Strong CP-problem & Axion

$$\mathcal{L} = -\frac{1}{2g^2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{32\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\,F_{\mu\nu}F_{\rho\sigma}$$

(or $\mathrm{tr}\,F\tilde{F}$)

total derivative, yet physical: $\sim \mathrm{tr}\,F\wedge F$

$$n = \frac{1}{64\pi^2}\int d^4x\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \quad \text{— Instanton \#.}$$

$\pi_d(M)$ — dth homotopy group (group of classes of maps $S^d \to M$)

$\pi_3(SU_N)$ — non-triv.

(easy ex.: $SU_2 \simeq S^3$)

Wick rotate to Eucl. space, let A_μ be pure gauge at inf. of Eucl. $\mathbb{R}^4$ (at the S^3 boundary): $A_\mu = \partial_\mu U U^{-1}$

where U is the map $S^3 \to SU_N$

(charact. by integer n).

But: $F_{\mu\nu} \neq 0$ inside S^3 $\Rightarrow$ instanton!

(suppr. in eucl. path int.: $e^{-8\pi^2|n|/g^2}$)

Nevertheless: must be incl. in $\int \mathcal{D}A$

"rel. weighting" by $e^{\# i\theta}$ physical!

Diff. perspective:

~~[illegible]~~ fix t; go to gauge $A_0 = 0$, $\mathbb{R}^3 \,\hat{=}\, S^3$

let $A_i(\vec{x}) = (\partial_i U(\vec{x}))\, U(x)^{-1}$

"diff." (U as before)

$\Rightarrow$ vacua related by "large gauge tf."
characterized by n

θ-vac.:

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta} |n\rangle$$

($n \to n+1$ can only change θ by phase!)

(The above inst. corr. to tunneling between ~~the~~ the diff. vacua)

Include massless quarks (e.g. QCD w. just u_L, u_R) and let $u \to e^{i\gamma_5 \eta} u$

(Dirac notation)

anomaly $\Rightarrow$ $\theta \to \theta - 2\eta$

$\Rightarrow$ θ unphysical

Our world: ~~θ — θ~~ θ equiv. to phase of mass: $\mathcal{L} \supset m \bar{u}_R u_L$; can change by phase!

more generally: $m \to M$ (mass matrix)

$\bar{\theta} = \theta - \arg\det M$ is physical!

$\bar{\theta} \neq 0 \Rightarrow$ CP violation

(n el. dipole moment $\Rightarrow \bar{\theta} \lesssim 10^{-9}$)

fine tuning!

Ways out: 1) $m_u = 0$

2) CP cons. fundamentally; broken spont. at high-scale to induce obs. CP-viol. in weak sector; $\bar{\theta}$ can be kept small in SUSY version of this

3) Axion (Peccei-Quinn; Kim; Dine/Fischler/Srednicki)

Here: only 3)

Idea: promote the $U(1)_A$ to spont. broken class. symm. (before anomaly)

Suppl. $\mathcal{L}$ by

$$-\frac{F_a^2}{2}(\partial_\mu a)(\partial^\mu a) + \frac{a}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\operatorname{tr} F_{\mu\nu}F_{\rho\sigma}$$

or

$$\underbrace{-\frac{1}{2}(\partial_\mu \tilde{a})(\partial^\mu \tilde{a}) + \frac{\tilde{a}}{32\pi^2 F_a}(\operatorname{tr} F \wedge F)_{\mu\nu}}_{\text{higher-dim. op.}}$$

(can absorb any present θ-term in def. of a)

$U(1)_A$ acts as shift symm. of a and is always broken spont.

The coeff. of the $F\tilde{F}$-term is now a dyn. question:
Which VEV does "a" take ~~...~~?

Simplest is low-energy limit of QCD: Chiral Lagr.:

$$\mathcal{L} = -\frac{F_\pi^2}{16}\,\mathrm{tr}\left(\partial_\mu \Sigma\, \partial^\mu \Sigma^{-1}\right) + \frac{v}{2}\,\mathrm{tr}\left(M\Sigma + \bar{M}\Sigma^{-1}\right)$$

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

$F_\pi = 184$ MeV (yet another conv.!)

$\boxed{\text{Srednicki Witten}}$

$v(m_u + m_d) = F_\pi^2 m_\pi^2 / 4$

including a: $M = \begin{pmatrix} m_u e^{ia} & 0 \\ 0 & m_d \end{pmatrix}$

extremize $V(\Sigma, a)$ in Σ at fixed a

$$\Rightarrow V(a) = \frac{a^2}{8} F_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

$$\Rightarrow m_a = \frac{F_\pi m_\pi}{2F_a} \cdot \frac{\sqrt{m_u m_d}}{m_u + m_d} \qquad (m_u/m_d \sim 1/1.8)$$

$$\Rightarrow m_a \simeq 5 \cdot 10^{-10}\,\mathrm{eV} \cdot \frac{10^{16}\,\mathrm{GeV}}{F_a} \Leftarrow \text{"natural value?"}$$

SN cooling etc. $\Rightarrow F_a \gtrsim 10^9$ GeV
Ax. DM overprod. $\Rightarrow F_a \lesssim 10^{12}$ GeV
} Ax. window

Srednicki/Witten: difficult to get in ST (But axion natural in general!)

Historically: Axion from chiral extension of SM

Peccei/Quinn: several Higgses allowing for $U(1)_A$; spont. broken at weak scale

(excluded exp.!)

Kim "invis. axion"

Fischler/Dine/Srednicki/Zhitnitsky

($U(1)_A$ broken spont. at scale $\gg M_Z$)

ST: $U(1)_A$ broken at M_{string}

PC: NMSSM

$$u Q H_u \;;\; d Q H_d \;;\; \underbrace{S}_{e^{-2i\alpha}} \underbrace{H_u H_d}_{e^{2i\alpha}} \qquad \cancel{\lambda S^3}$$

$e^{i\alpha}$ $\qquad$ $e^{i\alpha}$

? need $\langle S \rangle \gg M_Z$

needed to comp. chiral phase rot.
