7 Higgs-mass loop correction & MSSM fine tuning problem

7.1 Loop correction to the potential in a simplified model

For this analysis, we ignore down-type/lepton Yukawas and set $\tan\beta = \infty$, i.e. $b = 0$. Also, we assume $m_1^2 \gg |m_2^2|$. Then the tree-level potential reads

$$V_0 = m_2^2 |H_u|^2 + \lambda |H_u|^4 \quad \text{where} \qquad m_2^2 = -\frac{m_Z^2}{2}$$
$$\lambda = \frac{1}{8}(g_1^2 + g_2^2).$$

After electroweak symm. breaking, the Goldstones are removed, and we write the potential for a single real scalar $\phi = h_0$:

$$V_0 = m_2^2 \phi^2 + \lambda \phi^4 \quad \text{with minimum at } v.$$

We are interested in the loop corrections, which are dominated by top & stop:

$$m_t^2 = \lambda_t^2 \phi^2 \quad \& \quad m_{\tilde{t}}^2 = \lambda_t^2 \phi^2 + m_0^2$$

top-Yukawa. "soft mass"

We need to understand the correction to a potential (to the "effective potential") induced by a (ϕ-dependent) particle. (The reason why m_t & $m_{\tilde{t}}$ dominate is their strong dependence on ϕ, coming from the large top-Yukawa coupling.)

Recall that in QFT (we now focus on a single real scalar with mass m), Greens fcts. are generated by the path integral

$$\int D\varphi \exp\left(-\frac{1}{2}\varphi G^{-1}\varphi - S_{\text{interact.}}\right),$$

where G^{-1} is the operator $\partial^2 + m^2$ (we work in euclidean space, which is ok since we won't consider propagating particles, but just the potential).

- The effective action is defined by

$$\int D\varphi \exp\left(-\tfrac{1}{2}\varphi G^{-1}\varphi - S_{int.}\right) = \exp\left(-\underset{\substack{\uparrow \\ \text{as fct. of } m^2}}{S_{eff}}\right)$$

$$S_{eff} = (V \cdot T)\left[V_{eff} + \text{terms with derivatives}\right]$$

(irrelevant for our purposes).

- Thus: $(VT)V_{eff} = -\ln \int D\varphi \exp\left(-\tfrac{1}{2}\underbrace{\varphi G^{-1}\varphi}\right)$ at "1-loop".

$$\int d^4x\, d^4y\, \varphi(x)\, G^{-1}(x-y)\, \varphi(y)$$

or, in momentum space,

$$\int d^4k\, \varphi_k (k^2 + m^2) \varphi_k$$

- Replacing $\int D\varphi$ by $\prod_k \int d\varphi_k$, we get a product of integrals $\underbrace{\int dx\, e^{-\alpha x^2}}_{\sim \frac{1}{\sqrt{\alpha}}}$ with $\alpha = \frac{1}{2}(k^2 + m^2)$

(the prefactor is irrelevant since we will take the log.)

$$\Rightarrow (VT) V_{eff} = -\ln \prod_k \frac{1}{\sqrt{k^2 + m^2}}$$

(the "$\frac{1}{2}$" is again irrelevant)

$$= \text{"}\sum_k\text{"}\ \frac{1}{2} \ln(k^2 + m^2)$$

or $V_{eff} = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ln(k^2+m^2)$.

(we will only be interested in the dependence on m^2, since in our case m^2 will depend on ϕ. The present φ will be replaced, e.g., by d.o.fs of the stop etc.)

- We will calculate this integral in d dimensions taking the limit $d \to 4$ at the end ("dimensional regularization"):

$$I = \int \frac{d^dk}{(2\pi)^d} \ln(k^2+m^2) = \int dm^2 \int \frac{d^dk}{(2\pi)^d} \cdot \frac{1}{k^2+m^2}$$

$$= \int dm^2 \, \frac{\Omega_{d-1}}{(2\pi)^d} \int_0^\infty \frac{k^{d-1}dk}{k^2+m^2} \, .$$

