# 12 Low-Energy Effective Field Theories

## 12.1 11d SUGRA

- $d = 11$ – max. dim. allowing SUSY

(Rough reason: spinor grows $\sim 2^{d/2}$, while graviton d.o.f. $\sim d^2$
$\Rightarrow$ it becomes more and more difficult to find reprs. of

$$\{Q_\alpha, \bar{Q}^\beta\} = -2 P_\mu (\Gamma^\mu)_\alpha{}^\beta$$

in high dims. using graviton and <u>not</u> using fields with very high spin (for which no consistent FTs are known))

- in $d = 11$, SUGRA possible on the basis of:

– bosonic: $g_{\mu\nu}$ – d.o.f.: $\binom{d-2}{2} + (d-3) = \frac{9 \cdot 8}{2} + 8 = 44$

$C_3 \, (F_4)$ – d.o.f.: $\binom{d-2}{3} = \frac{9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3} = 84$

$$\left.\begin{matrix} 44 \\ 84 \end{matrix}\right\} \underline{\underline{128}}$$

– fermionic: $\psi_\mu$ – d.o.f.: $\left(2^{(d-1)/2} \cdot \underset{\text{complex}}{2} / \underset{\text{Majorana}}{2} / \underset{\text{1st. order EOMs}}{2}\right) \cdot \underset{\text{transversality \& } \Gamma_\mu \psi^\mu = 0}{(d-3)} = \frac{32}{2} \cdot 8 = \underline{\underline{128}}$

- Ignoring all numerical coefficients, the lagrangian reads:

$$\mathcal{L} \sim R(\omega) + \bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu\left(\tfrac{1}{2}(\omega + \hat{\omega})\right) \psi_\rho + |F_4|^2$$

$$+ \left(\bar{\psi}_\mu \Gamma^{\mu\nu\rho\sigma\lambda\tau} \psi_\tau + \bar{\psi}^\nu \Gamma^{\rho\sigma} \psi^\lambda\right) \cdot \left(F_4 + \hat{F}_4\right)_{\nu\rho\sigma\lambda}$$

$$+ \text{"} \sqrt{g}^{\,-1} F_4 \wedge F_4 \wedge C_3 \text{"}$$

This term is to be integrated as an 11-form.

where:

- $|F_p|^2 \equiv F_{\mu_1 \cdots \mu_p} F^{\mu_1 \cdots \mu_p}$

- $\Gamma^{\mu_1 \cdots \mu_n} \equiv \Gamma^{[\mu_1} \cdots \Gamma^{\mu_n]}$

- We freely use Einstein- or Lorentz indices (which can be translated using the vielbein $e_\mu^{\ a}$).

- $\omega$ is the (spin) connection defined by

$$\omega_{\mu ab} = \underset{\substack{\uparrow \\ \text{Riemannian connection}}}{\omega^0_{\mu ab}} + K_{\mu ab} \longleftarrow \text{"Contorsion tensor"}$$

(i.e. $D_\mu v_a = \partial_\mu v_a + \omega_{\mu ab} v^b$ is the usual covar. derivative)

$$K_{\mu ab} = \bar\Psi_c \Gamma_{\mu ab}{}^{cd} \Psi_d + 2\left(\bar\Psi_\mu \Gamma_b \Psi_a + \bar\Psi_\mu \Gamma_a \Psi_b + \bar\Psi_b \Gamma_\mu \Psi_a\right)$$

<u>Note:</u> This connection is "natural" since it arises from the EOMs if both connection & vielbein are varied independently (cf. also "Palatini formalism" or "First order formalism).

- $\hat\omega_{abc} = \omega_{abc} + \bar\Psi^d \Gamma_{dabce} \Psi^e$

("supercovariant connection")

- $\hat F_{\mu\nu\rho\sigma} = F_{\mu\nu\rho\sigma} + \bar\Psi_\mu \Gamma_{\nu\rho} \Psi_\sigma$

("supercovariant field strength", recall also that $(F_4)_{\mu\nu\rho\sigma} = \underset{\substack{\uparrow \\ \text{prefactor depends on conventions}}}{\sim} \partial_{[\mu} (C_3)_{\nu\rho\sigma]}$ )

- The action is invariant under:

$$\delta_\xi e_\mu{}^a = \bar{\xi}\Gamma^a\psi_\mu$$
$$\delta_\xi C_{\mu\nu\rho} = \bar{\xi}\Gamma_{[\mu\nu}\psi_{\rho]}$$
$$\delta_\xi \psi_\mu = D_\mu(\hat{\omega})\xi + (\Gamma_\mu{}^{\rho\sigma\tau\kappa} - \delta_\mu{}^\rho\Gamma^{\sigma\tau\kappa})\xi \cdot \hat{F}_{\rho\sigma\tau\kappa}$$

recall that we are ignoring the prefactors!

