Physics beyond the standard model – – from supersymmetry to extra dimensions

1 Introduction

1.1 Basic structure of the standard model

$$S = \int d^4x \, \mathcal{L}$$

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{fermions} + \mathcal{L}_{scalars} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{gauge} = - \sum_{i=1}^{3} \frac{1}{2g_i^2} \operatorname{tr}_i F_{\mu\nu}^{(i)} F^{(i)\mu\nu}$$

group: $\underbrace{SU_3 \times SU_2 \times U_1}_{G_{SM}}$; labelled by $i = 3, 2, 1$

$$\mathcal{L}_{fermions} = \sum_j \bar{\psi}_j \, i \not{D} \psi_j \qquad ; \qquad \not{D} = \gamma^\mu D_\mu$$

$$D_\mu = \partial_\mu + i A_\mu \quad ; \quad A_\mu \in \text{Lie}(G_{SM})$$

When applied to any charged field, e.g. ψ_j, one always uses the matrix $R(A_\mu)$, where R is the representation of the relevant field.

The fermions are labelled by

$$j \in \{ \{ q_L^a, u_R^a, d_R^a, \ell_L^a, e_R^a \}, \underbrace{a = 1,2,3}_{} \}$$

(We will frequently write q_L^a instead of $\psi_{q_L^a}$.)

a characterizes the 3 different families.

To give an example, q_L^a is a triplet of SU_3 ("3 of SU_3"), a doublet of SU_2, and has a Hypercharge ($\equiv$ charge under $U_1 = U_{1,Y}$) of $1/6$.

$$\mathcal{L}_{scalar} = -(D_\mu\phi)^+(D^\mu\phi) - V(\phi)$$

$$V(\phi) = -\mu^2\phi^+\phi + \lambda(\phi^+\phi)^2 \quad ; \quad \phi = \begin{pmatrix}\phi^1\\ \phi^2\end{pmatrix}$$

(complex) Higgs doublet

$$\phi^+\phi = (\bar{\phi}^1, \bar{\phi}^2)\cdot\begin{pmatrix}\phi^1\\ \phi^2\end{pmatrix} = |\phi^1|^2 + |\phi^2|^2$$

We use the metric $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ so that

$$\mathcal{L}_{scalar} = |\partial_0\phi|^2 + \cdots = |\partial_t\phi|^2 + \cdots$$

$$\mathcal{L}_{Yukawa} = \sum_{\substack{\text{all possible}\\ \text{gauge inv. terms}}} \lambda_{jk}\bar{\Psi}_j\Psi_k\phi + \tilde{\lambda}_{jk}\bar{\Psi}_j\Psi_k\bar{\phi} + \text{h.c.}$$

contractions of group indices, in particular SU_2-indices are implied. This implies that only <u>one</u> of the two fermions can be an SU_2 doublet

1.2 Higgs mechanism

$V(\phi)$ is chosen such that its minimum is at $\phi \neq 0$.

[One can always work in a gauge where $\phi^1 = 0$, i.e. $\phi = \begin{pmatrix}0\\ v\end{pmatrix}$ in the vacuum.]

Since ϕ is charged under SU_2 and U_1, some of the corresponding $4 = 3+1$ gauge bosons acquire a mass from terms of the type

$$(D_\mu \phi)^+ (D^\mu \phi) \supset \sim A_\mu A^\mu \phi^2 .$$

In fact, one of the gauge bosons stays massless (the photon). The others (W^+, W^-, Z) acquire a mass.
(experimentally $m_W \sim 80$ GeV ; $m_Z \sim 90$ GeV.)

Furthermore, the Yukawa interactions $\sim \bar\psi \psi \overset{(-)}{\phi}{}'$ obviously give rise to fermion masses if $\phi \neq 0$ in the vacuum. Only ν_L (the lower component of the SU_2-doublet l_L) remains massless.

To understand this structure more deeply, it is crucial to think in terms of mass dimensions of fields and couplings:

$$[S] = 0 \; ; \; [\int d^4x] = -4 \; ; \; [\mathcal{L}] = 4 ;$$

$$[\partial_\mu] = 1$$

$$[\phi] = 1 \quad (\text{from } \mathcal{L} \supset |D_\mu \phi|^2)$$

$$[\psi] = 3/2 \quad (\text{from } \mathcal{L} \supset \bar\psi \not{D} \psi)$$

$$[A_\mu] = 1 \quad (\text{from } D_\mu = \partial_\mu + A_\mu)$$

$$[g_i] = 0 \quad (\text{from } \mathcal{L} \supset \tfrac{1}{g^2} F^2 \text{ and } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \text{ e.g. for } U_1)$$

$$[\lambda_{ijk}] = 0 \quad (\text{since } [\bar{\psi}\psi\phi] = 4)$$
$$[\lambda] = 0 \quad (\text{since } [(\phi^+\phi)^2] = 4)$$
$$[\mu^2] = 2 \quad (\text{since } [\phi^+\phi] = 2)$$

Thus, μ^2 is the only dimensionful parameter of the SM. This implies that all particle masses (except those of bound states associated with the scale Λ_{QCD} which the SU_3-gauge coupling becomes $O(1)$; e.g. baryons & glueballs) are proportional to μ.