$\underline{\underline{\Omega_{d-1}}}$: $\int d^dx \, e^{-x^2} = \Omega_{d-1} \int_0^\infty x^{d-1} e^{-x^2} dx =$

$$= \Omega_{d-1} \frac{1}{2} \int_0^\infty dx^2 (x^2)^{\frac{d}{2}-1} e^{-x^2} = \Omega_{d-1} \frac{1}{2} \left(-\frac{\partial}{\partial\alpha}\right)^{\frac{d}{2}-1} \int_0^\infty dx^2 \, e^{-\alpha x^2} \Big|_{\alpha=1}$$

$$= \Omega_{d-1} \frac{1}{2} \left(-\frac{\partial}{\partial\alpha}\right)^{\frac{d}{2}-1} \frac{1}{\alpha}\Big|_{\alpha=1} = \cdots = \Omega_{d-1} \, \frac{1}{2} \Gamma\left(\frac{d}{2}\right) = \pi^{d/2}$$

$\underline{\underline{\int_0^\infty \frac{k^{d-1}dk}{k^2+m^2}}}$: $\int_0^\infty \frac{x^a dx}{(x^2+m^2)^b} = \frac{1}{2} \int_0^\infty \frac{x^{a-1} dx^2}{(x^2+m^2)^b} = \frac{1}{2} \int_0^\infty \frac{z^{\frac{a-1}{2}} dz}{(z+m^2)^b} =$

$$= \frac{1}{2} \left(\frac{\partial}{\partial\alpha}\right)^{\frac{a-1}{2}} \left(\int_0^\infty \frac{dz}{(\alpha z+m^2)^{b-\frac{a-1}{2}}} \right) \cdot \frac{1}{\left(-b+\frac{a-1}{2}\right)\left(-b+\frac{a-1}{2}-1\right)\cdots(-b+1)}$$

$$= \cdots = \frac{1}{2} (m^2)^{\frac{a+1}{2}-b} \cdot \Gamma\left(\frac{a+1}{2}\right) \Gamma\left(b-1-\frac{a-1}{2}\right) \Big/ \Gamma(b)$$

Now, with $a = d-1$ & $b = 1$, we find

$$\frac{1}{2}\, m^{d-2}\, \frac{\Gamma(\frac{d}{2})\,\Gamma(-\frac{d}{2}+1)}{\Gamma(1)} .$$

$$\Rightarrow\; I = \int dm^2\, \frac{1}{(2\pi)^d} \cdot \frac{\pi^{d/2}}{\frac{1}{2}\Gamma(d/2)} \cdot \frac{1}{2}\, m^{d-2}\, \Gamma(\tfrac{d}{2})\,\Gamma(-\tfrac{d}{2}+1)$$

$$= \underbrace{2\int dm\, m^{d-1}}_{\frac{2}{d} m^d} \cdot \frac{1}{(4\pi)^{d/2}} \cdot \underbrace{\Gamma(-\tfrac{d}{2}+1)}_{-\frac{d}{2}\Gamma(-\frac{d}{2})} = -\, m^d\, \frac{\Gamma(-\frac{d}{2})}{(4\pi)^{d/2}}$$

This would give $V_{eff} = -\frac{1}{2}\, m^d\, \frac{\Gamma(-d/2)}{(4\pi)^{d/2}}$.

We would like to set $d = 4-2\epsilon$ and consider the limit of V_{eff} for $\epsilon \to 0$.

We face the problem that the mass-dimensions of V changes with ϵ. This can be solved by redefining

$$V_{eff} \to \mu^{4-d}\, V_{eff} ,$$

where μ plays the role of the renormalization scale. (In a more careful treatment, we would apply the same procedure to all couplings, allowing us to define a proper renormalization scheme.)

Thus, we find
$$\underline{V_{eff} = -\frac{1}{2}\, m^4\, (m/\mu)^{d-4}\, \frac{\Gamma(-d/2)}{(4\pi)^{d/2}}} .$$

Next, we set $d = 4-2\epsilon$ and expand in ϵ.

