10 Consistent Superstring Theories – part 1

10.1 GSO projection

problems of open superstrings as developed so far:

1) tachyon
2) WS SUSY $\not\Leftrightarrow$ space-time SUSY

(at $M^2 = -\alpha'/2$: tachyon (boson) $\leftrightarrow$ no fermions
$M^2 = 0$: vector (NS) $\leftrightarrow$ spinor (R)
(8 d.o.f.) (16 = 32/2 d.o.f.)
for Dirac eq.*

* This is as in QED: electron $\hat{=}$ Dirac spinor
$\Rightarrow$ 8 real (4 complex) d.o.f. ; after quantization:
electron/positron ; left/right $\Rightarrow$ 4 real d.o.f.

3) (possibly perceived) problem of spin & statistics:
$|0,k\rangle$ & $\psi^\mu|0,k\rangle$ are both bosonic although ψ^μ is an anticommuting operator.

Solution proposed by Gliozzi, Scherk, Olive (GSO):

perform projection:

(let P be operator with $P^2 = P$ and $PH = HP$; restrict theory to states with $P|\phi\rangle = +|\phi\rangle$ (or $P|\phi\rangle = -|\phi\rangle$).)

here: discard states with odd fermion number:

$$P = (-1)^F \quad ; \quad FX^\mu = X^\mu F$$
fermion number (of F)
$$F\psi^\mu = -\psi^\mu F$$

We require that $P|\phi\rangle = |\phi\rangle$ for allowed states.

NS: define $(-1)^F|0\rangle = -|0\rangle$; the rest is then defined

R: define $(-1)^F|\alpha\rangle = |\beta\rangle\,(\Gamma)^\beta{}_\alpha$; the rest is then defined

$$\Gamma \equiv \Gamma^{11} \equiv \Gamma^0\Gamma^1 \cdots \Gamma^9$$

Aside on R-vacuum: We had found $\not{k}\,u(k) = 0$.
Choosing ($k^0 = k^1 \neq 0$ & other $k^i = 0$), we have

$$(\Gamma^0 + \Gamma^1)\,u = 0$$

or $(\Gamma^0\Gamma^1 + 1)\,u = 0$ (since $(\Gamma^0)^2 = 1$)

or $(S^0 - \frac{1}{2})\,u = 0$ where $S^0 = -\frac{1}{4}[\Gamma^0, \Gamma^1]$

half of d.o.f. are lost

from 32 down to 16

generator of rotation in (0,1)-plane ; eigenvalues $\pm 1/2$ for spin-1/2-state *

$\Gamma u = u$

from 16 down to 8

*check: $(S^0)^2 = \frac{1}{4}\Gamma^0\Gamma^1\Gamma^0\Gamma^1 = \frac{1}{4}(-1)\cdot(-1) = \frac{1}{4}$ ✓

$\Rightarrow$ 8 on-shell d.o.f. left in R vacuum

Finally: spectrum of open superstring:

$$\boxed{\text{massless vector + spinor (8 d.o.f.)}}$$

This corresponds to a familiar 10d SUSY FT:

10d Super Yang-Mills (SYM) Theory:

$$S = \int d^{10}x \left(-\frac{1}{4}F^2 + \frac{i}{2}\bar{\psi}\not{D}\psi\right)$$

$$F_{\mu\nu}^{\ a} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

$$(D_\mu \psi)^a = \partial_\mu \psi^a + g f^{abc} A_\mu^b \psi^c$$

Majorana-Weyl fermion in adjoint repres. of gauge group.

invariant under: $\delta_\epsilon A_\mu^{\ a} = \frac{i}{2}\bar{\epsilon}\Gamma_\mu \psi^a$

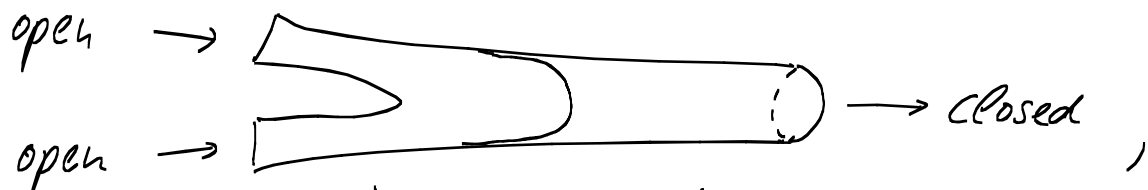
$$\delta_\epsilon \psi^a = -\frac{1}{4} F_{\mu\nu}^{\ a}\Gamma^{\mu\nu}\epsilon \qquad \left(\Gamma^{\mu\nu} = \frac{1}{4}[\Gamma^\mu, \Gamma^\nu]\right)$$

Counting of d.o.f.: $A_\mu \to$ 8 on shell ✓

$\psi \to 32$ complex $\xrightarrow[\text{Weyl}]{}$ 16 complex $\xrightarrow[\text{Majorana}]{}$ 16 real $\xrightarrow[\text{Dirac-eq.}]{}$ 8 on shell ✓

more on spinors: see Appendix of Polchinski, vol. II

Note: All of this was really only a "prelude" introducing the concept of the GSO projection. The open string theory we have derived is, by itself, inconsistent since we can always have processes



i.e., we must include closed strings.

