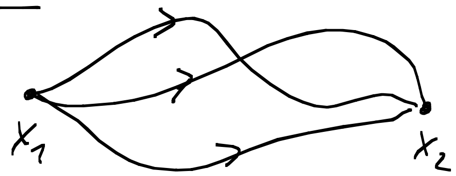
5 Modern Covariant Quantization

5.1 Polyakov Path Integral

- quantum-mechanical point particle:

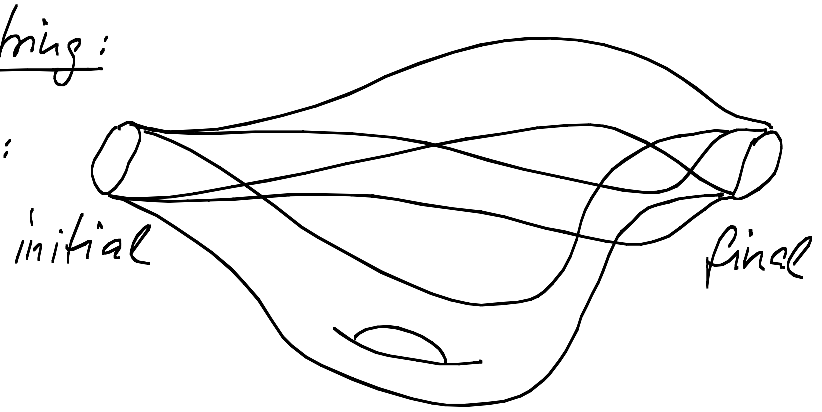
amplitude = "sum over histories" =



$$\langle x_2, t_2 | x_1, t_1 \rangle = \int\limits_{\substack{x(t_1) = x_1 \\ x(t_2) = x_2}} \mathcal{D}x(t)\, e^{iS[x]}$$

- quantized string:

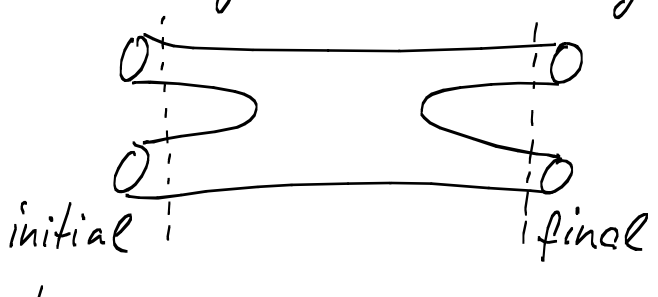
analogously:



$$\langle \text{final} | \text{initial} \rangle = \int \mathcal{D}X\, \mathcal{D}h\, \exp\{ i S_p [X, h] \}$$

all worldsheets bounded by the initial & final string configurations

(This can clearly include scattering processes, e.g.,



)

- more technically: We will not need initial / final states with fixed position but rather with fixed momentum. This will be described by the insertion of "vertex ops."

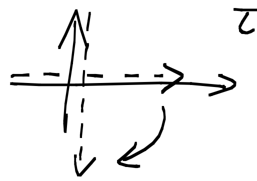
(already briefly mentioned above) in the "path integral" or "functional integral":

$$\int \mathcal{D}X \mathcal{D}h \, e^{iS_P[X,h]} \, V_1(X,h) \cdots V_4(X,h)$$

They replace the 2 initial & 2 final string configs. in the scattering picture above.

- as a first step: ignore the V's or the boundaries and consider simply $\qquad Z = \int \mathcal{D}X \mathcal{D}h \, e^{iS_P[X,h]}$
- convenient (though not strictly necessary):

Wick rotation :



$$\int_{-\infty}^{\infty} d\tau \longrightarrow \int_{i\infty}^{-i\infty} d\tau = \{\tau = -i\tau'\} = -i\int_{-\infty}^{\infty} d\tau' \,, \quad \text{then rename } \tau' \to \tau$$

Thus, the overall effect is:

$$i\int d\tau d\sigma \left[(\partial_\tau X)^2 - (\partial_\sigma X)^2 \right] \xrightarrow{\text{"}\tau \to -i\tau\text{"}} \int d\tau d\sigma \left[-(\partial_\tau X)^2 - (\partial_\sigma X)^2 \right]$$