Furthermore $v \sim \mu/\sqrt{\lambda}$ and $m_{gauge} \sim g \cdot v$.

More specifically: $v \sim 175$ GeV. $\qquad g \lesssim O(1)$

(And μ can not be much larger without making $\lambda \gg 1$, which would make the model uncalculable.)

The physical Higgs mass $m_H = \sqrt{2}\mu$ is known experimentally (from LEP) to be above ~ 115 GeV (excluding the opposite limit $\lambda \ll 1$ and $\mu \ll v$).

Thus, μ is roughly equal to (sets the) "electroweak scale" of ~ 100 GeV, which brings us to the

1.3 Hierarchy problem

Hierarchy problem 1: Since the full lagrangian is

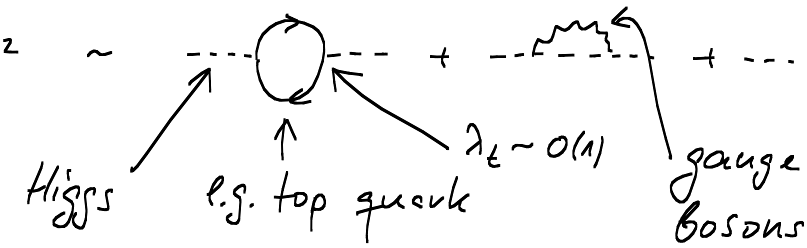
$$\mathcal{L} = \frac{1}{2}\underbrace{\bar{M}_P}_{\substack{\text{reduced Planck mass} \\ \sim 10^{18}\,\text{GeV}}}{}^2 \underbrace{R}_{\text{Ricci scalar}} + \mathcal{L}_{SM},$$

why is the electroweak scale so much smaller than $\bar{M}_p$? ($M_p = \sqrt{8\pi}\,\bar{M}_p \sim 10^{19}$ GeV)

(This is largely an aesthetical problem, which can not be further specified without embedding the SM in a finite theory of quantum gravity or some extension of the SM in which μ^2 is not just a parameter in $\mathcal{L}$.)

To be more specific, assume that we have such a finite theory of quantum gravity (e.g. superstring theory). Then μ^2 is calculable. There are contributions from loops:

$\delta\mu^2 \sim$



$$\delta\mu^2 \sim \int^{\Lambda} dk^2 \sim \Lambda^2$$

(Λ:) cutoff, expected to be $\sim M_p$

Thus, very roughly $\quad \mu^2 \sim \underbrace{\mu^2_{tree} + \Lambda^2}$

two very large numbers have to cancel with an unbelievable accuracy to produce the observed value $\mu^2 \sim (100\,\text{GeV})^2$.

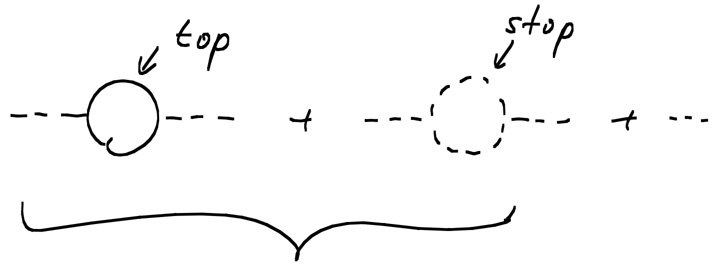
This is the <u>hierarchy problem 2</u>: The fine tuning problem.

[This is the main reason why we expect the LHC to find not just the Higgs but some "physics beyond the SM".]

Typically, models avoiding the fine tuning problem of the Higgs mass are of the following type:

Some extended theory $\mathcal{L}_{SM}'$ (which reduces to $\mathcal{L}_{SM}$ at low energies) has a symmetry implying $\mu = 0$ (forbidding the Higgs mass term). If this symmetry is non-anomalous ($\equiv$ respected by quantum corrections), the loop contributions driving μ^2 to $M_P{}^2$ will not arise. In practice, this frequently means that new particles cancelling the known loop contributions are present. For example, in supersymmetry

$\delta\mu^2 =$



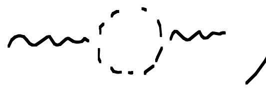
cancel exactly if SUSY is unbroken.