(Note also that, by contrast, a theory with only closed strings can be consistent.)

10.2 Type II superstrings

- We will find 2 consistent purely closed string theories with 2 gravitini (in 10d), i.e., $N=2$ SUSY in 10d (hence "type II").
- The main building block is the open string spectrum:

Sector	SO(8) repres.	
NS −	1	— tachyon: $m^2 = -1/2\alpha'$
NS +	8_v	3 massless sectors
R −	$8'$	3 massless sectors
R +	8	3 massless sectors

eigenvalue of $(-1)^F$

vector

l.h. & r.h. spinor

as prescribed by the general formula

$$m^2 = \frac{1}{\alpha'}(N-\nu) \quad \text{with } \nu = \begin{cases} 0 & (R) \\ 1/2 & (NS) \end{cases}$$

- recall:
  The GSO projection removes the "1" and "8'" (or, equivalently, the "8").
- as in the bosonic case, the closed string combines a left-moving (~) and right-moving sector, each identical to the open string.

- The phys. state conditions can be imposed independently for each sector, except for the

  <u>level matching condition</u>: $(L_0 - \tilde{L}_0)|phys\rangle = 0$, i.e.

  $$((N-\nu) - (\tilde{N}-\tilde{\nu}))|phys\rangle = 0 .$$

- The mass shell condition then reads

  $(L_0 + \tilde{L}_0)|phys\rangle = 0$, i.e. $\left(\frac{\alpha'}{2}p^2 + N + \tilde{N} - \nu - \tilde{\nu}\right)|phys\rangle = 0$,

  i.e. $m^2 = \frac{4}{\alpha'}(N-\nu) = \frac{4}{\alpha'}(\tilde{N}-\tilde{\nu}) \quad \left(\nu = \begin{cases} 0 & (R) \\ 1/2 & (NS) \end{cases}\right)$

<u>Note:</u> The level spacing differs by a factor of 4 between open and closed string (only relevant if open & closed strings are combined).

- <u>full (not yet GSO projected) spectrum:</u>

<u>Sector</u>	<u>SO(8) repr.</u>	
(NS−, NS−)	1	— tachyon: $m^2 = -\frac{2}{\alpha'}$
(NS+, NS+)	$8_v \times 8_v$	9 massless sectors
(R+, R+)	8×8	
(R+, R−)	$8 \times 8'$	
(NS+, R−)	$8_v \times 8$	
...	...	

all possible combinations of $(-1)^F$ & $(-1)^{\tilde{F}}$

- as in the open case, a projection to a phys. subset is again necessary.

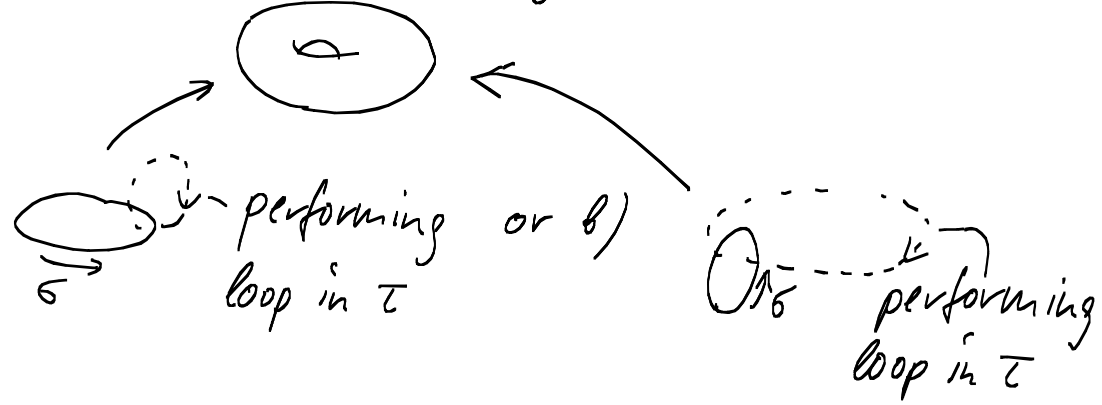
• We could, in principle, consider all possible combinations of sectors: $\underline{2^{10} \text{ options !}}$