$\Rightarrow \; Z = \int \mathcal{D}X \mathcal{D}h \, e^{-S_E}$ where the euclidean action is

$$S_E = \frac{1}{4\pi\alpha'} \underbrace{\int d^2\sigma \sqrt{h}}_{\text{euclidean WS}} \, \partial_a X^\mu \partial^a X^\nu \eta_{\mu\nu}$$

We will suppress this index in the future.

remains Minkowski

5.2 Gauge fixing: Fadeev-Popov-method

- recall: $h_{ab} \xrightarrow[\text{Weyl-rescaling}]{\text{diffeom.,}} h'_{ab} = e^{2\omega}\left(\frac{\partial\sigma^c}{\partial\sigma'^a}\right)\left(\frac{\partial\sigma^d}{\partial\sigma'^b}\right)h_{cd}$
- let the functional freedom of $\omega(\sigma)$ and $\sigma'(\sigma)$ be combined in a set of fcts. ζ so that

$$h_{ab} \xrightarrow[\text{gauge trf.}]{} h^{\zeta}_{ab}$$

- we know (at least locally) that if, for fixed $\hat{h}$, $\underline{\zeta \text{ runs over all gauge trfs.}}$ then $\underline{\hat{h}^{\zeta} \text{ runs over all metrics } h}$

$$\Rightarrow \int \mathcal{D}h\, F[h] = \int \mathcal{D}\zeta \det\left(\frac{\delta \hat{h}^{\zeta}}{\delta\zeta}\right) F[\hat{h}^{\zeta}]$$

$$\Rightarrow Z = \int \mathcal{D}X\mathcal{D}\zeta\, e^{-S[X,\hat{h}^{\zeta}]} \det\left(\frac{\delta\hat{h}^{\zeta}}{\delta\zeta}\right)$$

$$= \int \mathcal{D}\zeta\mathcal{D}X^{\zeta} e^{-S[X^{\zeta},\hat{h}^{\zeta}]} \det\left(\frac{\delta\hat{h}^{\zeta}}{\delta\zeta}\right)$$

$$= \int \mathcal{D}\zeta\mathcal{D}X\, e^{-S[X,h]} \underbrace{\det\left(\frac{\delta\hat{h}^{\zeta}}{\delta\zeta}\right)}_{\equiv \Delta_{FP}[\hat{h}^{\zeta}]}$$

here we used the invariance of the measure $\mathcal{D}X$ *

$\equiv \Delta_{FP}[\hat{h}^{\zeta}]$ (Fadeev-Popov determinant)

- $\underline{\text{about } \Delta_{FP}}$: $\Delta^{-1}_{FP}[h^{\zeta}] = \int \mathcal{D}\zeta'\,\delta[h^{\zeta'} - h^{\zeta}] = \int \mathcal{D}\zeta'\,\delta[h^{\zeta'\zeta^{-1}} - h]$

δ-fct. is gauge-invariant since gauge trf. is additive in ζ

Using the gauge-inv. of the ζ-measure we find

$$\Delta_{FP}^{-1}[h^{\zeta}] = \int \mathcal{D}(\zeta\zeta'^{-1})\,\delta[h^{\zeta\zeta'^{-1}} - h] = \int \mathcal{D}\zeta''\,\delta[h^{\zeta''} - h]$$

$$= \Delta_{FP}^{-1}[h]$$ (i.e. Δ_{FP} does not vary along the gauge orbit)

• Dropping an overall factor $\left(\int \mathcal{D}\zeta\right)$ we now have

$$Z = \int \mathcal{D}X\, e^{-S[X,\hat{h}]}\,\Delta_{FP}[\hat{h}]$$

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*) for finite-dim. integrals: norm $\longrightarrow$ measure

$$\|dx\|^2 = g_{ij}\,dx^i dx^j \longrightarrow \int \sqrt{g}\,d^n x$$

for our functional integral:

$$\underbrace{\|\delta X\|^2 = \int d^2\sigma \sqrt{h}\,\delta X^\mu \delta X_\mu} \longrightarrow \int \mathcal{D}X$$

This is diff. but not Weyl invariant

$\Rightarrow$ The step $\int \mathcal{D}X^{\zeta} \to \int \mathcal{D}X$ is in general incorrect.

