The Message from Inflation

→ In the previous lecture: slow-roll inflation can explain the homogeneity of the observed universe.

→ ~~Theoretically~~ From a theoretical point of view: is this the end of the story?

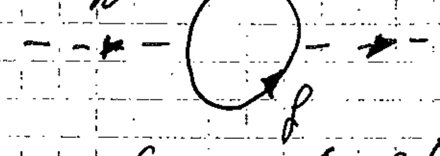
→ Slow-roll inflation uses a scalar field to produce an epoch of accelerated expansion in the early universe.

→ Scalar fields generically introduce problems of hierarchies. More precisely:

→ the running mass of a scalar particle takes contributions from the masses of any particle it couples to

→ m_ϕ renormalizes additively!

E.G.: the Higgs boson



$$\mathcal{L}_{YUK} = -y h \bar{f}_L f_R + h.c.$$

$$\delta m_h^2 \sim \sum_f \frac{y^2}{4\pi^2}\left[\Lambda_{UV}^2 - 6 m_f^2 \ln \frac{\Lambda_{UV}}{m_f}\right]$$

→ How does the problem appear for the inflaton?

Effective field theories

In order to answer this question we need to ~~know~~ investigate the influence of heavier degrees of freedom on ϕ.

→ We need a treatment in EFT:

→ Key point: dynamics at large length scales does not depend too much on microscopic scales

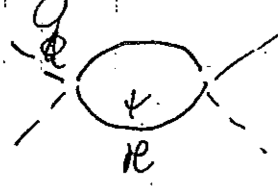
→ Two points of view: → TOP DOWN: we know the full UV theory

$$\mathcal{L} = \underbrace{\mathcal{L}_e[\phi]}_{\text{LIGHT FIELD(S)}} + \mathcal{L}_h[\psi] + \mathcal{L}_{eh}[\phi,\psi]$$

↓

We integrate out the heavy states:

e.g.



$|k| \gg E_\phi$ the loop is small in position space

It just looks pointlike with respect to the long wavelength of the ext. particle →

$\mathcal{L}_{eff}$ = ~~[illegible]~~ $+ \sum_i c_i(g) \frac{O_i[\phi]}{M^{d_i}} = \mathcal{L}_e + \dots$

with O_i of dim. $d_i + 4$

all operators allowed by the symmetries of the UV theory

→ Important: $c_i \sim O(1)$ if M is the characteristic scale of the UV theory. This is the

NATURALNESS CRITERION

→ <u>BOTTOM UP</u>: we don't know the full theory.

We assume some symmetries of the UV theory and write down all the operators allowed by those symmetries

$$\mathcal{L}_{eff}[\phi] = \cancel{\mathcal{L}_{UV}[\phi]} \sum_i c_i \frac{O_i[\phi]}{M^{d_i}}$$

→ <u>NATURALNESS</u>: ~~NP appears at some scale $M \lesssim M_{MAX}$~~ Parameters of $\mathcal{L}$ are natural as long as their measured values are larger than the loop corrections.

→ Used to "guess" M_{MAX}, max scale of validity of the EFT.

→ ~~What happ~~ When we also include gravity:

$$S_{eff}[\phi, g_{\mu\nu}] = \overset{1.}{S_{gr}} + S_{eff}[\phi] + \overset{2.}{S_{g,\phi}}$$

1. → $$S_{gr} = \int d^4x \sqrt{-g}\left[\Lambda^4 + \frac{M_{Pl}^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{1}{M_{(Pl)}^2}\left(d_1 R^3 + \dots\right) + \dots\right]$$

We will take:

$$S_{gr} \simeq \int d^4x \sqrt{-g}\, \frac{M_{Pl}^2}{2} R$$ ignoring Λ^4 and higher order terms.

2. → If the curv. is small in units of $M_{(Pl)}$:

$$S_{g,\phi} \simeq \int d^4x \sqrt{-g}\, \xi \phi^2 R$$

↳ dimensionless, can be set to 0 by Weyl rescaling.

~~UV sensit.~~

The problems of Inflation in EFT

Starting point: $S_{eff}[\phi] = \int d^4x \sqrt{-g} \left[\frac{M_{PL}^2}{2} R + \mathcal{L}_{LIGHT} + \right.$

$$+ \sum_i c_i \frac{O_i[\phi]}{M^{d_i}}$$

where O_i have dim $d_i + 4$, $d_i > 0$.