• <u>require in addition</u>: 1) no tachyon

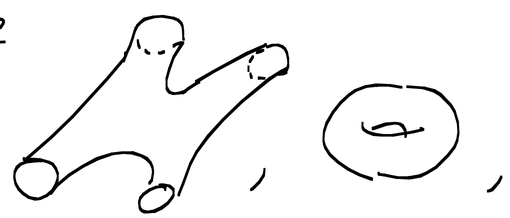
2) consistency of interacting theory (see more later)

3) modular invariance * (see more later)

*) related to possibility of viewing torus-WS



• 2) & 3) are related to existence of particular spin bundles on WSs like



, etc.

• 2) also includes the simple requirement that the selection made should be respected by scattering (i.e., it should be impossible to produce a forbidden state dynamically).

• for now, we just report the "fairly natural" result of this analysis:

- GSO projections: $\mathrm{IIB}: \; (-1)^F = (-1)^{\tilde F} = 1$

  $\mathrm{IIA}: \; (-1)^F = 1 \; ; \; (-1)^{\tilde F} = \begin{cases} 1 & (NS) \\ -1 & (R) \end{cases}$

- Explanation:

  a) Use $(-1)^F = 1$ for both NS-sectors to remove tachyon

  b) For R-sectors, $(-1)^F = +/-1$ corresponds to choice $8/8'$ (or "left"/"right" spinor). Thus, the physically distinct choices are only

  - same chirality for left-movers & right-movers
    $\to$ IIB ("chiral theory")
  - opposite chirality for left-movers & right-movers
    $\to$ IIA ("non-chiral theory")

(almost obvious: $\exists$ phys. equivalent IIA' & IIB' projections with $8 \leftrightarrow 8'$ on both sides, i.e. a sign-flip of $(-1)^{\tilde F'}$ for all R sectors)

Field content IIA

	SO(8)	tensor/spinor	dimensions
(NS+, NS+)	$8_v \times 8_v$	$= [0]_\phi + [2]_{B_2} + (2)_G$	$= 1 + 28 + 35$
(NS+, R−)	$8_v \times 8'$	$=$ spinor + vector-spinor (λ, χ'_μ)	$= 8 + 56'$
(R+, NS+)	$8 \times 8_v$	$=$ " (λ', χ_μ)	$= 8' + 56$
(R+, R−)	$8 \times 8'$	$= [1]_{C_1} + [3]_{C_3}$	$= 8_v + 56_t$

$\uparrow$ NS+ or R+ $\quad \uparrow$ NS+ or R−

Explanations:

- generic tensor: $t_{\mu\nu} = \eta_{\mu\nu} \cdot \phi + A_{\mu\nu} + S_{\mu\nu}$

  scalar (here: dilaton) — antisymm. tensor — traceless symm. tensor

- $[m] \equiv$ m-index, antisymm. tensor

  (special cases: $[0]$ - scalar, $[1]$ - vector)

- $(m) \equiv$ m-index, traceless symm. tensor

- $8_v \times 8' = 8 + 56'$ ($\to$ problems)

  spinor (dilatino λ) — vector-spinor (gravitino χ'_μ)

- Note: 56 & $56'$ are distinguished by chirality, as 8 & $8'$.

- $8 \times 8' =$ "bosonic repr." $= [1] + [3] = 8_v + 56_t$

  ($\to$ problems) — "tensor" (as opposed to the spinorial 56 & $56'$)

- counting d.o.f.'s of dynamical fields:

— dilaton ϕ — 1 d.o.f.

— NS-2-form-potential B_2: $B_2 = (B_2)_{\mu\nu} dx^\mu \wedge dx^\nu$;

3-form-field strength: $H_3 = dB_2$

as for the photon, only transverse components count (the rest is related to gauge freedom or unphysical).

$\Rightarrow (B_2)_{ij}$ with $i,j \in \{1,\dots,D-2\}$; $D=10 \Rightarrow$ d.o.f. $= \binom{8}{2} = \underline{\underline{28}}$

– metric $G_{\mu\nu} \to G_{ij}$, symm., $\underbrace{\text{traceless}}_{\text{related to extra gauge freedom of GR}}$ $\Rightarrow$ d.o.f. $= \binom{D-2}{2} + D - 3$

$$= \underline{\underline{\frac{D(D-3)}{2}}}$$

(for metric in general dim.s.)