However:
- in specific target-space dimensions the measure is Weyl invariant
- this corresponds to situations where Weyl-rescalings are not anomalous
- from the comparison of LCQ & OCQ (which involves Weyl rescaling) we expect this to happen in $D = 26$

To find Z, we need to evaluate Δ_{FP}:

$$\Delta_{FP}^{-1}[h] = \int d\zeta \, \delta[h^\zeta - h]$$

$$\left.\begin{array}{l}\text{infinit. Weyl-rescaling by } \omega \\ \text{diffeom. } \sigma^a \to \sigma^a + \epsilon^a\end{array}\right\} \Rightarrow (h^\zeta - h)_{ab} = 2\omega h_{ab} - (\nabla_a \epsilon_b + \nabla_b \epsilon_a)$$

covariant derivatives since h is now not necessarily the flat metric

Thus:

$$(h^\zeta - h)_{ab} = (2\omega - \nabla_c \epsilon^c) h_{ab} - \underbrace{\{\nabla_a \epsilon_b + \nabla_b \epsilon_a - (\nabla_c \epsilon^c) h_{ab}\}}_{\equiv 2(P\epsilon)_{ab}}$$

differential operator (defined to produce a <u>traceless</u> symm. tensor)

$$\Delta_{FP}^{-1}[h] = \int D\epsilon D\omega \, \delta[(2\omega - \nabla_c \epsilon^c) h - 2P\epsilon]$$

$$= \int D\epsilon D\omega D\beta \, \exp\left\{ i \int d^2\sigma \sqrt{h} \, \beta^{ab} [(2\omega - \nabla_c \epsilon^c) h_{ab} - 2(P\epsilon)_{ab}] \right\}$$

symm. tensors β^{ab}

recall: $\int d^3\bar{k} \, e^{i\bar{k}\bar{x}} \sim \delta^3(\bar{x})$

here: $x, y \longrightarrow \bar{x} \cdot \bar{y}$ is replaced by

$\beta^{ab}, \gamma_{ab} \longrightarrow \int d^2\sigma \sqrt{h} \, \beta^{ab} \cdot \gamma_{ab}$

$\int D\omega \longrightarrow \delta[\beta^a{}_a]$; The $\beta \cdot h$-term in exponent can be dropped.

$$\Rightarrow \Delta_{FP}^{-1}[h] = \int \mathcal{D}\epsilon\, \mathcal{D}\beta \, \exp\left\{ i \int d^2\sigma \sqrt{h}\, \beta^{ab} (P\epsilon)_{ab} \right\}$$

symm. traceless tensors β^{ab}

<u>Note:</u> ϵ_a & β^{ab} have the same # of d.o.fs. ;
P maps vectors to symm. traceless tensors
$\Rightarrow$ P can be viewed as an (infinite-dimensional) quadratic matrix

<u>recall:</u> $\int d^n x\, d^n y\, e^{-x^T M y} = \left\{ \begin{array}{l} y' = My \\ dy' = (\det M)\, dy \end{array} \right\} =$

$$= (\det M)^{-1} \int d^n x\, d^n y'\, e^{-x^T y'} \sim (\det M)^{-1}$$

$$\Rightarrow \Delta_{FP}(h)^{-1} \sim (\det P)^{-1}$$

overall constant irrelevant; only the h-dependence entering via $P = P[h]$ matters.

<u>recall:</u> Grassmann variables

$$\theta_i \; ; \quad \theta_i \theta_j = - \theta_j \theta_i \quad (\theta_i^2 = 0)$$

<u>Def:</u> $\int d\theta_i = 0 \qquad \int (d\theta_i)\theta_j = \delta_{ij}$

$$\Rightarrow \int d^n\theta\, d^n\psi\, e^{\theta^T M \psi} = \int d^n\theta\, d^n\psi\, (\theta^T M \psi)^n / n!$$

(use $\int d^n\theta\, \theta_{i_1} \cdots \theta_{i_n} = \epsilon_{i_1 \cdots i_n}$, which follows from the above definition of the integral)

$\Rightarrow$ by the definition of the determinant:

$$\int d^n\theta\, d^n\psi\, e^{\theta^T M \psi} = \det M$$

$$\Rightarrow \Delta_{FP}[h] = (\det P) = \int \mathcal{D}b\, \mathcal{D}c\, \exp\left[\int d^2\sigma \sqrt{h}\, b^{ab} (Pc)_{ab}\right]$$

