

5 The minimal supersymmetric SM (MSSM)

5.1 Supersymmetrizing the SM

- 3 (sets of) real SFs V_i ($i=1,2,3$ for U_1, SU_2, SU_3)
- chiral SFs Q, U, D, L, E, H_u, H_d

$\underbrace{Q, U, D}$	$\underbrace{L, E}$	$\underbrace{H_u, H_d}$
l.h.	r.h.	two Higgs doublets
quarks	leptons	

U_1 -charges: $1/6 \quad -2/3 \quad 1/3 \quad -1/2 \quad 1 \quad 1/2 \quad -1/2$

- lagrangian: gauge: $\sum_{i=1}^3 \left(\frac{1}{2g_i^2} \text{tr } W^2 \Big|_{\theta^2} + \text{h.c.} \right)$

Kähler: $Q^\dagger e^{2V} Q + U^\dagger e^{2V} U + \dots$

superpot.: $W = \mu H_u H_d$ ("μ-term")

(Yukawa couplings) $\rightarrow + \lambda_u Q H_u U + \lambda_d Q H_d D + \lambda_e L H_d E$
 (extra terms) $\rightarrow + a L H_u + b Q L D + c U D D + d L L E$

[only dim. 3 & 4 operators]

- Note: The second Higgs doublet, H_d , is needed since, in contrast to the SM, we can not write terms like $Q \bar{H}_u D$, $L \bar{H}_u E$ because of holomorphicity.
- To forbid the "extra" operators (which induce fast proton decay) we introduce R-symmetry.

$U_{1,R} : \phi(y, \theta) \rightarrow e^{in\alpha} \phi(y, e^{-i\alpha}\theta)$ [$n = R(\phi)$ is the "R-charge" of the SF ϕ .]

in components: $A \rightarrow e^{in\alpha} A$
 $\psi \rightarrow e^{i(n-1)\alpha} \psi$
 $F \rightarrow e^{i(n-2)\alpha} F$

- Invariance of terms in the Lagrangian:
 - $\phi^\dagger \phi / \theta^2 \bar{\theta}^2$ is invariant for any n
 - $\phi^2 / \theta^2 \rightarrow e^{2in\alpha} e^{-2i\alpha} \phi^2 / \theta^2$ is invariant only for $n=1$
 - more generally: $W(\phi) / \theta^2$ is invariant if the scalar component of $W(\phi)$ has R -charge 2
 - since e^{2V} appears in the Lagrangian, $R(V) = 0$.
 $\Rightarrow R(W_\alpha) = R(\bar{D}^2 D_\alpha V) = 2 - 1 + 0 = 1$
 $\Rightarrow W^\alpha W_\alpha / \theta^2$ is invariant.
- With the assignment

	Q	U	D	L	E	H_u	H_d
$R:$	1	1	1	1	1	0	0

 , the only terms of our $W(Q, U, \dots)$ which are allowed are the Yukawa couplings.
- This is unacceptable since we need a non-zero μ -term to give mass to the Higgsinos
- Way out: Together with SUSY-breaking, the U_1 R -symm. will be broken to its Z_2 subgroup (of the transformations $e^{i\alpha}$ ($\alpha \in [0, 2\pi)$), only those with $\alpha = 0$ & $\alpha = \pi$, i.e. 1 & -1, survive as symmetries).
- An independent reason for breaking $U_{1,R}$ to $Z_{2,R}$ is the need for a gaugino mass term $\sim (\lambda^a)^\alpha (\lambda^a)_\alpha$. (λ_α is the lowest component of W_α and hence has R -charge 1 $\Rightarrow \lambda^\alpha \lambda_\alpha$ is not inv. under $U_{1,R}$ but is invariant under $Z_{2,R} \subset U_{1,R}$.)

- Transformations under $Z_{2,R}$:

Higgs scalars	\rightarrow	+ Higgs scalars
Higgsinos	\rightarrow	- Higgsinos
fermions	\rightarrow	+ fermions
sfermions	\rightarrow	- sfermions
gauge bosons	\rightarrow	+ gauge bosons
gauginos	\rightarrow	- gauginos
- This Z_2 -symm. forbids all terms in $W(Q, U, \dots)$ except for the Yukawa couplings & the μ -term
- In addition, the lightest superpartner (LSP) will be absolutely stable since its decay to SM-particles would violate $Z_{2,R}$ ("R-parity")
- Next, we need to break SUSY to give large masses to the superpartners (except the Higgsinos, which already have a mass because of the μ -term).

