

Horizon problem

 $ds^2 = a(\tau)^2 (-d\tau^2 + d\mathbf{x}^2)$







Evolution of the horizon



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• Scalar perturbations: curvature perturbations induced by spatial fluctuation in scalar field.

Power spectrum:
$$\Delta_r^2(k) = rac{k^3}{2\pi^2} \langle |R_k|^2
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• **Tensor perturbations:** gravitational waves.

Power spectrum:
$$\Delta_h^2(k) = 2 \frac{k^3}{2\pi^2} \langle |h_{p,k}|^2 \rangle$$

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Power spectrum:
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Tensor-to-scalar ratio
(normalized amplitude)
$$\equiv \frac{\Delta_{h}^{2}(k)}{\Delta_{R}^{2}(k)} \simeq \left(\frac{V}{\left[2 \times 10^{16} \text{ GeV}\right]^{4}}\right)$$

T

 $(\Delta_R^2(k) \text{ is known from } \langle T^2(k) \rangle, \ H^2 \propto V)$











Why interesting?

Why interesting?



Why interesting?



Model-independent evidence of inflation!

A small motivation for indirect detection



arXiv:0707.3319 [gr-qc]

The CMB - A Screenshot of Primordial Gravitational Waves



GWs 🕞 CMB

- primordial GWs too faint to detect directly
- GWs generate anisotropies in the matter distribution
- at the time of decoupling this results in anisotropies of CMB
- indistinguishable from (more dominant) scalar quantum fluctuations
- however: **B-mode** polarization of CMB is **unique to GWs**!
- power of B-polarized CMB waves (in inflationary low frequency band) gives tensor-toscalar ratio *r*

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Detection of B-modes proves existence of primordial GWs!

What are E- and B-Modes?

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- Information in CMB: temperature $T(\hat{q}\hat{q})$ polarization as function of position $\hat{n} = (\theta, \phi)$
- Qr(\hat{n}) $U(\hat{n})$

• Polarization measured by symmetric, tracéless tensor:

$$\mathcal{P}_{ab}(\hat{n}) = \begin{pmatrix} Q(\hat{n}) & U(\hat{n}) \\ U(\hat{n}) & -Q(\hat{n}) \end{pmatrix}$$



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• Just like $V_a = \nabla_a E + \epsilon_{ab} \nabla_b B$ we decompose

$$\mathcal{P}_{ab} = \nabla_a \nabla_b E + \epsilon_{ac} \nabla_b \nabla_c B$$

(+ symmetrization - trace), where $E(\hat{n})$ is "the ${\bf gradient}$ " and $B(\hat{n})$ is "the ${\bf curl}$ "

 $Qr(\hat{n}) = U(\hat{n})$





Anisotropies in CMB induce two types of polarization: gradient E- and curl B-modes

Reminder: Thomson Scattering

Reminder: Thomson Scattering $\frac{I}{I' \Delta \Omega} = \frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi} |\hat{\epsilon}' \cdot \hat{\epsilon}|^2$









CMB with a monochromatic scalar perturbation

arXiv:1510.06042



CMB with a monochromatic GW

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(qualitatively)

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- with this one checks that $E(\hat{n}^{is})$ even and $B(\hat{n}^{is})$ odd
- since scalar perturbations give rise to (even) scalar functions like $T(\hat{n})$ they cannot source $B(\hat{n})$





Why are B-Modes only generated by GWs?

(qualitatively)

+U

Q - 0, U < 0

B-mode

100% O

+Q

O≻0:U÷0.

-0

 $\mathbf{Q} < \mathbf{D}; \mathbf{U} = \mathbf{D}$

E-mode

(grad)

- from the picture we read off that $Q(\hat{n})$ an even and $U(\hat{n})$ is an odd function under $\mathbb{P} = \{x \to -x\}$
- with this one checks that $E(\hat{n}^{is})$ even and $B(\hat{n}^{is})$ odd
- since scalar perturbations give rise to (even) scalar functions like $T(\hat{n})$ they cannot source
- consider x-polarized GW:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + 2h_{\times}\mathrm{d}x\mathrm{d}y$$

 $\stackrel{\mathbb{P}}{\to} -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 - 2h_{\times}\mathrm{d}x\mathrm{d}y$ read off: amplitude h_{\times} is odd under parity!

GWs are odd and can therefore source B-modes!



