10 Little Higgs Models

(Arkani-Hamed, Cohen, Georgi, 2001 ; we will follow the lectures of M. Schmaltz at TASI 2004)

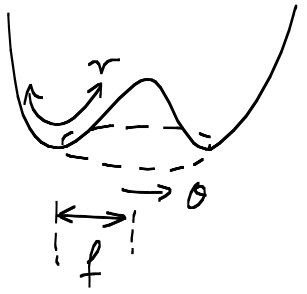
- Basic idea: – The Higgs is a Pseudo-Nambu-Goldstone Boson (similarly to technicolor)
  - The origin of the underlying symm. breaking is in many cases not specified (at least it is not ascribed to a fermion condensate).
  - This prevents the extrapolation to the GUT scale, but it allows for simpler and more calculable models addressing the little hierarchy problem than technicolor

10.1 Nambu-Goldstone Bosons

- Recall the U_1-case: $\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)^* - V(\phi,\phi^*)$

$$\phi(x) = \frac{1}{\sqrt{2}}\big(f + r(x)\big)e^{i\theta(x)/f}$$

ensure canonical kinetic term for r & θ.



- $e^{i\alpha/f} \in U_1$: $r \to r$, $\underbrace{\theta \to \theta + \alpha}_{U_1\text{-symm. is non-linearly realized}}$

- r can in principle be integrated out ; then just θ with flat potential protected by the non-lin. realized U_1 is left.

- Other example: consider breaking $SU_N \to SU_{N-1}$

$$(N^2-1) - ((N-1)^2-1) = \underbrace{2N-1}_{\#\text{ of NGB's}} .$$

- parametrization of NGBs:

$$\phi = \exp \frac{i}{f} \left(\begin{array}{c|c} \underbrace{\mathrm{diag}(-\pi_0/\sqrt{2})}_{N-1} & \begin{matrix} \pi_1 \\ \vdots \\ \pi_{N-1} \end{matrix} \\ \hline \pi_1^* \cdots \pi_{N-1}^* & \pi_0/\sqrt{2} \end{array} \right) \cdot \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f \end{pmatrix} \equiv \underbrace{e^{i\pi/f}\phi_0}_{\text{short-hand notation}}$$

($\pi_1 \cdots \pi_{N-1}$ complex; π_0 real)

The matrix in the exponent is the part of Lie(SU_N) broken by a fundamental VEV $(0, \cdots, 0, f)^T$.

- Yet another example (already familiar to us): $SU_N \times SU_N \to SU_N$ (diag. subgroup)

$$2(N^2-1) - (N^2-1) = N^2-1 .$$

- Consider ϕ transforming as $\phi \to U_L \phi U_R^\dagger$.
- VEV $\phi_0 = f \cdot \mathbb{1}$ is invariant only if $U_L = U_R = U$.
- NGBs parametrized by $\phi = \phi_0 e^{i\pi/f} = f e^{i\pi/f}$; π hermitian & traceless

Transformation of NGBs in $SU_N \to SU_{N-1}$ case

(one also says: NGBs parametrize the space SU_N / SU_{N-1}.)

- let $f = 1$ for simplicity
- $\phi = e^{i\pi}\phi_0 \xrightarrow{\text{unbroken } SU_{N-1}} U\phi = U e^{i\pi} U^\dagger U \phi_0$

$$= e^{iU\pi U^\dagger}\phi_0 ,$$

"$\begin{pmatrix} U & \\ \hline & 1 \end{pmatrix}$" $\nearrow$

i.e. $\pi \to U\pi U^+$ (SU_{N-1} linearly realized on the complex fields $\pi_1 \cdots \pi_{N-1}$: $\pi = \left(\begin{array}{c|c} 0 & \begin{smallmatrix}\pi_1\\ \vdots \\ \pi_{N-1}\end{smallmatrix} \\ \hline \pi_1^* \cdots \pi_{N-1}^* & 0\end{array}\right)$; fundam. repres.;

π^0 is a singlet.)