5.2 SUSY breaking

- Spont. breaking: S (or \hat{H}) - SUSY-inv.
 φ_0 (or $|0\rangle$) - not SUSY-inv.
 \uparrow
 (classical ground state)

F-term breaking

- recall: $\delta_{\xi} A = \sqrt{2} \xi \psi$
 $\delta_{\xi} \psi = i\sqrt{2} \sigma^{\mu} \bar{\xi} \partial_{\mu} A + \sqrt{2} \xi F$
 $\delta_{\xi} F = i\sqrt{2} \bar{\xi} \bar{\sigma}^{\mu} \partial_{\mu} \psi$
- Poinc.-invariance $\Rightarrow \varphi_0 = 0; \partial_{\mu} A_0 = 0$
 \Rightarrow r.h. side can only be non-zero if $F_0 = \text{const.} \neq 0$.

\Rightarrow $(A_0, \psi_0, F_0) = (0, 0, \text{const.})$ is the only way in which a chiral multiplet can break SUSY in a Lorentz-inv. way. SUSY is broken in this vacuum since $\int_{\Sigma} \psi_0 = \sqrt{2} \zeta F_0 = \sqrt{2} \zeta \text{const} \neq 0$

• The simplest renormalizable model of this type is the

O'Raifeartaigh model:

$$\mathcal{L} = \sum_{i=1}^3 \bar{\phi}_i \phi_i \Big|_{\theta^2 \bar{\theta}^2} + \left[\phi_1 \left(M^2 + \frac{\lambda}{2} \phi_3^2 \right) + \mu \phi_2 \phi_3 \right] \Big|_{\theta^2} + \text{h.c.}$$

scalar potential:

$$V(\phi, \bar{\phi}) = \sum_i |F_i|^2 = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2$$

Problem: check that this follows from the EOMs for F_i .

$$\Rightarrow V = \left| M^2 + \frac{\lambda}{2} \phi_3^2 \right|^2 + \left| \mu \phi_2 + \lambda \phi_1 \phi_3 \right|^2 + \left| \mu \phi_3 \right|^2$$

This is minimized by $\phi_2 = \phi_3 = 0$ and any value of ϕ_1 (which is a "flat direction"). Hence, $F_1 = M^2$ in the vacuum.

A simpler model with F-term breaking

(more generally, chiral SF models of this type are sometimes also referred to as O'Raifeartaigh models)

- Consider $\mathcal{L} = \bar{\phi} \phi \Big|_{\theta^2 \bar{\theta}^2} + c \phi \Big|_{\theta^2} + \text{h.c.}$
- Component form: $\mathcal{L} = F \bar{F} + c F + \text{h.c.} + \dots$
 $\Rightarrow F \neq 0$
- However, SUSY is not really broken since this is a free

theory with degenerate boson & fermion.

- Introducing interactions in the form $\phi^2|_{\theta^2}$ or $\phi^3|_{\theta^2}$ does not work since then the linear term can be absorbed in a shift of ϕ and SUSY will again be unbroken.
- This problem can be overcome by introducing interactions via $(\bar{\phi}\phi)^2|_{\theta^2\bar{\theta}^2}$ which, however makes the model non-renormalizable. If accept this, we have a nice model with F-term breaking:

$$\mathcal{L} = [\bar{\phi}\phi + (\bar{\phi}\phi)^2]|_{\theta^2\bar{\theta}^2} + \phi|_{\theta^2} + \text{h.c.}$$

(all prefactors are set to 1 for simplicity)

- Ignoring derivatives and fermions, we find

$$\mathcal{L} = F\bar{F} + 4F\bar{F}\phi\bar{\phi} + F + \bar{F} + \dots$$

$$\frac{\delta}{\delta\phi} \dots \Rightarrow \phi = 0$$

$$\frac{\delta}{\delta\bar{F}} \dots \Rightarrow F + 1 = 0 \Rightarrow F = -1 \neq 0$$