$\underbrace{\text{symm. traceless tensor} \quad \text{vector}}_{\text{both fermionic}}$ — arbitrary prefactor

finally:

$$\boxed{Z = \int \mathcal{D}X\, \mathcal{D}b\, \mathcal{D}c\, e^{-S_X - S_g} \quad ; \quad S_g = -\frac{i}{2\pi}\int d^2\sigma \sqrt{h}\, c_a \nabla_b b^{ba}}$$

"FP-ghosts"

(Note: $\int b(Pc) \sim \int b^{ab}(\nabla_a c_b + \nabla_b c_a - h_{ab}(\nabla_c c^c)) \sim \int b^{ab} \nabla_a c_b \sim \int c_a \nabla_b b^{ba}$)

5.3 Quantizing the FP ghosts

- Now we can quantize canonically without the gauge fixing problems we were facing in OCQ & LCQ.
- Choose $\underline{\hat{h}_{ab} = \delta_{ab}}$.
- X --- as before ...
- $\underline{b, c}$: rewrite S_g as $S_g = \frac{i}{\pi}\int d^2\sigma\, (c^+ \partial_- b_{++} + c^- \partial_+ b_{--})$
- a straightforward calculation gives

$$\{b_{++}(\tau,\sigma), c^+(\tau,\sigma')\} = 2\pi\delta(\sigma-\sigma')$$

$$\{b_{--}(\tau,\sigma), c^-(\tau,\sigma')\} = 2\pi\delta(\sigma-\sigma')$$

- action $\Rightarrow$ EOMs: $\partial_- c^+ = \partial_- b_{++} = 0$
  $\qquad\qquad\qquad\quad \partial_+ c^- = \partial_+ b_{--} = 0$

- mode decomposition — <u>closed string</u>

— all fields periodic $\rightarrow$ $c^+ = \sqrt{2} \sum_{-\infty}^{\infty} c_n e^{-2in\sigma^+}$

$$c^- = \sqrt{2} \sum_{-\infty}^{\infty} \tilde{c}_n e^{-2in\sigma^-}$$

and analogously $b_{++} \longrightarrow b_n$
$\qquad\qquad\qquad\quad\; b_{--} \longrightarrow \tilde{b}_n$

- mode decomposition — <u>open string</u>

— can be viewed as closed string on intervall $(0, 2\pi)$ under the restriction of a $\mathbb{Z}_2$-symmetry:

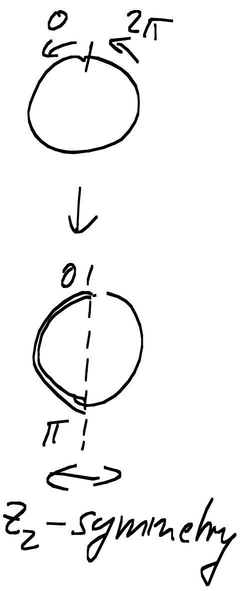
$\mathbb{Z}_2: (\tau, \sigma) \longrightarrow (\tau, -\sigma)$ <u>picture:</u>

or, equivalently,

$(\sigma^0, \sigma^1) \longrightarrow (\sigma^0, -\sigma^1)$

$X(\sigma^0, \sigma^1) \longrightarrow X(\sigma^0, -\sigma^1) \overset{!}{=} X(\sigma^0, \sigma^1)$

This is the requirement of $\mathbb{Z}_2$-symmetry



Since this $\mathbb{Z}_2$-symm. is a reflection of the WS, it acts non-trivially on the vector index of the ghost c^a:

$$c^0(\sigma^0, \sigma^1) \longrightarrow c^0(\sigma^0, -\sigma^1) \overset{!}{=} c^0(\sigma^0, \sigma^1)$$
$$c^1(\sigma^0, \sigma^1) \longrightarrow -c^1(\sigma^0, -\sigma^1) \overset{!}{=} c^1(\sigma^0, \sigma^1)$$

or, equivalently,

$$c^+(\sigma^+) \longrightarrow c^-(\sigma^-) \overset{!}{=} c^+(\sigma^+)$$

$\Rightarrow \quad c_n = \tilde{c}_n$

analogously, for the open string one has $b_{++}(\sigma^+) = b_{--}(\sigma^-)$

$\Rightarrow \quad b_n = \tilde{b}_n$

(i.e., as before, the open string modes correspond to "half of" the closed string modes)