The most important steps are :

$$\Gamma(-2+\epsilon) = \frac{\Gamma(-1+\epsilon)}{-2+\epsilon} = \frac{\Gamma(\epsilon)}{(-2+\epsilon)(-1+\epsilon)}$$

$$\Gamma(-\epsilon) = -\frac{1}{\epsilon} + \gamma_E + O(\epsilon) \quad ; \quad \gamma_E = 0.577\ldots$$

$$\left(\frac{m}{\mu}\right)^{\epsilon} = e^{\epsilon \ln(m/\mu)} = 1 + \epsilon \ln\frac{m}{\mu} + O(\epsilon^2)$$

The result reads:

$$V_{eff} = \frac{1}{4(4\pi)^2} m^4 \Big(\frac{1}{\epsilon} \underbrace{- \gamma_E + \ln(4\pi)}_{\substack{\text{is absorbed in } \mu, \\ \text{replacing } \mu \text{ by } \bar{\mu}.}} - \ln\left(\frac{m^2}{\mu^2}\right) + \frac{3}{2} + O(\epsilon)\Big)$$

(Hence, $\overline{MS}$-scheme)

"minimal subtraction" (for dropping $\frac{1}{\epsilon}$).

Better:

[absorb $\frac{1}{\epsilon}$ in redefinition of tree-level parameters, e.g. for $m^2 \sim \phi^2$; $m^4 \frac{1}{\epsilon} \sim \phi^4 \frac{1}{\epsilon}$ can be absorbed in $\lambda \phi^4$ by redefining λ]

finally: $$V_{eff} = \frac{1}{4(4\pi)^2} \cdot m^4 \left(\ln \frac{m^2}{\mu^2} - \frac{3}{2} \right)$$

(We drop the "$\bar{\ }$" on our "$\bar{\mu}$" for simplicity.)

In our case, we had a tree level potential depending on ϕ, $V_0(\phi) = m_2^2 \phi^2 + \lambda \phi^4$ and our V_{eff} becomes ΔV or V_1, to be added to V_0.

It is induced by the top and the stop, the fermion-loop giving an extra "−". Furthermore, we have 4 d.o.f. for each color (1 Dirac fermion or 2 complex scalars) and 3 colors:

$$V = V_0 + V_1$$

$$V_1 = \frac{3}{16\pi^2}\left[m_{\tilde{t}}^4\left(\ln\frac{m_{\tilde{t}}^2}{\mu^2} - \frac{3}{2}\right) - m_t^4\left(\ln\frac{m_t^2}{\mu^2} - \frac{3}{2}\right)\right]$$

7.2 The Higgs mass correction

- The physical Higgs mass is, to a very good approximation

$$\frac{1}{2}\frac{\partial^2 V}{\partial\phi^2}\bigg|_{\phi = v}.$$

(Recall that, coming from a model with the complex scalar H_u, we have not redefined the kinetic term to make it canonical for a real field, i.e.

$$|\partial H_u|^2 \rightarrow (\partial\phi)^2 = \frac{1}{2}(\partial_\mu \sqrt{2}\phi)(\partial^\mu \sqrt{2}\phi).$$

Thus, the mass is the coefficient of the term $\sim \delta\phi^2$ at the extremum, <u>not</u> of $\frac{1}{2}\delta\phi^2$.)

- The vacuum is at $\phi = v$, with v fixed by the Z-boson mass & the gauge couplings. This has to be maintained after including V_1. For our purposes, the easiest way to achieve this is by choosing μ appropriately, i.e.

$$\frac{\partial V_1}{\partial\phi}\bigg|_{\phi=v} = 0 \quad \left(\text{since we already have } \frac{\partial V_0}{\partial\phi}\bigg|_{\phi=v} = 0\right).$$

Note: • This might appear too simplistic since we actually encountered divergences $\sim \phi^2$ & $\sim \phi^4$, forcing us to renormalize two coefficients of the tree-level lagrangian. However, in SUSY (even if spontaneously broken) the divergences $\sim \phi^4$ cancel between t & $\tilde{t}$. We could have known this in advance since the only quartic term comes from the D-term-potential. The latter is proportional to the gauge couplings. The gauge couplings are only loop-corrected by charged particles. Hence, the loop corrections are suppressed by gauge couplings, while we only care about corrections $\sim \lambda_t$.

