Introduction to String Theory and String Phenomenology

→ www.thphys.uni-heidelberg.de/~hebecker

Literature: (CUP = Cambridge Univ. Press)

- Green, Schwarz, Witten: Superstring Theory I + II, CUP, '87
- Polchinski: String Theory I + II, CUP, '98
- Lüst, Theisen: Lectures on String Theory, Springer, '89
- Zwiebach: A First Course in String Theory, CUP, '04
- M. Kaku: Introduction to Superstrings & M-Theory Springer, '99
- Bailin, Love: Supersymm. Gauge Field Theory and String Theory, IOP Publishing, '94

....

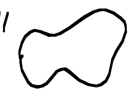
also: lecture notes of C. Ewerz in our library

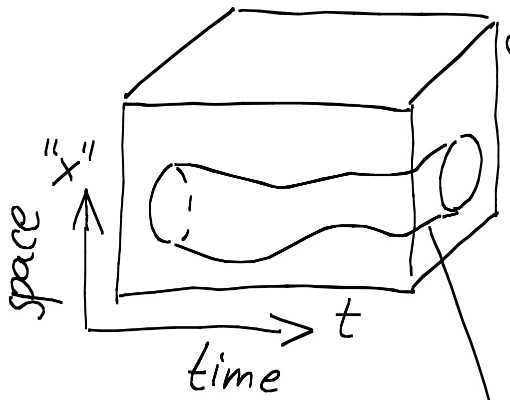
I am indebted to Jan Louis, whose String Theory course in Hamburg (2003) I attended. I will freely use my notes taken during these lectures.

String Theory

1. Introduction

1.1 Basic Ideas

- consider strings (open – "" or closed – "") as fundamental objects
- consider their motion in a given (fixed) space-time



$\mathcal{M}$ – target space

simplest possibility: D-dimensional Minkowski space with metric

$$\eta_{\mu\nu} = \mathrm{diag}(\underbrace{-1, 1, \dots, 1}_{D \text{ entries}})$$

propagating closed string
= string world sheet Σ
= 2-dim. submanifold of $\mathcal{M}$

- quantize this system
  (i.e. the motion of a single string in a fixed D-dim. background) .... of course, to do this we will need an action, which we will discuss shortly...
- a certain spectrum will emerge

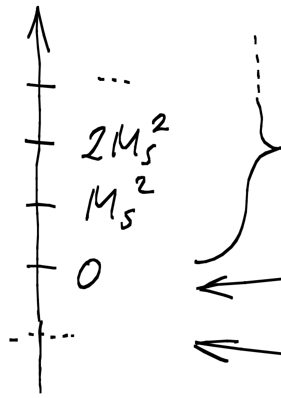
→ let M be Minkowski space, its symmetry is the Poincare group (essentially Lorentz group & translations), let the action respect this symmetry

→ <u>states will fill representations of D-dim. Poincare group</u>

- translations are generated by $\hat{P}^\mu$
- mass operator: $\hat{M}^2 = -\hat{P}^\mu \hat{P}_\mu$ commutes with all generators ("Casimir op.")

Note our metric convention!

$M^2 = \langle \psi | \hat{M}^2 | \psi \rangle$ for state $|\psi\rangle$



infinitely many massive states with $M^2 = n M_s^2$, $n \in \mathbb{N}$ (M_s – "string scale")

finite # of $M = 0$ states

Tachyon* (will appear in some models, making them (naively) inconsistent)

* recall: $m^2 c^4 = E^2 (1 - v^2/c^2)$; $v > c$ "⇒" $m^2 < 0$, hence the name of this state with $m^2 < 0$

The string scale M_s is derived from a dimensionfull parameter appearing in the world sheet (WS) action.

$M_s^2 \sim$ tension of string $\equiv$ energy / length

- $M=0$ states are further classified according to their transformations under the Lorentz group, i.e. their spin (helicity) s:

one finds: scalars ("$s=0$")
vectors ("$s=1$")
gravitons ("$s=2$")
...

This labelling by s is actually misleading, since it is the 4d-language, while $D \neq 4$ in general.