<u>here</u>: $D=10 \Rightarrow$ d.o.f. $= \underline{\underline{35}}$.

- λ – "dilatino"-spinor – 8 d.o.f. (see above)
- ψ_μ – as for photon $\mu \to i \in \{1,\dots,D-2\}$ for phys. d.o.f., <u>furthermore</u>: subtract one more of the possible values of μ since there is a condition $\gamma^\mu \psi_\mu = 0$ following from the EOMs.

<u>Thus</u>: d.o.f. = (d.o.f. of spinor) $\times (D-3)$,

for $D=10$: d.o.f. $= 8 \cdot 7 = \underline{\underline{56}}$

- C_1, gauge field ($F_2 = dC_1$)
- C_3, antisymm. tensor

$\Big\}$ RR-forms

$$C_3 = (C_3)_{\mu\nu\rho}\, dx^\mu \wedge dx^\nu \wedge dx^\rho$$

$$\text{d.o.f.} = \binom{D-2}{3} \underset{(D=10)}{=} \frac{8\cdot 7\cdot 6}{2\cdot 3} = \underline{\underline{56}} .$$

Overall, this field content corresponds to (known) 10 d SUSY FT:

$$\left.\begin{array}{l} G_{\mu\nu} \xleftrightarrow{\text{1st SUSY}} \psi_\mu \\ G_{\mu\nu} \xleftrightarrow{\text{2nd SUSY}} \psi_\mu' \end{array}\right\} \begin{array}{l} \text{2 superpartners} \\ \text{(2 gravitini)} \end{array}$$

$\Rightarrow$ $N=2$ SUSY

$\Rightarrow$ "type II".

<u>Next:</u>

<u>Field content IIB</u>

	<u>SO(8)</u>		<u>dimensions</u>
(NS+, NS+)	$8_v \times 8_v$	$= [0]_\phi + [2]_{B_2} + (2)_G$	$= 1+28+35$
(NS+, R+)	$8_v \times 8$	= spinor + vector-spinor	$= 8' + 56$
(R+, NS+)	$8 \times 8_v$	= — " —	$= 8' + 56$
(R+, R+)	8×8	$= [0]_{C_0} + [2]_{C_2} + [4]_{+\,C_4}$	$= 1 + 28 + 35_+$

$\uparrow$ NS+ or R+ $\qquad$ $\uparrow$ NS+ or R+

<u>Explanations:</u>

- $[4]_+$ – "+" means self-duality: $t_{i_1 \cdots i_4} = \varepsilon_{i_1 \cdots i_8} t^{i_5 \cdots i_8}$
- the corresponding FT has $F_5 = dC_4$ with self-duality imposed in 10d on F_5 (using $\epsilon_{\mu_1 \cdots \mu_{10}}$)

Counting of d.o.f. of C_4:
$$\underset{\substack{\uparrow \\ \text{self-duality!}}}{\frac{1}{2}} \binom{8}{4} = \frac{8 \cdot 7 \cdot \cancel{6} \cdot 5}{\cancel{2} \cdot 4 \cdot 3 \cdot \cancel{2}} = \underline{\underline{35}}$$

<u>important:</u> This theory is chiral (A certain definite chirality, namely that of the two gravitini, is preferred).

Note also: IIA – odd RR-form-potentials
IIB – even RR-form-potentials

10.3 Type I superstrings

Preliminary consideration: Orientation of the string

- When discussing diffeoms., we have so far excluded
$$\sigma' = \pi - \sigma, \; \tau' = \tau \quad (\text{for } \sigma \in (0,\pi) \text{ as in GSW})$$
(i.e. orientation change).

- At the quantum level, this symmetry is realized by an operator, which we call Ω ($\Omega^2 = 1$):
$$|\psi'\rangle = \Omega|\psi\rangle \quad (\text{for } |\psi\rangle \text{ any state of the 2d QFT}).$$

- Naturally, we expect (using π-periodicity in σ):
$$\langle\psi'|\hat{X}(\tau,\sigma)|\psi'\rangle = \langle\psi|\hat{X}(\tau,-\sigma)|\psi\rangle,$$
which implies
$$\Omega^{-1}\hat{X}(\tau,\sigma)\Omega = \hat{X}(\tau,-\sigma)$$
$$\text{or} \quad \Omega^{-1}\hat{X}(\sigma^+,\sigma^-)\Omega = \hat{X}(\sigma^-,\sigma^+).$$