• Furthermore, one might worry that corrections to the kinetic term $(\partial\phi)^2$ affect the prediction for the phys. mass. This does not happen since $\langle\phi\rangle = v$ [with v related to the Z-mass] implies that ϕ is canon. normalized.*)

• In summary, although there are in principle divergences (and hence renormaliz. conditions) for $(\partial\phi)^2$, ϕ^2 & ϕ^4, only that $\sim\phi^2$ arises in SUSY. Thus, there is just one renormalization condition that has to be imposed. We take this condition into account by choosing μ appropriately, i.e., ensuring $\partial V/\partial\phi| = 0$ at $\phi = v$.

$$\Rightarrow 0 \overset{!}{=} \frac{\partial V_1}{\partial\phi}\Big|_v = \frac{3}{16\pi^2}\left(\frac{\partial m_t^2}{\partial\phi}\right)\left[\frac{\partial}{\partial m_{\tilde{t}}^2}\left\{m_{\tilde{t}}^4\left(\ln\frac{m_{\tilde{t}}^2}{\mu^2} - \frac{3}{2}\right)\right\} - \frac{\partial}{\partial m_t^2}\left\{m_t^4\left(\ln\frac{m_t^2}{\mu^2} - \frac{3}{2}\right)\right\}\right]$$

*) Multiplicative renormalization of ϕ, m^2, λ gives

$$Z_{\phi^2}(\partial\phi)^2 + Z_{m^2}Z_{\phi^2}m^2\phi^2 + Z_\lambda Z_{\phi^2}^2\lambda\phi^4 .$$

As we already argued, there is no divergence $\sim\phi^4$ in the SUSY case, hence $Z_{\phi^2}^2 Z_\lambda = 1$. (The reason being that λ is actually a fct. of the gauge couplings.)

We fix the value of ϕ in the vacuum by using the term

$$Z_{\phi^2}\, g^2 A_\mu A^\mu \phi^2 \quad \text{(schematic)},$$

which can be viewed as part of the kinetic term ($\partial_\mu \to D_\mu$).

Note: No factors Z_{g^2} or Z_{A^2} are introduced since they would have to be of the type $1+O(g^2)$, while we only care about λ_t-corrections. Moreover, one can enforce $Z_{g^2}Z_{A^2}=1$, allowing only for a renormalization of the F^2-term.

Hence, by measuring $\langle\phi\rangle$ via the Z-boson mass & gauge coupling, we actually measure $\langle Z_{\phi^2}^{1/2}\phi\rangle$. Thus, we may as well think about $Z_{\phi^2}^{1/2}\phi$ as our basic field and never worry about Z_{ϕ^2}.

$\Rightarrow$ There is really only one relevant renormalization, Z_{m^2}, and this is taken care of by our choice of μ.

(Here we think of the Z's as finite quantities, left over after divergences are absorbed, which parameterize the freedom of redefining lagrangian parameters.)

$$0 \overset{!}{=} \frac{3}{16\pi^2}\left(\frac{\partial m_t^2}{\partial \phi}\right)\left[\left\{m_{\tilde t}^2\left(2\ln\frac{m_{\tilde t}^2}{\mu^2} - 3 + 1\right)\right\} - \left\{\tilde t \to t\right\}\right]\Big|_{\phi = v}$$

$$0 = \frac{3}{8\pi^2}\left(\frac{\partial m_t^2}{\partial \phi}\right)\cdot\left[m_{\tilde t}^2\left(\ln\frac{m_{\tilde t}^2}{\mu^2} - 1\right) - m_t^2\left(\ln\frac{m_t^2}{\mu^2} - 1\right)\right]\Big|_{\phi = v}$$

<u>This fixes μ!</u>

- Next, we evaluate the correction to the phys. Higgs mass:

$$\Delta m^2 = \frac{1}{2}\frac{\partial^2 V_1}{\partial \phi^2}\Big|_v = \frac{1}{2}\left(\frac{\partial m_t^2}{\partial \phi}\right)\frac{\partial}{\partial m_{t,\tilde t}^2}\left(\frac{\partial V_1}{\partial \phi}\right)\Big|_v$$

$$= \frac{1}{2}\frac{\partial m_t^2}{\partial \phi}\frac{\partial}{\partial m_{t,\tilde t}^2}\left\{\frac{3}{8\pi^2}\left(\frac{\partial m_t^2}{\partial \phi}\right)\left[m_{\tilde t}^2\left(\ln\frac{m_{\tilde t}^2}{\mu^2} - 1\right) - m_t^2\left(\ln\frac{m_t^2}{\mu^2} - 1\right)\right]\right\}$$