Here $\xi = \xi(x)$ is the Majorana spinor generating the 11d $N=1$ SUSY.

- Note: The "supercovariant" objects $\hat{\omega}$ & $\hat{F}$ are useful since $\delta_\xi\hat{\omega}$ & $\delta_\xi\hat{F}$ do not involve derivatives of $\xi(x)$ (which is not true for $\delta_\xi\omega$ & $\delta_\xi F$).

- The 11d theory has no free parameters. (Of course, we may call the overall prefactor $(M_{P,11})^9$ or $(\kappa_{11})^{-2}$.)

- Even though there is no known microscopic ST-realisation of this theory, it is linked to ST in many ways...

## 11.2 Type IIA

- Consider the 10d eff. theory arising from 11d SUGRA on $\mathbb{R}^{10}\times S^1$:

$$11d \xrightarrow{\quad S^1 \quad} 10d$$

$$\underset{(\mu\nu = 0\cdots 10)}{G_{\mu\nu}} \longrightarrow \underset{(\mu\nu = 0\cdots 9)}{G_{\mu\nu}} + 1\text{ vector} + \underset{(\text{dilaton})}{1\text{ scalar}}$$

or, more explicitly:

$$e_\mu{}^a \longrightarrow \left( \begin{array}{c|c} e_\mu{}^a & \phi(C_1)_\mu \\ \hline 0 & \phi \end{array} \right)$$

$C_3 \longrightarrow C_3, \ C_2 \to B_2$ (with field strength $F_3 \to H_3$

($\equiv C_3$-11d with one index taking the value "10")

(to avoid confusion with a different $C_2$ & $F_3$ arising in type IIB)

$\Psi_\mu \longrightarrow \Psi_\mu{}^L, \Psi_\mu{}^R, \Psi^L, \Psi^R$
gravitini   dilatini

opposite 10d chiralities are bound to arise since, in $d=11$, the Lorentz group can rotate $\Psi^L_{10d}$ into $\Psi^R_{10d}$ etc. (This is also the reason why there is no chirality in odd dimensions.)

- Thus, in agreement with our string-theoretic construction, we have
$$\underbrace{G_{\mu\nu}, \phi, B_2(H_3)}_{NS}, \underbrace{C_1(F_2), C_3(F_4)}_{R}, \Psi_\mu{}^{L,R}, \Psi^{L,R}$$
2 gravitini $\Rightarrow N=2$ in 10d
- Interesting fact: Dilaton $\phi$ arises as the radius of the 11th dim.
- Action follows from 11d action by assuming all fields indep. of $x^{10}$ (more precisely: no KK excitations).

- After some field redefinitions, the bosonic part reads:

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \Big\{ e^{-2\phi} \underbrace{\left(R + 4(\partial\phi)^2 - \tfrac{1}{2}|H_3|^2\right)}_{NS} - \tfrac{1}{2}\underbrace{\left(|F_2|^2 + |\tilde{F}_4|^2\right)}_{R}$$

$$- \frac{1}{4\kappa_{10}^2} \int \underbrace{B_2 \wedge F_4 \wedge F_4}_{} \Big\}$$

CS (≡ Chern-Simons)

<u>Note:</u> Product of forms does not need a $\sqrt{-G}$-prefactor to allow for a diff-invariant integration.

- $\tilde{F}_4 \equiv F_4 - C_1 \wedge H_3$

Comment on gauge invariance:

<u>in $d=11$:</u>
- diffeomorphisms
- $C_3 \to C_3 + d\lambda_2$ (with $F_4 = dC_3$ gauge inv.)