- The broken symmetries are non-linearly realized:

$$\phi = e^{i\pi}\phi_0 \longrightarrow U e^{i\pi} = \exp\left(i\begin{pmatrix}0 & \vec\alpha\\ \vec\alpha^+ & 0\end{pmatrix}\right)\exp\left(i\begin{pmatrix}0&\vec\pi\\ \vec\pi^+ & 0\end{pmatrix}\right)\phi_0$$

$$\equiv \exp\left(i\begin{pmatrix}0&\vec\pi'\\ \vec\pi'^+&0\end{pmatrix}\right)\underbrace{\begin{pmatrix}\tilde U & \vec 0\\ \vec 0^T & 1\end{pmatrix}\phi_0}_{=\phi_0}$$

(any SU_N tf. can be written as product of SU_N/SU_{N-1}-tf. & SU_{N-1} tf.)

$$\Big\downarrow \text{ at linear order}$$

$$\vec\pi \longrightarrow \vec\pi' = \vec\pi + \vec\alpha .$$

- Effective action: The only possible SU_N invariants made from $\phi = e^{i\pi/f}\phi_0$ (without derivatives) are:

$$\phi^+\phi = f^2 = \text{const.}$$

$$\& \quad \varepsilon^{i_1 \cdots i_N}\phi_{i_1} \cdots \phi_{i_N} = 0$$

$\Rightarrow$ flat potential

Leading term with derivatives: $f^2\,(\partial_\mu\phi)^+(\partial^\mu\phi)$

(similar to our QCD/pion discussion earlier)

10.2 Constructing the Higgs doublet from NGB's

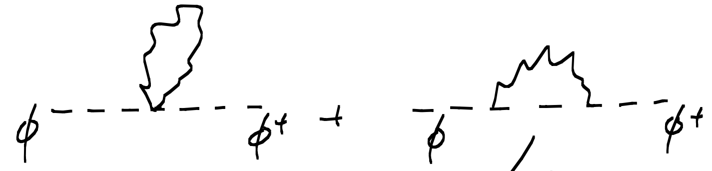
- Consider SU_3/SU_2 : $\pi = \left(\begin{array}{cc|c} -\eta/2 & 0 & \\ 0 & -\eta/2 & h \\ \hline & h^+ & \eta \end{array}\right)$ ← required SU_2 doublet

  ↖ singlet (ignore for the moment)

- Need to introduce SU_2 gauge-interact.s. for h.
  - "gauging" the whole SU_3 does not work since then the NGB's are "eaten" (& all vectors acquire a mass $\sim f$, while we want them to have a mass $\ll f$)
  - gauging just the SU_2 subgroup, i.e.

$$|\partial_\mu \phi|^2 \to |D_\mu \phi|^2 \quad \Rightarrow \quad \mathcal{L} \supset \left| g \left(\begin{array}{c|c} W_\mu & 0 \\ \hline 0 & 0 \end{array}\right) \phi \right|^2$$

leads to quadratic divergences:



$\swarrow$ from $\sum_i \left[\left(\begin{array}{c|c} \sigma^i & 0 \\ \hline 0 & 0 \end{array}\right) \left(\begin{array}{c|c} \sigma^i & 0 \\ \hline 0 & 0 \end{array}\right) \right]_{ab}$

$$\Rightarrow \frac{g^2}{16\pi^2} \cdot \Lambda^2 \cdot \phi^+ \left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & 0 \end{array}\right) \phi$$

$$= \frac{g^2}{16\pi^2} \Lambda^2 h^+ h + \cdots$$

... as bad as usual

(we have Pseudo-NGB-Higgs, but the "Pseudo" introduced by gauging just the SU_2 is too strong, it leads to a quadrat. div. mass)

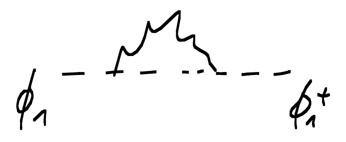
Better: Take two NGB-copies, ϕ_1 & ϕ_2, and gauge the whole SU_3. (The underlying idea is that not all of the NGB's will be eaten.)