<u>Note:</u> $\frac{\partial}{\partial m_{t,\tilde t}^2}$ does not need to be applied to $\frac{\partial m_t^2}{\partial \phi}$ since the term in square brackets vanishes at $\phi = v$.

$$\Delta m^2 = \frac{3}{16\pi^2}\left(\frac{\partial m_t^2}{\partial \phi}\right)^2\left[\ln\frac{m_{\tilde t}^2}{\mu^2} - \ln\frac{m_t^2}{\mu^2}\right] = \underline{\underline{\frac{3\lambda_t^4 v^2}{4\pi^2}\ln\frac{m_{\tilde t}^2}{m_t^2}}}$$

Going away from "$\tan\beta = \infty$" (i.e. allowing for $v_u \neq v$) and allowing for different masses for the scalars belonging to t_L & t_R (which are usually called $m_{\tilde t_1}$ & $m_{\tilde t_2}$), we find:

$$\underline{\underline{\Delta m_{h^0}^2 = \frac{3\lambda_t^4 v_u^2}{4\pi^2}\sin^4\beta\,\ln\frac{m_{\tilde t_1} m_{\tilde t_2}}{m_t^2}}}$$

(In fact, for large A-terms (the ~~SUSY~~ trilinear terms), further large corrections proportional to these A-terms may arise (and be used to lift m_{h^0}) – we won't discuss this.)

We focuss again on large $\tan\beta$ & $m_{\tilde{t}_1} = m_{\tilde{t}_2}$ to write:

$$\Delta m_{h_0}^2 \geq (114\,\text{GeV})^2 - (\underset{\substack{\uparrow \\ m_Z}}{91\,\text{GeV}})^2 \approx (69\,\text{GeV})^2$$

$$\Rightarrow \ln\left(m_{\tilde{t}}^2/m_t^2\right) \gtrsim (69\,\text{GeV})^2 \cdot \left(\frac{3\lambda_t^4 v^2}{4\pi^2}\right)^{-1}$$

$$\lambda_t^2 v^2 \approx m_t^2 = (173\,\text{GeV})^2$$

$$m_{\tilde{t}}^2 \gtrsim m_t^2 \exp\left(\frac{4\pi^2}{3} \cdot \left(\frac{69}{173}\right)^2\right)$$

$$m_{\tilde{t}} \gtrsim m_t \exp\left(\frac{2\pi^2}{3} \cdot \frac{1}{(2.5)^2}\right) \approx m_t \exp(1) \approx 500\,\text{GeV}$$

Actually: $m_t \approx \lambda_t v_u \left(1 + \frac{g_3^2}{3\pi^2} + \cdots\right)$

$$\Rightarrow \lambda_t v_u \approx 165\,\text{GeV} \quad \Rightarrow \quad \underline{\underline{m_{\tilde{t}} \gtrsim 620\,\text{GeV}}}$$

<u>Note:</u> Our simplified presentation followed the beautiful paper by Ellis, Ridolfi, Zwirner, Phys. Lett. <u>B257</u> ('91) 83. Of course, this has since been redone by many people with great precision.

7.3 The SUSY fine tuning problem

We have good reason to expect SUSY to be broken at a high scale:

- Unification of g_1, g_2, g_3 at $\sim 10^{16}$ GeV in MSSM
- simplicity of gravity mediation

In this case, the scale dependence (running) of all parameters, in particular of the soft terms, is crucial:

We only show the equations most important for our purposes, and use $g_1 \ll g_2, g_3$; $\lambda_i \ll \lambda_t$:

This may actually not hold for $\lambda_{b,\tau}$ at $\tan\beta \gg 1$.

This clearly doesn't hold near M_{GUT}, but it's suppressed by a group-theory factor.

