INFLATION

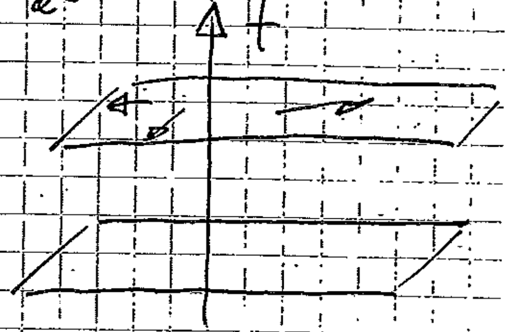
→ Homogeneity and isotropy at large scales ($\sim 10^6$ light-yrs $\sim 10^{24}$ cm) lead to the following metric:

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 \qquad \underline{\text{FRW metric}}$$

where

$$d\vec{x}^2 = \frac{d\bar{r}^2}{1 - \mathcal{K}\bar{r}^2} + \bar{r}^2 d\Omega_2 \quad \text{has curvature} \quad R^{(2)} = \frac{\mathcal{K}}{a^2}$$

$$\text{with } \mathcal{K} = \begin{cases} 0 & \text{FLAT} \\ 1 & \text{SPHERICAL} \\ -1 & \text{HYPERBOLIC SPACE} \end{cases}$$



→ This metric solves EE:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} - \Lambda g_{\mu\nu} \qquad (1)$$

if the scale factor $a(t)$ satisfies the Friedmann's eqs:

→ (1), take the (00) component and insert the stress-energy tensor for a perfect fluid:

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu} \quad \text{where}$$

I can take $u_\mu = (1,0,0,0)$ at rest, and obtain:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\mathcal{K}}{a^2} = \frac{8\pi G}{3} + \frac{\Lambda}{3} \qquad \underline{\text{FE}} \qquad (2)$$

$$\cancel{\left(\frac{\dot{a}}{a}\right)^2 + \frac{\mathcal{K}}{a^2}}$$

ON THE STR.-EN TENSOR

~~S_x σ_{xx} S_x σ_{yx} S_z σ_{zx}~~

densità di flusso
en.

DIGRESSION

→ (ij) comp. of (1) give other equations,

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{K}{a^2}=-8\pi G p+\Lambda \qquad (3)$$

→ A third eq., not independent from (2) and (3) is obtained by imposing the 1st law of thermodynamics:

$$\frac{\partial}{\partial t}\left[\rho a^3\right]+p\frac{\partial}{\partial t}(a^3)=0$$

$$\Rightarrow dE+p\,dV=0$$

→ A way to understand FEs is by obtaining their newtonian analogue: $K=0$, $\Lambda=0$, $p\ll\rho$.
Consider a sphere of radius $R(t)$ containing an ideal gas,

Conserv. of energy: $\frac{d}{dt}\left(\frac{4}{3}\pi a^3\rho\right)=0$

$$\Rightarrow 3\frac{\dot{a}}{a}\rho+\dot{\rho}=0 \qquad \text{1st law of thermody.}$$

E.O.M. for a particle on the sphere:

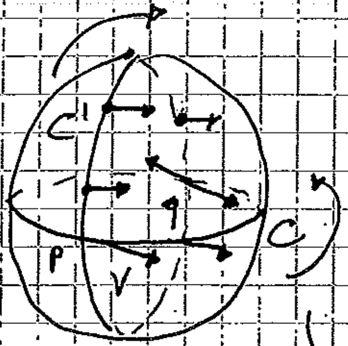
$$m\ddot{a}=-G\left(\frac{4}{3}\pi a^3\rho\right)\frac{m}{a^2}\Rightarrow\frac{\ddot{a}}{a}=-\frac{4}{3}\pi G\rho$$

which is the diff. between (2) and (3).

DIGRESSION

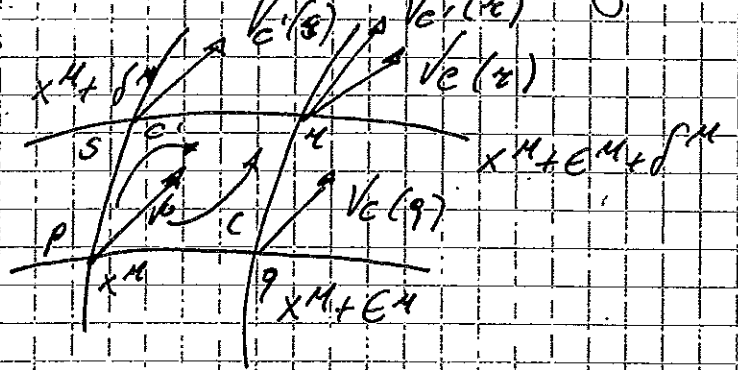
More on curvature

Let me explain curvature by looking at parallel transported vectors.



$\vec{V}$ parallel transported along C, C' if the angle that $\vec{V}$ makes with C, C' is kept fixed.

→ Result of the parallel transport along C, C' is different.



