4 Supersymmetry Breaking

4.1 General remarks

- We have seen that (in class. FT) $\delta_\xi = \xi Q + \bar{\xi}\bar{Q}$ is a symm. of S. It is generated by the diff. operator Q and it transforms integer-spin-fields $\Leftrightarrow$ half-int.-spin fields.

- Thus, after quantization, we expect that Q will be promoted to a Hilbert-space operator commuting with H and transforming bosons $\Leftrightarrow$ fermions.
- We also know $[Q_\alpha, P_\mu P^\mu] = 0$ (now for Hilb.-sp. operators), implying that $|\Psi\rangle$ & $Q_\alpha|\Psi\rangle$ have the same mass for any $|\Psi\rangle$. In other words, A & ψ (of Φ) and A_μ & λ_α (of V) have the exact same mass (and, obviously, the same gauge-group repres.). This is in clear contradiction to the SM where (almost) no field has an "oppos.-statistics" partner in the same gauge-group repres.
- Thus, if SUSY is a symm. of some fund. S, it is spontaneously broken, i.e. S (or $\hat{H}$) – SUSY-inv.

  φ_0 (or $|0\rangle$) – not SUSY-inv.

  (classical ground state)
- Now, if $|\Psi\rangle$ is a 1-part.-state, $Q|\Psi\rangle$ is <u>not</u> any more a 1-part. state. This solves the "problem" above.

4.2 F-term breaking

- recall:

$$\delta_\xi A = \sqrt{2}\,\xi\psi$$
$$\delta_\xi \psi = i\sqrt{2}\,\sigma^\mu\bar{\xi}\partial_\mu A + \sqrt{2}\,\xi F$$
$$\delta_\xi F = i\sqrt{2}\,\bar{\xi}\bar{\sigma}^\mu\partial_\mu\psi$$

- Poinc.-invariance $\Rightarrow$ $\psi_0 = 0$; $\partial_\mu A_0 = 0$

  $\Rightarrow$ r.h. side can only be non-zero if $F_0 = \text{const.} \neq 0$.

$\Rightarrow$ $(A_0, \psi_0, F_0) = (\dots, 0, \text{const.})$ is the only way in which a chiral multiplet can break SUSY in a Lorentz-inv. way. SUSY is broken in this vacuum since $\delta_\xi \psi_0 = \sqrt{2}\,\xi F_0 = \sqrt{2}\,\xi\,\text{const} \neq 0$

- The simplest <u>renormalizable</u> model of this type is the

<u>O'Raifeartaigh model:</u>

$$\mathcal{L} = \sum_{i=1}^{3} \bar{\Phi}_i \phi_i \Big|_{\theta^2\bar{\theta}^2} + \Big[\phi_1 \big(M^2 + \tfrac{\lambda}{2}\phi_3^2\big) + \mu \phi_2 \phi_3 \Big]\Big|_{\theta^2} + \text{h.c.}$$

scalar potential:

viewed as fct. of complex scalars ϕ_i (not of the corresponding SFs!)

$$V(\phi, \bar{\phi}) = \sum_i |F_i|^2 = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2$$

<u>Problem:</u> check that this follows from the EOMs for F_i

$$\Rightarrow V = \left|M^2 + \tfrac{\lambda}{2}\phi_3^2\right|^2 + |\mu\phi_2 + \lambda\phi_1\phi_3|^2 + |\mu\phi_3|^2$$

This is minimized by $\phi_2 = \phi_3 = 0$ and any value of ϕ_1 (which is a "flat direction"). Hence, $F_1 = M^2$ in the vacuum.

<u>A simpler model with F-term breaking</u>

(more generally, chiral SF models of this type are sometimes also referred to as O'Raifeartaigh models)

- Consider $\mathcal{L} = \bar{\Phi}\phi \big|_{\theta^2\bar{\theta}^2} + c\,\phi\big|_{\theta^2} + \text{h.c.}$
- Component form: $\mathcal{L} = F\bar{F} + cF + \text{h.c.} + \cdots$

$$\Rightarrow F \neq 0$$

- However, SUSY is not really broken since this is a free theory with degenerate boson & fermion.

- Introducing interactions in the form $\phi^2|_{\theta^2}$ or $\phi^3|_{\theta^2}$ does not work since since then the linear term can be absorbed in a shift of ϕ and SUSY will again be unbroken.

- This problem can be overcome by introducing interactions via $(\bar{\phi}\phi)^2|_{\theta^2\bar{\theta}^2}$ which, however makes the model non-renormalizable. If we accept this, we have a nice model with F-term breaking:

$$\mathcal{L} = [\bar{\phi}\phi - (\phi\bar{\phi})^2]|_{\theta^2\bar{\theta}^2} + \phi|_{\theta^2} + h.c.$$

(all prefactors are set to 1 for simplicity)

- Ignoring derivatives and fermions, we find

$$\mathcal{L} = F\bar{F} - 4F\bar{F}\phi\bar{\phi} + F + \bar{F} + \cdots$$

$$\frac{\delta}{\delta\bar{\phi}} \cdots \Rightarrow \phi = 0$$

$$\frac{\delta}{\delta\bar{F}} \cdots \Rightarrow F + 1 = 0 \Rightarrow F = -1 \neq 0$$