→ When do we cutoff?

EFT has to be valid at and below this scale ← $H \lesssim M \lesssim M_{PL}$ → always the maximal cutoff.

→ UV SENSITIVITY

1. ETA PROBLEM I: $\Delta m_\phi^2 \sim \lambda^2 M_{UV}^2$.

⇒ $\eta = \frac{M_{PL}^2 V''}{V}$ ⇒ $\Delta\eta \sim \frac{M_{UV}^2}{H^2}$ and taking

$M_{UV} \gtrsim H$ we have:

$$\boxed{\Delta\eta \gtrsim 1}$$ which violates the SLOW ROLL CONDITION

Possible solutions:

a) Fine tuning (?);

b) Symmetries; in particular, as we're familiar with from the Hierarchy problem:

→ SUSY : but ~~t~~ $\rho_{\sqrt{V_{AC}}}$ during inflation is positive!

⇒ SUSY ~~a~~. However the problem is ameliorated, because high frequency modes don't see spacetime curvature.

$$\Rightarrow \quad \Delta m^2 \sim \mathcal{H}^2 \quad \text{(only scale is Hubble)}.$$

$$\Rightarrow \underline{\text{still}} \ \underline{\Delta\eta \sim 1} \ !$$

How much tuning?

$$-6\epsilon + 2\eta = n_s - 1 \quad , \quad n_s \approx 0.96$$

→ <u>SMALL FIELD</u> : $\epsilon \ll \eta \Rightarrow |\eta| \approx \frac{1}{2}(n_s - 1) \approx 0.02$

<u>% level</u>.

~~→ LARGE FIELD : $\epsilon \sim \eta \Rightarrow -4\eta = n_s - 1$~~

~~$\epsilon \sim 10^{-2}$ ... $\Delta\eta = (n_s-1) + 6 \cdot 10^{-2}$~~

~~$\Rightarrow \eta = \frac{1}{2}(n_s - 1) + 6 \cdot 10^{-2}$~~

→ <u>GLOBAL SYMM.</u> : e.g. <u>SHIFT SYMM.</u> of the ren. part of the effective Lagrangian.

$$\phi \to \phi + \text{const}.$$

(~~coming~~ symptomatic of spont. broken symmetries).

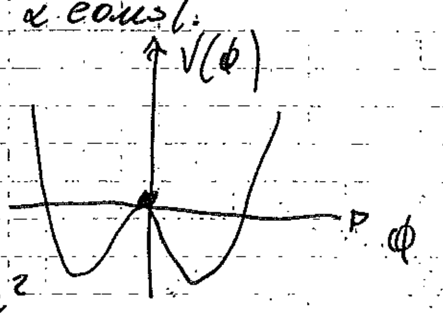
<u>DIGRESSION</u> : LINEAR SIGMA MODEL

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi + m^2 \phi \phi^* - \frac{\lambda}{4} \phi^2 \phi^{*2}$$

<u>GLOB. U(1)</u> : $\phi(x) \to e^{i\alpha} \phi(x)$, α const.

→ ~~$m \neq 0$~~ $m^2 > 0$

~~Unstable symmetry~~



→ $V(\phi)$ minimised by $|\phi|^2 = \frac{2m^2}{\lambda}$

→ Infinite number of equiv. vacua $|\Omega\rangle$.

Pick $|\Omega\rangle$ s.t. $\langle\Omega|\phi|\Omega\rangle$ is real $\Rightarrow$ $\langle\Omega|\phi|\Omega\rangle = v = \sqrt{\frac{2m^2}{\lambda}}$

→ Expand around v in terms of two real fields:

$$\phi(x) = \left(\sqrt{\frac{2m^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma(x)\right) e^{i\frac{\pi(x)}{f_\pi}}$$

⇒ $V(\phi)$ is independent of π ⇒ π is massless

→ $$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 + \left(\sqrt{\frac{2m^2}{\lambda}} + \frac{1}{\sqrt{2}}\sigma(x)\right)^2 \frac{1}{f_\pi^2}(\partial_\mu\pi)^2 + V(\sigma)$$

⇒ π enjoys a shift symmetry, the residual of the broken $U(1)$:
$$\phi \to e^{i\theta}\phi$$
$$\pi(x) \to \pi(x) + f_\pi\theta, \quad \sigma \to \sigma.$$

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2. __ETA PROBLEM II__ : is the continuos global symmetry ~~respected by the pre~~ preserved in the UV theory?