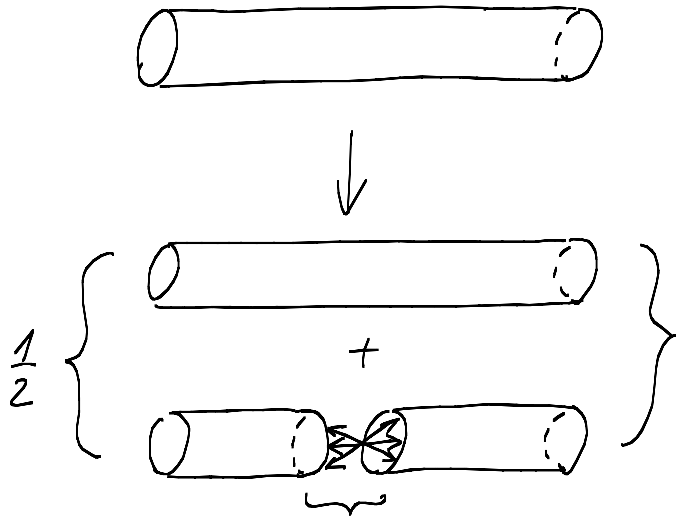
- Recalling that $\hat{X}(\sigma^+,\sigma^-) = \hat{X}_L(\sigma^+) + \hat{X}_R(\sigma^-)$
and that $\tilde{\alpha}_n$, α_n are the Fourier modes of X_L, X_R, we have
$$\Omega^{-1}\alpha_n\Omega = \tilde{\alpha}_n \quad \& \quad \Omega^{-1}\tilde{\alpha}_n\Omega = \alpha_n.$$

- Since $\Omega^2 = 1$, $P = \frac{1}{2}(1+\Omega)$ is a projection operator ($P^2 = P$) and consider a theory arising from this projection:

Unoriented closed string: $\Omega|\psi\rangle = |\psi\rangle$

(states with $\Omega|\psi\rangle = -|\psi\rangle$ are excluded)

- Geometrically this means (consider the string propagation, i.e. a WS):

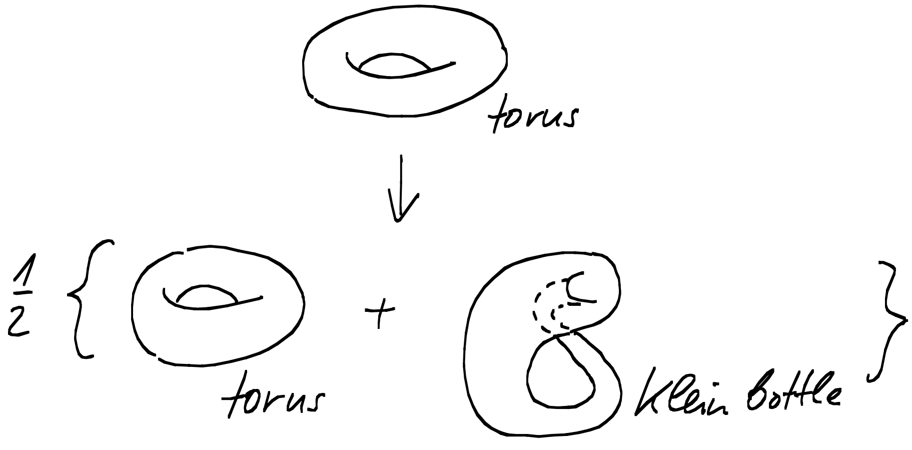


oriented

unoriented

glued together with changed orientation
$\hat{=}$ insertion of operator Ω

- One-loop amplitude:



oriented

unoriented

- Implication for the spectrum of the bosonic string:
  - at the massless level we have states $\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0,k\rangle$,
  - thus: $\Omega\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0,k\rangle = \tilde{\alpha}_{-1}^{\mu}\alpha_{-1}^{\nu}|0,k\rangle = \alpha_{-1}^{\nu}\tilde{\alpha}_{-1}^{\mu}|0,k\rangle$.

$\Rightarrow P = \frac{1}{2}(1+\Omega)$ corresponds to a symmetrizer, i.e.

$$\underbrace{G_{\mu\nu},\, B_{\mu\nu},\, \phi}_{\text{oriented}} \longrightarrow \underbrace{G_{\mu\nu},\, \phi}_{\text{unoriented}}$$

<u>Note</u>: This projection operation is sometimes called the "gauging" of Ω. By declaring Ω to be a gauge symm., we automatically ensure that the Hilbert space only contains Ω-invariant states.

