3. Yukawa Couplings, Flavor and Neutrino Masses

3.1 Yukawa couplings

For simplicity, we start with just one family. Convince yourself that the only gauge-inv. fermion-fermion-scalar couplings are:

$$\lambda_e \underbrace{l_i}_{2}\, e\, \underbrace{\Phi_i}_{\bar{2} \text{ of } SU_2} \;;\quad \lambda_d\, \underbrace{q_{ia}}_{3}\, \underbrace{d_a}_{\bar{3} \text{ of } SU_3}\, \Phi_i \;;\quad \lambda_u\, \underbrace{q_{ia}}_{2}\, u_a\, \underbrace{\phi_j}_{2 \text{ of } SU_2}\, \varepsilon^{ij}$$

(and their hermitian conjugates)

Note: Here and below we suppress Weyl indices using the standard convention $\quad le = l^\alpha e_\alpha$ etc.

Using this convention, we have $\psi\chi = \chi\psi$ in spite of anticommutation:

$$\psi\chi = \psi^\alpha \chi_\alpha = -\chi_\alpha \psi^\alpha = -\varepsilon_{\alpha\beta}\chi^\beta \varepsilon^{\alpha\gamma}\psi_\gamma$$

$$= \chi^\beta \psi_\beta = \chi\psi$$

Here we used $\varepsilon_{\alpha\beta}\varepsilon^{\alpha\gamma} = -\delta_\beta{}^\gamma \quad \left(\varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} ;\; \varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right)$

This is the convention of W/B. Some authors (e.g. Sohnius, Phys. Rep.) use other conventions).

Using $\phi = \begin{pmatrix} 0 \\ v \end{pmatrix}$, we now immediately find the mass lagrangian

$$\mathcal{L} \supset m_e e_L e_R + m_d d_L d_R + m_u u_L u_R + h.c.$$

with $m_e = \lambda_e v$ etc.

(Check that this is equivalent to Dirac mass terms of the type $m \bar{e}_D e_D$ with $e_D = \begin{pmatrix} e_L \\ \bar{e}_R \end{pmatrix}$.)

Note: The phase (in particular the sign) of a fermionic mass term is irrelevant since it can be changed by a phase redefinition of the Weyl fermions.

The generalization to 3 generations is straightforward; one simply replaces the λ's by matrices λ_{ab} with $a, b \in \{1,2,3\}$. Using matrix notation, we have

$$\mathcal{L} \supset \bar{e}_L^T M_e \bar{e}_R + \bar{d}_L^T M_d \bar{d}_R + \bar{u}_L^T M_u \bar{u}_R + h.c.$$

where $e_L = \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$ etc.

[Note: I used the dotted spinors to be consistent with some of literature writing

$$\mathcal{L} \supset \bar{e}_{D,L} M_e e_{D,R} + \cdots \qquad \text{with } e_{D,L} = \begin{pmatrix} e_L \\ 0 \end{pmatrix}; \; e_{D,R} = \begin{pmatrix} 0 \\ \bar{e}_R \end{pmatrix}.$$]

3.2 The CKM matrix

Important fact: Any complex matrix M can be diagonilized by a biunitary trf.:

$$L^+ M R = M^{diag.};$$

The unitary matrices L & R can be chosen such that all entries of $M^{diag.}$ are real & non-negative.

<u>Problem:</u> Prove this!

- Writing, e.g., $M_e = L_e M_e^{diag.} R_e^+$ and introducing new fields $\bar{e}_R' \equiv R_e^+ \bar{e}_R$; $\bar{e}_L'^T = \bar{e}_L^T L_e$ etc.

we obtain $\mathcal{L} \supset \bar{e}_L'^T M_e^{diag.} \bar{e}_R' + \bar{d}_L'^T M_d^{diag.} \bar{d}_R' + \bar{u}_L'^T M_u^{diag.} \bar{u}_R'$.

- In this generation-vector-notation, the kinetic term reads
$$\mathcal{L} \supset \bar{e}_L^T \underbrace{\not{\partial}}_{\equiv \bar{\sigma}^\mu \partial_\mu} e_L = \bar{e}_L^T \cdot \mathbb{1} \cdot \not{\partial} e_L .$$

Hence, its form is not changed by the above field redefinition.