$t = \ln\mu$: $\quad \frac{d}{dt}\alpha_i^{-1} = -\frac{b_i}{2\pi}$

$$b_i^{SM} = \left(\frac{41}{10}, -\frac{19}{6}, -7\right)$$

$$b_i^{MSSM} = \left(\frac{33}{5}, 1, -3\right)$$

$$\frac{d}{dt}\lambda_t = \frac{\lambda_t}{16\pi^2}\left(6|\lambda_t|^2 - \frac{16}{3}g_3^2 - 3g_2^2\right)$$

$$\frac{d}{dt}\mu = \frac{\mu}{16\pi^2}\left(3|\lambda_t|^2 - 3g_2^2\right)$$

(This is not in contradiction to our claim that the superpotential is not renormalized. $H_{u,d}H_{u,d}^+|_{\theta^2\bar\theta^2}$ is renormalized and adjusting it to be canonical (keeping always a canonical kinetic term by $\phi \to \alpha\cdot\phi$) induces the above running of μ.)

$$\frac{d}{dt}A_t = \frac{1}{16\pi^2}\left[A_t\left(18|\lambda_t|^2 - \frac{16}{3}g_3^2 - 3g_2^2\right) + \lambda_t\left(\frac{32}{3}g_3^2M_3 + 6g_2^2M_2\right)\right]$$

$$\frac{d}{dt}b = \frac{1}{16\pi^2}\left[b\left(3|\lambda_t|^2 - 3g_2^2\right) + \mu\left(6A_t\lambda_t^* + 6g_2^2M_2\right)\right]$$

$$\frac{d}{dt}m_{H_u}^2 = \frac{1}{16\pi^2}\left[6|\lambda_t|^2\left(m_{H_u}^2 + 2m_{\tilde t}^2\right) + 6|A_t|^2 - 6g_2^2|M_2|^2\right]$$

$$\frac{d}{dt}m_{H_d}^2 = \frac{1}{16\pi^2}\left[-6g_2^2|M_2|^2\right]$$

Note: positive coefficients on the r.h. side mean that a given parameter runs down if the energy goes down.

- In particular, $m_{H_u}^2$ is corrected by

$$\Delta m_{H_u}^2 \simeq -\frac{3|\lambda_t|^2 m_{\tilde{t}}^2}{4\pi^2} \ln \frac{\Lambda}{m_{\tilde{t}}}$$

cutoff

, if one comes from a certain high-scale value.

- Since $m_2^2 = |\mu|^2 + m_{H_u}^2$, this is also a leading correction to m_2^2.
- Since m_2^2 has to be negative & $O(m_Z^2)$ at low scales (in our preferred region of large $\tan\beta$), a large and positive high-scale value of m_2^2 has to be fine-tuned against the $\tilde{t}$-driven loop correction:

$$\text{tuning} = \frac{\Delta m_{H_u}^2}{m_Z^2} \simeq \frac{3|\lambda_t|^2 m_{\tilde{t}}^2}{4\pi^2 m_Z^2} \ln \frac{\Lambda}{m_{\tilde{t}}} .$$

- Using a very modest value for Λ, $\Lambda \simeq 100\,\text{TeV}$ (motivated, e.g., by the flavor problem), we find

$$\text{tuning} \simeq \frac{1}{12} \cdot \left(\frac{600}{90}\right)^2 \ln \frac{10^5}{600} \simeq \frac{100 \cdot 2^2}{12 \cdot 3^2} \cdot \overbrace{\ln \frac{10^5}{10^3}}^{\sim 4} \simeq 12$$

- Using instead $\Lambda \simeq 10^{16}\,\text{GeV}$, $\ln \frac{\Lambda}{10^3} = \ln 10^{13} \simeq 26$

$$\Rightarrow \text{tuning} \simeq \frac{100 \cdot 4 \cdot 26}{4 \cdot 3^3} \simeq \underline{\underline{100}}$$

$\Rightarrow$ "1%-level fine tuning"

7.4 Radiative electroweak symm. breaking and a more general perspective on fine-tuning

- recall the crucial renormaliz. group eqs.

$$\frac{d}{dt} m_{H_u}^2 = \frac{1}{16\pi^2}\left[6|\lambda_t|^2(m_{H_u}^2 + 2m_{\tilde t}^2) + 6|A_t|^2 - 6g_2^2|M_2|^2\right]$$

$$\frac{d}{dt} m_{H_d}^2 = \frac{1}{16\pi^2}\left[-6g_2^2|M_2|^2\right]$$

- in contrast to $m_{H_d}^2$, $m_{H_u}^2$ decreases with decreasing $t = \ln\mu$ because of the contribution $\sim |\lambda_t|^2$.

