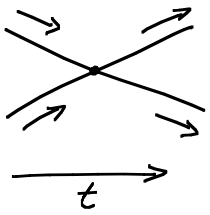
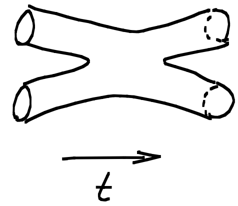
String theory & string phenomenology

1 Introduction / motivation

point-particles scatter:



strings scatter:



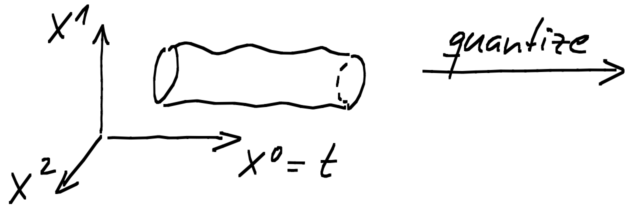
(no need to specify a "vertex")

- This removes UV divergence [It is automatically cut off at scale $M_s \sim 1/\ell_s$, where ℓ_s is the typical length of a string.]

- In particular, UV-div. of graviton-scattering is removed

  $\Rightarrow$ Model of quantum gravity with by far the best quantitative control [compared e.g. to LQG; triangulations; asympt. safety etc.].

- Quantization of single string:



Propagation of discrete set of quantum states ("vibration modes") with calculable masses, spins, interactions

- At low energies, only the lightest (e.g. massless) modes are relevant

- They play the role of the particles in a (very specific) QFT, including gravity!

- Bosonic string: Need $d = 26$ (and still, the vacuum of the 26d-QFT is unstable)

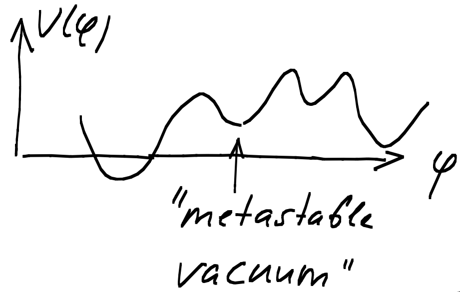
- Superstring: Need $d = 10$ (more or less unique 10d QFT: "10 supergravity")

- After "Compactification" (i.e. $\mathcal{M}_{10} \equiv \mathbb{R}_{1,3} \times \mathcal{M}_6$, with $\mathcal{M}_6$ a "compact" manifold) one can get <u>many</u> 4d effective theories.

  $\Rightarrow$ "landscape of string theory vacua" (or "string theory landscape")

  [<u>Ex. 1</u>: Many crystal structures emerging from a certain set of atoms and a unique theory of QM.

  <u>Ex. 2</u>: QFT with real scalar φ and complicated $V(\varphi)$:



  ]

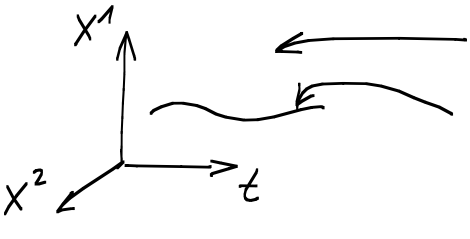
- However: Fundamental problems remain unsolved (Quantitative understanding of the "landscape"; How is it "populated"?; How to make predictions?)

- Nevertheless: The above logic makes ST the prime candidate for a "TOE" (theory of everything").

- Independent motivation: ST is an essential tool for the study of quantum gravity and QFT. [Because it provides a consistent "physical" regulator in pert. theory <u>and</u> in certain cases, it provides a non-pert. definition of certain QFTs via the "AdS/CFT correspondence". This includes 3d theories relevant for condensed matter physics ("AdS/CMT").]

2 Classical bosonic string

2.1 Relativistic point particle



"target space" $\mathcal{M}$ parametrized by X^μ $(\mu = 0, \dots, D-1)$

"worldline" γ parametrized by τ

- Embedding of γ in $\mathcal{M}$ specified by set of fcts. $X^\mu(\tau)$

- $S_{NG} = -m\int_\gamma ds$ with $ds^2 = -\eta_{\mu\nu}\, dX^\mu dX^\nu$; $\hbar = c = 1$

  "Nambu-Goto"

  i.e., we focus on $\mathcal{M} = \mathbb{R}_{1,D-1}$

- With $dX^\mu = \dot{X}^\mu d\tau$ we have

$$S_{NG} = -m\int d\tau \sqrt{-\eta_{\mu\nu}\dot{X}^\mu \dot{X}^\nu}$$

- S_{NG} is <u>by definition</u> invariant under reparametrizations, $\tau \to \tau'(\tau)$, with arbitrary fcts τ' [nevertheless useful to check this explicitly using the 2nd form of S_{NG}].