The difference between $V_{C'}$ and V_C is described by the Riemann curvature tensor:

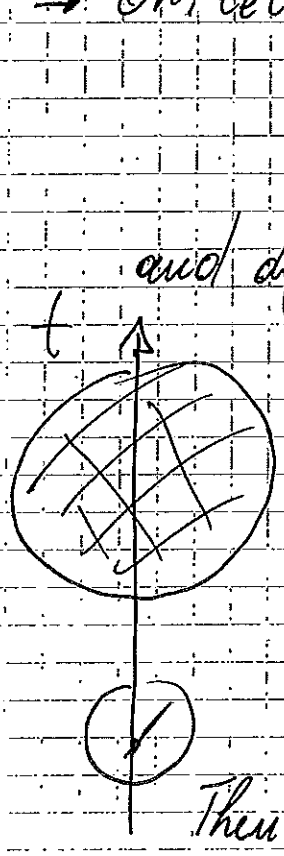
$$V_{C'}(r) - V_C(r) = V_0^{\kappa} R^{\mu}{}_{\kappa\lambda\nu}\,\epsilon^{\lambda}\delta^{\nu}.$$

Then: $R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$ and $R = g^{\mu\nu}R_{\mu\nu}$.

→ OK, let me start again from the FE:

$$\left(\frac{\dot a}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho(t)$$

and define $H \equiv \frac{\dot a}{a}$ and $M_{Pl}^2 = \frac{1}{8\pi G}$



H describes the rate of growth of e.g. a sphere

Let me define also:

$\rho_{crit} = \frac{3H^2}{8\pi G}$, ~~corr. to K~~ which is the en. density for $K=0$, and the abundancies

$$\Omega_i \equiv \frac{\rho_i}{\rho_{crit.}}$$

Then FE: $1 = \Omega_m + \Omega_r + \Omega_\Lambda + \Omega_K$.

Behavior of ρ during matter and radiation domination

Matter: # of particles ~~per unit volume~~ doesn't change, but the volume scales with $a(t)$, so ~~there~~

is $a \to a/2$, thus 8 more parts $\times$ 1 vol. ~~$a \to a/2$~~ ~~$\to$~~

$\Rightarrow \quad \rho a^3 = \text{const.}$ (density of particles goes like $1/V$)

Radiation: # of photons doesn't change. So $n_\gamma \sim \frac{1}{a^3}$, but also wavelengths scale as $\frac{1}{a}$ a and:

$$\to \quad \rho a^4 = \text{const.}$$

Cosm. const.: $\rho = \text{const.}$

$\to$ Evolution of the universe as dictated by these two energy densities:

Radiation domination $\to$ Matter domination.

- There is a time at which $\rho_\gamma \sim \rho_M$: t_{MR}.
- There is a time at which $T_{UNIV} \sim eV$ (atomic scales)

The mean free path of photons due to Thomson scattering with electrons is short:

$$\lambda \sim \frac{1}{n_e \sigma_T} \qquad \sigma_T \sim r_e^2 = \frac{\alpha}{m_e}$$

It turns out that the times at which the universe became ~~opaque~~ trans. and is almost the same as t_{MR}.

$$t_{\gamma DEC} \simeq 10^6 \text{ yrs}$$

$$t_{MR} \simeq 10^5 \text{ yrs.}$$

Shortcomings of Big FRW cosmology

→ Flatness problem: let me ~~int~~ define some energy density associated to curvature as:

$$\rho_K \approx -\frac{K}{a^2}$$

It scales much slower than matter and radiation.

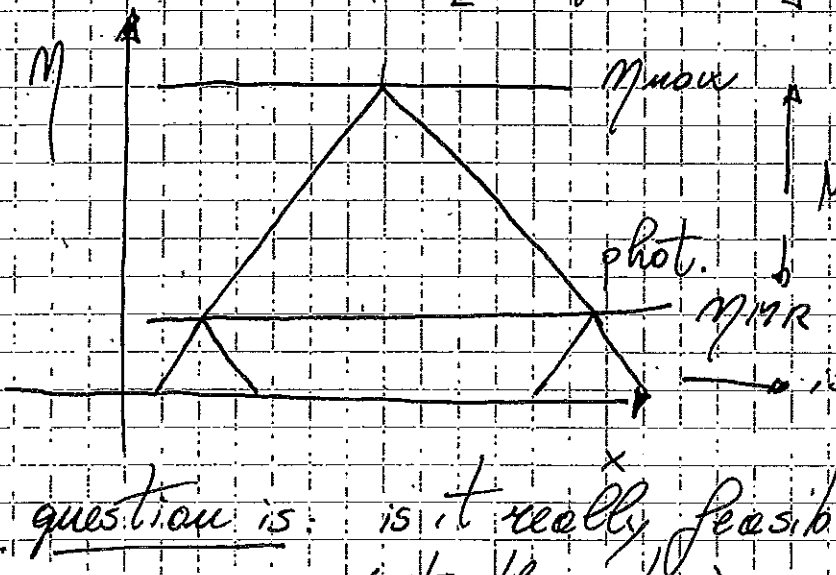
→ Why didn't curvature dominate then? → You need to assume that ~~it was~~ K was always $\ll H^{-1}$

(make comparison with c.c. problem)

→ Horizon problem: let me put $K=0$ from now on.

$$ds^2 = a^2(t)\left[-\frac{dt^2}{a^2(t)} + d\vec{x}^2\right] \quad \text{and define } d\eta = \frac{dt}{a(t)}$$

$$\rightarrow ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right] \quad \text{conformally flat}$$



→ is there enough time to be in contact?