<u>in $d=10$:</u>
- 10d-diffeomorphisms
- $C_3 \to C_3 + d\lambda_2$ as before
- <u>in addition:</u>

$x^{10} \longrightarrow x^{10} + \ldots \lambda_0(x^0 \cdots x^9)$ (reparam. of the 11th dim.)

$dx^{10} \longrightarrow dx^{10} + \cdots d\lambda_0$

$C_1 \longrightarrow C_1 + d\lambda_0$ (since $e^{10}_{\mu} dx^{\mu} = \phi (C_1)_{\mu} dx^{\mu}$ is diff.-inv. in 11-dims.)

$(C_3)_{\mu\nu\rho} \to (C_3)_{\mu\nu\rho} + \cdots \frac{\partial x^{10}}{\partial x^{\mu}} C_{10\nu\rho}$

$C_3 \longrightarrow C_3 + d\lambda_0 \wedge B_2$

$$\Rightarrow \tilde{F}_4 = dC_3 - C_1 \wedge H_3 \longrightarrow \tilde{F}_4 + \overbrace{d\lambda_0 \wedge dB_2 - d\lambda_0 \wedge H_3}^{=0}$$

$\Rightarrow \tilde{F}_4$ is indeed the proper gauge inv. object

[for more see Cremmer, Julia, Scherk, '78

## 12.3 Type II B

As we already know from string theory, there must be a similar theory with $\psi_\mu^L, \psi_\mu^{L'}, \psi^L, \psi^{L'}$. Indeed, the corresponding SUGRA has been constructed. [J.H. Schwarz, '83
As expected, it uses the 128 bosons

$$\underbrace{G_{\mu\nu},\ \phi,\ B_2\,(H_3)}_{R\ \text{(as before)}},\ \underbrace{C_0\,(F_1),\ C_2\,(F_3),\ C_4\,(F_5)}_{NS}\ \text{with}\ \tilde{F}_5 = *\tilde{F}_5$$

as a constraint

- Hodge-star: $*: \omega_p \longrightarrow (*\omega)_{d-p}$

$$(*\omega)_{\mu_1\cdots\mu_{d-p}} = \underbrace{\frac{1}{p!}\sqrt{-G}\ \underbrace{\epsilon_{\mu_1\cdots\mu_d}}_{\pm 1,0}}_{\text{tensor}}\ \underbrace{\omega^{\mu_{d-p+1}\cdots\mu_d}}_{\text{indices raised using } G^{\mu\nu}}$$

(Recall that $\sqrt{-G}\,dx^1\wedge\cdots\wedge dx^d \sim \sqrt{-G}\,\epsilon_{\mu_1\cdots\mu_d}dx^1\wedge\cdots\wedge dx^d$ is invariant. Some authors include $\sqrt{-G}$ in the definition of the symbol $\epsilon$.)

Action:

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left\{\underbrace{e^{-2\phi}\left(R+4(\partial\phi)^2-\frac{1}{2}|H_3|^2\right)}_{R}-\frac{1}{2}\underbrace{\left(|F_1|^2+|\tilde{F}_3|^2+\frac{1}{2}|\tilde{F}_5|^2\right)}_{NS}\right\}$$

$$-\frac{1}{4\kappa_{10}^2}\int C_4\wedge H_3\wedge F_3$$

where $\tilde{F}_3 = F_3 - C_0\wedge H_3$ ; $\tilde{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$

- As in the IIA-case:
  $F_3, F_5$ transform non-trivially under the gauge-tfs. of $C_0, C_2, B_2$ such that $\tilde{F}_3, \tilde{F}_5$ are gauge inv.

- Imposing $\tilde{F}_5 = * \tilde{F}_5$ on the action leads to

$$\int |\tilde{F}_5|^2 \sim \int \tilde{F}_5 \wedge * \tilde{F}_5 = \int \tilde{F}_5 \wedge \tilde{F}_5 = 0,$$

which does not give the correct dynamics. One has to first vary the action and then impose the constraint on the EOMs. (In this sense, the EOMs define the theory and the above action is only a useful mnemonic.)