$\rightarrow$ in normalization conventions of GSW:

$$c^+ = \sum_{-\infty}^{\infty} c_n e^{-in\sigma^+}$$

$$c^- = \sum_{-\infty}^{\infty} c_n e^{-in\sigma^-} \qquad \text{(no "tilde"!)}$$

analogously: $b_{++} \rightarrow b_n$

$\qquad\qquad\quad b_{--} \rightarrow b_n$

- Now, focussing exclusively on the open string, one has
  - canonical commut. relations $\Rightarrow \quad \{c_m, b_n\} = \delta_{m+n}$

$$\{c_m, c_n\} = \{b_m, b_n\} = 0$$

  - action $\Rightarrow$ energy-mom.-tensor $\Rightarrow$ Virasoro generators

$$L_m^{(c)} = \frac{1}{\pi}\int_{-\pi}^{\pi} d\sigma\, e^{im\sigma}\, T_{++}^{(c)} = \sum_{n=-\infty}^{\infty} (m-n)\, b_{m+n}\, c_{-n}$$

$$[L_m^{(c)}, L_n^{(c)}] = (m-n) L_{m+n}^{(c)} + A^c(m)\, \delta_{m+n}$$

with $A^c(m) = \frac{1}{6}(m - 13m^3)$

- Finally, we consider the combined action $S = S_X + S_g$ and, correspondingly,

$$L_m = \underset{\substack{\uparrow \\ \text{X-oscillators}}}{L_m^{(\alpha)}} + \underset{\substack{\uparrow \\ b,c \text{ oscillators}}}{L_m^{(c)}} - a\delta_m$$

here, we include the normal-ordering-constant in L_0 rather than keeping it separate as before

- The total anomaly of the L_m-algebra is then found to be

$$A(m) = \frac{D}{12}(m^3 - m) + \frac{1}{6}(m - 13m^3) + 2am$$

so that

$$A(m) = 0 \iff \left\{ \begin{matrix} D = 26 \\ a = 1 \end{matrix} \right\}$$

(as will become clear soon, this is deeply related to the conformal symmetry of the 2d-QFT and to the absence of a Weyl-rescaling-anomaly)

5.4 BRST quantization - general

(see advanced QFT texts for more details)

(→ Becchi, Rouet, Stora, Tyutin)

- let $Z = \int \mathcal{D}\phi_i \, e^{-S_\phi[\phi]}$ be the path integral for a gauge theory (in our case $\phi_i \to X^\mu(\sigma^a)$ and the index "i" stands for both "μ" and "σ^a".)

- Let the gauge conditions (labelled by A) be

$$F^A(\phi) = 0$$

(in our case this corresponds to $\mathrm{Tr}\, h^{ab} - \eta^{ab} = 0$)

- Then, up to overall factors, one can write

$$Z = \int \mathcal{D}\phi_i\, \mathcal{D}B_A\, \mathcal{D}b_A\, \mathcal{D}c^\alpha\, e^{-S_\phi[\phi] - S_{gf}[B,\phi] - S_g[b,c,\phi]}$$

where
$$S_{gf} = -iB_A F^A(\phi)$$
$$S_g = b_A\, c^\alpha \delta_\alpha F^A(\phi)$$

and α labels the generators of the group of gauge trfs.:

$$[\delta_\alpha, \delta_\beta] = f_{\alpha\beta}{}^\gamma \delta_\gamma .$$

(the ghosts c^α are labelled by the same index α; the ghosts b_A are labelled by the same index A as the gauge conditions F^A)

Note:
- Above, we have derived a simplified version of this were we had already carried out the B_A-integration to produce a δ-fct. and actually fix the gauge.
- In our previous calculation we have found just 2 ghosts (c^a with $a = 0, 1$) since we were able to explicitly perform the ω-integration, making β^{ab} and hence b^{ab} traceless.

- The full action is invariant under the trf. generated by
$$\delta \phi_i = -i \epsilon c^\alpha \delta_\alpha \phi_i$$
$$\delta B_A = 0$$
$$\delta b_A = \epsilon B_A$$
$$\delta c^\alpha = \frac{i}{2} \epsilon c^\beta c^\gamma f_{\beta\gamma}{}^\alpha \qquad (\epsilon - \text{infinitesimal}),$$
which can be seen as a generalized gauge trf. where the gauge parameter is chosen to be fermionic (c^α). This is the BRST trf.