Extension to the <u>open bosonic string</u>:

- Recall that $\hat{X} = \hat{x} + \hat{p}\tau + i\sum_{n\neq 0} \frac{1}{n}\alpha_n e^{-in\tau}\cos n\sigma$

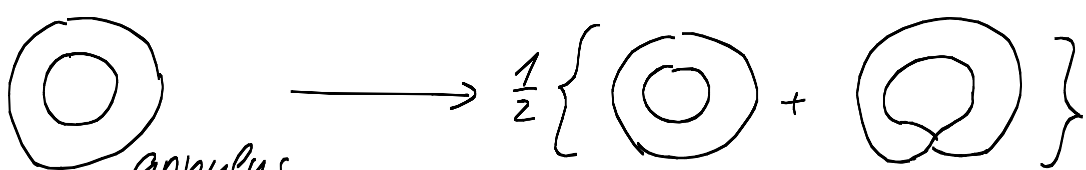
  ($\ell = 1$, index μ suppressed).

- $\Omega \hat{X}(\tau,\sigma)\Omega^{-1} = \hat{X}(\tau, \pi-\sigma) \Rightarrow \Omega \alpha_n \Omega^{-1} = (-1)^n \alpha_n$.

  (Since $\cos n(\pi-\sigma) = \frac{1}{2}\left(e^{in(\pi-\sigma)} + e^{-in(\pi-\sigma)}\right)$

  $= e^{in\pi}\frac{1}{2}\left(e^{in\sigma} + e^{-in\sigma}\right) = (-1)^n \cos n\sigma$.)

- Geometrical picture for, e.g., the 1-loop diagram:



annulus $\longrightarrow \frac{1}{2}\{$ annulus + Möbius strip $\}$

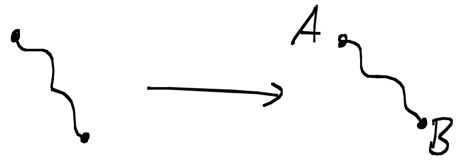
- Massless spectrum:

  $\alpha_{-1}^{\mu}|0,k\rangle \mathrel{\hat{=}}$ "photon" $A_\mu \longrightarrow$ nothing (more generally: $\Omega |N,k\rangle = (-1)^N |N,k\rangle$)

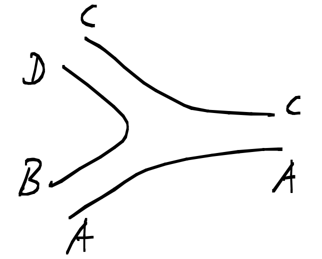
$\uparrow$ level

- There exists an important generalization of the open string where the situation is more interesting:

Introduce <u>Chan-Paton factors</u>:

 with $A, B \in \{1, \cdots, N\}$

(Each boundary has a label; when drawing WSs, the labels have to match.)

E.g. 

is only acceptable if $D = B$.

- General massless state: $\lambda_A{}^B \, \alpha_{-1}^\mu |0, A, B, k\rangle$

$n \times n$-matrix; labels of boundaries

(The states form an n^2-dimensional vector space.)

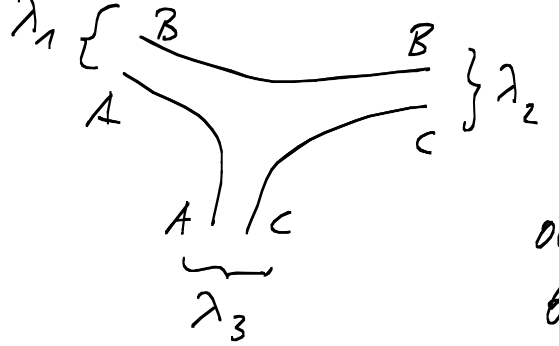
- We can write $\lambda_A{}^B = \sum_a c^a (T^a)_A{}^B$

complex coefficients; hermitian matrices, generators of the group $U(N) = SU(N) \times U(1)$

- We can guess that the interacting theory will be a $U(N)$ gauge theory. (Confirmed by calc. of scattering amplitudes.)

- Some motivation for this:

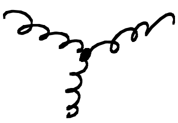
<u>3-point-function:</u>



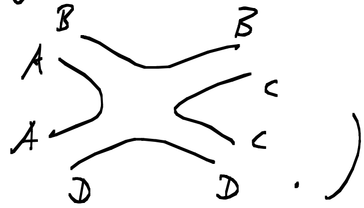
$\longrightarrow$ $\sim \mathrm{tr}(\lambda_1\lambda_2\lambda_3)$

need to sum over all possible boundaries

$\sim \underbrace{\zeta_1^a \zeta_2^b \zeta_3^c \, \mathrm{tr}(T^a T^b T^c)}$

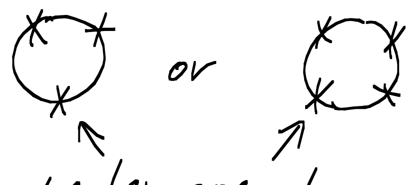
Same structure as in non-Abelian gauge theory vertex  !

