# 5 The minimal supersymmetric SM (MSSM)

## 5.1 Supersymmetrizing the SM

- 3 (sets of) real SFs $V_i$ ($i = 1,2,3$ for $U_1$, $SU_2$, $SU_3$)
- chiral SFs $\underbrace{Q}_{\text{l.h.}}, \underbrace{U, D}_{\text{r.h.}}, \underbrace{L}_{\text{l.h.}}, \underbrace{E}_{\text{r.h.}}, \underbrace{H_u, H_d}_{\text{two Higgs doublets}}$

quarks (l.h., r.h.); leptons (l.h., r.h.)

$U_1$-charges: $1/6 \quad -2/3 \quad 1/3 \quad -1/2 \quad 1 \quad 1/2 \quad -1/2$

- <u>lagrangian:</u> gauge: $\sum_{i=1}^{3} \left( \frac{1}{2g_i^2} \operatorname{tr} W^2 \big|_{\theta^2} + h.c. \right)$

Kähler: $Q^+ e^{2V} Q + U^+ e^{2V} U + \cdots$

superpot.: $W = \mu H_u H_d$ ("$\mu$-term")

(Yukawa couplings) $\longrightarrow$ $+ \lambda_u Q H_u U + \lambda_d Q H_d D + \lambda_e L H_d E$

(extra terms) $\longrightarrow$ $+ a L H_u + b QLD + c UDD + d LLE$

[only dim. 3 & 4 operators]

- <u>Note:</u> The second Higgs doublet, $H_d$, is needed since, in contrast to the SM, we can not write terms like $Q \bar{H}_u D$; $L \bar{H}_u E$ because of holomorphicity.

- To forbid the "extra" operators (which induce fast proton* decay) we introduce <u>R-symmetry</u>.

* see p. 31

$U_{1,R}$: $\phi(y,\theta) \longrightarrow e^{in\alpha} \phi(y, e^{-i\alpha}\theta)$ [$n = R(\phi)$ is the "R-charge" of the SF $\phi$.]

in components: 
$$A \to e^{in\alpha} A$$
$$\psi \to e^{i(n-1)\alpha} \psi$$
$$F \to e^{i(n-2)\alpha} F$$

- Invariance of terms in the Lagrangian:
  - $\phi^+\phi|_{\theta^2\bar\theta^2}$ is invariant for any $n$
  - $\phi^2|_{\theta^2} \longrightarrow e^{2in\alpha}e^{-2i\alpha}\phi^2|_{\theta^2}$ is invariant only for $n=1$
  - more generally: $W(\phi)|_{\theta^2}$ is invariant if the scalar component of $W(\phi)$ has R-charge 2
  - since $e^{2V}$ appears in the Lagrangian, $R(V)=0$.

    $\Rightarrow R(W_\alpha) = R(\bar D^2 D_\alpha V) = 2-1+0 = 1$

    $\Rightarrow W^\alpha W_\alpha|_{\theta^2}$ is invariant.

- With the assignment

| | $Q$ | $U$ | $D$ | $L$ | $E$ | $H_u$ | $H_d$ |
|---|---|---|---|---|---|---|---|
| R: | 1 | 1 | 1 | 1 | 1 | 0 | 0 |

  the only terms of our $W(Q,U,\dots)$ which are allowed are the Yukawa couplings.

- This is inacceptable since we need a non-zero $\mu$-term to give mass to the Higgsinos

- Way out: Together with SUSY-breaking, the $U_1$ R-symm.** will be broken to its $Z_2$ subgroup (of the transformations $e^{i\alpha}$ ($\alpha\in[0,2\pi)$), only those with $\alpha=0$ & $\alpha=\pi$, i.e. $1$ & $-1$, survive as symmetries).

see p.30.1

- An independent reason for breaking $U_{1,R}$ to $Z_{2,R}$ is the need for a gaugino mass term $\sim (\lambda^a)^\alpha(\lambda^a)_\alpha$.
  ($\lambda_\alpha$ is the lowest component of $W_\alpha$ and hence has R-charge 1 $\Rightarrow \lambda^\alpha\lambda_\alpha$ is not inv. under $U_{1,R}$ but <u>is</u> invariant under $Z_{2,R}\subset U_{1,R}$.)