The question is: is it really feasible that the universe (at 0th order) ~~seems~~ is homogeneous and isotropic? Is it feas. tha T is almost the same everywhere?

→ we should have some ~~way of~~ causal connection between diff. regions of space.

Scaling of ρ with time $\quad \left(\frac{\dot a}{a}\right)^2 \propto \rho$

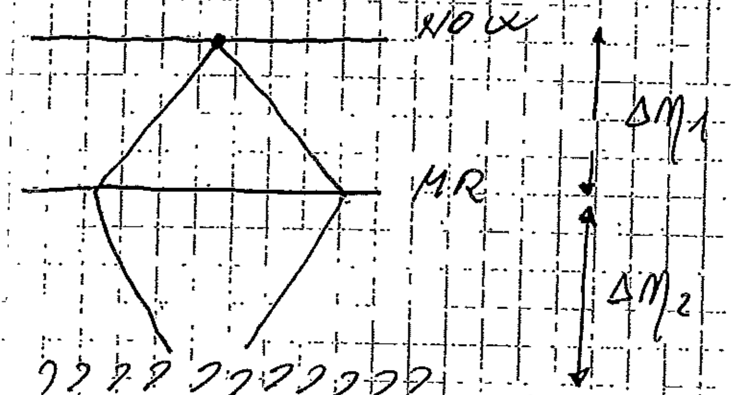
- $\rho a^3 = \text{const} \Rightarrow \frac{\dot a^2}{a^2} = \frac{c}{a^3} \Rightarrow a^{1/2}\dot a = \text{const}$

$$\Rightarrow \dot{\left(a^{3/2}\right)} = \text{const} \Rightarrow a \sim t^{2/3}$$

- $\rho a^4 = \text{const} \Rightarrow a \sim t^{1/2}$

- $\rho = \text{const} \Rightarrow \frac{\dot a}{a} = \text{const} \Rightarrow a \sim e^{ct}$

→ Is there enough conformal time between the start and η_{MR} to ~~be~~ have all the regions in causal contact?



Let us compute:

$$\Delta\eta_1 = \int_{t_{MR}}^{t_{now}} \frac{dt}{a} = \int \frac{dt}{a_0\left(\frac{t}{t_0}\right)^{2/3}} =$$

$$= \frac{t_0}{a_0}\left[1 - \left(\frac{t_{MR}}{t_0}\right)^{1/3}\right] \simeq \frac{t_0}{a_0}$$

The integral is dominated by late times!

$$\Delta\eta_2 \simeq \frac{t_{MR}}{a_{MR}} = \frac{t_{MR}}{a_0\left(\frac{t_{MR}}{t_0}\right)^{2/3}} = \left(\frac{t_0}{a_0}\right) \times \left(\frac{t_{MR}}{t_0}\right)^{1/3}$$

So it's clear that $\Delta\eta_2 \ll \Delta\eta_1$.

→ How many ~~independent regions~~ causally disconnected regions?

$$\#\text{ disc. regions} \sim \left(\frac{\Delta\eta_1}{\Delta\eta_2}\right)^3 \approx \frac{t_0}{t_{MR}} \sim \frac{10^{10}\text{ yrs}}{10^5\text{ yrs}} \sim 10^5\,!$$

→ No causal mechanism can explain ~~this~~ the observed homogeneity.

How to solve the Horizon problem

η_0

η_{MR}

↓ I want to extend this region

??????????

→ What do I need to make the int. dominated at ~~bat~~ early times?

$$\int_{a_{st}}^{a_{end}} dt/a = \int_{a_{st}}^{a_{end}} \frac{dt}{da}\frac{da}{a} = \int_{a_{st}}^{a_0} \frac{da}{a}\left(\frac{1}{Ha}\right)$$

$\frac{da}{a}$ gives a log div. while eq. contr. from st. to end

So I need to make Ha decrease when $a \to a_{st}$.

$$\Rightarrow \frac{d}{da}(Ha) = a\frac{dH}{da} + H = a\frac{dt}{da}\frac{dH}{dt} + H = \frac{\dot{H}}{H} + H$$

$$= \frac{a}{\dot{a}}\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) + \frac{\dot{a}}{a} = \frac{\ddot{a}}{aH}$$

$\Rightarrow$ we need $\ddot{a} > 0$ $\Rightarrow$ early accelerating period!

→ ~~I assume from now~~

Notice that if I consider $a \sim e^{Ht}$, then:

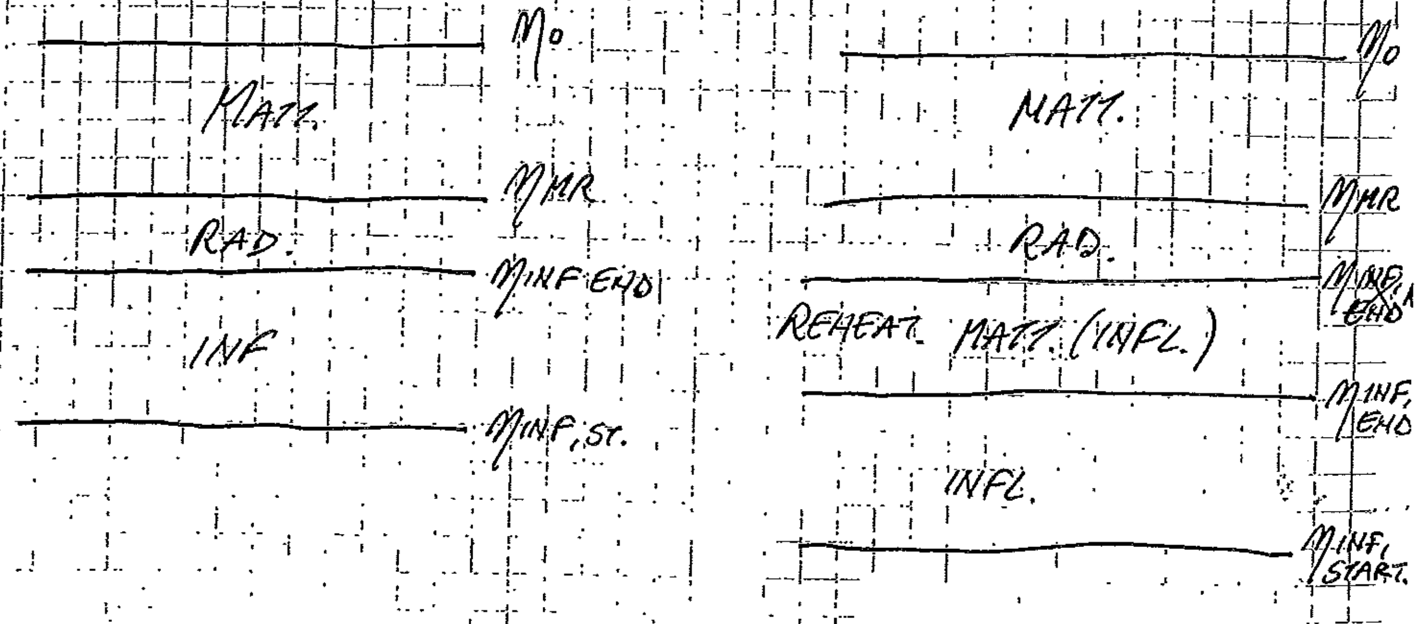
$$\dot{a} = H e^{Ht}, \quad \ddot{a} = H^2 e^{Ht} > 0$$

→ Inflation: I assume $a \propto e^{H_{inf} t}$

$$\Delta\eta_3 = \int d\eta = \int_{a_{st}}^{a_{end}} \frac{da}{a^2 H} \simeq \frac{1}{H_{inf}\, a_{st}} = \frac{1}{H_{inf}\, a_{end}} \left(\frac{a_{end}}{a_{start}}\right) = \frac{1}{H_{inf}\, a_{end}} e^{N_e}$$

where N_e is the number of e-foldings.

⇒ If we have N_e large enough then $\Delta\eta_3$ can win



A NOTE: actual def. of INFL. $\frac{d}{dt}(aH)^{-1} < 0$

$$\Rightarrow \epsilon = -\frac{\dot{H}}{H^2} < 1, \text{ for } \epsilon \to 0 \quad a(t) \propto e^{Ht}$$

~~I use~~ How many e-foldings? $\quad H_{inf} t_0 \sim H_{inf} \frac{a}{\dot a}$

→ We have to impose $\Delta\eta_{INF} = \frac{e^{N_e}}{H_{INF}\, a_{end}} = \frac{t_0}{a_0}$.

So let's rewrite a_{end}:

$$a_{end} \sim a_0 \times \left(\frac{t_{MR}}{t_0}\right)^{2/3} \times \left(\frac{t_{RH}}{t_{MR}}\right)^{1/2} \times \left(\frac{t_{INF}}{t_{RH}}\right)^{2/3}$$

So:

$$e^{N_e} \sim \left(\frac{H_{INF}}{H_0}\right)\left(\frac{H_0}{H_{MR}}\right)^{2/3}\left(\frac{H_{MR}}{H_{RH}}\right)^{1/2}\left(\frac{H_{RH}}{H_{INF}}\right)^{2/3}$$

$$= \left(\frac{H_{INF}}{H_0}\right)^{1/3}\left(\frac{H_{RH}}{H_{MR}}\right)^{1/6}$$

→ During rad. domination: $\rho \sim T^4$ so $H^2 \simeq \frac{\rho}{3M_{PL}^2}$

I can have a natural definition of temperature $\Rightarrow H \sim \frac{T^2}{M_{PL}}$

$$\longrightarrow e^{N_e} \sim \left(\frac{H_{INF}}{H_0}\right)^{1/3}\left(\frac{T_{RH}}{T_{MR}}\right)^{1/3}$$

$$= \left(\frac{10^{17}s}{10^{-43}\cdot 10^{6}s}\right)^{1/3}\left(\frac{10^{16}\,GeV}{10^{-9}\,GeV}\right)^{1/3}\left(\frac{H_{INF}}{10^{14}\,GeV}\right)^{+1/3} \times$$

$$\times \left(\frac{T_{RH}}{10^{16}\,GeV}\right)^{+1/3} \simeq \left\| 61 + \frac{1}{3}\log\left(\frac{H_{INF}}{10^{14}\,GeV}\right) + \frac{1}{3}\log\left(\frac{T_{RH}}{10^{16}\,GeV}\right)\right\|$$