The enhanced symmetry of $\mathcal{L}_{SM}'$ is broken near (slightly above) the electroweak scale. The new particles or interactions are characterized by this scale (e.g. have a corresponding mass).

This new scale is now the "effective cutoff" Λ of the top quark loop and

$$\delta\mu^2 \sim \underbrace{\frac{\lambda_t{}^2}{16\pi^2}}_{\text{"loop factor"}} \Lambda^2 \,.$$

Apparently (since $\lambda_t \sim O(1)$), Λ can be somewhat above the electroweak scale (i.e., the new physics is not necesserily excluded by present data) and everything is fine.

<u>However:</u> Indirect effects of the Higgs (e.g. in the W, Z - propagator: ) suggest that m_H is really small (near its experimental lower bound).

[$\to$ LEP precision data]

This forces Λ to be low (to avoid fine tuning) and then the same LEP precision data (as well as other data, e.g. "flavour changing neutral currents") rule out the postulated new physics in many concrete examples.

$\Rightarrow$ <u>hierarchy problem 3:</u> (the LEP problem or the little hierarchy problem)

LEP teaches us that the Higgs is light and that there is <u>no</u> new physics near the electroweak scale. This makes it hard to solve the fine tuning problem.

(Stated positively, it also means that the LHC is likely to see the new physics solving the fine tuning problem since its energy scale can not be too high, given that we expect the Higgs to be light.)

In summary, most of present research in "physics beyond the SM" focusses on solving the hierarchy problem.

1.4 The SUSY solution to the hierarchy problem

In essence SUSY transforms fermions $\leftrightarrow$ bosons and introduces a bosonic (fermionic) "SUSY partner" for every SM particle. Unfortunately the minimal SUSY extension of the SM predicts $m_h \leq m_Z$ for the lightest Higgs, which is ruled out. Loop corrections to this tree-level prediction are only sufficiently large if some SUSY-breaking "soft terms" (e.g. the stop mass) are large. This reintroduces a certain fine tuning problem (at the 1%-level) within SUSY.

On the positive side, low-energy SUSY-breaking is easy to realize in explicitly treatable perturbative models, in which also the the gauge couplings unify numerically and

$$SU_3 \times SU_2 \times U_1 \subset SU_5 \quad (\text{SUSY-GUTs}).$$

1.5 The Technicolor solution

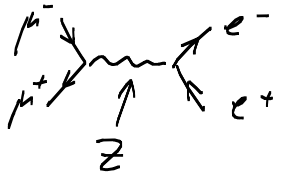
The Higgs is not fundamental but arises as a condensate (like the pion of QCD) due to some new strong gauge interaction (the technicolor group). It is easy to imagine that the electroweak scale is then realized by the logarithmic running of the new gauge coupling, which becomes strong near the electroweak scale (just like QCD becomes strong near 1 GeV).

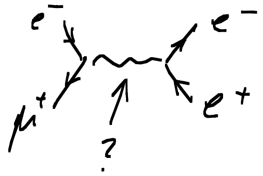
<u>Problems</u>: Imagine replacing the Higgs VEV by a fermion condensate: $\langle \bar f f \rangle \neq 0$. Then Yukawa couplings come from operators of the type $\frac{1}{M^2}\langle \bar f f \rangle \bar\psi \psi$

some high (i.e. above m_{EW}) energy scale — technifermions — SM fermions

It is then difficult to get a sufficiently large top quark mass. It is difficult to understand the large hierarchy of quark & lepton masses. Given that M can not be extremely high, it is hard to avoid operators of the type $\frac{\psi^4}{M^2}$, e.g. $\frac{\bar\mu e \bar e e}{M^2}$ leading to

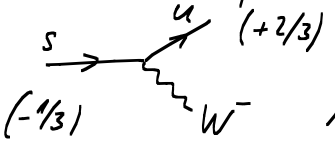
$\mu^- \to e^- + e^+ + e^-$ ("flavor changing neutral currents"), which are very strongly constrained experimentally

[Origin of the name: 

vs. 

$\bar\mu\gamma^\mu\mu$ is a flavor conserving neutral current

This would be flavor changing neutral current

(flavor changing <u>charged</u> currents are present in the SM, e.g. due to vertices like 

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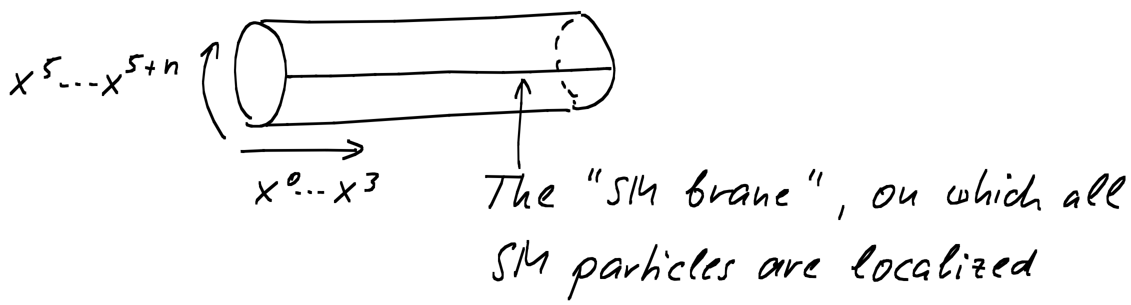
Furthermore, having many new particles (in particular fermions) near the electroweak scale tends to affect electroweak precision observables.