**) In the simplest case, one might assume that the $F$-term $F_S$ of some chiral SF $S$ with $R(S) = 0$ can do this job (indeed $R(F_S) = -2$, breaking $U_{1,R}$ to $Z_{2,R}$). However, such a spont. breaking leads to a Goldstone boson (the R-axion) which is ruled out even if it couples very weakly.

Ways out:
- Break R-symm. also explicitly (e.g. by a constant $W$: $W \to W + W_0$). Note that this does not necessarily destroy R-symm.-based arguments forbidding certain terms in the Lagrangian (superpot. is not renormalized, so some sectors may still respect R-symm.)

- Related to this: $U_{1,R}$ may be broken to $Z_{2,R}$ ($Z_{n,R}$) by compactification of string theory or higher-dim. SUGRA
- $U_{1,R}$ may be gauged (in which case the R-axion is "eaten" by the gauge boson). This requires SUGRA.

Note: Much of the discussion in the literature is related not to $U_{1,R}$ of SSM but $U_{1,R}$ required by spont. SUSY:

$\rightarrow$ Nelson / Seiberg hep-ph/9309299
(see also: hep-ph/9405345 ; hep-ph/9606388 ; 0705.1359)

Finally: One may get the desired $Z_{2,R}$ from a different (non-R) $U_1$-symm. (e.g. in $SO_{10}$-GUTs, see later)

- Transformations under $Z_{2,R}$:

| | | |
|---|---|---|
| Higgs scalars | $\to$ | + Higgs scalars |
| Higgsinos | $\to$ | − Higgsinos |
| fermions | $\to$ | + fermions |
| sfermions | $\to$ | − sfermions |
| gauge bosons | $\to$ | + gauge bosons |
| gauginos | $\to$ | − gauginos |

- This $Z_2$-symm. forbids all terms in $W(Q, U, \cdots)$ except for the Yukawa couplings & the $\mu$-term
- In addition, the lightest superpartner (LSP) will be absolutely stable since its decay to SM-particles would violate $Z_{2,R}$ ("R-parity")
- Next, we need to break SUSY to give large masses to the superpartners (except the Higgsinos, which already have a mass because of the $\mu$-term).

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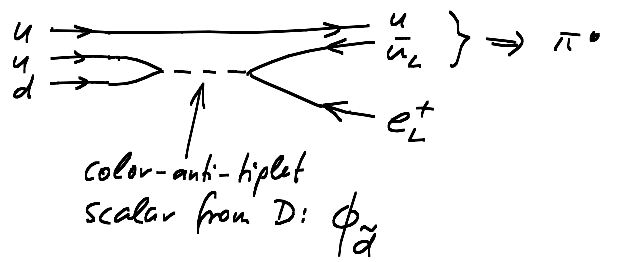
*) proton decay using UDD, QLD & h.c.:

$$\overline{\psi}^c_{D,u}\,\psi_{D,d}\,\phi^*_{\tilde d} \;\;(\text{h.c.}) \qquad \overline{\psi}^c_{D,L}\,\psi_{D,Q}\,\phi_{\tilde d}$$

recall: $\psi_D = \begin{pmatrix} \psi_L \\ \bar\psi_R \end{pmatrix}$

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## 5.3 ~~SUSY~~ in the supersymmetric SM

- We know that SUSY must be (spont.) broken. In the spirit of the mass sum rule of Sect. 4.4, one can show that this is impossible (at tree level) in the SSM (cf. Theorem in Dimopoulos, Georgi, Nucl. Phys. B193 ('81) 150). Idea: focus on $SU_3$-triplets; note: $m_{1/2}$ unchanged; $m_0$ split up/down by ~~SUSY~~ $\Rightarrow$ one always finds a light scalar triplet, which is clearly not observed in nature.
- The simplest solution is to break SUSY spontaneously in a "hidden sector" (a sector coupled to the SM just via non-

renormalizable operators). The transmission of ~~SUSY~~ to the SM is realized by these operators, which involve the non-zero F-term-VEVs of the hidden sector & SUSY-SM fields.

