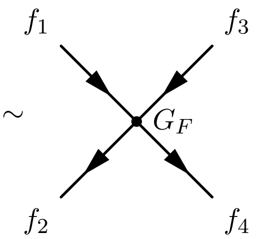
$$\mathcal{M}_{\text{full}} \sim$$



$$\sim \frac{g^2}{p^2 - m_W^2} \tag{2}$$

In the effective model, there are no W bosons, just a direct interaction between four fermions with coupling constant $G_F \propto g^2/m_W^2$:

$$\mathcal{M}_{\text{EFT}} \sim$$



$$\sim G_F \propto -\frac{g^2}{m_W^2} \tag{3}$$

The EFT turns the W propagator into a contact interaction between the fermions, shrinking the distance bridged by the virtual W to zero. Clearly, the two amplitudes agree as long as the momentum transfer through the vertex is small, $E^2 = p^2 \ll \Lambda^2 = m_W^2$:

$$\frac{g^2}{p^2 - m_W^2} = -\frac{g^2}{m_W^2}\left(1 + \frac{p^2}{m_W^2} + \mathcal{O}\left(p^4/m_W^4\right)\right) \approx -\frac{g^2}{m_W^2}\,. \tag{4}$$

One process described by this interaction is muon decay. Its typical energy scale $E \approx m_\mu$ is well separated from $\Lambda = m_W$. Fermi theory will describe the process quite accurately. The relative **EFT error**, i. e. the mistake we make when calculating an observable with the EFT rather than with the full model, should be of order

$$\Delta_{\text{EFT}} = \frac{\sigma_{\text{EFT}}}{\sigma_{\text{full}}} \sim E^2/\Lambda^2 \sim m_\mu^2/m_W^2 \approx 10^{-6}\,. \tag{5}$$

In proton collisions at the LHC the same interaction takes place, but at potentially much larger momentum transfer $E < 13$ TeV. The EFT error increases with E. For $E \gtrsim m_W$, the full model allows on-shell W production, a feature entirely missing in the EFT. Here the two descriptions diverge and Fermi theory is no longer a valid approximation of the weak interaction.

Down and up the theory ladder

In reality there are of course more than two theories, and the notion of underlying and effective model becomes relative. Take the example given above. On the one hand, we treated the Standard Model as the full theory. But the SM itself is not valid up to arbitrary large energies: it does not explain dark matter, the matter-antimatter asymmetry, inflation, or gravity. It is probably also internally inconsistent since at some very large energy the quartic coupling λ and the coupling constant g' seem to hit Landau poles, i. e. become infinite. So the SM is an effective theory with validity range $E \ll \Lambda \leq M_{Pl}$ and has to be replaced by some other description at larger energies. On the other hand, going to energies lower than a few GeV, the relevant physics changes again and we can (but do not have to) adapt a new effective theory. In this way, all theories can be thought of as a series of EFTs, where the model valid at one scale is the underlying model for the effective theory at the next lower scale.

If you think you know a theory that describes our world at sufficiently large energies, then in principle there is no need to use effective theories: you can calculate every single observable in your full model.[1] This will however make hard calculations necessary even for the simplest low-energy processes. One can save a lot of computational effort and focus on the relevant physics by dividing the phase space into regions with different appropriate effective descriptions.

Starting from a high energy scale where the parameters of the fundamental theory are defined, these parameters are run to lower energies until the physics changes substantially. At this **matching scale** an effective theory is constructed from the full model,

[1] This may not be true if the full model becomes non-perturbative at some energy, such as QCD at $E \sim \Lambda_{QCD}$. Then you will have to use an effective theory valid at $E \ll \Lambda_{QCD}$.

and its coefficients are determined from, or matched to, the underlying model. Then the coefficients of this EFT are run down to the next matching scale, where a new EFT is defined and its parameters are calculated, and so on. This is the **top-down** view of EFTs. For instance, we can start from the SM and construct Fermi theory as a simpler model valid at low energies. While we can certainly use the SM to calculate the muon lifetime, it is not necessary, and a calculation in Fermi theory is quite accurate and simpler.

But often we do not know the underlying theory. As mentioned above, there has to be physics beyond the SM, and there is still hope it will appear around a few TeV. If we want to parametrize the effects of such new physics on electroweak-scale observables, we do not know how the full model looks like. But even without knowing the underlying model, we can still construct an effective field theory based on a few very general assumptions. We will go through these ingredients in the next section. For this **bottom-up** approach, an effective theory is not only useful, but actually the only way we can discuss new physics without choosing a particular model of BSM physics.

High-energy physics can be seen as the field of working ourselves up a chain of EFTs to ever higher energies. But how does this chain end? Does it end at all? Even if we one day find a consistent theory that can explain all observations to date, how would we check if it indeed describes Nature up to arbitrarily high energies? Understanding all theories as effective, these questions do not matter! The EFT framework provides us with the tools to do physics without having to worry about the far ultraviolet.

2.2 Effective operators

Reminder: operators and power counting

EFTs are especially useful in the framework of quantum field theory (QFT). Before going into details, let us recapitulate how QFTs are organized. We will stick to local theories in $d = 4$ flat space-time dimensions.

The basic object describing such a QFT is the action

$$S = \int \mathrm{d}^4 x \, \mathcal{L}(x) \,. \tag{6}$$

The Lagrangian $\mathcal{L}(x)$ is a sum of couplings times operators, where the operators are combinations of fields and derivatives evaluated at one point x. These are either kinematic terms, mass terms or represent interactions between three or more fields. For instance, the Lagragian

$$\mathcal{L} = \mathrm{i}\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} - m_i \bar{\psi}_i \psi_i + m_V^2 V_\mu V^\mu - g \bar{\psi}_i \gamma_\mu \psi_i V^\mu \tag{7}$$

with implicit sum over i describes fermions ψ_i, a massive vector boson V_μ, and an interaction between them with coupling g.

A key property of each coupling or operator is its **mass dimension**. In simple terms this can be formulated as the following question: if you assign a value to a quantity, which power of a mass unit such as GeV would this value carry? Since we work in units with $\hbar = c = 1$, length and distance dimensions are just the inverse of mass dimensions. We will denote the mass dimension of any object with squared brackets, where $[\mathcal{O}] = D$ means that $\mathcal{O}$ is of dimension mass^D, or mass dimension D.

In QFT, the action can appear in exponentials such as $e^{\mathrm{i}S}$, so it must be dimensionless: $[S] = 0$. The space-time integral in Eq. (6) then implies $[\mathcal{L}] = d = 4$, so every term in the Lagrangian has to be of mass dimension 4. Applying this to the kinetic terms, we can calculate the mass dimension of all fields. This then allows us to calculate the mass dimension of operators and couplings in the theory.

In the example in Eq. (7), the kinetic term for the fermions contains one space-time derivative, $[\partial] = 1$. To get $[\bar{\psi}\partial\psi] = 4$, the fermion fields must have dimension $[\psi_i] = 3/2$. Similarly, the field strength $V_{\mu\nu}$ contains a derivative, so we end up with $[V_\mu] = 1$. With these numbers we can check the other operators. In addition to the expected $[m] = [m_V] = 1$, we find $[\bar{\psi}\psi V^\mu] = 4$ or $[g] = 0$.

