String Theory and String Phenomenology
Prof. Dr. Arthur Hebecker
Sebastian Kraus

Institute for Theoretical Physics
Heidelberg University
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Problem Sheet 5

(Due: November 21, 2012)

Problem 5.1:
Compute the anomaly term in the commutator

$$\left[J^{i-}, J^{j-}\right] = -\frac{1}{(p_+)^2} \sum_{m=1}^{\infty} \Delta_m \left(\alpha_{-m}^i \alpha_m^j - \alpha_{-m}^j \alpha_m^i\right),$$

where

$$\Delta_m = m\frac{26-D}{12} + \frac{1}{m}\left(\frac{D-26}{12} + 2(1-a)\right).$$

Problem 5.2 (*)**:**
Apply the Faddeev-Popov procedure to the electromagnetic field.

- Start from the partition function

$$Z = \int \mathcal{D}A \; e^{iS[A]}.$$

- Proceeding in analogy to the lecture and using the invariance of the action under the gauge transformations

$$A_\mu(x) \to A_\mu^\alpha(x) = A_\mu(x) + \partial_\mu \alpha(x),$$

  show that the partition function can be rewritten as

$$Z = \int \mathcal{D}\alpha \int \mathcal{D}A \det\left(\frac{\delta G(A)}{\delta\alpha}\right) e^{iS[A]} \delta\left(G(A)\right), \tag{1}$$

  where $G(A^\alpha)$ is some function chosen such that setting $G(A)$ to zero is the gauge fixing condition.

- Determine $G(A^\alpha)$ for the Lorentz gauge.

- Show that $\det\left(\frac{\delta G(A^\alpha)}{\delta\alpha}\right)$ is independent of A and thus only contributes an unimportant normalization factor to (1). (Note that for a non-abelian gauge theory this is not true anymore.)

- Now choose $G(A) = \partial^\mu A_\mu(x) - \omega(x)$ which gives a generalization of the Lorentz gauge condition.

- As the partition function (1) is independent of the choice of $\omega(x)$, we can equivalently integrate over all different $\omega(x)$ with some weighting function which we can choose to be a Gaussian

$$N(\xi) \int \mathcal{D}\omega \exp\left[-i \int \mathrm{d}^4x \frac{\omega^2}{2\xi}\right] \ldots$$

  Perform this integral and show that, effectively, in this procedure we have added a term $-\left(\partial^\mu A_\mu\right)^2/(2\xi)$ to the Lagrangian.