Simplest realization:

- Let the hidden sector contain a chiral SF $S$ with $\langle S \rangle = \theta^2 F_S = \theta^2 M_S^2$ ( "S" stands for "spurion"; $M_S$ is the SUSY breaking scale ).

[How SUSY breaking is in detail realized in the hidden sector will not be relevant for what follows. One may imagine some O'Raifeartaigh-like model. All masses in the hidden sector are assumed to be large, so that the dynamics of this sector is irrelevant at low energies. The F-term-VEV of $S$ is all that we need to know.]

- Let $Q$ be a generic chiral SM SF and $V$ (with corresponding $W_\alpha$) be a generic SM real SF.

- Assume that the following non-renormalizable couplings exist:

$$① \quad \frac{1}{M^2}\bar{Q}Q\bar{S}S\Big|_{\theta^2\bar{\theta}^2} \quad \text{or} \quad \frac{1}{M^2}Q^2\bar{S}S\Big|_{\theta^2\bar{\theta}^2}$$

$$② \quad \frac{1}{M}Q^3 S\Big|_{\theta^2} \quad + \text{h.c.}$$

$$③ \quad \frac{1}{M}W^2 S\Big|_{\theta^2} \quad + \text{h.c.}$$

- Effects: ① $\Rightarrow$ $\frac{M_S^4}{M^2}|A_Q|^2 \equiv M_0^2|A_Q|^2$ or $M_0^2 A_Q^2$

"squark, slepton and soft Higgs masses"

(the holomorphic version appears only in the Higgs sector)

(2) $\Rightarrow \quad \frac{M_s^2}{M} A_Q^{\ 3} \equiv A \cdot A_Q^{\ 3}$

↑ "trilinear couplings"

(3) $\Rightarrow \quad \frac{M_s^2}{M} \lambda^\alpha \lambda_\alpha \equiv M_{1/2} \lambda^\alpha \lambda_\alpha$

↑ "gaugino masses"

(Of course, $M_s^2/M = F_s/M$)

- For $M \sim M_p \sim 10^{19}$ GeV ("gravity mediation") and $M_s \sim 10^{11}$ GeV we find $M_{1/2} \sim A \sim M_0 \sim 1$ TeV, as required.

- Clearly, a large set of new parameters are introduced in this way (3 gaugino masses, as many trilinear terms as there are entries in the Yukawa matrices; 3x3 matrices (in generation space) of squark & slepton masses for every matter multiplet, ...).

  – FCNCs & other effects rule out most generic structures –

- Need to avoid (forbid by symmetries) renormalizable terms like $Q^2 S|_{\theta^2}$, since this would give large ~~SUSY~~ masses.

- Note that the above set of higher-dim. operators (1 ... 3) is not complete. (For example, terms like $\bar{Q} Q S|_{\theta^2\bar{\theta}^2}$ may also be present.) However, they are sufficient to generate all so-called "soft ~~SUSY~~ parameters" or "soft terms" in the low-energy eff. theory.

- The adjective "soft" means that these terms do not destabilize the hierarchy between the electroweak scale (= SUSY breaking scale) and some high scale.

- In the absence of SUSY breaking, the only mass parameter (which appears in the Higgs potential and hence has to be of the order of the electroweak scale) is the $\mu$-term.
  It is automatically stable (i.e. receives no loop corrections $\sim \Lambda$ (= cutoff)) due to the

  <u>non-renormalization theorem:</u>
  The superpotential receives no loop corrections.
  (proof see later)

- The terms $\sim W^2|_{\theta^2}$ ; $\sim \bar{Q}Q|_{\theta^2\bar{\theta}^2}$ receive only logarithmically divergent corrections since their coefficients are dimensionless (as in renormalizable non-SUSY QFT).

- The dimensionful SUSY-breaking terms all come (within the present approach) from higher-dim. operators. Such operators are generically less divergent than renormalizable operators (they can not receive loop corrections $\sim \Lambda^n$ $(n>0)$ for dimensional reasons).

- <u>In summary:</u> If we consider a SUSY-SM with SUSY-breaking terms, with $\mu \sim M_S^2/M \sim m_{EW}$, then the electroweak scale (defined in this way) is not destabilized by loop corrections.