4.3 Systematic treatment of general chiral-SF models

The full lagrangian of such more general models of the type

$$\mathcal{L} = K(\phi^i, \bar{\phi}^{\bar{\imath}})|_{\theta^2\bar{\theta}^2} + W(\phi^i)|_{\theta^2} + h.c.$$

can be given in an elegant form using the definitions

$$g_{i\bar{\jmath}} = \frac{\partial}{\partial\phi^i}\frac{\partial}{\partial\bar{\phi}^{\bar{\jmath}}} K(\phi, \bar{\phi})$$ (Here we treat $\phi^i, \bar{\phi}^{\bar{\jmath}}$ as complex variables, not SFs)

$$\Gamma_{ik}{}^{m} = g^{m\bar{j}}\, g_{i\bar{j},k} \qquad \left((\dots)_{,k} = \partial_k (\dots) = \frac{\partial}{\partial \phi^k}(\dots) \right)$$

$$R_{i\bar{j}k\bar{l}} = g_{m\bar{l}}\, \partial_{\bar{j}}\, \Gamma_{ik}{}^{m}$$

(Kähler metric, Kähler geometry, hence: Kähler potential)

$$\Rightarrow \mathcal{L} = - g_{i\bar{j}}\, \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} - i g_{i\bar{j}}\, \bar{\psi}^{\bar{j}} \bar{\sigma}^\mu D_\mu \psi^i$$

$$+ \frac{1}{4} R_{i\bar{j}k\bar{l}} (\psi^i \psi^k)(\bar{\psi}^{\bar{j}} \bar{\psi}^{\bar{l}}) - \frac{1}{2} (D_i D_j W) \psi^i \psi^j + \text{h.c.}$$

$$- g^{i\bar{j}} (D_i W)(D_{\bar{j}} \bar{W})$$

where $D_\mu \psi^i = \partial_\mu \psi^i + \Gamma_{jk}{}^{i} (\partial_\mu \phi^j) \psi^k$

$D_i W = \partial_i W \quad ; \quad D_i \partial_j W = \partial_i \partial_j W - \Gamma_{ij}{}^{k} \partial_k W$

and $g^{i\bar{j}}$ is defined by $g^{i\bar{j}} g_{i\bar{k}} = \delta^{\bar{j}}_{\bar{k}}$.

Applying this to our simple 1-field-model we find the scalar potential

$$V(\phi, \bar{\phi}) = g^{\phi\bar{\phi}} |\partial_\phi W|^2 = g^{\phi\bar{\phi}} = (g_{\phi\bar{\phi}})^{-1} = \frac{1}{1 - 4\phi\bar{\phi}},$$

which indeed has a minimum at $\phi = 0$.

SUSY breaking is visible since $F^i = -g^{i\bar{j}} \partial_{\bar{j}} \bar{W} \neq 0$.

4.4 D-term Breaking

For a $U(1)$ SUSY gauge theory with $V \hat{=} (A_\mu, \lambda, D)$ we have

$$\delta_\xi A_\mu = i \xi \sigma_\mu \bar{\lambda} + \text{h.c.}$$

$$\delta_\xi \lambda = i \xi D + \xi \sigma^{\mu\nu} F_{\mu\nu}$$

$$\delta_\xi D = \bar{\xi} \bar{\sigma}^\mu \partial_\mu \lambda + \text{h.c.}$$

- $\cancel{\text{SUSY}}$ is again signalled by a non-zero auxiliary field:
  $D = \text{const.} \neq 0$.

- Consider $\mathcal{L} = \frac{1}{2g^2} W^2 \Big|_{\theta^2} + \underbrace{2\kappa V \Big|_{\theta^2\bar\theta^2}}$

  Fayet-Iliopoulos or "FI" term

  (Note: such a term is only allowed in the U_1-case since it would not be gauge-inv. for non-abelian models)

- In Components: $\mathcal{L} = \frac{1}{2g^2} D^2 + \kappa D + \cdots$

  $$\Rightarrow D = -\kappa g^2 \neq 0 \Rightarrow \cancel{\text{SUSY}}$$

- However, the actual dynamical model is still supersymmetric since, because of missing interactions, the spectrum is not affected by $D \neq 0$.

- $\cancel{\text{SUSY}}$ becomes apparent in SUSY-QED:

$$\mathcal{L} = \frac{1}{2g^2} W^2 \Big|_{\theta^2} + \bar\Phi_1 e^{2V} \phi_1 \Big|_{\theta^2\bar\theta^2} + \bar\Phi_2 e^{-2V} \phi_2 \Big|_{\theta^2\bar\theta^2}$$

$$+ m \phi_1 \phi_2 \big|_{\theta^2} + \text{h.c.} + 2\kappa V \Big|_{\theta^2\bar\theta^2}$$

$\Rightarrow D \neq 0$; fermionic masses unaffected by $D \neq 0$,

$$m_{1,2}^2 = m^2 \pm \kappa g^2 \text{ for charged bosons.}$$

- This feature of a "symmetric mass splitting" induced by SUSY breaking has an important generalization to all renormalizable models with spontaneous $\cancel{\text{SUSY}}$:

<u>Mass sum rule:</u> $\operatorname{str} M^2 \equiv \operatorname{tr}(M_0^2) - 2\operatorname{tr}(M_{1/2}^+ M_{1/2}) + 3\operatorname{tr}(M_1^2) \overset{!}{=} 0$

"supertrace" of the mass matrix

scalar mass matrix (for real scalars in the sense of $\operatorname{Re}\phi, \operatorname{Im}\phi$ or $\phi, \bar\phi$ as indep. degrees of freedom

fermionic mass matrix (for Weyl spinors ψ; note that mass terms are holomorphic (or anti-holom.) in ψ

vector boson mass matrix

<u>In words:</u> The sum of all squared particle masses, taking spin multiplicities into account and giving fermionic contributions an opposite sign, vanishes.