- Choosing $\tau \equiv \{\text{proper time of } \gamma\} \equiv s$, one easily derives the EOM $\ddot{X}^\mu = 0$ from $\delta S_{NG} = 0$ [$\to$ problems].

- Non-relativistic limit: $S_{NG} \simeq \int dt \left(\frac{m}{2}\bar{v}^2 - m\right)$ [$\to$ problems]

- Recall the general notion of a manifold with metric (coordinates y^a; $a = 1\cdots n$; $ds^2 = g_{ab}\,dy^a dy^b$). Crucial: The integral $\int d^n y \sqrt{-\det(g_{ab})}\, f(y) = \int d^n y \sqrt{-g}\, f$ is reparametrization invariant (diff.-invariant) if f is scalar fct. For $f = 1$, we get the volume.

- It is convenient to treat γ as such a manifold, with a <u>new</u> metric degree-of-freedom h: $ds_\gamma^2 = h_{\tau\tau}\, d\tau^2$.

- A "natural" action is then

$$S_P = -\frac{m}{2}\int d\tau \sqrt{-h}\left(\underbrace{h^{\tau\tau}\frac{dX^\mu}{d\tau}\frac{dX_\mu}{d\tau}}_{\text{1-dim. analogue of } g^{ab}\frac{\partial X^\mu}{\partial y^a}\frac{\partial X_\mu}{\partial y^b}} + 1\right) = S_P[X,h]$$

"Polyakov"

(Note: $h^{\tau\tau} = h^{-1}_{\tau\tau}$; $h = h_{\tau\tau}$)

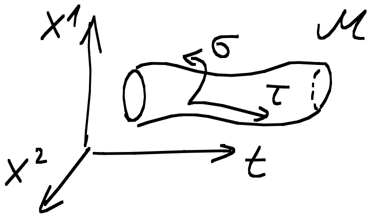
- $\frac{\delta S_P}{\delta h} = 0 \Rightarrow h_{\tau\tau} = \dot{X}^\mu \dot{X}_\mu$ (EOM for h)

- Easy to check: $S_P[X, h_{\tau\tau} = \dot{X}^2] = S_{NG}[X]$ [→ problems]

- S_P & S_{NG} are classically equivalent. S_P has the crucial advantage of being polynomial in $X(\tau)$ (no root!).

[For much more on this, especially the quantization of the point particle, see Zwiebach's book.]

2.2 Bosonic string



The embedding of the WS Σ (parametrized by τ, σ) is specified by fcts. $X^\mu(\tau,\sigma)$.

$$S_{NG} = -\underbrace{T}_{\text{string tension}} \underbrace{\int_\Sigma dA}_{\text{area of } \Sigma \text{ as a submanifold of } \mathcal{M}}$$

- To write this out more explicitly, let $(\tau, \sigma) \equiv (\xi^0, \xi^1) \equiv \xi$. An infinites. vector $d\xi$ in Σ induces an infinites. dX in $\mathcal{M}$, with length-square
$$ds^2 = \eta_{\mu\nu} dX^\mu dX^\nu = \eta_{\mu\nu} \left(\frac{\partial X^\mu}{\partial \xi^a} d\xi^a\right)\left(\frac{\partial X^\nu}{\partial \xi^b} d\xi^b\right)$$
$$= \underbrace{\eta_{\mu\nu} \partial_a X^\mu \partial_b X^\nu}_{\text{"induced metric" on } \Sigma} d\xi^a d\xi^b \equiv G_{ab} d\xi^a d\xi^b$$

$$\Rightarrow S_{NG} = -T \int_\Sigma d^2\xi \sqrt{-G} \qquad (G \equiv \det(G_{ab}))$$

- In analogy to our discussion of point-particles, we can introduce, as a new degree of freedom a "World sheet (WS) metric" h_{ab} and propose:

$$S_P = -\frac{T}{2}\int_\Sigma d^2\xi \sqrt{-h}\; h^{ab}\,\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

[Note the absence of a constant term $\sim \int d^2\xi \sqrt{-h}\cdot 1$. This will become more clear in the problems.]