Aside:

4d, massive states: go to rest frame, classify according to repres. of $SO(3) \simeq SU(2)$; these repr. are labelled by the spin, "s"

4d, massless: go to frame where, e.g.,

$$p \sim (\underset{\text{time}}{1}, \underbrace{1, 0, 0}_{\text{space}})$$

(last two components can be freely rotated)

$\Rightarrow$ classify according to repres.s of $SO(4-2) = SO(2)$

(under certain extra conditions associated with the embedding in the full Poincare group, the "charges" under this $SO(2) = U(1)$ are quantized, i.e.
we can again label by a single discrete number "s" or "h", the <u>helicity</u>

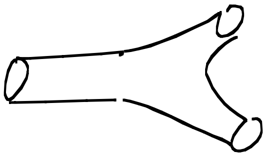
<u>$D \neq 4$, massless:</u> chose $p \sim (1, 1, \underbrace{0, \ldots, 0}_{})$

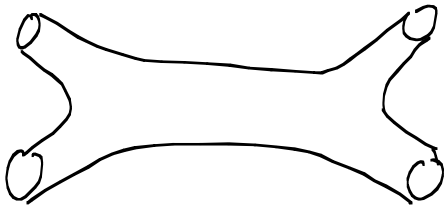
$SO(D-2)$ rotations must be represented on states

- e.g. fundamental (F) of $SO(D-2)$ $\rightarrow$ vector ("s=1")
- or $F \times F$ $\begin{cases} \text{symm. part} \longrightarrow \text{graviton ("s=2")} \\ \text{antisymm. part} \quad \ldots \end{cases}$

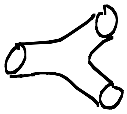
cf. the 4d case, where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, with the field $h_{\mu\nu}$ (with 2 symmetrized fundamental indices) being the graviton field (with $s = 2$)

<u>Crucial:</u> the graviton ("s=2") representation always arises in string theory

- <u>interactions:</u> decay:  $\sim g_s$

  scattering:  $\sim g_s^2$

g_s = string coupling = dimensionless parameter
= VEV of a scalar field (dilaton) that is also present in the spectrum discussed above (mass = 0)

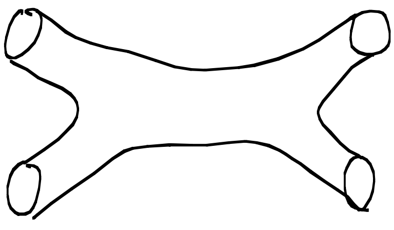
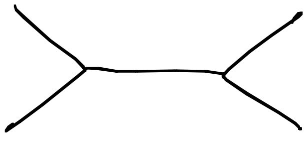
- the powers of g_s are assigned on a topological basis, e.g. "one g_s for each three-state-vertex " or, more correctly:

 $\sim g_s^{2+2n}$

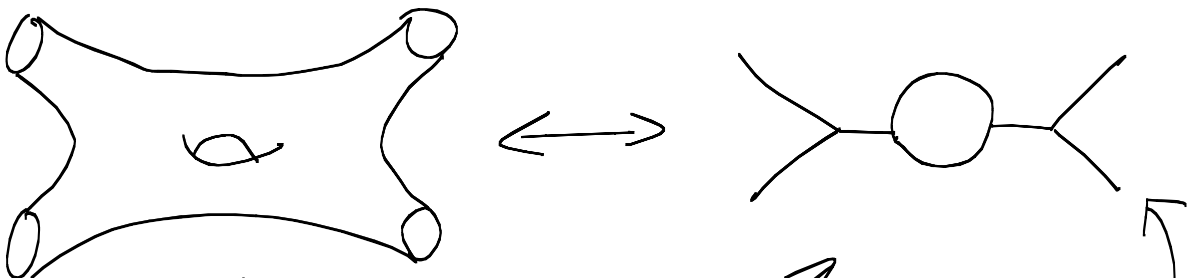
n holes

(We will see later on that this rule is justified by the WS action)

- <u>Relation to field theory:</u>

 $\longleftrightarrow$ 

(2 → 2 scattering of standard QFT)



(Note: string states appear point-like at small energies)

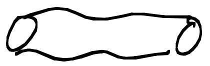
The "infinity" of QFT due to the integration region where the loop shrinks to zero size is absent in ST since "lines" have finite thickness $\sim M_s^{-1}$.