(This also applies to the "4-gluon-vertex" interpreted as



.)

<u>Comment:</u> Such amplitudes can be mapped to



and are therefore called "disk amplitudes".

- The above amplitudes are obviously symmetric under

$$\lambda_A{}^B \longrightarrow U_A{}^C \lambda_C{}^D (U^{-1})_D{}^B ,$$

corresponding to a global $U(N)$ symmetry.

- We are guaranteed to find a $U(N)$ gauge theory by the general statement that any symm. of string theory appears as a gauge symm. at the field theory level.

(This follows from a built-in locality at low energy: The whole FT is just the result of elementary strings propagating everywhere & forming condensates.)

- Now, indeed, the $\Omega = +1$ projection is not trivial:

$$\Omega \lambda_A{}^B \alpha_{-1}^\mu |0,A,B,k\rangle = -\lambda_A{}^B \alpha_{-1}^\mu |0,B,A,k\rangle$$
$$= -(\lambda^T)_A{}^B \alpha_{-1}^\mu |0,A,B,k\rangle$$

implies $\underline{\underline{\lambda = -\lambda^T}}$ $\Rightarrow$ unoriented open string has gauge group $SO(N)$.

- <u>In</u> fact, we can generalize Ω by demanding instead

$$\Omega \lambda_A{}^B \alpha_{-1}^\mu |0,A,B,k\rangle = -(M\lambda^T M^{-1})_A{}^B \alpha_{-1}^\mu |0,A,B,k\rangle .$$

The requirement $\Omega^2 = 1$ restricts the possible choices for M allowing only $M = 1$ (see above) + $\underline{\underline{1 \text{ other choice}}}$

$\downarrow$

This leads to a subgroup of $U(N)$ different from $SU(N)$.

<u>problem:</u> Determine the other choice for M and the corresponding subgroup.
(Use Schur's lemma!)

<u>Final comment</u>: When coupling open & closed strings, either both sectors have to be oriented, or both unoriented.

(<u>Reason</u>: Orientation is a feature of the whole WS. Imagine the oriented-string WS having a "black" and a "white" side. Clearly, once introduced, this feature has to be present on the <u>whole</u> WS, including sections with boundaries (open-string parts).)

Return to the Superstring

A condensed way of deriving the IIA/IIB spectrum is:

<u>IIA</u>: $(8_v+8)\times(8_v+8') = (1+28+35+8+56)_{\text{Bosonic}}$
$+ (8+8'+56+56')_{\text{Fermionic}}$

<u>IIB</u>: $(8_v+8)\times(8_v\times 8') = (1+28+35+28+35_+)_{B}$
$+ (8'+8'+56+56)_F$

- As before, making this unoriented corresponds to requiring symmetry between left- & right-movers.
- Only possible for IIB, since in IIA $l \leftrightarrow r|_{WS}$ is linked to $l \leftrightarrow r|_{\text{target space}}$.
- <u>Special new feature</u>: When exchanging fermionic l.-moving & r.-moving states, an extra "$-$" has to be introduced. ("graded symmetrization").
(To understand this deeper, one needs to discuss vertex operators in the R sector ...)

- Thus, we find the <u>unoriented version of type IIB:</u>

$$(8_v+8)\times(8_v+8)\Big|_{\substack{\text{graded}\\ \text{symm.}}} = (8_v\times 8_v)_{\text{symm.}} + (8_v\times 8) + (8\times 8)_{\text{antisymm.}}$$

$$= (1 + \underset{C_2}{28} + 35_G)_B + (8' + 56)_F$$

<u>Comments:</u>

- To understand that just C_2 survives from the (8×8), recall $8\times 8 = [0]_{C_0} + [2]_{C_2} + [4]_{+\,C_4} = 1 + 28 + 35_+$ and observe that the antisymm. part of an 8×8 matrix has 28 independent parameters.
- Only $N=1$ SUSY left, no B_2-field!
- The NS-NS form B_2 is very fundamental since it is the "gauge potential" sourced by the fundamental string itself. The coupling is $\int_{WS} B_2 \subset S$.

  2d-surface $\to$ WS $\quad$ 2-form

  However, without an orientation, this doesn't make sense. (The unoriented string is not "charged.")

<u>Problem:</u> This "restricted" theory is not consistent. One way to see this is via the chiral gravitational anomaly of 10d low-energy FT.