$$\phi_1 = e^{i\pi_1/f}\begin{pmatrix}\vec{0}\\ f\end{pmatrix} \; ; \quad \phi_2 = e^{i\pi_2/f}\begin{pmatrix}\vec{0}\\ f\end{pmatrix} \; ; \quad \mathcal{L} = \underset{\substack{\uparrow \\ \text{includes } g|(\underset{\substack{\uparrow \\ \text{from full } SU_3\text{-Lie-alg.}}}{A_\mu})\phi|^2}}{|D_\mu\phi_1|^2} + |D_\mu\phi_2|^2$$

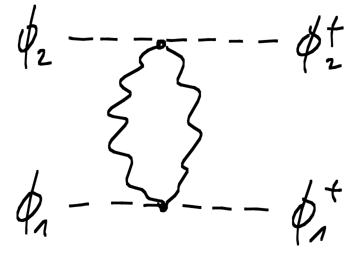
The above loop diagrams give:

$$\underset{\substack{\uparrow \\ \text{e.g.}}}{\frac{g^2}{16\pi^2}\Lambda^2\,\phi_1^+\,\mathbb{1}_{3\times3}\,\phi_1} + \underset{\substack{\uparrow \\ \text{e.g.}}}{\{\phi_1\to\phi_2\}} = \frac{g^2}{16\pi^2}\Lambda^2(f^2+f^2)$$

no potential induced!



- However, more complicated diagrams (involving both ϕ_1 & ϕ_2) give a contribution:



$$\Rightarrow \frac{g^4}{16\pi^2}\ln\Big(\underset{\substack{\uparrow \\ \text{cutoff}}}{\Lambda^2}/\underset{\substack{\uparrow \\ \text{typical} \\ \text{phys. scale}}}{\mu^2}\Big)\;|\phi_1^+\phi_2|^2$$

- Explicitly:

$$\phi_1 = \exp i\begin{pmatrix}0 & \overset{\text{"}\to\text{" etc.}}{k}\\ k^+ & 0\end{pmatrix}\Big/f \cdot \exp i\begin{pmatrix}0 & h\\ h^+ & 0\end{pmatrix}\Big/f \cdot \begin{pmatrix}0\\ f\end{pmatrix}$$

$$\phi_2 = \exp i\begin{pmatrix}0 & k\\ k^+ & 0\end{pmatrix}\Big/f \cdot \exp -i\begin{pmatrix}0 & h\\ h^+ & 0\end{pmatrix}\Big/f \cdot \begin{pmatrix}0\\ f\end{pmatrix}$$

(k - eaten, h - physical)

- $\phi_1^+\phi_2 = (\vec{0}\ f)\exp\left(-\frac{2i}{f}\begin{pmatrix}0 & h\\ h^+ & 0\end{pmatrix}\right)\begin{pmatrix}\vec{0}\\ f\end{pmatrix} = f^2\,(\vec{0}\ 1)\left(\cdots\right)\begin{pmatrix}\vec{0}\\ 1\end{pmatrix}$

$$= \left(f^2\cdot\mathbb{1} - 2fi\begin{pmatrix}0 & h\\ h^+ & 0\end{pmatrix} - 2\begin{pmatrix}hh^+ & \\ & h^+h\end{pmatrix} + \cdots\right)_{33}$$

$$= f^2 - 2h^+h + \cdots$$

$\Rightarrow\ m_{Higgs} \sim m_{Weak} \sim \frac{g^4}{16\pi^2}\cdot\ln\left(\frac{\Lambda^2}{\mu^2}\right)\cdot f^2$, i.e.

parametrically smaller than f ($\sim$ TeV).