⇒ __Consider__ e.g. $O_6 = c\,\underbrace{V_L(\phi)}_{\text{ren. potential}}\,\frac{\phi^2}{\Lambda^2}$ ⇒ $\Delta V \ll V(\phi)$ if $\langle\phi\rangle \ll \Lambda$

⇒ However:
$$\Delta\eta = \frac{M_{Pl}^2(\Delta V)''}{\Delta V} \approx 2c\,\frac{M_{Pl}^2}{\Lambda^2}$$ and if $c \sim O(1)$, $\Lambda < M_{Pl}$

⇒ __SLOW-ROLL CONDITION VIOLATED__

$$\boxed{\text{Generically: } \ldots + \frac{\phi^6}{M_{Pl}^6} + \ldots}$$

→ <u>QUESTION</u>: can these higher dim. operators be absent (suppressed) in the UV?

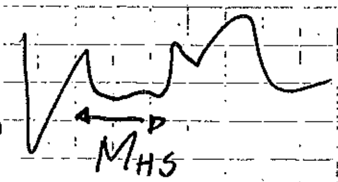
→ The answer requires a knowledge of <u>leading</u> <u>corrections to the</u> <u>infl. Lagrangian</u>.

~~We need This motivates~~

→ Concerning <u>large field inflation</u>

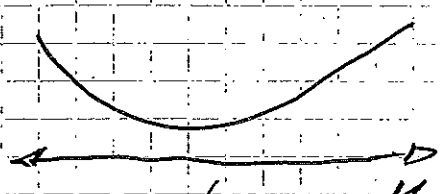
→ Assume a UV theory with ~~new~~ some additional heavy d.o.f. The EFT of ϕ is charact. by higher-order ops. & in ϕ, suppressed by powers of $M_{\substack{HEAVY \\ STATES\,UV}}$.

→ So generically the inflationary potential will exhibit ~~an~~ ~~approx~~ structures at scales $\Delta\phi \sim M_{\substack{HEAVY \\ STATES}}$



M_{HS}

→ but for quadratic inflation we need



$\Delta\phi \approx 5\text{-}10\, M_{Pl}$

→ Also kinetic-term gets corrected.

→ These issues can be avoided using a <u>shift</u> <u>symmetry</u> $\phi \to \phi + \text{const.}$

→ GOAL FOR ~~H~~ LARGE FIELD INFL. <u>IN STRING THEORY</u>.

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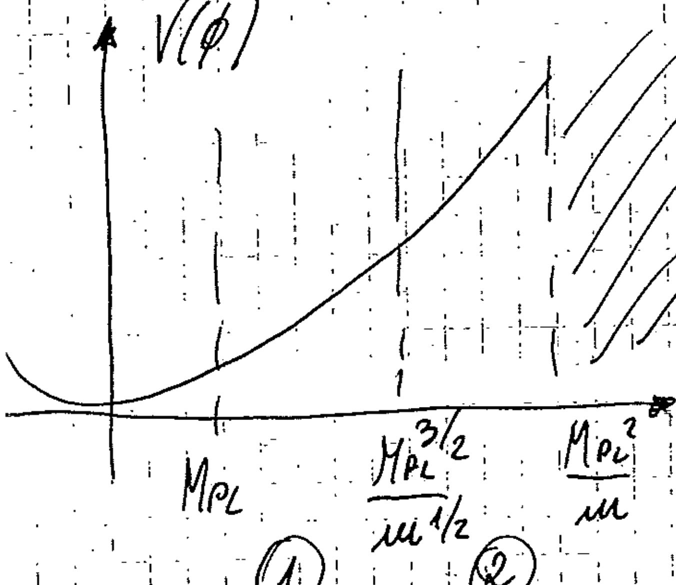
DIGRESSION : more about $m^2\phi^2$ ~~am~~

$V \sim m^2\phi^2 \sim H^2 M_{PL}^2$



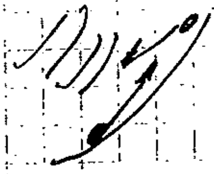
$$\frac{\delta\rho}{\rho} \sim \frac{H}{M_{PL}\sqrt{\epsilon}} \sim \frac{H\varphi}{M_{PL}^2} \sim \frac{m\,\varphi^2}{M_{PL}^3}$$

$\rightarrow \frac{\delta\rho}{\rho} \sim 1$ for $\varphi \sim \frac{M_{PL}^{3/2}}{m^{1/2}}$

$\rightarrow$ Planckian en. densities are reached for:

$$m^2\phi^2 \sim M_{PL}^4$$
$$\Rightarrow \phi \sim \frac{M_{PL}^2}{m}$$

$\downarrow$ Eternal inflation: $\frac{\delta\rho}{\rho} \sim 1$, perturbations are comp. to the cross. traj.