1.6 The large extra dimensions solution ("ADD")



Here the large coefficient $M_{P,4}^2$ of R arises as

$$M_{P,4}^2 \sim M_{P,4+n}^{2+n} \, L^n$$

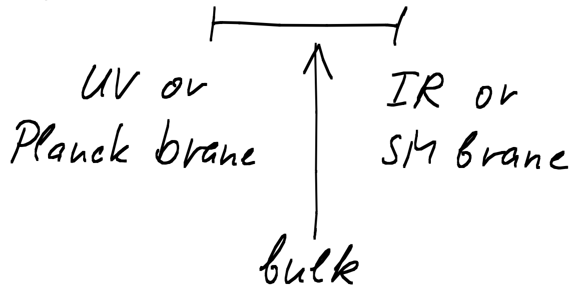
size of compact space.

($n = 1$ excluded ; $n = 2$ disfavoured ; $n \geq 3$ OK)

This mainly addresses the "large hierarchy problem" as an aesthetical problem. It may solve the fine tuning problem if $L \gg m_{EW}^{-1}$ can be explained dynamically. It does not address the little hierarchy problem.

1.7 The warped extra dimensions solution ("RS I")

Instead of S^1 (see above) use interval for compact space:



Metric in bulk is not flat, e.g.

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

<u>Crucial:</u> This "warped" metric is a solution of Einstein's eqs. if one allows for a negative 5d cosmol. constant. The smallness of m_{EW} is explained by the small factor $e^{-2k(y_{max}-y_{min})}$, which suppresses the metric on the IR brane. Since this effect is exponential, it is easy to realize $m_{EW} \sim 10^{-16} M_P$ with $O(10)$ ratios between fundamental constants in the 5d action. Such models nicely solve the large hierarchy problem & fine tuning problem, but have difficulties with the little hierarchy problem. In modern versions, some of the SM fields "live" in the bulk. [<u>Note:</u> This solution is related to technicolor via the "AdS/CFT correspondence".]

1.8 The "TeV-scale extra dimensions" solution

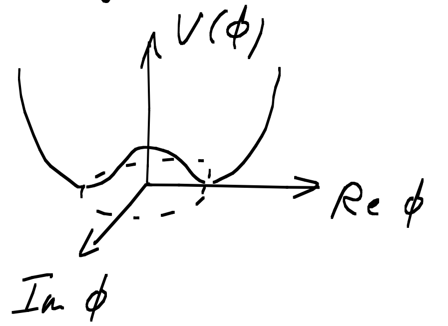
This approach does <u>not</u> address the large hierarchy problem, but attempts to solve the little hierarchy problem by realizing the Higgs scalar as A_5 or $A_5 + iA_6$.

extra dim. components of a gauge field.

The Higgs is then massless at tree level because 5d or 6d gauge invariance forbids a mass term. The mass term arises at one-loop level. It is, however, difficult to realize an appropriate Higgs potential and a proper top Yukawa coupling.

1.9. The "little Higgs" solution

This proposal also only addresses the little hierarchy problem. Here the Higgs is light (or "little") compared fundamental lagrangian mass parameters because it is a "pseudo Goldstone Boson". Recall: A massless Goldstone boson arises if a global symm. is spontaneously broken. Simplest example:



"bottle neck" or "mexican hat" potential.

(The angular d.o.f. of ϕ is exactly massless.)

In the case of a "pseudo Goldstone Boson", the potential still respects the symmetry but the full lagrangian does not. A Higgs mass is then introduced by loop corrections and is correspondingly small.

Interesting point: The dangerous top quark loop contributions to the Higgs mass are cancelled by particles of the same statistics (as opposed to the case of SUSY).

Even though this is a very nice idea, realistic models look "contrived" (at least to some people). (As usual, LEP precision data tends to be a problem.)

Note: proton decay and flavor violation are not as nicely avoided in low-scale models as in the pure SM.

1.10 Grand Unification (not a solution to hier. problem)

$G_{SM} \subset SU_5 \subset SO_{10}$; (3x) 16 of SO_{10} contains all SM fermions + r.h. neutrinos, works best with SUSY