3aumann, McAllisto

• any anisotropy in spatial photon power density creates polarization:

$$Q(\hat{n}) - iU(\hat{n}) \propto \int \mathrm{d}\theta' \mathrm{d}\phi' \, \sin^2 \theta' \, e^{2i\phi'} \, I'_{(1)}(\hat{n}, \theta', \phi')$$

• can be decomposed into E- and B-modes after plugging into and into **spherical harmonics** $Y_{lm}(\theta,\phi)$

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- density (scalar) quadrupole rotationally symmetric about plane wave axis $\propto Y_{20}$ GW (tensor) quadrupole not rotationally symmetric $\propto Y_{22}$

 ${\cal P}_{ab}$

• LHS of (1) contains E and B, RHS of (1) contains sources such as GWs; find: B-modes only sourced by GWs $\propto Y_{22}$

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 ${\cal P}_{ab}$

- LHS of (1) contains E and B, RHS of (1) contains sources such as GWs; find: B-modes only sourced by GWs $\propto Y_{22}$
- difficulty: B-mode creation after decoupling via gravitational lensing

What we learn about GWs

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- spectrum of GWs extracted from B-modes test theories of primoridal GWs, this not only includes GWs from fluctuations during inflation, but also phase transitions* (eternal inflation, EW phase transition)
- correlations larger than horizon at the time of decoupling are a test of inflation

*see Thomas' talk

What we learn about GWs

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What we learn about our Universe today

 we also learn about where and how much dust is in the universe, see BICEP2

*see Thomas' talk

Phase transitions in the early Universe



Phase transitions in the early Universe



QCD phase transition: T~O(100) MeV

- deconfinement: " $SU(3)_C \times U(1)_Q \rightarrow U(1)_Q$ "
- chiral SB: $SU(3)_L \times SU(3)_R \to SU(3)_{L+R}$
- → Second order phase transition!

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Electroweak phase transition: T~O(100) GeV

• Higgs mechanism:



Second order PT:

• real scalar field in Mexican hat

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{\lambda}{4} (\phi^2 - v^2)^2$$

- ground state (T=0): $\lambda(\phi^2 v^2)\phi = 0$
- @ high T: symmetry restoration



[T.Prokopec, Lecture notes for cosmology, 2008]

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- field decomposition: $\phi(x) \rightarrow \phi_0 + \delta \phi(x)$
 $$\begin{split} \lambda(\phi_0^2 + 3\langle\delta\phi^2\rangle - v^2)\phi_0 &= 0\\ \text{thermal contribution:} \quad \langle\delta\phi^2\rangle \simeq \frac{T^2}{12} + \mathcal{O}(T) \end{split}$$
- effective scalar potential:

$$V_{\text{eff}}(\phi_0) = \lambda \left(\frac{\phi_0^4}{4} + \frac{\phi_0^2}{2} \left(\frac{T^2}{4} - v^2 \right) \right)$$



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[T.Prokopec, Lecture notes for cosmology, 2008]

- continuous in the order parameter
- thermal equilibrium
- no bubbles for T<T_c

First order PT:

• scalar QED:

$${\cal L} = D_\mu \phi^* D^\mu \phi - rac{\lambda}{4} (\phi^* \phi - v^2)^2 - rac{1}{4} F_{\mu
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• field decomposition(wlog): $\phi_1 = -\phi_0 + \delta \phi_1$

$$\phi_2 = -\delta\phi_2$$



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$$V_{\rm eff}(\phi_0) = \frac{1}{2} \left[\left(\frac{\lambda}{3} + \frac{g^2}{4} T^2 - \lambda v^2 \right) \right] \phi_0^2 - \frac{g^3 T}{4\pi} \phi_0^3 + \frac{\lambda}{4} \phi_0^4 - \frac{g^2 T}{4\pi} \phi_0^3 + \frac{\lambda}{4} \phi_0^4 - \frac{g^2 T}{4\pi} \phi_0^3 + \frac{\lambda}{4} \phi_0^4 - \frac{g^2 T}{4\pi} \phi_0^4 + \frac{\lambda}{4} \phi_0^4 - \frac{g^2 T}{4\pi} \phi_0^4 + \frac{\lambda}{4} \phi_0^4 - \frac{g^2 T}{4\pi} \phi_0^4 + \frac{\lambda}{4} \phi_0^4 + \frac{\lambda}{4} \phi_0^4 - \frac{g^2 T}{4\pi} \phi_0^4 + \frac{\lambda}{4} \phi_0^$$



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[T.Prokopec, Lecture notes for cosmology, 2008]

- T<T_c: maxima develop <u>barriers</u>
- T<T_n: barriers become smaller,
 - tunnelling probability increases
- bubble formation in background of false vacuum

1. bubble nucleation into low-T phase

tunnelling or thermal fluctuations

1. bubble expansion & bubble collision:

latent heat rises $\mathrm{T}_{_{Plasma}}\&~\mathrm{E}_{_{Kin}}$ of bubble wall and bulk



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Bubble formation & collision



Youtube: https://www.youtube.com/watch? v=Ggs2fQL0ICU0

Bubble collision and origin of gravitational waves

GW production through 1OPT can be devided into three stages:

1. initial collision of scalar field shells (generally subdominant)