The mass dimension of an operator has two important consequences. First, the renormalization group flow of a theory, i. e. the running of the couplings between different energy scales, largely depends on the mass dimensions of the operators. Operators with mass dimension $D < d$ ("relevant" operators) receive large quantum corrections when going from large energies to low energies. This is the argument behind several finetuning problems such as the hierarchy problem or the cosmologocial constant problem. On the other hand, operators with $D > d$ ("irrelevant" ones) are typically suppressed when going to lower energies. Operators with $D = d$ are called "marginal".

The second consequence of the mass dimension affects the renormalizability of a theory. Theories with operators with $D > d$ are **non-renormalizable**:[2] particles in loops with energies $E \to \infty$ will lead to infinities in observables, and they are too many to be hidden in a renormalization of the parameters.

Building blocks

From now on we will only consider EFTs realized as a local QFT in 4 space-time dimensions, an approach that has proven very succesful in high-energy physics so far.

EFTs are then defined as a sum of operators $\mathcal{O}_i$, each with a specific mass dimension D_i. We can split the coupling in front of each operator into a dimensionless constant, the **Wilson coefficient** c_i, and some powers of a mass scale, for which we use the scale of heavy physics Λ:

$$\mathcal{L} = (\text{kinetic and mass terms}) + \sum_i \frac{c_i}{\Lambda^{D_i - d}} \mathcal{O}_i \,. \tag{8}$$

Why do we force Λ to appear in front of the operators like this? If we do not know anything about the underlying model at scale Λ, our best guess is that it consists of dimensionless couplings $\sim \mathcal{O}(1)$ and masses $\sim \mathcal{O}(\Lambda)$. Indirect effects mediated by this high-energy physics should therefore be proportional to a combination of these factors,

[2] The opposite is not true, by the way. Some theories contain only operators with $D \leq d$, but are still not renormalizable.

as given in Eq. (8) with couplings $c_i \sim \mathcal{O}(1)$. This is certainly true in Fermi theory, where the effective coupling G_F is suppressed by $\Lambda^2 = m_W^2$.

How the operators $\mathcal{O}_i$ look like might be clear in a top-down situation where we know the underlying theory. In a bottom-up approach, however, we need a recipe to construct a list of operators in a model-independent way. It turns out that this is surprisingly straightforward, and the list of operators we need to include in the EFT is defined by three ingredients: the particle content, the symmetries, and a counting scheme that decides which operators are relevant at the scale of interest. We will go through them one by one.

1. **Particle content**: one has to define the fields that are the dynamical degrees of freedoms in the EFT, i. e. that can form either external legs or internal propagators in Feynman diagrams. At least all particles with masses $m \ll \Lambda$ should be included. The operators are then combinations of these fields and derivatives.

2. **Symmetries**: some symmetry properties of the world have been measured with high precision, and we can expect that a violation of these symmetries has to be extremely small or happens at very high energies. These can be gauge symmetries (such as the $SU(3) \times SU(2) \times U(1)$ of the SM), spacetime symmetries (such as Lorentz symmetry), or other global symmetries (such as flavour symmetries). Requiring that the effective operators do not violate these symmetries is well motivated and can reduce the complexity of the theory significantly.

3. **Counting scheme**: with a set of particles and some symmetry requirements we can construct an infinite tower of different operators. We therefore need some rule to decide which of the operators we can neglect. Here the dimensionality of the operators becomes important. As argued above, we expect an operator with mass dimension $D > d$ to be suppressed by a factor of roughly $1/\Lambda^{D-d}$. Operators of higher mass dimension are therefore more strongly suppressed. Setting a maximal operator dimension is thus a way of limiting the EFT to a finite number of operators that should include the leading effects at energies $E \ll \Lambda$.

One property that is often required of theories is missing in this list: an EFT (with its intrinsic UV cutoff Λ) does not have to be renormalizable. In fact, most EFTs include operators with mass dimension $D > d$ and are thus non-renormalizable.

An EFT of weak interactions

As an example, let us pretend to not know anything about the SM, and construct an EFT of the weak interaction around or below a few GeV.

1. Above Λ_{QCD}, the particle content is given by the leptons and quarks, excluding the top. For a general EFT at these energy scales we would have to include photons and gluons as well, but for simplicity we will leave them out here. For energies below Λ_{QCD} we should in principle write down a different EFT based on baryons and mesons, but this does not really change the result.

2. The low-energy symmetries observed at these energies are Lorentz invariance as well as the conservation of electromagnetic charge, lepton number, and baryon number. Since we already leave out the gluons, we will pretend colour charges do not exist. [3]

3. Finally, let us only keep the operators with the lowest mass dimension (not counting kinetic and mass terms).

The kinetic and mass terms for the fermions read

$$\mathcal{L} \supset \mathrm{i}\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - m_i \bar{\psi}_i \psi_i \,. \tag{9}$$

As before, we can calculate the mass dimension of all objects and find

$$[\psi] = \frac{3}{2} \quad \text{and} \quad [\partial] = 1\,. \tag{10}$$

Adding operators composed of two fermion fields will only give us more kinetic and mass terms and not change anything. Operators with three fermion fields violate both fermion number conservation and Lorentz invariance. So the lowest-dimensional operators that we can write down include four fermion fields and no derivatives:

$$\begin{aligned}\mathcal{L} \supset\, & \frac{c_{1\,ijkl}}{\Lambda^2}\left(\bar{\psi}_i\psi_j\right)\left(\bar{\psi}_k\psi_l\right) + \frac{c_{2\,ijkl}}{\Lambda^2}\left(\bar{\psi}_i\gamma_5\psi_j\right)\left(\bar{\psi}_k\psi_l\right) \\ & + \frac{c_{3\,ijkl}}{\Lambda^2}\left(\bar{\psi}_i\gamma_5\psi_j\right)\left(\bar{\psi}_k\gamma_5\psi_l\right) + \frac{c_{4\,ijkl}}{\Lambda^2}\left(\bar{\psi}_i\gamma_\mu\psi_j\right)\left(\bar{\psi}_k\gamma^\mu\psi_l\right) \\ & + \frac{c_{5\,ijkl}}{\Lambda^2}\left(\bar{\psi}_i\gamma_5\gamma_\mu\psi_j\right)\left(\bar{\psi}_k\gamma^\mu\psi_l\right) + \frac{c_{6\,ijkl}}{\Lambda^2}\left(\bar{\psi}_i\gamma_5\gamma_\mu\psi_j\right)\left(\bar{\psi}_k\gamma_5\gamma^\mu\psi_l\right) \\ & + \frac{c_{7\,ijkl}}{\Lambda^2}\left(\bar{\psi}_i\gamma_\mu\gamma_\nu\psi_j\right)\left(\bar{\psi}_k\gamma^\mu\gamma^\nu\psi_l\right)\,,\end{aligned} \tag{11}$$

Here some entries of the Wilson coefficient matrices c_1 to c_7 have to be zero to conserve lepton and baryon number. We will drop these flavour indices from now on.