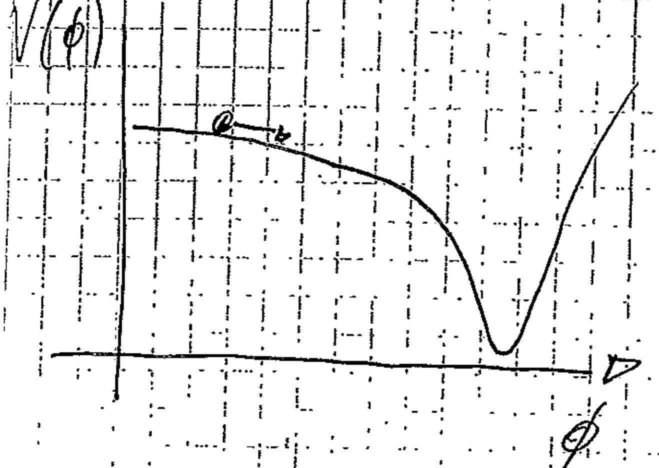
$$\Longrightarrow \boxed{N_e \approx 50-60 \quad 10\,\text{MeV} \lesssim T_{RH} \lesssim 10^{16}\,\text{GeV}}$$

10 MeV ~ temp. of nucleosynthesis, and here assume that Big Bang nucleosynthesis holds

Slow-roll inflation

→ As we have just seen, we need $p \approx$ const early in the universe to have accelerated expansion.

→ We will use a scalar field to realise inflation (from the point of view of EFT ~~it is natural~~ to there is a scalar field associated around the background that we want.)



→ something like this.

We take: $\phi \equiv \phi(t)$

$$S = \int d^3x\, dt\, a^3(t) \left[\frac{1}{2}\dot{\phi}^2 - V(\phi)\right]$$ ignoring the spatial dependence of ϕ.

E.O.M. $\quad \partial_t \left(a^3 \dot{\phi}\right) + a^3 V'(\phi) = 0$

$$\Downarrow$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

↳ Friction!

Slow roll: 1) $V \gg \dot{\phi}^2$ we want to move slowly along the ~~traject~~ potent.

2) $\ddot{\phi} \ll H\dot{\phi}$ so that friction determines the rolling velocity

$$\Rightarrow \dot{\phi} = -\frac{V'}{3H}$$

The ϵ and η parameters

In terms of the potential:

1) $\dot{\phi}^2 \ll V \Rightarrow \frac{V'^2}{H^2} \ll V$ and since $H^2 \sim \frac{V}{M_{PL}^2}$

$$\Rightarrow \boxed{\frac{M_{PL}^2 V'^2}{V^2} \ll 1}, \quad \boxed{\epsilon = \frac{1}{2}\frac{M_{PL}^2 V'^2}{V^2}}$$

2) $\ddot{\phi} \ll H\dot{\phi} \Rightarrow \boxed{\frac{V'' M_{PL}^2}{V} \ll 1}, \quad \boxed{\eta = \frac{M_{PL}^2 V''}{V}}$

Slow-roll conditions: $\epsilon, \eta \ll 1$

# of e-foldings: $e^{N_e} = \frac{a_{end}}{a_{st.}} \simeq \frac{e^{Ht_{end}}}{e^{Ht_{st.}}} = e^{H\Delta t}$

$$\Rightarrow \boxed{dN_e \sim H dt} = \frac{H}{\dot{\phi}} d\phi \simeq \frac{d\phi}{M_{PL}} \frac{3H^2}{V'} \cdot \frac{d\phi}{M_{PL}}$$

$$= d\phi \frac{V}{V' M_{PL}^2} = \frac{d\phi}{M_{PL}} \frac{1}{\sqrt{2\epsilon(\phi)}}$$

ROUGH ESTIMATE

$$\Rightarrow N_e = \int_{\phi_{IN.}}^{\phi_{end}} \frac{d\phi}{M_{PL}} \frac{1}{\sqrt{2\epsilon(\phi)}} \sim \frac{\Delta\phi}{M_{PL}} \frac{1}{\sqrt{\epsilon}}$$

RK: obviously ϵ cannot be const., because infl. has to stop.

→ This leads to the following classification:

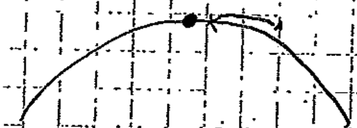
$\epsilon \sim 10^{-2}$, $\Delta\phi \gtrsim 10\, M_{PL}$. 1. LARGE FIELD MODELS

e.g. $V(\phi) = \frac{1}{2} m^2 \phi^2$ (Linde '83)
$= c\,\phi^p$

$\epsilon \ll 10^{-2}$, $\Delta\phi \ll M_{PL}$ 2. SMALL FIELD MODELS

e.g. ϵ very small, we need to be close to $V' = 0$

$\boxed{\eta < 0 \text{ from DATA}}$



e.g. $V = V_0\left[1 - \alpha \frac{\phi^2}{M^2}\right]$

$\epsilon \ll \eta$

1. LARGE FIELD

$$\epsilon = \frac{1}{2} \frac{M_{PL}^2 V'^2}{V^2} = \frac{1}{2} \frac{M_{PL}^2\, p^2}{\phi^2}, \quad \eta = p(p-1)\left(\frac{M_{PL}}{\phi}\right)^2$$

$$N_e = \int \frac{d\phi}{M_{PL}} \frac{1}{\sqrt{2\epsilon}} = \int \frac{d\phi}{M_{PL}} \frac{1}{p} \frac{\phi}{M_{PL}} = \frac{1}{2p}\left(\frac{\phi_{IN}^2}{M_{PL}^2}\right)$$