A brief excursion

The full Lagrangian of such more general models of the type

$$\mathcal{L} = K(\phi^i, \bar{\phi}^{\bar{j}})|_{\theta^2\bar{\theta}^2} + W(\phi^i)|_{\theta^2} + \text{h.c.}$$

can be given in an elegant form using the definitions

$$g_{i\bar{j}} = \frac{\partial}{\partial\phi^i} \frac{\partial}{\partial\bar{\phi}^{\bar{j}}} K(\phi, \bar{\phi}) \quad (\text{Here we treat } \phi^i, \bar{\phi}^{\bar{j}} \text{ as complex variables, not SFs})$$

$$\Gamma_{ik}^m = g^{m\bar{j}} g_{i\bar{j},k} \quad ((\dots)_{,ik} = \partial_k (\dots) = \frac{\partial}{\partial \phi^k} (\dots))$$

$$R_{i\bar{j}k\bar{e}} = g_{m\bar{e}} \partial_{\bar{j}} \Gamma_{ik}^m$$

(Kähler metric, Kähler geometry,
hence: Kähler potential)

$$\begin{aligned} \Rightarrow \mathcal{L} = & -g_{i\bar{j}} \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} - i g_{i\bar{j}} \bar{\psi}^{\bar{j}} \bar{\sigma}^\mu \partial_\mu \psi^i \\ & + \frac{1}{4} R_{i\bar{j}k\bar{e}} (\psi^i \psi^k) (\bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{e}}) - \frac{1}{2} (D_i D_{\bar{j}} W) \psi^i \bar{\psi}^{\bar{j}} + \text{h.c.} \\ & - g^{i\bar{j}} (D_i W) (D_{\bar{j}} \bar{W}) \end{aligned}$$

$$\text{where } D_\mu \psi^i = \partial_\mu \psi^i + \Gamma_{jk}^i (\partial_\mu \phi^j) \psi^k$$

$$D_i W = \partial_i W \quad ; \quad D_i \partial_{\bar{j}} W = \partial_i \partial_{\bar{j}} W - \Gamma_{i\bar{j}}^k \partial_k W$$

$$\text{and } g^{i\bar{j}} \text{ is defined by } g^{i\bar{j}} g_{i\bar{k}} = \delta_{\bar{k}}^{\bar{j}}.$$

Applying this to our simple 1-field-model we find the scalar potential

$$V(\phi, \bar{\phi}) = g^{\phi\bar{\phi}} |\partial_\phi W|^2 = g^{\phi\bar{\phi}} = (g_{\phi\bar{\phi}})^{-1} = \frac{1}{1-4\phi\bar{\phi}},$$

which indeed has a minimum at $\phi = 0$.

SUSY breaking is visible since $F^i = -g^{i\bar{j}} \partial_{\bar{j}} \bar{W} \neq 0$.

D-term breaking

For a $U(1)$ SUSY gauge theory with $V \hat{=} (A_\mu, \lambda, D)$ we have

$$\delta_\xi A_\mu = i \xi \sigma_\mu \bar{\lambda} + \text{h.c.}$$

$$\delta_\xi \lambda = i \xi D + \xi \sigma^{\mu\nu} F_{\mu\nu}$$

$$\delta_\xi D = \xi \bar{\sigma}^\mu \partial_\mu \lambda + \text{h.c.}$$

- SUSY is again signalled by a non-zero auxiliary field:

$$D = \text{const.} \neq 0.$$

- Consider $\mathcal{L} = \frac{1}{2g^2} W^2|_{\theta^2} + \underbrace{2kV|_{\theta^2\bar{\theta}^2}}$

Fayet-Iliopoulos or "FI" term

(Note: such a term is only allowed in the U_1 -case since it would not be gauge-inv. for non-abelian models)

- In Components: $\mathcal{L} = \frac{1}{2g^2} D^2 + kD + \dots$

$$\Rightarrow D = -kg^2 \neq 0 \Rightarrow \text{SUSY}$$

- However, the actual dynamical model is still supersymmetric since, because of missing interactions, the spectrum is not affected by $D \neq 0$.
- SUSY becomes apparent in SUSY-QED:

$$\mathcal{L} = \frac{1}{2g^2} W^2|_{\theta^2} + \bar{\phi}_1 e^{2V} \phi|_{\theta^2\bar{\theta}^2} + \bar{\phi}_2 e^{-2V} \phi_2|_{\theta^2\bar{\theta}^2} \\ + m \phi_1 \phi_2|_{\theta^2} + \text{h.c.} + 2kV|_{\theta^2\bar{\theta}^2}$$

!