In this bottom-up approach, all remaining coefficients are free parameters and have to be determined by experiment. With the measurement of the muon lifetime, beta decay, and parity violation it turns out that the c_5 and c_6 coefficients are equal and of opposite sign, while the others are zero (ignoring Z and H interactions):

$$\mathcal{L} = \mathrm{i}\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i - m_i\bar{\psi}_i\psi_i + \frac{c}{\Lambda^2}\left(\bar{\psi}_i(1-\gamma_5)\gamma_\mu\psi_j\right)\left(\bar{\psi}_k(1-\gamma_5)\gamma^\mu\psi_l\right)\,. \tag{12}$$

This is exactly Fermi theory, with $G_F = \sqrt{2}c/\Lambda^2 = 1.16 \cdot 10^{-5}\ \text{GeV}^{-2}$.

The dimension-6 operators in this theory are not renormalizable, so Fermi theory cannot be valid at arbitrary large energies. But even knowing the Wilson coefficients, one cannot determine the scale Λ where the EFT breaks down. By postulating that the underlying theory is perturbative, $c \lesssim 4\pi$, one can set an upper limit $\Lambda \lesssim \sqrt{4\pi/G_F} \approx 1040$ GeV, much larger than the observed $\Lambda = m_W = 80$ GeV.

[3] Based on the experience with electromagnetism, and without taking into account the measurement of P and C violation, one might be tempted to also prescribe P and C invariance, which would lead to the wrong EFT.

Why is the sky blue?

Finally, here is one last simple example for how the EFT framework lets us estimate physics effects even when we do not know the full theory. It is taken from Ref. [2].

Why does the sky appear blue to us? In other words, why is blue light coming from the sun scattered more strongly by particles in the atmosphere than red light? A full derivation of this takes some time and requires knowledge of the underlying electrodynamic interactions.

Let us instead try to answer this question by writing down an effective field theory for this process, Rayleigh scattering. The only thing we have to know are the basic scales of the process: photons with energy E_γ scatter off basically static nuclei characterized by an excitation energy ΔE, mass M and radius a_0. Looking at these numbers, we see that these scales are clearly separated:

$$E_\gamma \ll \Delta E, a_0^{-1} \ll M\,. \tag{13}$$

This is good news, since such a scale hierarchy is the basic requirement for an EFT. We are interested in elastic scattering, so we set the cutoff of the EFT as[4]

$$\Lambda \sim \Delta E, a_0^{-1}\,. \tag{14}$$

With this we can put together the building blocks for our EFT as discussed in Sec. 2.2:

1. As fields we will need photons and atoms, where we can approximate the latter as infinitely heavy.

2. The relevant symmetries are the $U(1)_{em}$ and Lorentz invariance. At these energies we will also not be able to create or destroy atoms, which you can see as another symmetry requirement on the effective Lagrangian.

3. We will include the lowest-dimensional operators that describe photon-atom scattering.

The kinetic part of such an EFT reads

$$\mathcal{L}_{\text{kin}} = \phi_v^\dagger \mathrm{i} v^\alpha \partial_\alpha \phi_v - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\,, \tag{15}$$

where ϕ_v is the field operator representing an infinitely heavy atom at constant velocity v, and $F_{\mu\nu}$ is the photon field strength tensor. Boosting into the atom's rest frame, $v = (1, 0, 0, 0)$ and the first term becomes the Lagrangian of the Schrödinger equation.

The usual power counting based on $[\mathcal{L}] = 4$ gives the mass dimensions

$$[\partial] = 1\,, \quad [v] = 0\,, \quad [\phi] = \frac{3}{2} \quad \text{and} \quad [F_{\mu\nu}] = 2\,. \tag{16}$$

[4] In reality there are two orders of magnitude between ΔE and a_0^{-1}, but this does not affect the line of argument at all and we choose to ignore this fact.

The interaction operators must be Lorentz-invariant combinations of $\phi^\dagger\phi$, $F_{\mu\nu}$, v_μ, and ∂_μ. Note that operators directly involving A_μ instead of $F_{\mu\nu}$ are forbidden by gauge invariance, and single instances of ϕ correspond to the creation or annihilation of atoms which is not possible at these energies. The first such operators appear at mass dimension 7,

$$\mathcal{L}_{\text{int}} = \frac{c_1}{\Lambda^3}\phi_v^\dagger\phi_v F_{\mu\nu}F^{\mu\nu} + \frac{c_2}{\Lambda^3}\phi_v^\dagger\phi_v v^\alpha F_{\alpha\mu} v_\beta F^{\beta\mu} + \mathcal{O}\left(1/\Lambda^4\right) , \tag{17}$$

and we expect them to contain the dominant effects of Rayleigh scattering at energies $E_\gamma \ll \Lambda$.

The scattering amplitude of light off the atmospheric atoms should therefore scale as $\mathcal{M} \sim 1/\Lambda^3$, which means that the cross section scales with $\sigma \sim 1/\Lambda^6$. Since the cross section has the dimension of an area,

$$[\sigma] = -2 , \tag{18}$$

and the only other mass scale in this low-energy process is the photon energy E_γ, we know that the effective cross section must be proportional to

$$\sigma \propto \frac{E_\gamma^4}{\Lambda^6}\left(1 + \mathcal{O}\left(E_\gamma/\Lambda\right)\right) . \tag{19}$$

In other words, blue light is much more strongly scattered than red light. Our effective theory, built just from a few simple assumptions, explains the colour of the sky!

Finally, we should check the validity range of our EFT. We expect it to work as long as

$$E_\gamma \ll \Lambda \sim \Delta E \sim \mathcal{O}\left(\text{eV}\right) \tag{20}$$

which is equivalent to wavelengths above $\mathcal{O}\left(100 \text{ nm}\right)$. Our approximation is probably safe for visible light! But already in the near ultraviolet we expect deviations from the E_γ^4 proportionality.

2.3 Matching

Full and effective descriptions of physics

Let us go back to the simple picture of one full and one effective theory and summarize the typical differences between the two setups.

- The full model contains heavy particles with mass $\gtrsim \Lambda$ that are not part of the EFT. In the effective model their effects are mapped onto additional higher-dimensional operators involving only the light fields.
- We pretend that the full model is valid at all energies (even though no such theory exists so far). The EFT, in any case, is only valid at $E \ll \Lambda$. Only in this low-energy region the two descriptions agree.

- The full model is renormalizable, while the EFT is not.
- An interaction mediated by heavy fields in the full model is described by the higher-dimensional operators in the EFT, see for instance Eqs. (2) and (3). This means that the non-local interaction in the full model is approximated as a local contact interaction in the EFT.

Matching full and effective theory

The final missing puzzle piece is now the link between a full model and its EFT: if we know an underlying model, how do we map it to the Wilson coefficients of an EFT? This procedure is known as **matching** and plays an important role in a top-down approach to EFTs. Note that it cannot be reversed into a bottom-up matching — one cannot uniquely reconstruct a full theory only based on the EFT.

The core idea of the matching procedure that at a matching scale the predictions of the underlying model and the EFT agree up to corrections suppressed by some power of $1/\Lambda$.

We will now sketch a straightfoward recipe based on Feynman diagrams. For a derivation and detailed discussion, see the QFT textbook of your choice or Ref. [3].