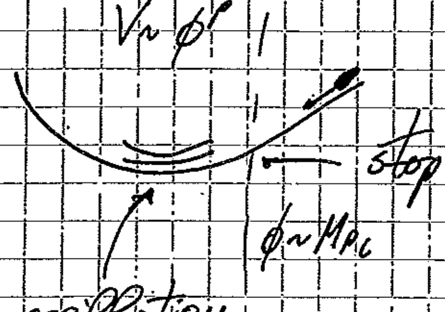
if $\phi_f \simeq 0$

So we can rewrite ϵ and η as functions of N_e:

$$\epsilon = \frac{p}{4N_e}, \quad \eta = \frac{p-1}{2N_e} \quad (\epsilon \sim \eta \sim 0.01)$$

Also Lyth bound: $\frac{\Delta\phi}{M_{PL}} \gtrsim \left(\frac{r}{0.01}\right)^{1/2}$

→ **Reheating** (ie $m^2\phi^2$ ...)



of a massive part. with $\omega \sim m_{inf}$

Hybrid infl.
1 more sc field which start has $m^2 < 0$ at the end of infl.
↙ go in fall in a new min.
↳ release all energy into SM.

DIGRESSION

$T_{RH} \sim V_{INF}$ ←

Reheating through decay

$\Gamma \sim$ ($\phi \to \psi\psi$) $\sim g^2 \frac{m}{32\pi}$

$\Gamma \sim H$ for out of eq.

$\rho_{INFL} \sim H^2 M_{PL}^2 \sim \Gamma^2 M_{PL}^2 \sim T_{RH}^4$

$\Rightarrow T_{RH} \sim \sqrt{\Gamma M_{PL}} \sim \sqrt{\frac{g^2}{32\pi}} \sqrt{\frac{m}{H}} \times \underbrace{\sqrt{\frac{H}{M_{PL}}}}_{T_{RH}^{max}}$

T_{RH} generally less than T_{GUT}.

DENSITY PERTURBATIONS

ADIABATIC: perturbs which look universal for every comp. of energy.
ISOCURVATURE: doesn't change the en. density, but moves it through the comp.

Today: $\frac{\delta\rho}{\rho} \sim 1$

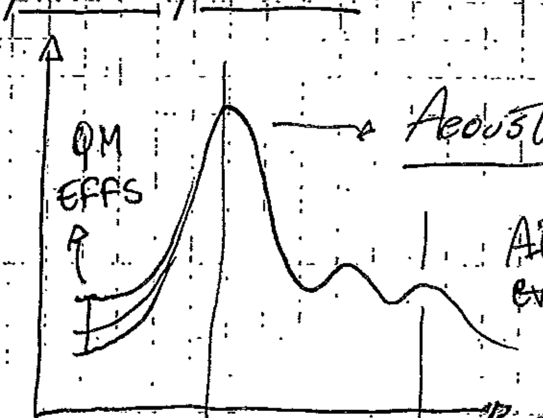
Early in the universe: almost homogeneous, but some places were hotter than others. (we obs. $\left(\frac{\delta\rho}{\rho}\right) \sim 10^{-5}$)

Then the universe cools and becomes matter dom. Hotter regions have higher ρ. Gravity then makes those regions collapse under their own gravit. pull. Inhomogeneities grow, until today.

MEASUREMENT: CMB ANISOTROPIES

$$T(\theta,\varphi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\varphi)$$ Temperature field in spherical harmonics.

Estimator for the power spectrum $\hat{C}_\ell = \frac{1}{2\ell+1} \sum_m |a_{\ell m}|^2$



→ Acoustic peaks

~~Also on~~ ev. sign of inflation

Cosmic variance

~~Var~~

$$\mathrm{Var}(\hat{C}_\ell) = \langle \hat{C}_\ell \hat{C}_\ell \rangle - \langle \hat{C}_\ell \rangle^2 = \frac{2}{2\ell+1} C_\ell^2$$

where $C_\ell = 2\pi \int_{-1}^{1} d\cos\theta \, C(\theta) P_\ell(\cos\theta)$

→ This error arises from having only $2\ell+1$ modes at each ℓ

to estimate the variance of their distribution.

<u>QM.</u> : n modes, $\frac{1}{\sqrt{n}}$ uncertainty.

<u>RK</u>: CMB anis. span super-hor. scales at recomb. and have coherent phases.

<u>2-point correlation fcts</u>

$$\rightarrow \left\langle \frac{\delta\rho(\vec{x})}{\rho} \frac{\delta\rho(\vec{y})}{\rho} \right\rangle = \int d^3k \, P(\vec{k}) \, e^{i\vec{k}\cdot(\vec{x}-\vec{y})}$$

↳ Power spectrum.