Aside: <u>Chiral anomalies</u> (very superficially)

- Consider the effect of integrating out chiral fermions from a gauge theory ($D_\mu = \partial_\mu + iA_\mu$; $A_\mu \in Lie(G)$):

$$Z[A] = \int D\psi D\bar\psi \, e^{-\int_x \bar\psi \not{D} \psi}$$

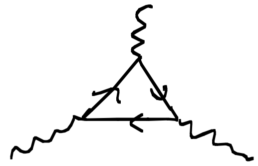
- In contrast to naive expectations, under a gauge trf. δ

$$\psi \to \psi + \delta\psi \simeq e^{i\alpha(x)}\psi \quad ; \quad \alpha(x) \in Lie(G),$$

we have

$$0 \neq \delta \ln Z \sim \int_x tr(F\tilde{F}\alpha) \qquad (\text{in } D=4)$$

- This is a different way of saying that problems with the UV-regularization of



make it impossible to define a generic chiral gauge theory at the quantum level. The condition

$$tr(\{T^A, T^B\}T^C) = 0$$

has to be imposed. (Which is indeed fulfilled in the SM.)

- This extends to gravity (as an $SO(1,D-1)$ gauge theory) and is formalized further as follows:

- Define $Z[A_\mu, e_\mu^m]$ as above, making D_μ both gauge- and gravitationally covariant. Then

$$\delta \ln Z \sim \int_x I_D(F_2, \underset{\substack{\uparrow}}{R_2}) \qquad (D = \text{space-time-dim.})$$

(curvature as an $so(1,D-1)$-valued 2-form)

where I_D is defined indirectly via

$$dI_D = \delta I_{D+1} \; ; \; dI_{D+1} = I_{D+2} \equiv I.$$

- $I = I(F_2, R_2)$ is the <u>anomaly polynomial</u> and can be calculated for all theories using very general methods.
- <u>Specifically:</u> For Majorana-Weyl fermions in $D=10$ (our "8" of type IIA/B SUGRA or of our 10d SYM theory) one has

$$I^8(F_2, R_2) = -\frac{Tr(F_2^6)}{1440} + \frac{n\, tr(R_2^6)}{725760} + \frac{Tr(F_2^4)\, tr(R_2^2)}{2304} + \cdots$$

where Tr – trace in gauge group repr.
tr – trace in tangent space
$n \equiv Tr(\mathbb{1})$ – dimension of gauge group repr.

- <u>Focus on the term $\sim tr(R_2^6)$:</u>

$$I^8 = \frac{n}{c} tr(R_2^6) + \cdots \quad , \quad I^{8'} = -I^8$$

$$I^{56} = -\frac{495}{c} tr(R_2^6) + \cdots \quad , \quad I^{56'} = -I^{56}$$

$$\underset{\substack{\uparrow \\ \text{self-dual 4-form}}}{I^{SD}} = \frac{992}{c} tr(R_2^6) + \cdots$$

$\Rightarrow$ <u>IIA:</u> trivially anomaly free since non-chiral

<u>IIB:</u> $SD + 2(56 + 8') \Rightarrow 992 + 2(-495-1) = 0$ ✓

<u>IIB unoriented:</u> $56 + 8' \Rightarrow -495 - 1 = -496 \neq 0$

problem!

<u>But:</u> If we could add 496 Maj.-Weyl fermions, everything would be OK (e.g., use a SYM theory with $\dim(G) = 496$!).

<u>Note:</u> $\dim(SO(N)) = \frac{N(N-1)}{2}$; thus, for $N=32$, we find precisely $32 \cdot 31/2 = 496$

- Thus, consider open superstring with Chan-Paton-factors $\in \{1, \ldots, 32\}$
  $\Rightarrow$ $U(32)$-SYM-theory.
- Apply projection $\Omega = +1$ subspace $\Rightarrow$ $SO(32)$-SYM-theory.
- Combine with unoriented part of type IIB superstring

$\Rightarrow$ <u>Type I superstring:</u> massless spectrum

$$\left(1_\phi + 28_{C_2} + 35_G\right)_B + 496\,(\underset{\substack{\uparrow\\ \text{gauge fields}}}{8_v} + \underset{\substack{\uparrow\\ \text{gauginos}}}{8})$$

$$+ (8' + 56)_F$$

("type I" because of $N=1$ SUSY)

<u>Outlook:</u> All 10d consistent superstring theories:

$$I,\ IIA,\ IIB,\ \underbrace{\text{het. } SO(32),\ \text{het. } E_8 \times E_8}_{\text{still missing so far}}$$