1. Start with the particle content of the full model. Choose Λ and divide the particles of the full model into light and heavy fields. Light fields, which should include at least those with masses below Λ, will make up the particle content of the effective theory. Heavy fields will be **integrated out**, that is, removed as dynamical degrees of freedom in the EFT.

2. Based on the particles and interactions of the full model, draw all connected Feynman diagrams in which
   - all external legs are light fields, and
   - all internal lines are heavy fields.

   Using the Feynman rules of the full model, calculate the expressions for these diagrams. Do not treat the external legs as incoming or outgoing particles, but keep the field operator expressions.

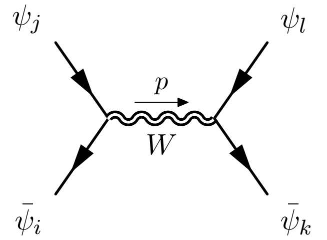
3. Express quantities of the full model in terms of Λ. Truncate this infinite series of diagrams at some order in $1/\Lambda$, depending on the dimension of the operators that you want to keep. Together with kinetic and mass terms for the light fields, these form the Lagrangian of the EFT.

Fermi theory again

Let us apply this top-down procedure to our standard example of Fermi theory. For simplicity, we do not take the full SM, but just the interactions between massive W bosons and fermions as the underlying theory. The Lagrangian of these interactions is similar to that given in Eq. (7).

1. Our full model consists of the quarks and leptons and the W boson. We want to analyse weak interactions below the W mass, so we set $\Lambda = m_W$. So all quarks and leptons except for the top are the light particles of the EFT, while the W boson and the top quark are heavy and have to be integrated out.

2. The only diagram with the requested features that has only one heavy propagator has the form



(21)

Double lines denote a heavy field. There are additional diagrams with W self-interactions or W loops, but they involve at least two W propagators, which means that all contributions from them will be of order $\mathcal{O}\left(1/\Lambda^4\right)$, which we will neglect.

This diagram evaluates to

$$\begin{aligned} &\left(\bar{\psi}_i \frac{\mathrm{i}g}{\sqrt{2}} \frac{1-\gamma_5}{2} \gamma^\mu \psi_j\right) \frac{-g_{\mu\nu}}{p^2 - m_W^2} \left(\bar{\psi}_k \frac{\mathrm{i}g}{\sqrt{2}} \frac{1-\gamma_5}{2} \gamma^\nu \psi_l\right) \\ =\ &\frac{g^2 \left(\bar{\psi}_i (1-\gamma_5)\gamma^\mu \psi_j\right) \left(\bar{\psi}_k (1-\gamma_5)\gamma_\mu \psi_l\right)}{8(p^2 - m_W^2)} \end{aligned} \tag{22}$$

3. The only dimensionful parameter is $m_W = \Lambda$, and for the EFT to be valid we assume $p^2 \ll \Lambda^2$. We can then expand this expression as

$$\frac{g^2}{8m_W^2} \left(\bar{\psi}_i (1-\gamma_5)\gamma^\mu \psi_j\right) \left(\bar{\psi}_k (1-\gamma_5)\gamma_\mu \psi_l\right) + \mathcal{O}\left(1/\Lambda^4\right) . \tag{23}$$

With this, we again rediscover the dimension-6 EFT matched to the weak interactions of the SM:

$$\mathcal{L} = \mathrm{i}\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - m_i \bar{\psi}_i \psi_i + \frac{c}{\Lambda^2} \left(\bar{\psi}_i (1-\gamma_5)\gamma_\mu \psi_j\right) \left(\bar{\psi}_k (1-\gamma_5)\gamma^\mu \psi_l\right) . \tag{24}$$

with heavy scale $\Lambda = m_W$ and Wilson coefficient $c = g^2/8$. Replacing c/Λ^2 by $G_F/\sqrt{2} = g^2/(8m_W^2)$ restores the historic form of Fermi theory.

A few words on operator mixing

So far we have neglected that like all parameters in a QFT, the Wilson coefficients of an EFT depend on the energy scale. Running the model from one energy to a different one, operators will generally mix: loop effects from one operator will affect the coefficients of other operators. If the Wilson coefficients are given at the matching scale Λ (we use this symbol since the matching scale is usually chosen only slightly below the EFT cutoff), at the scale of interest E they will take on values of the form

$$c_i(E) \sim c_i(\Lambda) \pm \sum_j \frac{g^2}{16\pi^2} \log \frac{\Lambda^2}{E^2} c_j(\Lambda) , \tag{25}$$

where g are the typical couplings in the loops.

If the matching scale is not too far away from the energy scale of interest and if all Wilson coefficients are already sizable at the matching scale, this is often negligible. There is an important consequence, though: even if an operator is zero at the matching scale, operator mixing will give it a small but non-zero value at lower energies. So regardless of what the underlying model is, it can be expected that eventually all effective operators allowed by the symmetries will receive contributions from it.

3 Higgs effective field theory at the LHC

3.1 Relevance

Let us leave historic and toy examples behind and focus on a more exciting area of physics: that explored at the electroweak scale of a few hundred GeV. So far the LHC experiments overwhelmingly show agreement with the SM. Still, there are a few reasons to expect new physics at TeV energies, such as the hierarchy problem or the WIMP "miracle" of dark matter. Unfortunately these (purely aesthetic) arguments do not tell us how exactly such physics should look like. There are many more or less motivated models, including supersymmetric ones, extended gauge groups, extra dimensions, composite Higgses, and the list goes on and on.

This leaves us with a question highly relevant for upcoming ATLAS and CMS analyses: what is the best language to discuss indirect signs of new physics at the electroweak scale, in particular in the Higgs sector?

Directly interpreting measurements in complete models of new physics is impractical: for $o \gg 1$ observables and $m \gg 1$ models this requires om limits to be derived. Also, the paramater space of such models (think of the relatively simple MSSM) can be huge, and many of the features of these models do not matter at the electroweak scale at all. It makes more sense to define an intermediate framework that can be linked both to measurements and to full theories, so only o sets of limits plus m translation rules from complete theories to the intermediate language have to be calculated. Such a framework should include all necessary physics, but no phenomena irrelevant at this scale. . . sounds familiar?

One such possibility is to dress the SM Lagrangian with form factors, as done for instance in the κ (or Δ) framework of Higgs physics. Such an approach is well suited to parametrize shifts in total rates, but unable to account for new structures that are not present in the SM, visible as changes in distributions. For better or worse, this framework is also agnostic about correlations between different couplings: there is no clear way how to combine the results from triple gauge vertices and Higgs properties, for instance.