- The same is true for the Z & A-couplings since they originate from the A^3 & B-couplings, which are governed by the diagonal matrices T^3 and $Y \cdot \mathbb{1}$.

- It is <u>not</u> true for the $W^\pm$-couplings, which have their origin in the non-diagonal generators $T^{1,2}$ mixing, e.g., up- & down-type quarks.

- These "charged bosons" couple to fermions via
$$\mathcal{L} \supset -\frac{g_2}{\sqrt{2}} \left(J_\mu^+ W^{+\mu} + h.c. \right) \equiv \mathcal{L}_{cc} ,$$
where $J_\mu^+ = \bar{\nu}_L^T \bar{\sigma}_\mu e_L + \bar{u}_L^T \bar{\sigma}_\mu d_L$ (This is called the "charged current", as opposed to the neutral current coupling to Z.)

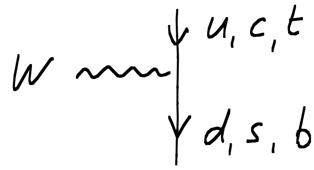
- In terms of the mass eigenstates
$$\bar{u}_L'^T = \bar{u}_L^T L_u \quad ; \quad d_L' = L_d^+ d_L \quad ; \quad e_L' = L_e^+ e_L$$

we have $$J_\mu^+ = \bar{\nu}_L^{\,T} \bar{\sigma}_\mu L_e e_L' + \bar{u}_L'^{\,T} \bar{\sigma}_\mu \underbrace{L_u^+ L_d}_{\equiv V_{CKM}} d_L' .$$

This matrix can simply be absorbed in a redefinition of the vector ν_L.

($\to$ Cabbibo, Kobayashi, Maskawa, '73)

This matrix governs flavor changes in charged current transitions, e.g.



The actual numbers:

Masses: (in GeV)

family	u	d	e
1	0.003	0.005	0.0005
2	1.2	0.1	0.1
3	175	4.2	1.7

(very rough) pattern: [slightly better after running to $M_{GUT} \sim 10^{16}$ GeV, where it can be "explained" in certain SU_5 models]

$\alpha^4 m_t$	$\alpha^2 m_{b,\tau}$	with
$\alpha^2 m_t$	$\alpha\, m_{b,\tau}$	$\alpha \sim 1/200$
m_t	$m_{b,\tau}$	

CKM-matrix:

$$V_{CKM} \simeq \begin{pmatrix} 0.975 & \cdots & \cdots \\ 0.22 & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

to be multiplied by $O(1)$-numbers ("Wolfenstein parameterization)

3.3 CP-violation

Let us count the parameters of V_{CKM} for the general case of N flavors:

- a $U(N)$-matrix has N^2 real parameters
- demanding reality, we have the smaller group of $O(N)$-matrices with $N(N-1)/2$ real parameters (angles).

$\Rightarrow$ $U(N)$-matrices have $N^2 - N(N-1)/2 = N(N+1)/2$ phases.

- The fermions can absorb $2N-1$ phases.

  (an overall phase does not affect V_{CKM})

$\Rightarrow$ V_{CKM} has $N(N+1)/2 - (2N-1) = (N-1)(N-2)/2$ physical phases. (zero for $N=2$, one for $N=3$)

Phases, being truely complex, physical lagrangian parameters, induce CP-violation.

- To understand this statement, recall first that, for a charged Dirac fermion $\begin{pmatrix}\psi\\ \bar{\chi}\end{pmatrix} \xrightarrow{C} \begin{pmatrix}\chi\\ \bar{\psi}\end{pmatrix}$,

  or, in the language of Weyl fermions, $\psi, \chi \xrightarrow{C} \chi, \psi$

- P (parity) by definition exchanges l.h. & r.h. fields, i.e.

$$\begin{pmatrix}\psi\\ 0\end{pmatrix}, \begin{pmatrix}0\\ \bar{\chi}\end{pmatrix} \xrightarrow{P} \begin{pmatrix}0\\ \bar{\chi}\end{pmatrix}, \begin{pmatrix}\psi\\ 0\end{pmatrix}.$$

(This can not be formulated as a trf. of a set of Weyl fermions.)