$\Rightarrow D \neq 0$; fermionic masses unaffected by $D \neq 0$,

$$m_{1,2}^2 = m^2 \pm kg^2 \text{ for charged bosons.}$$

- This feature of a "symmetric mass splitting" induced by SUSY breaking has an important generalization to all renormalizable models with spontaneous SUSY:

Mass sum rule: $\text{str } M^2 \equiv \text{tr}(M_0^2) - 2\text{tr}(M_{1/2}^+ M_{1/2}) + 3\text{tr}(M_1^2) \stackrel{!}{=} 0$

"supertrace" of the mass matrix

↑ scalar mass matrix (for real scalars in the sense of $\text{Re } \phi, \text{Im } \phi$ or $\phi, \bar{\phi}$ as indep. degrees of freedom)

↑ fermionic mass matrix (for Weyl spinors ψ_i ; note that mass terms are holomorphic (or anti-holom.) in ψ)

↑ vector boson mass matrix

In words: The sum of all squared particle masses, taking spin multiplicities into account and giving fermionic contributions an opposite sign, vanishes.

- This sum rule is one of the reasons for the fact that:
It is impossible to break SUSY spontaneously (in a phenomenologically acceptable way) within the supersymmetric SM as a renormalizable theory. (There are very many light fermions and far fewer light bosons.)

Other reasons are the limited availability of F-term VEVs (since chiral SFs are charged) and the limited availability of D-terms (just $U_{1,Y}$).

5.3 SUSY in the supersymmetric SM

The simplest solution is to break SUSY spontaneously in a "hidden sector" (a sector coupled to the SM just via non-

renormalizable operators). The transmission of SUSY to the SM is realized by these operators, which involve the non-zero F-term-VEVs of the hidden sector & SUSY-SM fields.

Simplest realization:

- Let the hidden sector contain a chiral SF S with $\langle S \rangle = \theta^2 M_s^2$ ("S" stands for "spurious"; M_s is the SUSY breaking scale).

[How SUSY breaking is in detail realized in the hidden sector will not be relevant for what follows. One may imagine some O'Raifeartaigh-like model. All masses in the hidden sector are assumed to be large, so that the dynamics of this sector is irrelevant at low energies. The F-term-VEV of S is all that we need to know.]

- Let ϕ be a generic chiral SM SF and V (with corresponding W_α) be a generic SM real SF.
- Assume that the following non-renormalizable couplings exist:

$$\begin{aligned} \textcircled{1} \quad & \frac{1}{M^2} \phi \phi \bar{S} S \Big|_{\theta^2 \bar{\theta}^2} \quad \text{or} \quad \frac{1}{M^2} \phi^2 \bar{S} S \Big|_{\theta^2 \bar{\theta}^2} \\ \textcircled{2} \quad & \frac{1}{M} \phi^3 S \Big|_{\theta^2} + \text{h.c.} \\ \textcircled{3} \quad & \frac{1}{M} W^2 S \Big|_{\theta^2} + \text{h.c.} \end{aligned}$$

- Effects: $\textcircled{1} \Rightarrow \frac{M_s^4}{M^2} |A_\phi|^2 \equiv M_0^2 |A_\phi|^2$ or $M_0^2 A_\phi^2$
 "squark, slepton and soft Higgs masses"
 (the holomorphic version appears only in the Higgs sector)

$$\textcircled{2} \Rightarrow \frac{M_s^2}{M} A \phi^3 \equiv A \cdot A \phi^3$$

↑
"trilinear couplings"

$$\textcircled{3} \Rightarrow \frac{M_s^2}{M} \lambda^\alpha \lambda_\alpha \equiv M_{1/2} \lambda^\alpha \lambda_\alpha$$

↑
"gaugino masses"

- For $M \sim M_p \sim 10^{19}$ GeV ("gravity mediation") and $M_s \sim 10^{11}$ GeV we find $M_{1/2} \sim A \sim M_0 \sim 1$ TeV, as required.
- Clearly, a large set of new parameters are introduced in this way (3 gaugino masses; as many trilinear terms as there are entries in the Yukawa matrices; 3×3 matrices (in generation space) of squark & slepton masses for every matter multiplet, ...).
 - FCNCs & other effects rule out most generic structures -
- Need to avoid (forbid by symmetries) renormalizable terms like $\phi^2 S|_{\theta^2}$, since this would give large ~~SUSY~~ masses.
- Note that the above set of higher-dim. operators ($\textcircled{1} \dots \textcircled{3}$) is not complete. (For example, terms like $\bar{\Phi} \phi S|_{\theta^2 \bar{\theta}^2}$ may also be present.) However, they are sufficient to generate all so-called "soft ~~SUSY~~ parameters" or "soft terms" in the low-energy eff. theory.
- The adjective "soft" means that these terms do not destabilize the hierarchy between the electroweak scale (= SUSY breaking scale) and some high scale.