- To discover an important symmetry:

$$G_{E\,\mu\nu} = e^{-\phi/2} G_{\mu\nu}, \quad \tau = C_0 + i e^{-\phi}, \quad G_3 = F_3 - \tau H_3$$

"Einstein frame" ($G_{E\,\mu\nu}$)  "String frame" ($G_{\mu\nu}$)

$$\Rightarrow S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G_E} \left\{ R - \frac{|\partial \tau|^2}{2(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{2\,\mathrm{Im}\,\tau} - \frac{|\tilde{F}_5|}{4} \right\}$$

$$+ \frac{1}{8 i \kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\mathrm{Im}\,\tau}$$

- This is inv. under $\tau \to \frac{a\tau + b}{c\tau + d}$; $G_3 \to \frac{G_3}{c\tau + d}$

with $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$ (Integers with $ad - bc = 1$).

→ problems

Jumping ahead:
This symm. includes in particular $\tau \to -1/\tau$ or $g_s = e^{\phi} \to g_s^{-1} = e^{-\phi}$. This an S-duality or

strong–weak-coupling duality which, among other things, exchanges fund. strings (F-strings) with D1-branes (non-pert. objects or solitons or D-strings also present in this theory). More generally, the symm. also allows to exchange the fund. string with, e.g., a bound state of c D-strings & d F-strings. This is why only $SL(2,\mathbb{Z})$ is a symm. of string theory (while the above action is inv. under $SL(2,\mathbb{R})$.

## 12.4 Type I

- Recall that type I arose from type IIB via the projection $\Omega$ making the WS unoriented. The FT equivalent of $\Omega$ is the projection on states inv. under parity. This removes 1/2 of the spinors as well as $C_0$, $B_2$ and $C_4$.
- For consistency (we argued from anomaly cancellation, but more "stringy" arguments exist), we have then to add 32 $\underbrace{\text{D9-branes}}$.

non-pert. objects on which strings can end.
(D9-brane → 10d-object → fills out all of target-space → corresponds to adding open strings with Chan-Paton-labels 1 ... 32)

- This adds a gauge theory sector with group $SO(32)$
- Action: $$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\, e^{-2\phi}\left(R + 4(\partial\phi)^2 - \frac{1}{2}|\tilde{F}_3|^2\right)$$
$$-\frac{1}{2g_{10}^2}\int d^{10}x\sqrt{-G}\, e^{-\phi}\, \underbrace{\mathrm{Tr}_V}_{\text{vector-repr.}}\, (\underbrace{|F_2|^2}_{so(32)\text{-valued 2-form}})$$

with
$$\tilde{F}_3 = dC_2 - \frac{\kappa_{10}^2}{g_{10}^2}\omega_3$$
and
$$\omega_3 = \mathrm{Tr}_V\left(A_1\wedge dA_1 + \frac{2}{3}A_1\wedge A_1\wedge A_1\right).$$
(CS-3form; this plays an important role in the cancellation of <u>reducible</u> anomalies via the Green-Schwarz-mechanism, which was a discovery that started the "1st ST revolution".)

<u>Gauge-transformations:</u> $$A_1 \to A_1 + d\underbrace{\lambda_0}_{so(32)\text{-valued!}} - i[A_1,\lambda_0]$$
$$C_2 \to C_2 + \frac{\kappa_{10}^2}{g_{10}^2}\mathrm{Tr}_V(\lambda F_2)$$
(This unexpected trf. is needed to compensate the trf. of $\omega_3$ thereby making $\tilde{F}_3$ gauge-inv.)

## 12.5 Heterotic

From our string-theoretic discussion, we know the massless modes:
$$\underbrace{G_{\mu\nu},\ \phi,\ B_2\,(H_3) + 1\text{ gravitino} + 1\text{ dilatino}}_{N=1\text{ SUGRA}}$$

$$+ \underbrace{(A_1, \text{gaugino}) \text{ in adjoint of } SO(32) \text{ or } E_8\times E_8}_{10d\ N=1\ SYM}$$

- Focussing on the $SO(32)$ case, the action is

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\, e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2}|\tilde{H}_3|^2\right] - \frac{1}{2g_{10}^2}\int d^{10}x\sqrt{-G}\, e^{-2\phi}\, \mathrm{Tr}_v(|F_2|^2)$$

where $\tilde{H}_3 = dB_2 - \frac{\kappa_{10}^2}{g_{10}^2}\omega_3 \leftarrow$ CS-form (defined in terms of gauge fields as in type I above)

<u>Note:</u> This differs from the type I case in two ways:

1) – purely formal at FT level – $\underset{\text{(heterotic)}}{B_2\,(H_3)} \longleftrightarrow \underset{\text{(type I)}}{C_2\,(F_3)}$

(The different naming reflects the fact that $C_2$ arises string-theoretically, like the C-forms of type II, from the RR sector. In contrast, $B_2$ arises string-theoretically in a way similar to the NS-NS fields of type IIB.)