- Specifically for a (Dirac) mass term:

$$m\,\bar{\psi}_{D,L}\,\psi_{D,R} \xrightarrow{P} m\,\bar{\psi}_{D,R}\,\psi_{D,L}. \quad \left(\text{where } \psi_{D,L} = \begin{pmatrix}\psi\\0\end{pmatrix} \;\&\; \psi_{D,R} = \begin{pmatrix}0\\ \bar{\chi}\end{pmatrix}\right)$$

- This can be translated in "Weyl language" as

$$m\,\bar{\psi}\bar{\chi} \xrightarrow{P} m\,\chi\psi\,.$$

- Promoting ψ & χ to vectors of Weyl fermions and introducing corresponding mass matrices we thus have

$$\mathcal{L}_M = \psi^T M \chi + \bar{\psi}^T \bar{M} \bar{\chi}$$

$$\downarrow P$$

$$\mathcal{L}_M = \bar{\chi}^T M \bar{\psi} + \chi^T \bar{M} \psi$$

$$\downarrow C$$

$$\mathcal{L}_M = \bar{\psi}^T M \bar{\chi} + \psi^T \bar{M} \chi$$

$$= \psi^T \bar{M} \chi + \bar{\psi}^T M \bar{\chi}$$

Thus, $\mathcal{L}_M$ is "CP-invariant" if M is a real matrix. (Of course, this is only meaningful if the remaining part of $\mathcal{L}$ prevents us from simply absorbing any phases in ψ & χ.)

- This can now be immediately applied to the SM case: Go to the formulation with generic mass matrices & no "V_{CKM}" in the gauge part (gauge eigenstates). The physical phase of V_{CKM} then finds its way into the mass Lagrangian and induces CP-violation according to the above argument.

Note: CP is also referred to as "particle-antiparticle symmetry". [C does this job for scalar fields, but in the fermionic case it also changes the handedness. CP is, by contrast, the true "particle-antiparticle" symm. of generic Lagrangians, e.g.

$$\begin{pmatrix} e_L \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \bar{e}_R \end{pmatrix} \xrightarrow{P} \begin{pmatrix} 0 \\ \bar{e}_R \end{pmatrix}, \begin{pmatrix} e_L \\ 0 \end{pmatrix} \xrightarrow{C} \begin{pmatrix} 0 \\ \bar{e}_L \end{pmatrix}, \begin{pmatrix} e_R \\ 0 \end{pmatrix} \Big]$$

3.4 GIM mechanism

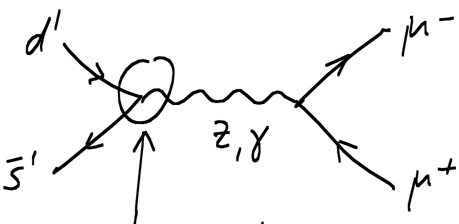
Recall: At tree level in the SM there are no FCNC's since

$W^{\pm}_\mu$ couple to $\bar{u}_L{}^T \bar{\sigma}_\mu d_L$ ($\rightarrow$ FC after going to mass eigenstates)

γ, Z_μ couple to $\bar{u}_L{}^T \bar{\sigma}_\mu u_L$, $\bar{d}_L{}^T \bar{\sigma}_\mu d_L$ ($\rightarrow$ no FC after going to mass eigenstates since rotation matrix drops out)

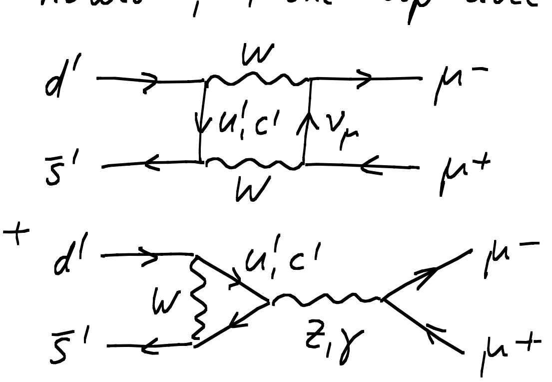
Example:

$K \longrightarrow \mu^+\mu^-$ does not arise at tree level since
$(d'\bar{s}')$



no such vertex exists!