By dim. analysis $P(k) \sim \frac{1}{k^3}$ what fits the data is

$$\boxed{P(k) \sim \frac{(10^{-5})^2}{k^3} \quad \text{scale-invariant spectrum}}$$

→ In position space:

$$\left\langle \frac{\delta\rho(x)}{\rho} \frac{\delta\rho(y)}{\rho} \right\rangle \doteq (10^{-5})^2 \ln|\vec{x}-\vec{y}|$$

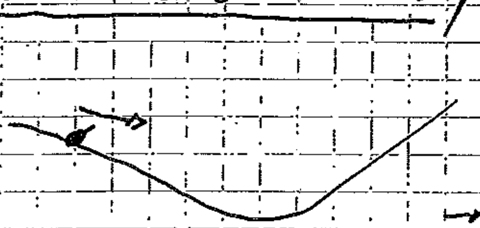
<u>Observationally</u>:

→ <u>Gaussanity</u>: 1. $\left\langle \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \right\rangle \simeq 0$

2. $\left\langle \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \right\rangle \approx \left\langle \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \right\rangle \times \left\langle \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \right\rangle$

<u>Scalar and tensor perturbations</u> : rough estimate.

<u>Slow-roll</u> V'' $\eta \ll 1$.



→ So we can cons. think of ϕ as a massless field. ? V''

$m_{\mathrm{inf}} \ll H_{\mathrm{inf}}$

→ The only scale in the problem is set by H_{inf}.

to estimate the variance of their distribution.

QM. : n modes, $\frac{1}{\sqrt{n}}$ uncertainty.

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2-point correlation fcts

$$\rightarrow \quad \left\langle \frac{\delta\rho}{\rho}(\vec{x}) \quad \frac{\delta\rho}{\rho}(\vec{y}) \right\rangle = \int d^3k \, P(k) \, e^{i\vec{k}\cdot(\vec{x}-\vec{y})}$$

$\hookrightarrow$ Power spectrum.

By dim. analysis $P(k) \sim \frac{1}{k^3}$ what fits the data is

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$\rightarrow$ In position space:

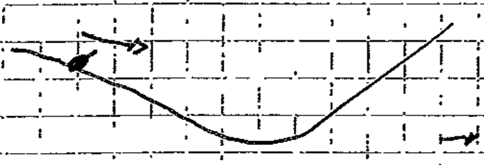
$$\left\langle \frac{\delta\rho(x)}{\rho} \quad \frac{\delta\rho(y)}{\rho} \right\rangle = (10^{-5})^2 \ln|\vec{x}-\vec{y}|$$

Observationally:

$\rightarrow$ Gaussianity: 1. $\left\langle \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \right\rangle \approx 0$

2. $\left\langle \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \right\rangle \sim \left\langle \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \right\rangle \times \left\langle \frac{\delta\rho}{\rho} \frac{\delta\rho}{\rho} \right\rangle$

Scalar and tensor perturbations : rough estimate.



Slow-roll $\eta \ll 1$. (V'')

$\rightarrow$ So we can ~~cons.~~ think of ϕ as a massless field. (V'')

$m_{inf} \ll H_{inf}$

$\rightarrow$ The only scale in the problem is set by H_{inf}.

$\rightarrow \delta\phi \sim H_{inf}$

How does this translate into $\delta\rho$?

Observationally, we need adiabatic perturbations
? fluct. in ~~Ł~~

$\rightarrow$ The picture is:

1. Classically: 1 time t at which infl. ends.

2. QMlly: the ~~time~~ surface at which infl. ends fluctuates

Magnitude of the perts.

$$\left(\frac{\delta\rho}{\rho}\right) \sim \delta N_e \sim H\delta t \sim \frac{H\delta\phi}{\dot\phi}, \quad \delta\phi \sim H, \quad \dot\phi \sim -\frac{V'}{3H}$$

$$V \simeq H^2 M_{PL}^2$$

so:

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot\phi} \simeq \frac{H^3}{V'} \sim \frac{M_{PL}H^3}{V\sqrt{\epsilon}} \sim \frac{H}{M_{PL}}\frac{1}{\sqrt{\epsilon}}$$

$$\Rightarrow k^3 P(k) \sim \left(\frac{H^2}{M_{PL}^2\epsilon}\right)$$

$n_s = 0.9603 \pm 0.0073$

$n_s = 1$ excl. at 6σ

Tilt of the scale invariant spectrum

$\rightarrow$ The spectrum is not exactly scale invariant, because H is changing slowly.

$$\rightarrow [k^3 P_s(k)] = \left(\frac{H_0^2}{M_{PL}^2\epsilon_0}\right)\left(\frac{k}{k_0}\right)^{\overbrace{(n_s-1)}^{\delta_s}}$$

$\rightarrow$ some ref. scale

n_s

where $\delta_s = n_s - 1 = \frac{d}{d\log k}\log(k^3 P_s)$

Let's express δ_S at leading order in the slow roll params.

$$\mathcal{H}\frac{d}{d\mathcal{H}} \to \frac{1}{H}\frac{d}{dt} = \frac{\dot{\phi}}{H}\frac{d}{d\phi}$$
(like $\frac{d\rho}{\rho} \sim H dt$)

$$\epsilon = \frac{1}{2} M_{Pl}^2 \frac{V'^2}{V^2}$$

$$H^2 M_{Pl}^2 \simeq V$$

so:

$$\delta_S = \frac{\dot{\phi}}{H}\frac{d}{d\phi}\log\left(\frac{H^2}{M_{Pl}^2\epsilon}\right) = \frac{\dot{\phi}}{H}\frac{d}{d\phi}\log\left(\frac{V^3}{V' M_{Pl}^6}\right)$$

$$= -\frac{V'}{3H^2}\frac{d}{d\phi}\ln\frac{V^3}{V' M_{Pl}^6} = -\frac{V'}{3H^2}\left[\frac{3V'}{V} - 2\frac{V''}{V'}\right] =$$

$$= \boxed{-6\epsilon + 2\eta = n_s - 1}$$ (few %) ($n_s \sim 0.96$

RK: not both ϵ and η can be small in to agree with data, also, $\eta < 0$.