Of course in this lecture the answer instead has to be an effective field theory. We will construct this Higgs EFT (or SM EFT) in the next section. In Sec. 3.3 we will analyse the phenomenology of its effective operators, and it will turn out that they can indeed parametrize changes in distributions as well as allow us to combine different observables in a global fit. It seems that we have found the perfect language for our problem — until we remember in Sec. 3.5 that for effective field theories one always has to check the validity…

3.2 Dimension-6 operators

Building blocks

Since we do not know what physics lays beyond the SM, we have to construct Higgs EFT from a bottom-up perspective. As discussed in Sec. 2.2, this means we have to write down all operators based on a set of particles that are compatible with certain symmetries and are important according to some counting scheme. Let us go through these one by one:

1. Our EFT will contain all SM fields. There is one subtlety: in the SM, the physical Higgs particle h and the Goldstone bosons w_i that become part of the W and Z bosons during electroweak symmetry breaking are combined in a $SU(2)_L$ doublet

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} w_1 + \mathrm{i} w_2 \\ v + h + \mathrm{i} w_3 \end{pmatrix} , \tag{26}$$

   where $v = 246$ GeV is the vacuum expectation value (VEV) of ϕ in its potential. We choose to use ϕ as the fundamental building block of our EFT, which is motivated by the fact that the SM is in great agreement with all observations so far. This is often called the **linear** Higgs EFT.[5]

2. All operators have to be invariant under Lorentz transformations and under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. They also should conserve lepton and baryon number.

3. We again assume that higher-dimensional operators will be suppressed by more powers of the cutoff scale Λ. We will keep those up to mass dimension 6.

Simple dimensional analysis of the kinetic terms of the SM fields tells us the mass dimensions of all building blocks:

$$[f] = \frac{3}{2}\,, \quad [V_\mu] = 1\,, \quad [V_{\mu\nu}] = 2\,, \quad [\phi] = 1\,, \quad [\partial_\mu] = 1 \quad \text{and} \quad [D_\mu] = 1\,. \tag{27}$$

Here f refers to the lepton and quark fields; V_μ to the gauge bosons G^i_μ, W^i_μ, and B_μ; $V_{\mu\nu}$ to the corresponding field strength tensors; and D_μ to the covariant derivative that combines a partial derivative with couplings to gauge bosons.

[5] As an alternative, it is also possible to use h as the fundamental building block and to not assume the SM doublet structure of Eq. (26). This is called the **non-linear** Higgs EFT. The terms "linear" and "non-linear" by the way refer to the behaviour under custodial transformations.

Operators and bases

There are no dimension-5 operators,[6] so our EFT will have the form

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{f_i}{\Lambda^2} \mathcal{O}_i \tag{28}$$

with unknown cutoff scale Λ and Wilson coefficients f_i.

On the one hand, note that the object $\phi^\dagger \phi$ is a scalar gauge singlet. We can simply attach it to any dimension-4 operator of the SM to get a dimension-6 operator of our EFT:[7]

$$\mathcal{O}_{\phi 3} = \frac{1}{3} (\phi^\dagger \phi)^3 \,, \tag{29}$$

$$\mathcal{O}_{\phi 4} = (\phi^\dagger \phi) \, (D_\mu \phi)^\dagger (D^\mu \phi) \,, \tag{30}$$

$$\mathcal{O}_{GG} = (\phi^\dagger \phi) \, G^a_{\mu\nu} \, G^{\mu\nu\, a} \,, \tag{31}$$

$$\mathcal{O}_{BB} = -\frac{g'^2}{4} \, (\phi^\dagger \phi) \, B_{\mu\nu} \, B^{\mu\nu} \,, \tag{32}$$

$$\mathcal{O}_{WW} = -\frac{g^2}{4} \, (\phi^\dagger \phi) \, W^k_{\mu\nu} \, W^{\mu\nu\, k} \,, \tag{33}$$

$$\mathcal{O}_f = (\phi^\dagger \phi) \, \bar{F}_L \phi f_R + \text{h.\,c.} \,. \tag{34}$$

Other operators have more complicated structures, often involving derivatives:

$$\mathcal{O}_{\phi 1} = (D_\mu \phi)^\dagger \phi \; \phi^\dagger (D^\mu \phi) \,, \tag{35}$$

$$\mathcal{O}_{\phi 2} = \frac{1}{2} \, \partial^\mu (\phi^\dagger \phi) \, \partial_\mu (\phi^\dagger \phi) \,, \tag{36}$$

$$\mathcal{O}_{BW} = -\frac{g \, g'}{4} \, (\phi^\dagger \sigma^k \phi) \, B_{\mu\nu} \, W^{\mu\nu\, k} \,, \tag{37}$$

$$\mathcal{O}_B = \mathrm{i} \frac{g}{2} \, (D^\mu \phi^\dagger)(D^\nu \phi) \, B_{\mu\nu} \,, \tag{38}$$

$$\mathcal{O}_W = \mathrm{i} \frac{g}{2} \, (D^\mu \phi)^\dagger \sigma^k (D^\nu \phi) \, W^k_{\mu\nu} \,. \tag{39}$$

There are many more operators one can write down, and we will not list all of them. The first complete set of 80 operators (not counting flavour structure and Hermitian conjugation) was published in 1985 [4]. It was soon noticed that not all of these operators are actually independent: they can be linked through equations of motions of the SM fields, integration by parts, or field redefinitions. Taking these effects into account, a (complete and not redundant) basis for dimension-6 operators consists of 59 operators (again up to flavour and Hermitian conjugation). It took until 2010 for someone to write down such a basis [5]. Three different conventions have become popular: in addition to

[6] After adding right-handed neutrinos to the SM, one can construct the dimension-5 "Weinberg operator". It generates a Majorana mass term for the neutrinos, but is entirely irrelevant for the physics we will discuss.

[7] $\mathcal{O}_f$ looks slightly different for up-type quarks, just as in the SM Yukawa couplings.

the “Warsaw” basis [5], there is the SILH convention [6] and the HISZ basis [7]. Here we stick to the latter.

If we count all possible flavour structures for three fermion generations, there are 2499 operators in the dimension-6 basis, so in practice we will always assume flavour-diagonal or even flavour-universal Wilson coefficients. And just in case you are wondering why we are stopping at dimension 6: using the counting that gave 59 independent dimension-6 operators, there are $\mathcal{O}(1000)$ ones at dimension 8 and $\mathcal{O}(10\,000)$ dimension-10 operators.

3.3 Operator phenomenology

As discussed above, we need to think about Higgs EFT from two perspectives, linking it to observables and complete theories of new physics. Let us start with the first connection and discuss the phenomenology of two example operators. For a more complete treatment, see e. g. Ref. [8].

$\mathcal{O}_{\phi 2}$: rescaled Higgs couplings

Ignoring the Goldstones,

$$\phi^\dagger \phi = \frac{v^2 + 2v\tilde{h} + \tilde{h}^2}{2} \tag{40}$$

where we use a tilde on the Higgs boson field for reasons that will become clear soon.