However, at one-loop-level this process can occur:



$+ \cdots$

[We restrict ourselves to a 2-generation-SM, which is a good approx. since the t-quark is very heavy and mixes very weakly.]

- This amplitude can be written as

$$A \sim V_{du} \bar{V}_{su} f(\Lambda, m_u, m_W) + V_{dc} \bar{V}_{sc} f(\Lambda, m_c, m_W).$$

- Since it calculates the coeff. of a higher-dim. operator (of the type $(\bar{\psi}\psi)^2$), which is not part of the tree-level lagrangian, renormalizability requires A to be finite.

- This is indeed the case since $m_u \neq m_c$ is irrelevant at high energies and

$$V_{du} \bar{V}_{su} + V_{dc} \bar{V}_{sc} = \sum_{i=u,c} V_{di} \bar{V}_{si} = (VV^+)_{ds} = 0$$

(since V is unitary).

- Once we know that A is finite, we can ask about its behaviour in the limit $m_u \to m_c$. By the same cancellation as above, A must vanish in this limit.

- The precise behaviour is $A \sim m_c^2 - m_u^2$ for $m_u \to m_c$ (see book by G. Ross on unified theories for details), implying

$$A \sim \frac{m_c^2 - m_u^2}{m_W^4}$$ for dimensional reasons.

- This extra suppression of FCNC's at 1-loop in the SM is known as <u>GIM mechanism</u> (Glashow, Iliopoulos, Maiani, '70). It occurs in many rare processes and is crucial for the correct description of flavor physics. BSM-models usually have difficulties to maintain this suppression.

<u>Comment:</u> The GIM paper <u>postulated</u> the c-quark to realize this suppression.

3.5 Neutrino masses

If the SM is only a low-energy eff. field theory (valid below some scale M), we expect higher-dim. operators even at tree level. The only such operator at mass-dim. 5 is

$$\mathcal{L} \supset \frac{1}{M}(\ell\cdot\phi)^2 = \frac{1}{M}\,\ell_i^\alpha \ell_{\alpha j}\,\varepsilon^{ik}\varepsilon^{jl}\,\underset{\uparrow}{\phi_k}\,\underset{\uparrow}{\phi_l}$$

(and its h.c.). $\qquad SU_2$ - indices

- $\phi_{vac} = \binom{0}{v} \;\Rightarrow\; \mathcal{L} \supset \frac{v^2}{M}\,\nu^\alpha \nu_\alpha + h.c.$

- This "Majorana mass term" can also be written using a "Majorana fermion" $\nu_M = \binom{\nu}{\bar\nu}$: $\quad \mathcal{L} \supset \frac{v^2}{M}\,\bar\nu_M \nu_M$.

- Interesting fact: $m_\nu \approx 5\cdot 10^{-2}\,eV$ (expected value)

  implies $M \approx 1.2\cdot 10^{15}\,GeV$ (hint at GUT scale?)

- In any case: The SM must break down at that scale (or earlier).

3.6 See-saw mechanism

(Minkowski, '77; Yanagida; Gell-Mann/Ramond/Slansky, ~'79)

- The above operator naturally arises if a total singlet ν_R (the "r.h. neutrino") is integrated out:

$$\mathcal{L} \supset \lambda\,\ell\phi\nu_R - \frac{1}{2}M\nu_R\nu_R + h.c. \;\Rightarrow\; \mathcal{L} \supset -m_D\,\nu_R\nu_L - \frac{1}{2}M\nu_R\nu_R + h.c.$$

$$\xrightarrow[\text{varying } \nu_R]{} \quad \delta\mathcal{L} \supset -\delta\nu_R\,(m_D\nu_L + M\nu_R)$$

$$\Rightarrow \nu_R = -\frac{m_D}{M}\nu_L \quad \Rightarrow \quad \mathcal{L} \supset \frac{1}{2}\underbrace{\frac{m_D^2}{M}}_{\equiv -m_\nu} \cdot \nu_L \nu_L$$

$\Rightarrow$ tiny ν-mass induced by "normal" m_D and large M through "see-saw".