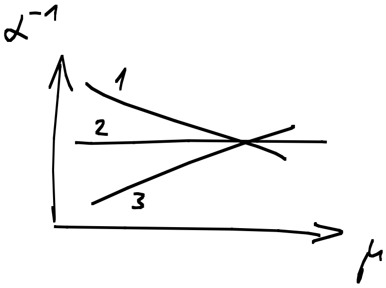
We can be a bit more precise: $e = g_2\sin\theta$ $\quad(\theta = \theta_W)$

$$\alpha_{EM} = \alpha_2\sin^2\theta$$

$$\cos^2\theta = \frac{m_W^2}{m_Z^2} \simeq \frac{80^2}{90^2} \simeq (1-0.11)^2 \simeq 1 - \underbrace{0.22}_{\sin^2\theta}$$

$$\Rightarrow\ \alpha_2 \simeq \frac{1}{128\cdot 0.22} \simeq \frac{1}{26}$$

Alternatively: Recall (in SUSY)



, i.e.

α_2 runs very weakly.

Thus, the well-known value $\alpha_{GUT} \simeq \frac{1}{25}$ approximately applies to $\alpha_2(m_Z)$.

$\Rightarrow\ m_{Weak}^2 \simeq \left(\frac{1}{25}\right)^2 f^2 \ \Rightarrow\ f \sim 2\,\text{TeV}$ (or 1 TeV allowing for $\ln\frac{\Lambda}{\mu}\sim 2$.)

It is easy to give the symmetry reasons for the observed absence of quadratic divergences:

- We have started with a $[SU_3/SU_2]^2$-model (before gauging), with $2\,(8-3) = 10$ NGBs

- Gauging both SU_3s with the same gauge field (i.e. gauging the diagonal SU_3) introduces terms

$$\mathcal{L} \supset |gA_\mu \phi_1|^2 + |gA_\mu \phi_2|^2 \, .$$

- This breaks part of the $(SU_3)^2$ symm. (only the diag. survives):

$$\mathcal{L} \xrightarrow{U_1, U_2} |gA_\mu U_1 \phi_1|^2 + |gA_\mu U_2 \phi_2|^2 \, ;$$

can be undone by $A_\mu \to A_\mu' = UA_\mu U^+$
only if $U_1 = U_2 \equiv U$.

- Clearly, if either ϕ_1 or ϕ_2 were not coupled to A_μ, the full $(SU_3)^2$-symm. would survive and the 5 surviving NGB's would remain massless. Hence their mass can only be produced by diagrams involving both the coupling $\phi_1 - A_\mu$ & $\phi_2 - A_\mu$. There are no quadratically divergent 1-loop diagrams of this type.

$\Rightarrow$ || The little Higgs is a PNGB of a collectively broken approximate global symm. ||

10.3 Including the top quark

(This is a crucial and non-trivial step since in the SM the top contributes dominantly to the Higgs mass divergence because of its large Yukawa coupling.)

Proposal: $Q = \begin{pmatrix} t_L \\ b_L \end{pmatrix} \longrightarrow \psi = \begin{pmatrix} t_L \\ b_L \\ T_L \end{pmatrix}$ (3 of SU_3)

$$t_R, b_R \longrightarrow t_{1R}, t_{2R}, b_R$$

$$\mathcal{L} \supset \lambda_1 \phi_1^+ \psi t_{1R} + \lambda_2 \phi_2^+ \psi t_{2R}$$

(Basically, we have simply included both NGB's symmetrically.)

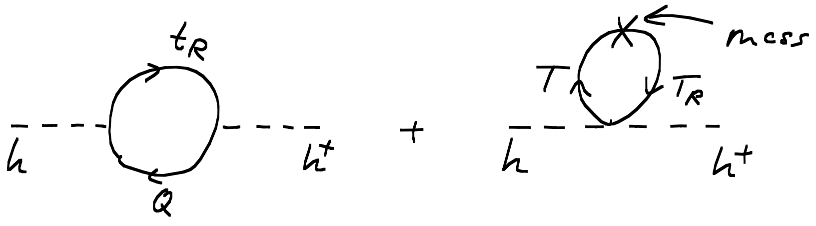
For simplicity, let $\lambda_1 = \lambda_2 \equiv \lambda/\sqrt{2}$; $T_R \equiv (t_{1R} + t_{2R})/\sqrt{2}$

$t_R \equiv -i(t_{1R} - t_{2R})/\sqrt{2}$

After expanding ϕ_1, ϕ_2 in terms of h, one finds:

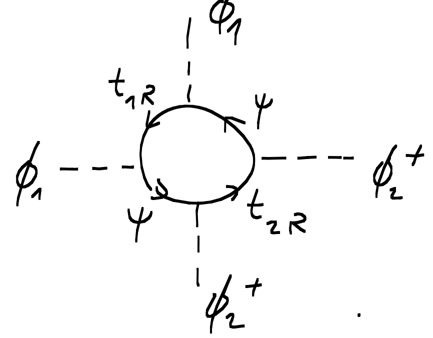
$$\mathcal{L} \supset \lambda f \left(1 - \frac{1}{2f^2} h^+ h\right) T T_R + \lambda h^+ Q t_R \,.$$

The obvious (quadratically divergent) one-loop contributions cancel:



[In contrast to the SUSY-case, the dangerous top-loop contribution is cancelled by a fermionic partner.]

A log-divergent contribution comes from



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Symmetry reasons:

As before, if either λ_1 or λ_2 is zero, the symm. is $(SU_3)^2$. If both are present, the symm. is just SU_3 (since ψ appears in both Yukawa terms, just like A_μ before). Hence, both λ_1 & λ_2 need to appear in any dangerous loop diagram. One may suspect that 1-loop diagrams $\sim \lambda_1 \lambda_2$ exist, but this is not the case for the following reason:

Introduce a "spurious" U_1-symm.: $t_{1R} \to e^{i\alpha} t_{1R}$

$\lambda_1 \to e^{-i\alpha} \lambda_1$

(One can think of λ_1 as of a VEV breaking the U_1; hence λ_1 is the "spurion", in analogy to our discussion of soft ~~susy~~.)

Any eff. operator contributing to the Higgs potential must be indep. of t_{1R} & U_1-invariant. Hence it can depend on λ_1 only via $|\lambda_1|^2$. The same holds for λ_2.

$\Rightarrow \sim |\lambda_1 \lambda_2|^2 \Rightarrow$ only log. divergence.

Some more details for the generic case $\lambda_1 \neq \lambda_2$ & $f_1 \neq f_2$:

$$m_T = \sqrt{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}$$

$$\lambda_t = \lambda_1 \lambda_2 \frac{\sqrt{f_1^2 + f_2^2}}{m_T} .$$

This is important since m_T can now be much smaller than the larger of the two f's (and hence smaller than the mass of the extra heavy vector bosons*). This is required if we want to keep the (finite) corrections to the Higgs mass small, allowing for a naturally light Higgs.

* which needs to be large because of EWPT.

10.4 Hypercharge & Quartic Higgs coupling

Hypercharge is not a problem:

$SU_3 \longrightarrow SU_3 \times U_1$; ϕ_i "=" 3 $\longrightarrow$ ϕ_i "=" $3_{-1/3}$

$\langle \phi \rangle \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow Y \equiv -\frac{1}{\sqrt{3}} T^8 + \underset{\substack{\uparrow \\ \text{generator} \\ \text{of } U_1}}{X}$ remains unbroken,

$$\left[T^8 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \right]$$

leading correct $SU_2 \times U_1$ quantum numbers of Higgs & fermions.

- The quartic Higgs coupling $\underline{\underline{\text{is}}}$ a problem:
  - the coupling generated at 1-loop is too small
  - at tree level, one could add operators $\sim (\phi_1 \phi_2^+)^n$, but they always introduce a (too large) mass together with the quartic coupling $\Rightarrow$ fine tuning required (but just 10% level ...)
  - a "model-building" solution: (→ Kaplan & Schmaltz)

$$SU_2,\ \underset{(2}{\phi_1},\ \underset{2)}{\phi_2} \longrightarrow SU_4,\ \underset{(4}{\phi_1},\ \underset{\cdots}{\cdots},\ \underset{4)}{\phi_4}$$

$\Rightarrow$ 2 "little Higgs" doublets are left after symm. breaking, a tree-level quartic potential can be written down (similarly to SUSY).