2) – (apparently) more real – $\mathcal{L} \supset e^{-\phi}\,\mathrm{tr}\,F^2$ (type I)

vs.

$\mathcal{L} \supset e^{-2\phi}\,\mathrm{tr}\,F^2$ (heterotic).

(This can only be understood from string-theoretic scattering amplitudes. See, however, the simple argument below.)

<u>Crucial fact:</u> In spite of the above, there is in fact <u>only one</u> 10d SYM theory with group $SO(32)$.

- To see this, take type I action and let

$$G_{\mu\nu} = e^{\phi}\, G'_{\mu\nu}\,:$$

$$\sqrt{-G}\, e^{-2\phi} R \longrightarrow \sqrt{-G'}\,(e^{\phi})^5 e^{-2\phi} \underbrace{R' e^{-\phi}} + \cdots$$

since $R = G^{\mu\nu} R_{\mu\nu}$ and $R_{\mu\nu}$ is inv. under a rescaling of $G_{\mu\nu}$
($R_{\mu\nu} = R_{\mu\rho\nu}{}^{\rho} = \partial_\rho \Gamma^{\rho}_{\mu\nu} + \cdots$ ; $\Gamma$ contains $G^{\mu\nu}$ & $G_{\mu\nu}$)

$$= \sqrt{-G'}\, e^{2\phi} R'$$

$$\downarrow \quad \phi = -\phi'$$

$$= \sqrt{-G'}\, e^{-2\phi'} R' \quad \text{as in heterotic case!}$$

also: $\sqrt{-G}\, e^{-\phi} F_{\mu\nu} F_{\rho\sigma} G^{\mu\rho} G^{\nu\sigma}$

$$\longrightarrow \sqrt{-G'}\,(e^{\phi})^5 e^{-\phi} F_{\mu\nu} F_{\rho\sigma} G'^{\mu\rho} G'^{\nu\sigma} e^{-2\phi}$$

$$= \sqrt{-G'}\, e^{2\phi} F^2 = \sqrt{-G'}\, e^{-2\phi'} F^2 \quad \text{as in het. case!}$$

• Thus, we have an <u>S-Duality</u>:

$$\left\| \begin{array}{l} G_{I\,\mu\nu} = e^{-\phi_h} G_{h\,\mu\nu} \; ; \; \phi_I = -\phi_h \\ \tilde{F}_{3I} = \tilde{H}_{3h} \; ; \; A_{1I} = A_{1h} \end{array} \right\|$$

$\Rightarrow$ <u>One</u> and the same phys. theory (namely 10 SUGRA + SYM with group SO(32)) has <u>two</u> string theoretic UV-completions:

1) type I – perturbative for $g_{sI} = e^{\phi_I}$ small
2) heterotic – perturbative for $g_{sh} = e^{\phi_h} = e^{-\phi_I}$ small

$\Rightarrow$ The different stringy descriptions are useful in different regimes (small $\phi_I$ vs. large $\phi_I$).

(The name – S-duality – comes from <u>strong-weak-coupling-duality</u>.)

## 12.6 More on couplings

**type II:**
- one dim. ful parameter: $\kappa_{10} \sim \alpha'^2$ $\left(\frac{1}{\kappa_{10}^2} \sim M_{P,10}^{\,8}\right)$
- the exact relation of $\kappa_{10}$ & $\alpha'$ depends on the definition of $\phi$:

(One can check that type I actions are inv. under

$$\phi \to \phi + c\,, \quad \kappa_{10} \to \kappa_{10} e^{-c}\,, \quad F_p \to F_p e^{c}\,.)$$

- also: The appearance of $\frac{1}{(\kappa_{10} e^{\phi})^2}$ as prefactor of $R$ tells us that small $\phi$ corresponds to weak gravit. coupling (for $\phi = 0$ $\kappa_{10}$ is the gravit. coupling).