- <u>Comment:</u> We argued that in renormalizable SUSY models no divergences $\sim \Lambda^n$ $(n>0)$ arise. The argument is roughly that the Kähler potential has dimensionful parameters (only terms $\sim \bar{Q}Q$ are allowed) while the superpotential is not renormalized for fundamental reasons. There is, however, one exception to this rule:

Terms of the type $\kappa V|_{\theta^2\bar\theta^2}$ have the same structure as Kähler terms ( $\theta^2\bar\theta^2$-projection) and can indeed receive loop corrections $\sim \Lambda^2$. Hower, in the MSSM only one such term (related to $U_{1,Y}$) can be written down. For this term, the $\Lambda^2$-divergence has a vanishing coefficient becaus of a cancellation between the various contributions (from $U_{1,Y}$ – charged matter). The deeper reason for this cancellation is the fact that $U_{1,Y}$ is <u>non-anomalous</u>.

- <u>Comment:</u> We can now be more specific concerning the idea that $U_{1,R}$ should be broken to $\mathbb{Z}_{2,R}$: let $R(S)=0$. At the component level this means $R(A_S)=0$ ; $R(F_S)=-2$. Thus,

$$U_{1,R}: \quad F_S \to e^{-2i\alpha} F_S \,,$$

and $\langle F_S \rangle \neq 0$ breaks $U_{1,R}$ to $\mathbb{Z}_{2,R}$ (only $U_{1,R}$-trfs. with $\alpha = 0$ & $\alpha = \pi$ leave the vacuum invariant). This is consistent with our earlier claim that gaugino masses require the breaking of $U_{1,R}$ to $\mathbb{Z}_{2,R}$: For $R(S)=0$ both

$W^2|_{\theta^2}$ & $\frac{1}{M} S W^2|_{\theta^2}$ are allowed. The second term induces an operator

$\frac{F_S}{M}\lambda^2$, which indeed breaks $U_{1,R}$ to $\mathbb{Z}_{2,R}$.

- <u>Comment:</u> As we will see later, we will need $\mu \sim \frac{M_S^2}{M} \sim m_{EW}$

phenomenologically. This seems unnatural since, a priori, the two scales $\mu$ & $M_s^2/M$ are completely unrelated. However, if $U_{1,R}$ is indeed a symmetry of the underlying theory, $\mu = 0$.
If we allow for an operators

$$\frac{1}{M}\bar{S} H_u H_d \Big|_{\theta^2\bar\theta^2} \quad \& \quad \frac{1}{M^2}\bar{S}S H_u H_d \Big|_{\theta^2\bar\theta^2},$$

we find at a (partial) component level

$$\frac{\bar F_S}{M}\underbrace{H_u H_d}_{\text{superfields}}\Big|_{\theta^2} \quad \& \quad \frac{|F_S|^2}{M^2}\underbrace{H_u H_d}_{\text{A-terms of Higgs-superfields}}$$

$\Rightarrow$ $\mu H_u H_d|_{\theta^2}$ with $\mu \sim M_s^2/M$ ("$\mu$-term")

& $(B\mu) H_u H_d$ with $(B\mu) \sim (M_s^2/M)^2$ ("holomorphic Higgs mass term or "$B\mu$-term")

(This is just a name – there is no deep reason for writing the coefficient of this operator as a product of $\mu$ with some new parameter $B$.)

This way of generating the supersymmetric $\mu$-term and the SUSY-breaking $B\mu$-term with the same mass scale is known as the "<u>Giudice-Masiero mechanism</u>".

<u>Note:</u> $\bar S H_u H_d|_{\theta^2\bar\theta^2}$ is $U_{1,R}$-invariant since $R(S) = R(H_{u,d}) = 0$ and the projection on $\theta^2\bar\theta^2$ does not change the R-charge.

## 5.4 Soft terms

SUSY breaking in the MSSM can also be viewed from a different perspective:

- Work out the component lagrangian of the SUSY version of the SM (The $\mu$-term is the only dim.-ful parameter).
- Add all SUSY-breaking terms which do not introduce power-like divergent loop-corrections ("soft terms").
- Without proof, these terms are given in $\mathcal{L}_{soft}$ below.