- Thus, starting with equal & positive $m_{H_u}^2$, $m_{H_d}^2$ at some high scale, it is natural to expect that $m_{H_u}^2$ will become negative at low energy scales and thereby trigger electroweak symm. breaking:

|| "radiative electroweak symm. breaking" ||.

[$m_2^2 = |\mu|^2 + m_{H_u}^2$; $m_{H_u}^2 < 0$ will imply $m_2^2 < 0$ if $|\mu|$ is not too large; this is natural since $|\mu|$ runs only proportionally to $|\mu|$ (also: b runs prop. to b & μ).]

- <u>This allows for a more general perspective on the SUSY fine tuning problem:</u> (→ Giudice, Rattazzi, hep-ph/0606105)

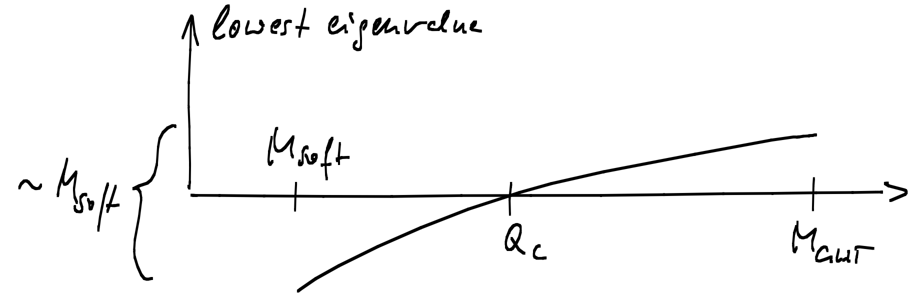
- For electroweak symm. breaking, it is crucial that the mass matrix

$$\begin{pmatrix} m_1^2 & -b \\ -b & m_2^2 \end{pmatrix}$$

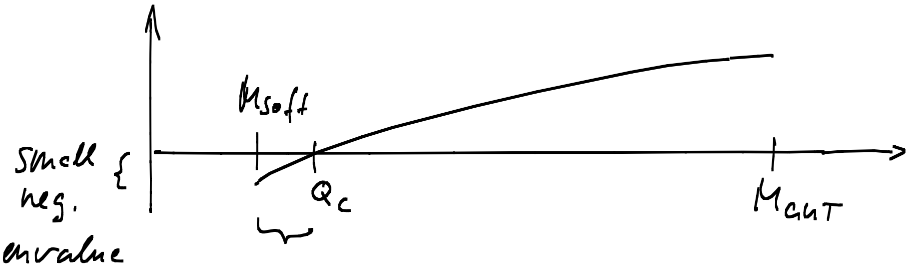
has a negative eigenvalue.

- Let's assume that at some high scale (e.g. M_P or M_{GUT}) all ~~SUSY~~ parameters & μ are $O(M_{soft})$, where M_{soft} is the generic scale of soft terms.

- The decisive question is whether the mass matrix at scale M_{soft} will have a neg. eigenvalue:



- As we have explained, having such a neg. eigenvalue is natural.
- However, to have $m_Z \ll M_{soft}$, the size of this neg. eigenvalue needs to be small, i.e.



The closeness of these two unrelated scales is necessary for $m_Z \ll M_{soft}$ and represents the SUSY fine tuning problem.

7.5 The NMSSM as a possible solution to the SUSY fine tuning problem

- Introduce an extra singlet S and

$$\Delta W_{NMSSM} = \lambda S H_u H_d + \frac{1}{6} k S^3 .$$

(No μ-term!)

- μ-term and $B\mu$-term are induced if $\langle S \rangle \neq 0$ & $\langle F_S \rangle \neq 0$ in the vacuum (which is easy to realize).
- The lightest Higgs mass can be larger than m_Z even at tree-level if λ & k are sufficiently large.
- Problem: λ runs to larger values with increasing energy scale and tends to reach a Landau pole below M_{GUT}.