($M_s/M_{P,10}$ (with $M_s \sim \frac{1}{\sqrt{\alpha'}}$) is small/large for small/large $g_s = e^{\phi}$. $\Rightarrow$ Pert. ST does not solve the problem of Planck-scale gravity; it simply changes the theory <u>before</u> the strong-coupling regime is reached.)

**type I:**
- analogously for the gravit. sector
- <u>new</u>: Apparently, there is a new dim-ful parameter $g_{10}^2$. $\left(\mathcal{L} \supset \frac{1}{g_{10}^2} F^2 \Rightarrow [g_{10}] = (\text{length})^3\right)$
- <u>however</u>: The dim. less ratio $\kappa_{10} g_{10}^{-4/3}$ changes if $\phi \to \phi + c$ and appropriate factors $e^c$ are absorbed in $\kappa_{10}$ & $g_{10}$ $\Rightarrow$ <u>not really a new parameter!</u>
- $\alpha'$ comes in via $g_{10,I}^2 / \kappa_{10,I} = 2\,(2\pi)^{7/2} \alpha'_I$

<u>heterotic:</u> • Analogously, but because of the $e^{-2\phi}$-prefactor of $F^2$, the ratio $\kappa_{10} g_{10}^{-4/3}$ changes in a diff. way under $\phi \to \phi + c$.

• $\alpha'$ enters via $g_{10,h}^2 / \kappa_{10,h}^2 = 4/\alpha'_h$

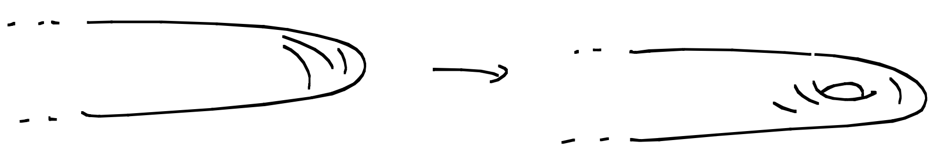
<u>Finally:</u> The WS action contains term $\sim \phi \int d^2\sigma R$

Part of target-space ($\equiv$ "background); in general $\phi = \phi(X)$, but we focus on constant $\phi$ (as we have focussed on $G_{\mu\nu}(X) = \eta_{\mu\nu}$, although the general action is $G^{\mu\nu}(X)\, \partial_a X_\mu \partial^a X_\nu$).

$\Rightarrow$ performing the $d^2\sigma$-integration, $S \supset -\phi \cdot \chi$

Euler # of surface

• Here we only need to know that $\chi \to \chi - 2$ if a "handle" is added:



$\chi \longrightarrow \chi' = \chi - 2$

• Since amplitude $\sim \int e^S$, an extra handle ($\equiv$ extra loop) corresponds to an extra factor $e^{2\phi} = g_s^2$.

($\Rightarrow$ factor $g_s^2$ for closed string loop)

• Open string loop corresponds to "1/2 of closed string loop:

($\Rightarrow$ factor $e^{\phi} = g_s$ for open-string loop)

Thus: We can finally understand the appearance of

$e^{-\phi}F^2$ vs. $e^{-2\phi}F^2$ in type I vs. heterotic

(and of $e^{-2\phi}R$ in both theories).

## 12.7 D-branes

- From our knowledge so far, het. & type I strings appear to be much more interesting phenomenologically than type II because of the non-abelian gauge groups.

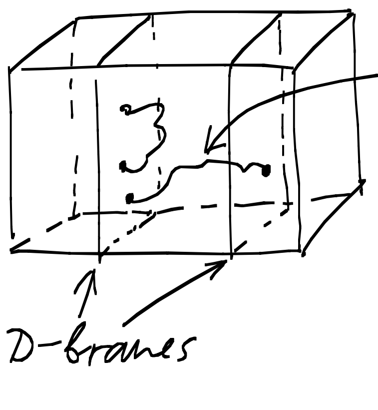
  (Indeed: most early phenom. work focussed on het. $E_8 \times E_8$ on $CY_3$-manifolds; see below)

- But: Recall that one can view type I as type IIB (with $\Omega$-projection) + 32 D9-branes.

  more generally: One can add (in the sense of considering backgrounds including ...) Dp-branes ($p = 0 \cdots 9$) to type II string theories.