1. wave of kinetic energy within plasma

1. shocks & turbulence (typical time- and length-scales?)

$$\Omega_{\rm GW} = \Omega_{\rm env} + \Omega_{\rm sw} + \Omega_{\rm turb}$$



[[]D.J.Weir, Phil.Trans.Roy.Soc.Lond. A376, 2018]

Bubble collision and origin of gravitational waves

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$$\begin{split} h^2 \Omega_{\rm env}(f) \simeq 1.65 \cdot 10^{-5} \Delta(v_w) \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_{\phi} \ \alpha_{T_*}}{1 + \alpha_{T_*}}\right)^2 S_{\rm env} \left(\frac{f}{f_{\rm env}}\right) \\ f_{\rm env} \simeq 16.5 \ \mu {\rm Hz} \ \left(\frac{f_*}{\beta}(v_w)\right) \left(\frac{\beta}{H_*}\right) \end{split}$$

1. wave of kinetic energy within plasma $h^2 \Omega_{\rm ew}(f) \simeq 8.5 \cdot 10^{-6} \overline{U_f} (\kappa_f, \alpha_{T_*})^4 \left(\frac{H_*}{\beta}\right) v_w S_{\rm sw} \left(\frac{f}{f_{\rm sw}}\right)$ $f_{\rm sw} \simeq 8.9 \ \mu {\rm Hz} \ \left(\frac{1}{v_w}\right) \left(\frac{\beta}{H_*}\right)$

1. shocks & turbulence (typical time- and length-scales?) $h^2 \Omega_{\text{turb}}(f) \simeq 3.35 \cdot 10^{-4} \left(\frac{H_s}{\beta}\right) \left(\frac{\kappa_{\text{turb}} \sigma_{T_s}}{1 + \alpha_{T_s}}\right)^{\frac{3}{2}} v_{w} S_{\text{turb}} \left(\frac{f}{f_{\text{turb}}}\right)$ $f_{\text{turb}} \simeq 27 \ \mu \text{Hz} \ \left(\frac{1}{v_w}\right) \left(\frac{\beta}{H_s}\right)$

$$\Omega_{\rm GW} = \Omega_{\rm env} + \Omega_{\rm sw} + \Omega_{\rm turb}$$



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- Change of bubble nucleation $\frac{\beta}{H_*}$ ate= $T_* \frac{d}{dT} \begin{pmatrix} S_5 \\ T \end{pmatrix}\Big|_{T_*}$
 - free energy of critical bubble:

$$S_3 = 4\pi \int r^2 dr \left(\frac{1}{2} \left(\frac{d\phi_b}{dr} \right)^2 + V(\phi_b, T) \right)$$

• bounce solution:

1)

$$\frac{d^2\phi_b}{dr^2} + \frac{2}{r}\frac{d\phi_b}{dr} - \frac{\partial V}{\partial\phi_b} = 0 \quad \text{with} \quad \frac{d\phi_b}{dr}\Big|_{r=0} = 0, \ \phi_b|_{r=\infty} = 0$$



Change of bubble nucleation $\left. \begin{array}{c} \beta \\ rate=T_{*} \ d \\ dT \end{array} \right|_{T_{*}} \left. \begin{array}{c} S_{5} \\ T \end{array} \right|_{T_{*}}$

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2) Latent heat to energy density rafts = $\frac{\epsilon}{\rho_{\text{rad}}}$

• two contributions to latent heat:

$$\epsilon = -\Delta V - T\Delta s = \left(-\Delta V + T\frac{\partial V}{\partial T}\right)_T$$



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GW are insensitive to:

- internal bubble structure
- small scale field configuration in the collision region



Detection possibilities of future experiments

- LISA sensitive to 10 TeV
- LIGOIII, LISA & BBO will probe T~100-107 GeV
- GW from PTs around 10-100 TeV could entirely screen signals from inflation!





[http://web.mit.edu/klmitch/classes/8.224/project/lisa.html]

[C.Grojean & G.Servant, Phys.Rev. D75, 2007]

Phase transitions & BSM physics

Search: GW from first order PT

- 1. Modification of electroweak PT
 - electroweak baryogenesis
 - Higgs portals

- 1. First order PT in new (yet hidden) sectors
 - new scalars
 - new forces & symmetry breaking



[W.Buchmüller, Acta Phys.Polon. B43, 2012)]

Conclusion:

- If inflation took place, GWs produced during that time, would actually survive untill CMB and can be even observed.
- Primordial GWs would be a model-independent evidence of the concept of inflation.
- GWs leave a unique imprint on the CMB in the form of polarization
- The extracted spectrum can give hints to their origins (e.g. low frequencies for inflationary GWs)
- GW from particle physics PT directly related to associated scalar potential
- "new"/old tool for future particle phenomenology!