[ $\tilde{g}$ is the superpartner of the gluon $g$ (i.e. a Weyl fermion); $\tilde{e}$ is the superpartner of the r.h. electron (i.e. a scalar); etc.]

$$\begin{aligned}
\mathcal{L}_{soft} = & -\frac{1}{2}\left(M_1 \tilde{B}\tilde{B} + M_2 \tilde{A}\tilde{A} + M_3 \tilde{g}\tilde{g}\right) + h.c. \\
& - \left(\tilde{u}^T A_u \tilde{q} H_u + \tilde{d}^T A_d \tilde{q} H_d + \tilde{e}^T A_e \tilde{l} H_d\right) + h.c. \\
& - \left(\tilde{q}^+ m_Q^2 \tilde{q} + \tilde{l}^+ m_L^2 \tilde{l} + \tilde{u}^+ m_U^2 \tilde{u} + \tilde{d}^+ m_D^2 \tilde{d} + \tilde{e}^+ m_E^2 \tilde{e}\right) + h.c. \\
& - m_{H_u}^2 \bar{H}_u H_u - m_{H_d}^2 \bar{H}_d H_d - (\underset{\substack{\uparrow \\ b \,\equiv\, B\mu \text{ for simplicity}}}{b} H_u H_d + h.c.)
\end{aligned}$$

($M_1, M_2, M_3$ are the bino, $\underset{\substack{\uparrow \\ \text{because of its relation to the W-bosons.}}}{\text{wino}}$ & gaugino mass)

- As we have shown above, all these terms can originate from higher-dim. operators coupling the SUSY-SM to a hidden sector with F-term breaking. This is the conceptual reason why these terms do not introduce power-like divergences. (recall that higher-dim. operators are less divergent than renorm. oper.)

<u>Comment:</u> This argument may appear to be not very convincing since, in the eff. theory below $M_S$, one simply has given certain scalars a ~~SUSY~~ mass term, and it is not clear why this mass should not receive corrections $\sim \Lambda$.

- To understand this more deeply, one may perform the following (advanced) exercise:

- Consider

$$\mathcal{L} = \underbrace{\left(S\bar{S} - c_1 S\bar{S}\right)\Big|_{\theta^2\bar{\theta}^2} + c_2 S\Big|_{\theta^2} + \text{h.c.}}_{\text{hidden sector}} + \underbrace{\bar{\phi}\phi\Big|_{\theta^2\bar{\theta}^2}}_{\text{"Higgs"}} + \underbrace{\frac{1}{M^2}\bar{\phi}\phi S\bar{S}\Big|_{\theta^2\bar{\theta}^2}}_{\cancel{\text{SUSY}}\text{-mediating operator}} .$$

- Work out the general form of loop corrections (SUSY will <u>not</u> be broken by loop corrections $\Rightarrow$ the general superfield structure will be respected; no superpotential for $\phi$ will be induced; all Kähler terms may receive small loop corrections; new terms like $\bar{\phi}\phi S|_{\theta^2\bar{\theta}^2}$ etc. may be induced (but they will be suppressed by $\frac{1}{M}$))

- Work out the low-energy spectrum (i.e. the mass of $\phi$) before & after these loop corrections.

- The ~~SUSY~~ mass of $\phi$ will be $\sim \frac{1}{M}$ in both cases (i.e. the hierarchy will <u>not</u> be destabilized).

<u>Note:</u> For this to work it is important that no linear terms in $\phi$ (like $\bar{S}\phi|_{\theta^2\bar{\theta}^2}$) are induced.

This can be achieved by requiring a symmetry like $\phi \to e^{i\alpha}\phi$, which is in fact present in the above lagrangian. In our simplistic model this symmetry is, unfortunately, anomalous. However, in the SM all chiral SFs are protected by non-anomalous gauge symmetries. Thus, no terms linear in the light fields ("tadpoles") are induced. <u>The above argument goes through</u>!

(See the review of Bagger referenced at my SUSY/SUGRA lecture Web page for more details.)