- One motivation: Consider 10d space with $(p+1)$-dim. hypersurfaces:

e.g.




Consider open string with Dirichlet bound. cond.s on branes;

e.g. $(X^{p+1}, \dots, X^9(0)) = (a^{p+1}, \dots, a^9)$

$(X^{p+1}, \dots, X^9(6)) = (6^{p+1}, \dots, 6^9)$

(Here $\bar{a}$ & $\bar{b}$ specify the positions of the two branes in the dim.s which they do not fill out.)

- Open string quantization gives:

  D9 $\to$ 10d SYM theory ($N=4$ SUSY in 4d language)
  D8 $\to$ 9d SYM theory ( — " — )
  ⋮
  D3 $\to$ 4d SYM theory with $N=4$ (4 gauginos)

- <u>Field content</u>: 10d — $A_\mu$, $\lambda_{10d}$ (Maj.-Weyl)
  9d — $A_\mu$, 1 scalar ($\hat{=} A_9$), $\lambda_{9d}$ (Maj.)
  ⋮
  5d — $A_\mu$, 5 scalars ($\hat{=} A_5 \cdots A_9$), $\lambda^1_{5d}$, $\lambda^2_{5d}$ (Dirac)
  (D3-brane) 4d — $A_\mu$, 6 scalars, $\lambda^1_{4d} \cdots \lambda^4_{4d}$ (Weyl)

- Geometrically: Scalars parametrize the position of the brane in (transverse part of) target space.

- <u>A different motivation of branes:</u> (in eff. FT framework)

  D-branes are the sources for the RR-fields:

- Reconsider electrodynamics from a geometrical perspective: (suppressing coupling constants)

$$S = \frac{1}{2}\int_{\text{space-time}} F \wedge *F + \int_{\substack{\text{world-line}\\ \text{of charged particle}}} A$$

— a p-form is naturally integrated over a p-dim. submanifold: volume-element specified by

$p$ linearly indep. infinit. vectors:

$$(\bar{\xi}_1, \dots, \bar{\xi}_p) \underset{\substack{\uparrow \\ \text{natural "application"}}}{\cdot} (F_p) \sim (F_p)_{\mu_1 \dots \mu_p} dx^{\mu_1}(\bar{\xi}_1) \cdots dx^{\mu_p}(\bar{\xi}_p)$$

where $\bar{\xi}_1 = \xi_1^\mu \frac{\partial}{\partial x^\mu}$ and $dx^\nu(\bar{\xi}) = \xi^\nu$

since $dx^\nu\left(\frac{\partial}{\partial x^\mu}\right) = \delta^\nu_\mu$.

The infinit. scalar obtained in this way is then summed over all infinit. volume elements of the submanifold.

($\Rightarrow$ it is natural to integrate $\underset{\substack{\uparrow \\ \text{2-form}}}{F} \wedge \underset{\substack{\uparrow \\ (d-2)\text{-form}}}{*F}$ ($d$-form) over space-time & $A$ (1-form) over the world-line.)

- If we allow the charged particle to be "smeared out", $\int A$ has to be replaced by $\int \underset{\substack{\uparrow \\ \text{current density}}}{j} \wedge A$.

($j$ is naturally a $(d-1)$-form: Given a current of particles, we can ask how many particles pass a transverse hyperplane element $(\bar{\xi}_1 \cdots \bar{\xi}_{d-2})$ in a given infinit. time $(\bar{\xi}^t)$ $\Rightarrow$ $j$ is dual to $(d-1)$-volume element.)

$$\Rightarrow \quad S = \int \left(\tfrac{1}{2} F \wedge * F + j \wedge A\right), \quad F = dA$$

$$\delta S = \int \delta A \wedge d*F - \delta A \wedge j \Rightarrow \left\| \begin{array}{l} d*F = j \\ dF = 0 \end{array} \right\| \text{ Maxwell eqs.}$$

- This naturally generalizes to p-forms:

$$F_p = dA_{p-1}$$

with $S = \int \frac{1}{2} F_p \wedge (* F_p) + \underbrace{\int A_{p-1}}_{\text{World-volume}}$

Should be $(p-1)$-dimensional, i.e. describing a $(p-2)$-dim. object moving in time:

- $F_p$ is sourced by $(p-2)$-brane
(or $F_{p+2}$ is sourced by $p$-brane.)