- <u>string amplitude</u>:

$$\mathcal{A} = \sum_n \mathcal{A}^{(n)} g_s^{2+2n}$$

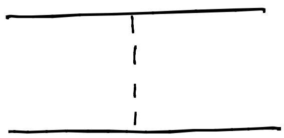
contribution with n holes (finite, as far as we know; the all order finiteness proof is incomplete)

<u>Note</u>: This (to a large extent) <u>defines</u> ST; There is nothing "of which" this is an expansion. Such a "something" would be <u>string field theory</u> of which, however, very little is known.

- given that "" includes "$h_{\mu\nu}$", this defines (loop corrections to) <u>quantum gravity</u>

Aside: Why is this such a big achievement?

- consider 1-graviton exchange:



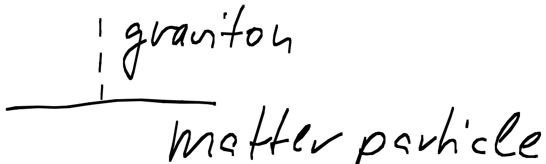
$\sim G_N$

recall that the gravitational energy is

$$E \sim \frac{G_N m_1 m_2}{r} \quad \text{(in 4 dimensions)}$$

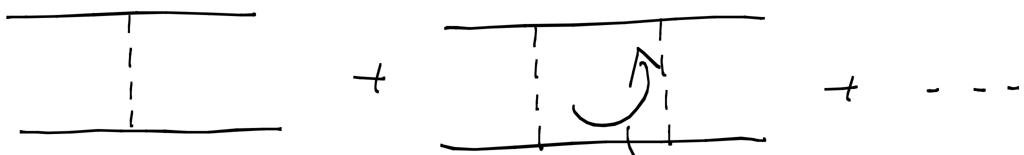
(let $\hbar = c = 1 \Rightarrow$ [time] = [length] = [momentum^{-1}]
= [energy^{-1}] = [mass^{-1}])

$\Rightarrow [G_N] = [\text{mass}^{-2}]$

$\Rightarrow$ 

$\sim \sqrt{G_N}$

- consider perturbation theory:



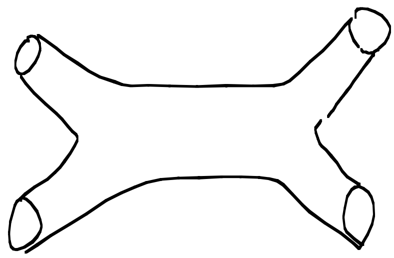
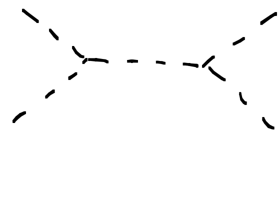
$$\sim G_N \left(1 + G_N \int dE \cdot E + \dots\right)$$

momentum or energy E in loop

this (positive) power of E is required by the mass dimension of G_N

$\Rightarrow$ Higher orders in perturbation theory involve worse and worse divergences (unlike, e.g., QED, where $[e]=1$)

$\Rightarrow$ Gravity is "sick" (non-renormalizable by power counting) as a QFT.

- consider  is ST  in GR

$\longleftrightarrow$ Comparing these two amplitudes allows one to identify G_N in terms of M_s:

$$G_{N,D} \sim M_{P,D}^{-(D-2)} \sim M_s^{-(D-2)}$$

- $\mathcal{M}$ (the target space) can be curved, i.e. have non-trivial $g_{\mu\nu}$. How does this relate to the stringy graviton state we discussed above?

<u>We know:</u> In electromagnetism, $E_{class.}$ can be understood as a coherent state or condensate of photons

Far less well understood: $g_{\mu\nu}$ of $\mathcal{M}$ must be related to a condensate of the single-string states (graviton states) corresponding to $h_{\mu\nu}$

$\Rightarrow$ interesting (hand waving) implication: There is no conventional geometry at distance scales $< M_s^{-1}$.

1.2 The Superstring / M-Theory

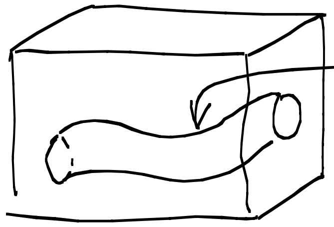
What has been discussed so far is the "bosonic string".

It requires
- $D = 26$ (otherwise inconsistent!)
- has tachyon
- no fermions

To do better: $\underbrace{\text{Super}}_{}$string

from "supersymmetry" (fermion-boson-symmetry)

recall:



described by a set of functions $X^\mu(\tau,\sigma)$, where $\mu = 0 \cdots D-1$ and (τ,σ) parameterize the WS Σ.