DIGRESSION : again $U(1)$ GOLDSTONE's (~AXIONS)

$$\mathcal{L} = \lambda\left(|\psi|^2 - F^2\right)^2 \qquad \psi \to e^{i\alpha}\psi$$

$$\rightarrow \psi = (F+h)\,e^{i\theta}$$

$$\rightarrow m_h^2 \sim \lambda F^2, \quad m_\theta = 0$$

$$\rightarrow \mathcal{L}_\theta = F^2(\partial\theta)^2$$

$\rightarrow$ Let's ~~breal~~ break it weakly to $\mathbb{Z}_2$:

$$\mathcal{L} = \lambda\left(|\psi|^2 - F^2\right) - m^2\left(\psi^2 + \psi^{*2}\right)$$

$$\Rightarrow \mathcal{L}_\theta = F^2(\partial\theta)^2 + m^2F^2(1-\cos 2\theta)$$

→ Inflaton: $\varphi = F\theta$

$$\mathcal{L}_\varphi = (\partial\varphi)^2 + m^2 F^2 \left(1 - \cos\frac{2\varphi}{F}\right) \sim$$

$$\sim (\partial\varphi)^2 - m^2\varphi^2 + \ldots$$

STRING COMPACTIFICATIONS

GF

→ String theory

1. Motivation: SM is ~~an~~ to be considered an EFT.

→ inclusion of gravity makes it non-renorm.

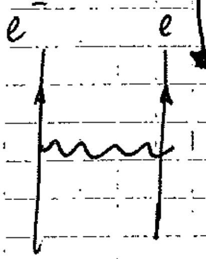
$\alpha_G = G m_e^2 = \left(\frac{m_e}{m_{Pl}}\right)^2 \sim 10^{-45}$

Mass

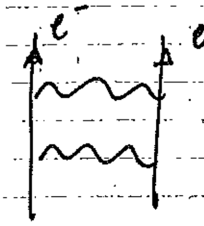
→ SM alone is renormalis., but not asympt. free. (h) sector bos. coupling grows log with energy. Becomes infinite at $M_L \sim m_W e^{b/\alpha}$

$E \sim M_L$ pert. descr. breaks down. (but $M_L \gg M_{Pl}$!)

→ Things ~~that~~ ~~in an~~ the spirit of EFT: $d = 0$:



$= A_1$



$= A_2$

↳ One more power of $G_N \sim M_{Pl}^{-2}$

→ $\frac{A_2}{A_1} \sim G_N \int dE' E'$ ⇒ diverges quadratically in the UV

→ More loops, more divergences!

→ But divergences become large only at $E \sim M_{Pl}$!

→ SM has its own th. problems.

→ Hierarchy probl.: $\mathcal{L}^{d=2} = a\,\mu^2\, \underbrace{H^+ H}$

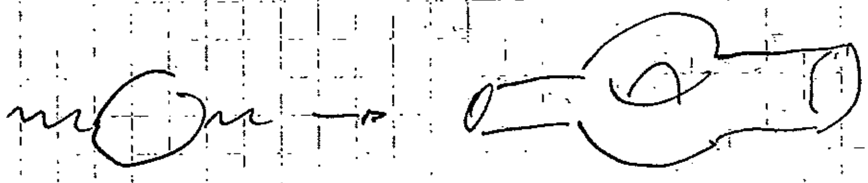
↳ $O(1)$ in units of the cutoff.

→ c.c. probl. $\mathcal{L}^{d=0} = c\,\Lambda^4 \to O(1)$ in units of the cutoff

→ ~~SUSY is~~

→ Solutions: SUSY, compositness, extra-dim. ...

→ String theory offers a unified framework of the elementary forces. Cures the divergences by smearing the interactions.



→ ~~Many~~ solutions of HP, c.c. find easily an incarn. in ST.