Numerical Minimization

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Naming Scheme

- many names for a similar concept:
 - mathematical minimization / maximization
 - modern: mathematical optimization
 - old: programming
 - linear programming
 - non-linear programming



- all:
 - find extremum of a function (given some constraints)

Applicability

- large class of problems:
 - optimize buisness decisions (\rightarrow LP)
 - minimize production cost
 - minimize travel for delivery
 - minimize X² value between theory and experiment
 - maximize the likelihood function given the data
 - minimize cost-function for multivariate classification

Problem definition

- Given some function f find the value x for which f takes ist smallest (largest) value
- "Rules"
 - f not known analytically
 - df not known analytically



- repeatedly evaluate f until a minimum is found
- there might be constraints on x

- Best method: fewest function calls
 - no best method available

Additional rules

- very large field
 - very hard problems

- focus on techniques / ideas
 - not particular problems
 - leave the ugly details out
- some simplifying assumptions:
 - only local extrema
 - sufficiently nice functions



Test function: Rosenbrock



Steepest Descent



Powells Conjugate Direction Set



Quasi-Newton



Test function: Goldstein-Price



Test function: Eggholder



Decision guideline

- no knowledge of gradient:
 - generally BFGS
 - well-conditioned: direction-set, Simplex Nelder-Mead
- knowledge of gradient:
 - BFGS
 - conjugate gradient if evaluation is very cheap
- noisy measurement:
 - Simplex Nelder-Mead