- In the absence of SUSY breaking, the only mass parameter (which appears in the Higgs potential and hence has to be of the order of the electroweak scale) is the μ -term. It is automatically stable (i.e. receives no loop corrections $\sim \Lambda$ ($= \text{cutoff}$)) due to the non-renormalization theorem:
The superpotential receives no loop corrections.
(proof see later)
- The terms $\sim W^2|_{\mathcal{O}_2}$; $\sim \tilde{\Phi}\Phi|_{\mathcal{O}_2\bar{\mathcal{O}}_2}$ receive only logarithmically divergent corrections since their coefficients are dimensionless (as in renormalizable non-SUSY QFT).
- The dimensionfull SUSY-breaking terms all come (within the present approach) from higher-dim. operators. Such operators are generically less divergent than renormalizable operators (they can not receive loop corrections $\sim \Lambda$ for dimensional reasons).
- In summary: If we consider a SUSY-SM with SUSY-breaking terms, with $\mu \sim M_s^2/M \sim m_{EW}$, then the electroweak scale (defined in this way) is not destabilized by loop corrections.
- Comment: We argued that in renormalizable SUSY models no divergences $\sim \Lambda^n$ ($n > 0$) arise. The argument is roughly that the Kähler potential has dimensionful parameters (only terms $\sim \tilde{\Phi}\Phi$ are allowed) while the superpotential is not renormalized for fundamental reasons. There is, however, one exception to this rule:

Terms of the type $\kappa V|_{\partial^2\bar{\partial}^2}$ have the same structure as Kähler terms ($\partial^2\bar{\partial}^2$ -projection) and can indeed receive loop corrections $\sim \Lambda^2$. However, in the MSSM only one such term (related to $U_{1,Y}$) can be written down. For this term, the Λ^2 -divergence has a vanishing coefficient because of a cancellation between the various contributions (from $U_{1,Y}$ -charged matter). The deeper reason for this cancellation is the fact that $U_{1,Y}$ is non-anomalous.

- Comment: We can now be more specific concerning the idea that $U_{1,R}$ should be broken to $\mathbb{Z}_{2,R}$: let $R(S) = 0$. At the component level this means $R(A_S) = 0$; $R(F_S) = -2$. Thus,

$$U_{1,R}: F_S \rightarrow e^{-2i\alpha} F_S,$$

and $\langle F_S \rangle \neq 0$ breaks $U_{1,R}$ to $\mathbb{Z}_{2,R}$ (only $U_{1,R}$ -kfs with $\alpha = 0$ & $\alpha = \pi$ leave the vacuum invariant). This is consistent with our earlier claim that gaugino masses require the breaking of $U_{1,R}$ to $\mathbb{Z}_{2,R}$: For $R(S) = 0$ both

$W^2|_{\partial^2}$ & $\frac{1}{M} SW^2|_{\partial^2}$ are allowed. The second term induces an operator

$$\frac{F_S}{M} \lambda^2, \text{ which indeed breaks } U_{1,R} \text{ to } \mathbb{Z}_{2,R}.$$

- Comment: As we will see later, we will need $\mu \sim \frac{M_S^2}{M} \sim M_{EW}$

phenomenologically. This seems unnatural since, a priori, the two scales μ & M_s^2/M are completely unrelated. However, if $U_{1,R}$ is indeed a symmetry of the underlying theory, $\mu = 0$. If we allow for an operators

$$\frac{1}{M} \bar{S} H_u H_d \Big|_{\theta^2 \bar{\theta}^2} \quad \& \quad \frac{1}{M^2} \bar{S} S H_u H_d \Big|_{\theta^2 \bar{\theta}^2},$$

we find at a (partial) component level

$$\frac{\bar{F}_S}{M} \underbrace{H_u H_d}_{\text{superfields}} \Big|_{\theta^2} \quad \& \quad \frac{|F_S|^2}{M^2} \underbrace{H_u H_d}_{\text{A-terms of Higgs-superfields}}$$

$$\Rightarrow \mu H_u H_d \Big|_{\theta^2} \quad \text{with} \quad \mu \sim M_s^2/M \quad (\text{"}\mu\text{-term"})$$

$$\& (B\mu) H_u H_d \quad \text{with} \quad (B\mu) \sim (M_s^2/M)^2 \quad (\text{"holomorphic$$

\uparrow Higgs mass term or "B μ -term")

(This is just a name - there is no deep reason for writing the coefficient of this operator as a product of μ with some new parameter B.)