Tensor perturbations: gravitons are produced during inflation (1 grav. every Hubble).

$$\mathcal{H}^3 P_{gr}(\mathcal{H}) \sim \frac{H^2}{M_{Pl}^2}$$ (ϵ det. the size of $\frac{\delta\rho_s}{\delta\rho_T}$)

Similar calc.

$$\boxed{n_T - 1 = -2\epsilon}$$

Tensor to scalar ratio: $r = \frac{P_{GRAV.}}{P_{SCAL}} \sim 16\epsilon$

$\hookrightarrow$ just convention

RK: 2 params, 3 rels. $\Rightarrow$ Consistency condition

$$\boxed{r = -8(n_T - 1)}$$

Lyth bound: $N_e \sim \frac{\Delta\phi}{M_{Pl}}\frac{1}{\sqrt{\epsilon}}$ &

$\frac{\Delta\phi}{M_{Pl}} \gtrsim \left(\frac{r}{0.01}\right)^{1/2}$ from $N_e \gtrsim 30 \div 50$

For large field mdl. $V \sim \phi^p$

$$\delta_s = -6\epsilon + 2\eta = -\frac{(p+2)}{2N_e}$$

$$r = 16\epsilon = \frac{4p}{N_e}$$ depends on T_{RH}.

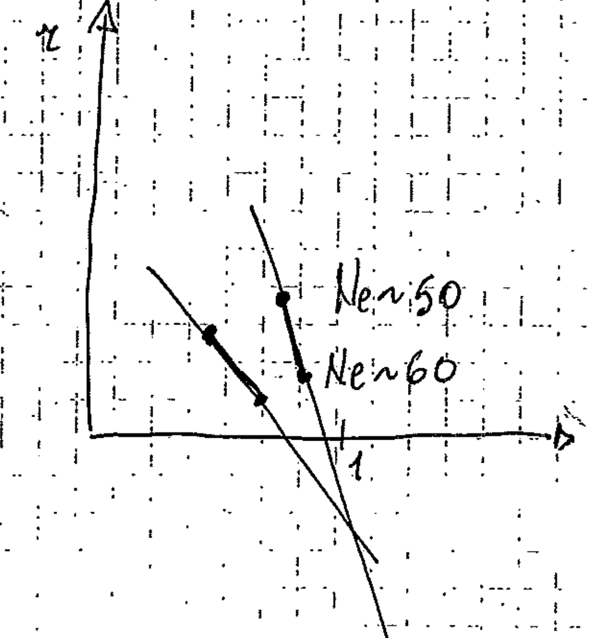
E.g.

$p=2$ concave up — $r = .13 \to .16$ ($N_e=60$, $N_e=50$) — $n_s - 1 = .97 - .96$

$p=1$ FLAT — $r = .065 - .08$ — $.975 - .97$

$\frac{r}{n_s}$ is indep of N_e (T_{RH})

$$r = -\frac{8p}{p+2}(n_s - 1) \Rightarrow p = \frac{2}{8\left(-\frac{\delta s}{r}\right) - 1}$$



Show plot from Planck

NB

Some more numbers:

$$r \sim .1 - .2 \Rightarrow \epsilon \sim 10^{-2}$$

$$\frac{H^2}{M_{PL}^2 \epsilon} \sim 10^{-10} \Rightarrow H_{INF} \sim 10^{14}\ GeV.$$

$$V \sim (10^{16}\ GeV)^4 \sim M_{SUSY\ GUT}$$

$\to V \sim m^2 \varphi^2$, $\varphi \sim 10\, M_{Pl}$ $\rightsquigarrow$ $m_{inf} \sim 10^{13}$ GeV

<u>Conclusions</u>:

$$\begin{array}{ll} \rule{2cm}{0.4pt} & 10^{19}\ \text{GeV} \\ \rule{2cm}{0.4pt} & 10^{16}\ \text{GeV} \\ H\ \rule{2cm}{0.4pt} & 10^{14}\ \text{GeV} \\ m_{inf}\ \rule{2cm}{0.4pt} & 10^{13}\ \text{GeV} \end{array}$$

<u>IMPORT. NUMBS.</u>

n_s $\quad 0.963 \pm 0.019$ $\quad$ without running
r $\quad < 0.115$

Ω_K $\quad -0.072 \pm 0.081$ (incl. it in the fit)

<u>Bicep?</u> $\quad r = 0.2^{+0.07}_{-0.05}$ $\quad$ without foregr. subtr.

<u>Future tests</u>
- $n_T = 2\frac{\dot{H}}{H^2}$, consistency cond.
- running n_s
- Non-gaussianity