- Redoing this calculation for 3 generations and N r.h. neutrinos, we find:

$$m_\nu = -\underset{\substack{\uparrow \\ 3\times N \text{ matrix}}}{m_D}\; \underset{\substack{\uparrow \\ \text{symm.} \\ N\times N \text{ matrix}}}{M^{-1}}\; \underset{\substack{\uparrow \\ N\times 3 \text{ matrix}}}{m_D^T}$$

3.7 MNS-matrix & ν-oscillations

(analogue of "CKM" ; Maki, Nakagawa, Sakata, '62)

- The complex symm. matrix m_ν can be diagonalized by a unitary trf.: $\quad m_\nu = L\, m_\nu^{diag} L^T .$

- Thus, we can write the lagrangian as

$$\mathcal{L} \supset -\bar{e}_L{}^T M_e \bar{e}_R - \frac{1}{2}\bar{\nu}^T m_\nu \bar{\nu} - \frac{g_2}{\sqrt{2}} W^{+\mu}\bar{\nu}^T \bar{\sigma}_\mu e_L + h.c.$$

$$= -\bar{e}_L{}^T L_e M_e^{diag} R_e^+ \bar{e}_R - \frac{1}{2}\bar{\nu}^T L_\nu m_\nu^{diag.} L_\nu^T \nu$$

$$- \frac{g_2}{\sqrt{2}} W^{+\mu}\bar{\nu}^T L_\nu \bar{\sigma}_\mu L_\nu^+ L_e L_e^+ e_L \quad + h.c.$$

$$= -\bar{e}_L'^T M_e^{diag} \bar{e}_R' - \frac{1}{2}\bar{\nu}'^T m_\nu^{diag}\bar{\nu}'$$

$$- \frac{g_2}{\sqrt{2}} W^{+\mu}\bar{\nu}'^T \bar{\sigma}_\mu \underbrace{V_{MNS}}_{\text{origin of } \nu\text{-mixing};\ \equiv L_\nu^+ L_e} e_L' \quad + h.c.$$

The actual numbers:

(unfortunately, only mass-square-differences are presently accessible)

solar $\nearrow$ $\Delta m_{12}^2 = m_2^2 - m_1^2 = (8.0 \pm 0.3) \cdot 10^{-5}\ eV^2$

atmosph. $\longrightarrow |\Delta m_{23}^2| = \quad .. \quad = (2.5 \pm 0.2) \cdot 10^{-3}\ eV^2$

$$V_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\varphi} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\varphi} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

<u>Note</u>: There are <u>in addition</u> two physical phases in $m_\nu^{diag.}$

Here: $s_{12} \equiv \sin\theta_{12}$ etc. <u>Data</u>: $\tan^2\theta_{12} = 0.45 \pm 0.05$

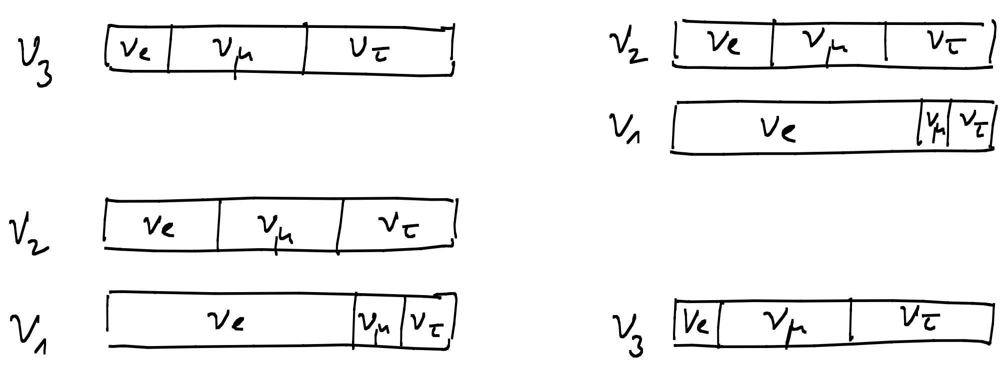
$\sin^2 2\theta_{23} = 1.02 \pm 0.04$

$\sin^2 2\theta_{13} = 0 \pm 0.05$

Theoretical speculations: • Is $\theta_{23} = 45°$ for "deep reason"

• Is $\theta_{13} = 0$ (or parametrically small)?

• Frequently used illustration:



("normal spectrum") ("inverted spectrum")