- Let Q be the corresponding Noether charge (also known as "BRST charge" or Q_{BRST}).
- easy to check: $\delta(b_A F^A) = i\epsilon (S_{gf} + S_g)$
- Consider a variation of the gauge choice:
$$F^A \longrightarrow F'^A = F^A + \underbrace{\delta_g}_{} F^A$$
This has nothing to do with the BRST transformation $\delta = \delta_{BRST}$!

- correspondingly, let
$$S_{gf}(F'^A) + S_g(F'^A) - S_{gf}(F^A) - S_g(F^A) = \delta_g (S_{gf} + S_g)$$
- phys. amplitudes must be independent of the gauge choice, i.e.
$$\int_{\text{initial/final}} D\phi \, DB \, Db \, Dc \; e^{-(S_\phi + S_{gf} + S_g)}$$
should not change if $(S_{gf} + S_g)(F^A) \longrightarrow (S_{gf} + S_g)(F'^A)$.

- This implies $\int\limits_{\text{initial/final}} \mathcal{D}\phi \cdots \mathcal{D}c\, e^{-S}\, \delta_g (S_{gf} + S_g) = 0$.

- In operator language, one has
$$\langle \text{final} |\, \delta_g (S_{gf} + S_g) \,| \text{initial} \rangle = 0$$
$$\Rightarrow \langle \text{final} |\, \delta_g\, \delta_{BRST} (b_A F^A) \,| \text{initial} \rangle = 0 \,, \quad \text{with}$$
$$\delta_g \delta_{BRST} (b_A F^A) = \delta_{BRST} (b_A\, \delta_g F^A) \sim \{Q, b_A \delta_g F^A\}$$

- Thus, we need $\langle \text{final} | \{Q, b_A \delta_g F^A\} | \text{initial} \rangle = 0$ for all $\delta_g F^A$. This can be realized by demanding
$$\| \; Q\, |\text{phys}\rangle = 0 \; \| \,,$$
which we take as a definition of phys. states.

(Note: Since $Q^\dagger = Q$, $\langle \text{phys}| Q = 0$ follows from $Q |\text{phys}\rangle = 0$)

- We can check that $Q^2 = 0$
(e.g. $\delta_{\epsilon'} \delta_\epsilon b_A = \delta_{\epsilon'} (\epsilon B_A) = 0$ since $\delta_{\epsilon'} B_A = 0$, and similarly for the other fields).

- $\Rightarrow$ any state $Q|\chi\rangle$ is physical $(Q(Q|\chi\rangle) = Q^2 |\chi\rangle = 0)$; such states are also orthogonal to all phys. states:
$\langle \text{phys} | Q | \chi \rangle = 0$ since $\langle \text{phys} | Q = 0$.

$\Rightarrow$ states of the form $Q|\chi\rangle$ are "null".

- We conclude that the "true" (pos. norm) Hilbert space is $\mathcal{H}_{BRST} = \dfrac{\mathcal{H}_{closed}}{\mathcal{H}_{exact}}$ ($\equiv$ space of equivalence classes),

where $\mathcal{H}_{closed} = \{\text{all } |\psi\rangle \text{ with } Q|\psi\rangle = 0\}$

$\mathcal{H}_{exact} = \{\text{all } |\psi\rangle \text{ with } |\psi\rangle = Q|\chi\rangle \text{ for some } |\chi\rangle\}$.

Thus, $\mathcal{H}_{BRST}$ is the cohomology of Q. (Recall the cohomology of the exterior derivative d and consider $Q \leftrightarrow d$.)

5.5 BRST quantization of the bosonic string

- Noether theorem $\rightarrow$ Q explicitly given in terms of fields,
  $\rightarrow$ Q explicitly given in terms of oscillators.

$$Q = \sum_{-\infty}^{\infty} :\left(L_{-m}^{(\alpha)} + \frac{1}{2} L_{-m}^{(c)} - a\delta_m\right) c_m :$$

- One can check that, with $L_m = L_m^{(\alpha)} + L_m^{(c)} - a\delta_m$,

$$Q^2 = \frac{1}{2}\{Q, Q\} = \frac{1}{2} \sum_{m,n=-\infty}^{\infty} \underbrace{\left([L_m, L_n] - (m-n)L_{m+n}\right)}_{\sim \text{ anomaly of Virasoro alg.}} c_{-m} c_{-n}$$

$\Rightarrow$ $\boxed{\text{The BRST program, which requires } Q^2 = 0 \text{ also at the quantum level, forces us to demand } D = 26 \text{ and } a = 1.}$

- recall that $H = L_0$; L_0 now contains a contribution $L_0^{(c)}$; the latter reads:

$$L_0^{(c)} = \sum_{-\infty}^{\infty} (-n)\, b_n c_{-n} = \sum_{n=1}^{\infty} \left(n b_{-n} c_n + n c_{-n} b_n \right) + \cdots$$

This specifies that c_n, b_n $(n>0)$ should be viewed as annihilators (with b_{-n}, c_{-n} the corresponding creators).