The operator $\mathcal{O}_{\phi 2}$ then becomes

$$\begin{aligned} \mathcal{L}_{\text{EFT}} &\supset \frac{f_{\phi 2}}{2\Lambda^2}\, \partial^\mu(\phi^\dagger\phi)\, \partial_\mu(\phi^\dagger\phi) \\ &= \frac{f_{\phi 2}}{2\Lambda^2}\, \frac{(2v\partial_\mu\tilde{h} + 2\tilde{h}\partial_\mu\tilde{h})^2}{4} \\ &= \frac{f_{\phi 2} v^2}{2\Lambda^2}\, \partial_\mu\tilde{h}\partial^\mu\tilde{h} + \mathcal{O}\left(h^3\right) . \end{aligned} \tag{41}$$

The h^3 (and higher) interactions are only important for Higgs pair production. The first term, on the other hand, presents a rescaling of the kinetic term of the Higgs boson:

$$\mathcal{L}_{\text{EFT}} \supset \left(1 + \frac{f_{\phi 2} v^2}{\Lambda^2}\right) \frac{1}{2}\, \partial_\mu\tilde{h}\partial^\mu\tilde{h}\,. \tag{42}$$

The usual Feynman rules (and all Monte-Carlo software) assume a canonical normalization of this term, so we have to renormalize $\tilde{h}$ as

$$h = \sqrt{1 + \frac{f_{\phi 2} v^2}{\Lambda^2}}\, \tilde{h}\,. \tag{43}$$

This restores the canonical form of the kinetic term, but also rescales all other instances of the Higgs field in the Lagrangian, for instance the Higgs couplings to all other particles

$$g_{hxx} = \frac{1}{\sqrt{1+\frac{f_{\phi 2}v^2}{\Lambda^2}}}\, g_{hxx}^{\rm SM} = \left(1 - \frac{f_{\phi 2}v^2}{2\Lambda^2}\right) g_{hxx}^{\rm SM} + \mathcal{O}\left(v^4/\Lambda^4\right) . \tag{44}$$

A non-zero Wilson coefficients $f_{\phi 2}$ will thus universally rescale all couplings of the Higgs boson and lead to some more complicated effects in the Higgs self-coupling. The first effect would lead to shifts in the measured signal strengths in all Higgs measurements, while the distributions stay the same as in the SM (neglecting interference effects). The latter will be visible in Higgs pair production, both in the total rate and in distributions.

$\mathcal{O}_W$: new Lorentz structures

As a second example, consider $\mathcal{O}_W$. The covariant derivative acting on the Higgs doublet is defined as

$$D_\mu \phi = \partial_\mu \phi - {\rm i} g \frac{\sigma^k}{2} W_\mu^k \phi - {\rm i}\frac{g'}{2} B_\mu \phi\,, \tag{45}$$

and the field strength tensor reads

$$W_{\mu\nu}^k = \partial_\mu W_\nu^k - \partial_\nu W_\mu^k + g\varepsilon^{kmn} W_\mu^m W_\nu^n\,. \tag{46}$$

Expanding $\mathcal{O}_W$ and only keeping the pieces that will affect the hWW coupling, we find

$$\begin{aligned}
\mathcal{L}_{\rm EFT} &\supset \frac{f_W}{\Lambda^2}\,{\rm i}\frac{g}{2}\,(D^\mu\phi)^\dagger \sigma^k (D^\nu \phi)\, W_{\mu\nu}^k \\
&= \frac{f_W}{\Lambda^2}\,{\rm i}\frac{g}{2}\left(\partial^\mu \phi^\dagger + {\rm i}\frac{g}{2} W^{m\,\mu} \phi^\dagger \sigma^m + {\rm i}\frac{g'}{2} B^\mu \phi^\dagger\right)\sigma^k \\
&\qquad\qquad \cdot \left(\partial^\nu \phi - {\rm i}g\frac{\sigma^n}{2} W^{n\,\nu}\phi - {\rm i}\frac{g'}{2}B^\nu \phi\right) W_{\mu\nu}^k \\
&\supset \frac{f_W}{\Lambda^2}\,{\rm i}\frac{g}{2}\left\{\frac{\partial^\mu h}{\sqrt{2}}\,\left[\sigma^k\sigma^n\right]_{22}\,\frac{-{\rm i}g}{2}\,W^{n\,\nu}\,\frac{v}{\sqrt{2}} + \frac{{\rm i}g}{2}\,W^{m\,\nu}\,\frac{v}{\sqrt{2}}\,\left[\sigma^m\sigma^k\right]_{22}\,\frac{\partial^\mu h}{\sqrt{2}}\right\} W_{\mu\nu}^k \\
&= \frac{f_W}{\Lambda^2}\,\frac{g^2 v}{8}\,\left[\sigma^k,\sigma^n\right]_{22}\,(\partial^\mu h)\,W^{n\,\nu}\,W_{\mu\nu}^k \\
&= \frac{f_W}{\Lambda^2}\,\frac{g^2 v}{8}\,2{\rm i}\varepsilon^{knm}\,\sigma_{22}^m\,(\partial^\mu h)\,W^{n\,\nu}\,W_{\mu\nu}^k \\
&= \frac{f_W}{\Lambda^2}\,\frac{{\rm i}g^2 v}{4}\,\varepsilon^{nk3}\,(\partial^\mu h)\,W^{n\,\nu}\,W_{\mu\nu}^k\,.
\end{aligned} \tag{47}$$

With

$$m_W = \frac{gv}{2} \tag{48}$$

and

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp {\rm i}W_\mu^2)\,, \tag{49}$$

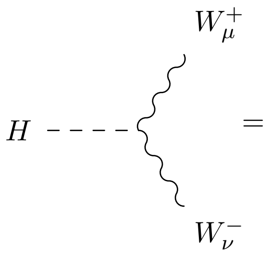
this finally gives

$$\mathcal{L}_{\text{EFT}} \supset \frac{f_W}{\Lambda^2}\,\frac{\mathrm{i}gm_W}{2}\,(\partial^\mu h)\left(W^{+\,\nu}\,W^-_{\mu\nu} + W^{-\,\nu}\,W^+_{\mu\nu}\right) . \tag{50}$$

This is another contribution to the hWW vertex. But unlike the SM-like coupling

$$\mathcal{L}_{\text{SM}} \supset gm_W\,hW^{+\,\mu}\,W^-_\mu\;, \tag{51}$$

the $\mathcal{O}_W$ term includes derivatives. This means that the interaction gains a momentum dependence:



$$= \mathrm{i}gm_W\left[g_{\mu\nu} + \frac{f_W}{2\Lambda^2}\,p_H^2\,g_{\mu\nu} + \frac{f_W}{2\Lambda^2}\left(p^H_\mu p^+_\nu + p^-_\mu p^H_\nu\right)\right], \tag{52}$$

where $p^\pm_\mu$ and p^H_μ are the incoming momenta of the $W^\pm$ and the H, respectively.

Two features of the EFT approach stand out. First, $\mathcal{O}_W$ does not only affect the hWW vertex, but also hZZ interactions and triple-gauge couplings such as WWZ. This means that the dimension-6 operator language allows us to combine different measurements in a global fit.

Second, $\mathcal{O}_W$ changes the shape of distributions. One example is the Higgs-strahlung process at the LHC,

$$pp \to Zh\,. \tag{53}$$

Here a Z boson is produced off-shell and radiates off a Higgs. The intermediate Z can carry arbitrary large energy and momentum, which we can measure for instance as the invariant mass of the final Zh system. From Eq. (52) we expect that the effects from $\mathcal{O}_W$ will grow with m_{Zh}.

In Fig. 1 we demonstrate this by comparing the distribution of m_{Zh} based on only the SM couplings, on the dimension-6 operator $\mathcal{O}_W$ only, and on the interference between the two components. Indeed we see that $\mathcal{O}_W$ contributes mostly in the high-energy tail of the distribution. This and a few other operators thus allow us to describe new physics effects in distributions, not only in total rates. Or from the opposite perspective, we will have to analyse kinematic distributions to disentangle different operators.