- add to this: a set of functions $\psi^\mu(\tau,\sigma)$ which anticommute ("The string also moves in an additional, fermionic target space.")
- the full action (see much later) is required to possess a certain extra symmetry (supersymmetry or SUSY) under which $\delta X \sim \psi$ & $\delta\psi \sim X$ (schematic).

<u>One now finds:</u>
- $D = 10$
- no tachyons in 5 models
- 10d-fermions ($\equiv$ string states transforming in the spinor repres. of $SO(1, D-1)$).

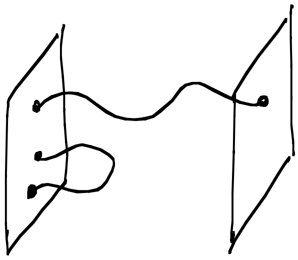
<u>The 5 models are:</u>

type I, type IIA, type IIB, heterotic SO_{32}, heterotic $E_8 \times E_8$

exceptional Lie group (gauge group of this model)

The differences between these 5 models are related to the way in which the WS fermions are introduced.

- ST includes certain "non-perturbative objects" called D-branes; open strings can end on D-branes:



(D stands for Dirichlet since the ends of the string are attached to the brane by a Dirichlet boundary condition.)

- The branes are dynamical; they can move, bend, ...
- The discovery of branes was crucial for the discovery of <u>dualities</u>, e.g.,

type I with $\langle A_\mu \rangle \neq 0$ $\quad$ type IIB with D-branes

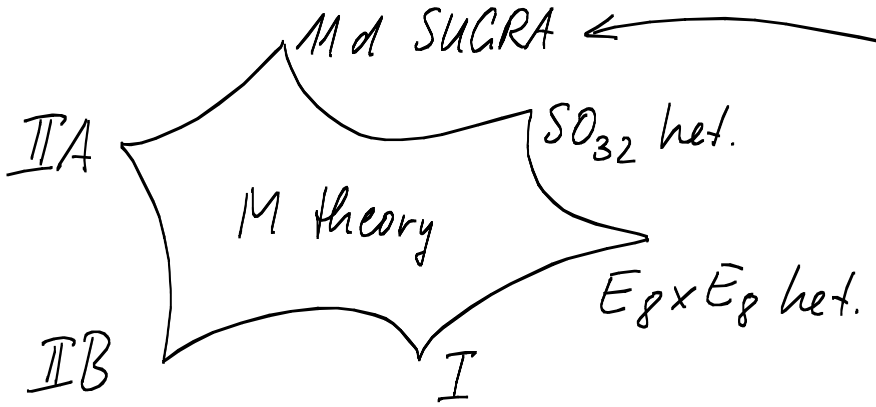
$\searrow \quad \swarrow$

different descriptions of exactly the same physical theory

(different Lagrangians, different fields, but exactly the same <u>spectrum of physical states</u>)

- all the 5 consistent 10d models (and one extra 11d field theory) are linked by dualities:

$\Rightarrow$ (picture of) M theory



sometimes the word M theory is used more specifically for this 11d Supergravity model.

<u>general idea</u>: moving in this diagram corresponds to varying the background geometry (including g_s)

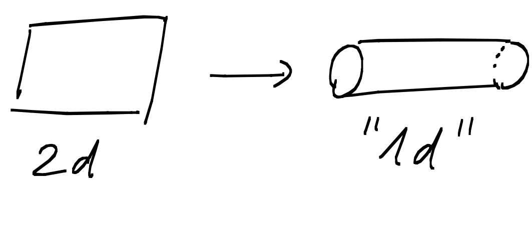
(Unfortunately, not much is known except the "corners" and a number of dualities linking the. The "middle" is largely unknown.)

- <u>To relate to observations</u>

all this is 10d (10d FT with gravity, spinors, ...)

$\Big\downarrow$ compactify 6 dimensions

(e.g.



)

4d FT with gravity, fermions, gauge fields, scalars.

In principle, this is all we need for the standard model. However, finding "the right" compactification turns out to be difficult.

(Note: A "compactification" should be a solution of all equations of motion (EOMs) of the 10d theory.)

Recently: Very large numbers of such compactifications ($\sim 10^{300}$!?) have been discovered (the "String Landscape"), yet the actual standard model remains to be found ...