- The above also motivates the definition of a <u>ghost number operator</u>:

$$U = \sum_{-\infty}^{\infty} : c_{-n} b_n : = \frac{1}{2}(c_0 b_0 - b_0 c_0) + \sum_{n=1}^{\infty} \left(c_{-n} b_n - b_{-n} c_n \right),$$

which counts the b_{-n} (c_{-n})-excitations with positive (negative) sign.

- <u>Note</u>: There is absolutely no way to determine which of c_0 and b_0 is the creator and which the annihilator (since they don't appear in H).

- <u>The c_0, b_0-subalgebra:</u> $c_0^2 = b_0^2 = 0$; $\{c_0, b_0\} = 1$

- <u>Its smallest representation:</u> 2 states: $|\uparrow\rangle$, $|\downarrow\rangle$ with

$$c_0 |\downarrow\rangle = |\uparrow\rangle, \quad b_0 |\uparrow\rangle = |\downarrow\rangle, \quad c_0 |\uparrow\rangle = b_0 |\downarrow\rangle = 0$$

furthermore: $U |\uparrow\rangle = \frac{1}{2} |\uparrow\rangle$; $U |\downarrow\rangle = -\frac{1}{2} |\downarrow\rangle$

(This corresponds to the (arbitrary) choice of an additive constant in the definition of U above.)

- Thus, we have <u>two</u> natural vacua:

  1) $|0,\uparrow\rangle$ – annihilated by all annih. operators & by c_0
  2) $|0,\downarrow\rangle$ – — " — & by b_0

- The <u>physically</u> correct choice is $|0,\downarrow\rangle$:

  – let $|\chi\rangle$ be a state built on $|0,\downarrow\rangle$ using α-creation ops. only (i.e. $c_n|\chi\rangle = b_n|\chi\rangle = 0$ for $n>0$)
  – then $Q|\chi\rangle = 0$ implies

$$0 = \sum_{-\infty}^{\infty} :\left(L_{-m}^{(\alpha)} + \tfrac{1}{2} L_{-m}^{(c)} - a\delta_m\right) c_m : |\chi\rangle$$

$$= \left[\left(L_0^{(\alpha)} - 1\right) c_0 + \sum_{m>0} c_{-m} L_m^{(\alpha)}\right] |\chi\rangle$$

  – this requires that $(L_0^{(\alpha)}-1)|\chi\rangle$ and $L_m^{(\alpha)}|\chi\rangle$ $(m>0)$ vanish, which agrees with our results in QCQ.

- <u>More generally</u>:
  States now include ghost excitations, e.g., at level 1:

$$|\psi\rangle = \left(\epsilon_\mu \alpha_{-1}^\mu + \beta b_{-1} + \gamma c_{-1}\right)|0,k,\downarrow\rangle$$

$Q|\psi\rangle = 0 \Rightarrow$ <u>mass shell condition</u>

(more specifically: $Q|\psi\rangle = 0$ & $b_0|\psi\rangle = 0$ imply $L_0|\psi\rangle = 0$) $\Rightarrow$

$$\alpha' m^2 = \alpha'(-k^2) = \sum_{n=1}^{\infty} n\left(c_n b_{-n} + b_n c_{-n} + \alpha_n^\mu \alpha_{-n,\mu}\right) - 1$$

$Q|\psi\rangle = 0 \implies$ further conditions:

$$k \cdot \varepsilon = 0 \quad \text{and} \quad \beta = 0$$

- The gauge freedom resides in excitations of type

$$k_\mu \cdot \alpha_{-1}^\mu \quad \text{and} \quad c_{-1} .$$

In summary:

Compared to OCQ, we have enlarged the Fock space by ghosts to be able to write a simpler phys.-state-condition: $Q|\psi\rangle = 0$. This (together with the equivalence relation) takes us back to the phys. Hilbert space of OCQ.

For an alternative (simpler!) proof of the no ghost theorem in this framework, see Polchinski's Book.