On a side note, the interference terms contribute at order $\mathcal{O}\left(1/\Lambda^2\right)$ to the cross section, while the squared operators only contribute at $\mathcal{O}\left(1/\Lambda^4\right)$. So the latter appear at the same order as the leading (neglected) dimension-8 operator effects. Some people argue that for this reason one should not include the square of dimension-6 operators in calculations. But the interference between the SM amplitudes and the dimension-6 operators can be destructive, as shown in the right panel of Fig. 1, and push the differential cross section to negative values when the squared operators are neglected.

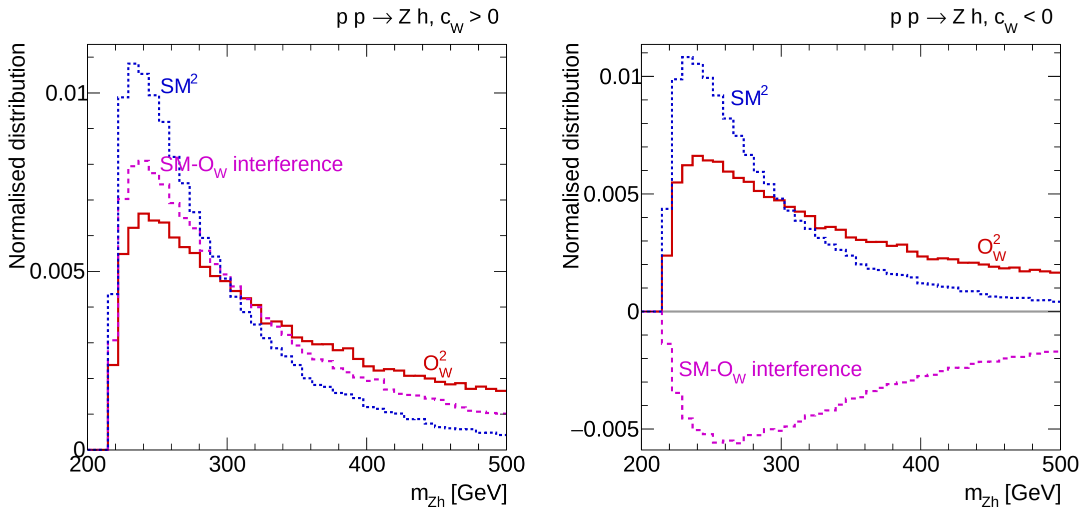


Figure 1: Distribution of the Zh invariant mass in the Higgs-strahlung process $pp \to Zh$ at LHC conditions. The contribution from the dimension-6 operator $\mathcal{O}_W$ is enhanced at larger momentum transfer compared to the SM amplitudes. The interference between them can be constructive (left) or destructive (right). Note that this plot is based on a different operator basis, which is why the operator $\mathcal{O}_W$ used in this plot is closely related, but not identical to that defined in Eq. (39).

3.4 From new physics models to Higgs EFT

A proper discussion of the other perspective to Higgs EFT, the matching of complete models of new physics to the dimension-6 operators, is beyond the scope of lecture. One rather handwaving example will have to suffice. For more details see for instance [3, 9].

Maybe the simplest model of new physics is the extension of the SM by one real scalar singlet, also known as a "Higgs portal". This new field s only couples to the Higgs doublet ϕ proportional to its mass m_s times a coupling λ_s. Identifying $\Lambda = m_s$ and

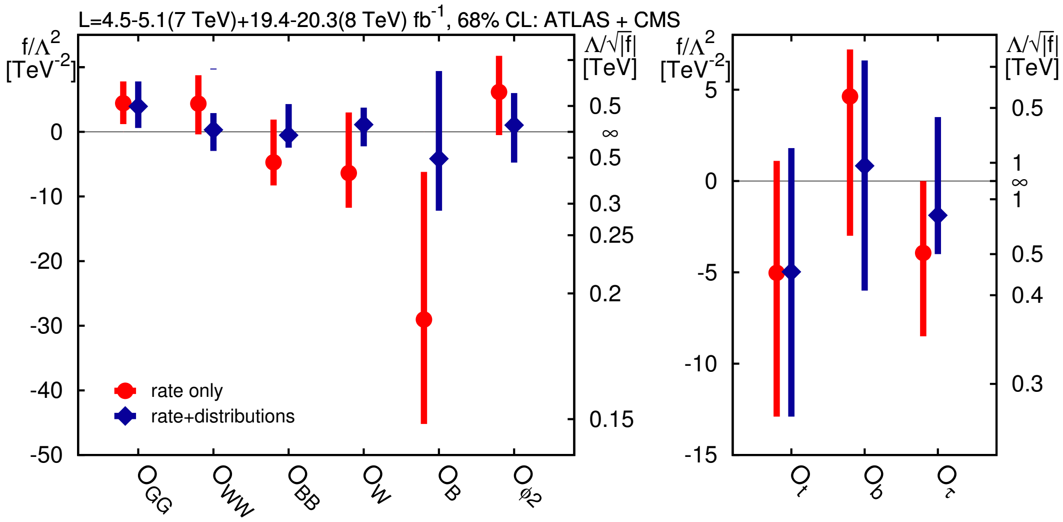
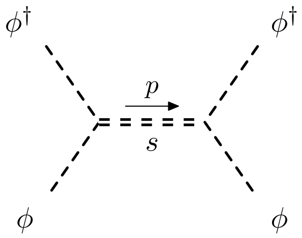


Figure 2: Current 68% CL limits on the dimension-6 Wilson coefficients. The blue lines take into account kinematic distributions in Higgs-strahlung and Higgs production in weak boson fusion. Figure taken from Ref. [10]. Note that a few of these operators have a different normalization from that in Eqs. (29) to (39).

integrating out the singlet gives rise to the diagram



$$\sim (\phi^\dagger\phi)\frac{m_s^2\lambda_s^2}{p^2-m_s^2}(\phi^\dagger\phi)$$

$$\begin{aligned}
&\sim \lambda_s^2(\phi^\dagger\phi)\left(1+\frac{p^2}{\Lambda^2}+\mathcal{O}\left(1/\Lambda^4\right)\right)(\phi^\dagger\phi) \\
&\sim \lambda_s^2(\phi^\dagger\phi)\left(1-\frac{\partial^2}{\Lambda^2}+\mathcal{O}\left(1/m_s^4\right)\right)(\phi^\dagger\phi) && (54) \\
&\sim \lambda_s^2(\phi^\dagger\phi)^2+\frac{\lambda_s^2}{\Lambda^2}\,\partial_\mu(\phi^\dagger\phi)\partial^\mu(\phi^\dagger\phi)\,. && (55)
\end{aligned}$$

The first term is an unobservable renormalisation of an SM operator, but the second one is just $\mathcal{O}_{\phi 2}$ with Wilson coefficient $f_{\phi 2}=2\lambda_s^2$. At tree level, this is the only operator generated by this model.