- A completely equivalent ("dual") point of view on the $F_p$, $A_{p-1}$ system is provided by taking $(* F_p) \equiv \tilde{F}_{d-p}$ as fundamental. The free action has the same form ($\sim \int \tilde{F}_{d-p} \wedge (* \tilde{F}_{d-p})$) and the corresponding potential is introduced via $\tilde{F}_{d-p} = d \tilde{A}_{d-p-1}$. It couples to objects of dimension $(d-p-1)$ or $(d-p-2)$-branes:

$$\int \tilde{A}_{d-p-1}$$

World volume of $(d-p-2)$-brane.

(<u>Example:</u> Electrodynamics in $d=4$ : $d=4$; $p=2$
$d-p-2 = 0 \Rightarrow$ point-like-object = magn. monopole)

<u>EOMs:</u>

$$dF = \tilde{j} \qquad\qquad d * \tilde{F} = \tilde{j}$$
$$d * F = j \qquad \text{or} \qquad d\tilde{F} = j$$

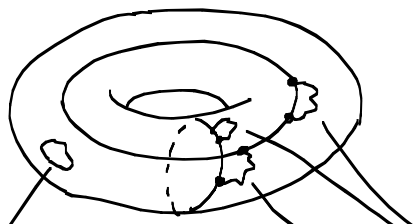
## Back to string theory:

- $B_2\,(H_3)$ – sourced by F-string or NS5-branes (non-pert. objects) (type II & het.)

- $C_1\,(F_2)$, $C_3\,(F_4)$ – sourced by D0, D2, D4, D6-branes (type IIA)

- $C_0\,(F_1)$, $C_2\,(F_3)$, $C_4\,(F_5)$ – sourced by D1, D3, D5, D7, D9-branes (type IIB)

$\Rightarrow$ generic D-brane model of type II:



gravity & form-fields in "bulk" (closed string sector)

gauge fields
matter at brane intersections } (open string sector)

(Semi-realistic models with field content close to SM & $N=1$ SUSY in 4d have been found in these "intersecting D-brane constructions.)

## Comments:

- The explicit brane-coupling is introduced in type II actions as

$$S \supset \underbrace{\int}_{\text{world-volume of p-brane}} \Big( -\underbrace{T_p}_{\text{brane-tension}}\, dVol. + \underbrace{\mu_p}_{\text{charge}}\, C_{p+1} \Big)$$

$$T_p = |\mu_p|\, e^{(p-3)\phi/4} \quad ; \quad \mu_p^2 = \frac{1}{\alpha'(4\pi^2\alpha')^p}$$

both signs possible ( Dp & $\bar{D}p$ branes)
(anti-branes)

(in conventions where $C_{p+1}$ is dim-less and $|F_{p+2}|^2$ has mass-dim. 2, just like $R$; using Einstein-frame metric )

- In flat compactifications, both D-brane charge & tension have to cancel $\Rightarrow$ need, in addition to Dp-branes, Op-planes (orientifold planes) with opposite charge & <u>tension</u>. They arise from the generalization of our $\Omega$-proj: $\mathrm{II}B \to \mathrm{I}$ at the fixed-points of the corresponding proj.-operations (which in general also act on space-time).
- The dynamics of the brane itself is, in the FT limit, governed by the Dirac-Born-Infeld (DBI) action:

$$S_{Dp} = -\mu_p \underbrace{\int d^{p+1}x}_{\text{over brane-world-volume}}\, e^{-\phi} \sqrt{-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})}\,.$$

projection parallel to brane

U(1)-field strength on brane, mass-dim. 2 as in 4d

(<u>Note</u>: Letting $B = F = 0$ and going to Einstein fram by $G_{\mu\nu} \to e^{\phi/2} G_{\mu\nu}$ we recover our simplified formula $\sim T_p \int dVol.$ given above.)