This way of generating the supersymmetric μ -term and the SUSY-breaking B μ -term with the same mass scale is known as the " Giudice-Masiero mechanism".

Note: $\bar{S} H_u H_d \Big|_{\theta^2 \bar{\theta}^2}$ is $U_{1,R}$ -invariant since $R(S) = R(H_{u,d}) = 0$ and the projection on $\theta^2 \bar{\theta}^2$ does not change the R-charge.

5.4 Soft terms

SUSY breaking in the MSSM can also be viewed from a different perspective:

- Work out the component Lagrangian of the SUSY version of the SM (The μ -term is the only dim.-ful parameter).
- Add all SUSY-breaking terms which do not introduce power-like divergent loop-corrections ("soft terms").
- Without proof, these terms are given in $\mathcal{L}_{\text{soft}}$ below.

[\tilde{g} is the superpartner of the gluon g (i.e. a Weyl fermion);
 \tilde{e} is the superpartner of the r.h. electron (i.e. a scalar); etc.]

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & -\frac{1}{2} (M_1 \tilde{B}\tilde{B} + M_2 \tilde{A}\tilde{A} + M_3 \tilde{g}\tilde{g}) + \text{h.c.} \\ & - (\tilde{u}^T A_u \tilde{q} H_u + \tilde{d}^T A_d \tilde{q} H_d + \tilde{e}^T A_e \tilde{l} H_d) + \text{h.c.} \\ & - (\tilde{q}^+ m_Q^2 \tilde{q} + \tilde{l}^+ m_L^2 \tilde{l} + \tilde{u}^+ m_u^2 \tilde{u} + \tilde{d}^+ m_D^2 \tilde{d} + \tilde{e}^+ m_E^2 \tilde{e}) + \text{h.c.} \\ & - m_{H_u}^2 \bar{H}_u H_u - m_{H_d}^2 \bar{H}_d H_d - (b H_u H_d + \text{h.c.}) \\ & \qquad \qquad \qquad \uparrow \\ & \qquad \qquad \qquad b \equiv B\mu \text{ for simplicity} \end{aligned}$$

(M_1, M_2, M_3 are the bino, wino & gaugino mass)

↑
because of its relation to the W -bosons.

- As we have shown above, all these terms can originate from higher-dim. operators coupling the SUSY-SM to a hidden sector with F -term breaking. This is the conceptual reason why these terms do not introduce power-like divergences. (Recall that higher-dim. operators are less divergent than renorm. oper.)

Comment: This argument may appear to be not very convincing since, in the eff. theory below M_s , one simply has given certain scalars a ~~SUSY~~ mass term, and it is not clear why this mass should not receive corrections $\sim \Lambda$.

- To understand this more deeply, one may perform the following (advanced) exercise:
- Consider

$$\mathcal{L} = \underbrace{(S\bar{S} - c_1 S\bar{S}) \Big|_{\theta^2\bar{\theta}^2}}_{\text{hidden sector}} + \underbrace{c_2 S \Big|_{\theta^2}}_{\text{"Higgs"} + \text{h.c.}} + \underbrace{\frac{1}{M^2} \phi\bar{\phi} S\bar{S} \Big|_{\theta^2\bar{\theta}^2}}_{\text{SUSY-mediating operator}}.$$

- Work out the general form of loop corrections (SUSY will not be broken by loop corrections \Rightarrow the general superfield structure will be respected; no superpotential for ϕ will be induced; all Kähler terms may receive small loop corrections; new terms like $\phi\bar{\phi} S \Big|_{\theta^2\bar{\theta}^2}$ etc. may be induced (but they will be suppressed by $\frac{1}{M}$))
- Work out the low-energy spectrum (i.e. the mass of ϕ) before & after these loop corrections.
- The ~~SUSY~~ mass of ϕ will be $\sim \frac{1}{M}$ in both cases (i.e. the hierarchy will not be destabilized).

Note: For this to work it is important that no linear terms in ϕ (like $\bar{S}\phi \Big|_{\theta^2\bar{\theta}^2}$) are induced.

This can be achieved by requiring a symmetry like $\phi \rightarrow e^{i\alpha} \phi$, which is in fact present in the above Lagrangian. In our simplistic model this symmetry is, unfortunately, anomalous. However, in the SM all chiral SFs are protected by non-anomalous gauge symmetries. Thus, no terms linear in the light fields ("tadpoles") are induced. The above argument goes through!

(See the review of Bagger referenced at my SUSY/SUGRA lecture Web page for more details.)