3.5 Higgs EFT at its limits

Results from the LHC, LEP, and other experiments constrain the Wilson coefficients of Higgs EFT. So far the experimental collaborations have not published such limits themselves, but a number of phenomenologists are working on it. Fig. 2 shows results from a fit of dimension-6 operators to LHC run I data [10]. In addition to total rates, this study takes into account distributions in the Higgs-strahlung and weak boson fusion channels.[8]

It turns out that analyzing kinematic distributions indeed tightens the constraints on some operators. This constraining power comes in particular from the high-energy bins (remember the discussion around Fig. 1). Overall, no significant deviation from the SM (all dimension-6 coefficients zero) has been found. The upper limits on the Wilson coefficients are roughly around

$$\frac{|f|}{\Lambda^2} \lesssim 5 \text{ TeV}^{-2} . \tag{56}$$

Wait. . . what?

This number should set alarm bells ringing. Inverting it yields

$$\Lambda \gtrsim \sqrt{|f|} \cdot 400 \text{ GeV} . \tag{57}$$

For a strongly coupled underlying system, we expect large Wilson coefficients f, and Λ must at least be around a few TeV. But if physics beyond the SM is weakly coupled, we expect Wilson coefficients $|f| \lesssim \mathcal{O}(1)$, and Λ can be as low as a few hundred GeV. Compare this to the typical energy scale of the processes we describe. Total Higgs production rates at the LHC probe a typical energy of $E \sim m_h$. More importantly, we just argued that fits rely on the high-energy bins of distributions, which contain events with a typical momentum transfer of up to

$$E \sim 500 \text{ GeV} . \tag{58}$$

If the underlying model is weakly coupled, LHC measurements are currently only sensitive to scenarios with $\Lambda \sim E$. The lack of a separation between E and Λ means that the EFT will not necessarily provide a good approximation to full models of new physics! The EFT validity is only guaranteed for more strongly coupled scenarios. Designed to be as model-independent as possible, the EFT description of Higgs physics only seems justified for certain classes of models.[9]

Still, as we argued above Higgs EFT has many desirable features, and there is no obvious alternative for a model-independent parametrization of electroweak physics. If and how the dimension-6 Lagrangian is useful for Higgs physics at the LHC is quite a hot topic right now. In Refs. [9, 11] we have tackled this question by explicitly comparing the Higgs phenomenology of complete models of new physics and the corresponding EFT.

We find that the dimension-6 Lagrangian is surprisingly powerful: its operators can describe nearly all effects expected in Higgs physics from typical weakly interacting SM

[8] Regrettably, ATLAS and CMS have published only a few of the distributions of interest.

[9] The dark matter community has been discussing a similar problem. In that field EFTs work very well to describe direct detection experiments, in which the momentum transfer is very low, and annihilation in the early universe. But dark matter production at the LHC probes much higher energy scales, and it turns out that the usual effective theories are not valid anymore.

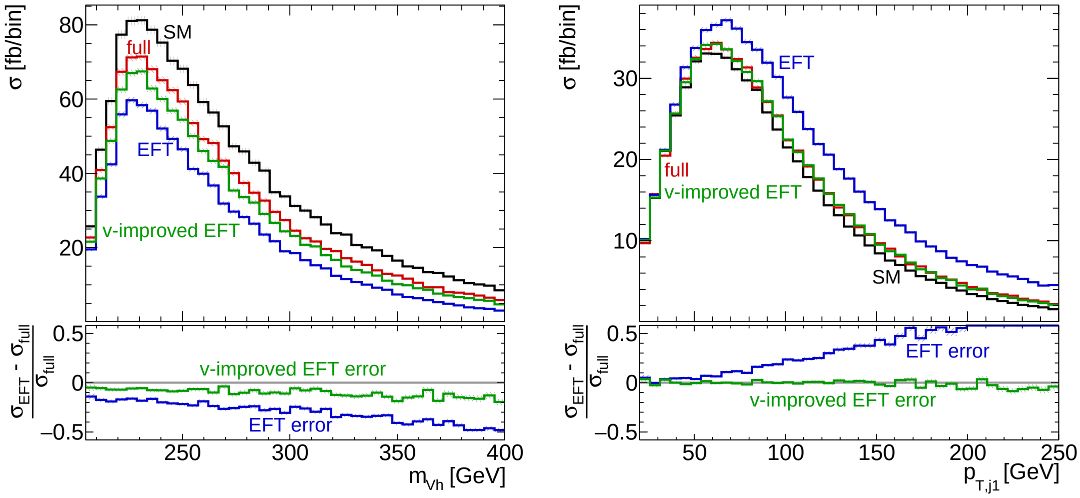


Figure 3: Effects from an additional heavy vector boson on Higgs production. Left: distribution of the invariant mass in Higgs-strahlung. Right: transverse momenta of the tagging jets in weak boson fusion. The predictions of the full model are compared to those of two different EFT constructions. The "v-improved" EFT includes effects from the nonzero Higgs vacuum expectation value that formally correspond to a dimension-8 operator. Figure taken from Ref. [9].

extensions. Resonances from new light particles are an obvious exception. Higgs pair production is problematic because intricate cancellations between amplitudes are not well reproduced by the EFT. Total rates and distributions in single Higgs production, however, are well described by the dimension-6 Lagrangian. Here the lack of a scale hierarchy between E and Λ leads to one complication in the matching procedure: subleading terms suppressed by powers of v^2/Λ^2 can be important even though they correspond to dimension-8 operator effects. Including these terms in the dimension-6 Wilson coefficients, a procedure dubbed "v-improvement", is then necessary to guarantee that Higgs EFT captures the new physics effects. This subtlety is no problem for a fit of dimension-6 operators to experimental data, it only plays a role when translating limits on Wilson coefficients to parameters of complete models.

Let us illustrate this with one example. As new physics scenario, consider a heavy vector boson added to the SM. Through mixing with the W and Z, it affects the Higgs-gauge interactions, changing kinematic distribution in addition to total rates. In the EFT this is mapped onto (among others) the operator $\mathcal{O}_W$, which was discussed in Sec. 3.3. Fig. 3 shows how this scenario affects distributions in two of the main Higgs production modes, Higgs-strahlung and weak boson fusion. The predictions of the full model are reasonably well approximated by the EFT based on v-improved matching, while the naive matching leads to gross errors and is ultimately useless.

4 Summary

There is nothing strange or complicated about effective field theories. They simply provide an organized way of doing what we always do in physics: neglecting effects that do not matter for a given question. EFTs in the form of quantum field theories consist of a set of (typically non-renormalizable) operators. You have seen how this framework allows us to start with a full theory and constructive an effective approximation from the top down, and how it even allows us to construct an approximate description of physics even if we do not know the underlying theory.

At TeV energies find ourselves in the latter bottom-up situation. There has to be physics beyond the standard model, and it better be accessible by the LHC experiments, but we do not know what it is. Higgs effective field theory is designed as a model-independent language that captures the effects of such new physics on electroweak-scale observables. Its minimal version consists of 59 dimension-6 operators, some of which parametrize changes in kinematic structures in the interactions of Higgs and gauge bosons. A global fit to these operators works fine, especially if distributions are included. Concerns about the validity of this effective theory have to be taken seriously, but so far it seems that this language is the way to go as long as no new light particles are discovered.

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