- The classical equivalence of S_P & S_{NG} follows as before:

Vary w.r.t. h: $\quad 0 \stackrel{!}{=} \delta_h\{\sqrt{-h}\, h^{ab} G_{ab}\} = -\frac{\delta h}{2\sqrt{-h}}\, h^{ab} G_{ab} + \sqrt{-h}\, \delta h^{ab} G_{ab}$

Fact: (see problems for proof)

For a generic matrix A, $\quad \delta \det A = (\det A)\, \mathrm{tr}\,(A^{-1}\delta A)$

$$\delta h = h\, h^{ab}\delta h_{ab} = -h\, h_{ab}\,\delta h^{ab} \qquad (\text{since } \delta(h^{ab}h_{bc}) = 0)$$

$$\Rightarrow\ 0 \stackrel{!}{=} \delta h^{ab}\left[\frac{h}{2\sqrt{-h}}\, h_{ab}\, h^{cd} G_{cd} + \sqrt{-h}\, G_{ab}\right]$$

$$\frac{h}{\sqrt{-h}} = -\sqrt{-h} \quad \Rightarrow \quad \boxed{\frac{1}{2} h_{ab}\, h^{cd} G_{cd} = G_{ab}} \quad \text{EOM for } h$$

$$\Rightarrow\ h_{ab} = \alpha G_{ab} \quad (\text{any } \alpha\,!)$$

$$S_P = -\frac{T}{2}\int d^2\xi \sqrt{-h}\; h^{cd} G_{cd} = -\frac{T}{2}\int d^2\xi \sqrt{-\alpha^2 G}\; 2\alpha^{-1} = S_{NG}$$

✓

2.3 Symmetries & EOM

It will be convenient to work with S_P and view it as a QFT on a 2d-space-time with metric h and D scalar fields X^μ:

$$S_P = -\frac{T}{2}\int d^2\xi \sqrt{-h}\,(\partial X)^2 \quad ; \quad (\partial X)^2 \equiv h^{ab}\,\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

Symmetries:

1) Diffeomorphisms: $\xi^a \to \xi'^a(\xi^0,\xi^1)$
2) D-dim. Poincaré-invariance (as an "internal symmetry of our 2d QFT):

$$X^\mu \to X'^\mu = \Lambda^\mu{}_\nu X^\nu + V^\mu \; ; \quad \Lambda \in SO(1,D-1)$$

3) Weyl-rescalings: $h_{ab}(\xi) \to h'_{ab}(\xi) = \underbrace{\varphi(\xi)}_{\text{arbitrary scalar fct.}} h_{ab}(\xi)$

[1) & 2) also hold generic "p-branes" with world-volumes parameterized by $\xi^0, \xi^1, \dots, \xi^p$. Here $p=0$ is the point-particle; $p=1$ the string; for $p \geq 2$ see problems & later. 3) holds only for $p=1$.]

- Recall that the "energy-momentum tensor" T^{MN} is an important object in QFT & GR on a space parameterized by coordinates x^M:

$$T^{MN} \equiv \frac{2}{\sqrt{-g}} \cdot \frac{\delta S}{\delta g_{MN}} \qquad \left(\text{or } T_{MN} \equiv \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{MN}} \right)$$

[$T_{MN} = \text{diag}(\rho, p, \dots, p)$ for isotropic fluid.]

- Here: $T^{ab} \equiv \frac{-4\pi}{\sqrt{-h}} \cdot \frac{\delta S_P}{\delta h_{ab}} = \frac{-4\pi}{\sqrt{-h}} \cdot \frac{\partial \mathcal{L}_P}{\partial h_{ab}}$ ("stringy" normalization convention)

- Explicitly: $T^{ab} = -2\pi T \left(G^{ab} - \frac{1}{2} h^{ab} h^{cd} G_{cd} \right)$ [$\Rightarrow$ problems]

$$= -\frac{1}{\alpha'} \left(G^{ab} - \frac{1}{2} h^{ab} h^{cd} G_{cd} \right)$$

[$\alpha' \equiv \frac{1}{2\pi T}$ is the "Regge slope". This name goes back to ST as a model of hadronic physics. $\sqrt{\alpha'} \sim l_s$ (conventions for the factor of proportionality vary.]

- EOM for h $\hat{=}$ stationarity of S_P w.r.t. h $\hat{=}$ $\boxed{T^{ab} = 0}$

- Note: $T^a{}_a = 0$ holds as an identity (without using EOM).
  (Problem: Derive this from the symmetries of the action!)

- As in GR, diff.-invariance implies $D_a T^{ab} = 0$
  $\uparrow$ covariant derivative

- EOMs for X are as in QFT.

$$\text{Summary:} \quad \underbrace{T^{ab} = 0}_{h\text{-EOM}} \;;\; \underbrace{\Box X^\mu = 0}_{X\text{-EOM}} \qquad (\Box = D^a \partial_a)$$

2.4 Gauge choice

- Coord. choice on Σ & Weyl rescalings of h_{ab} are redundancies introduced in the Polyakov formulation. They do not affect the "physical" embedding of Σ into $\mathcal{M}$. We thus declare them to be gauge symmetries (i.e., different choices describe the same physics).

- Crucial claim: (at least locally) on Σ we can use Diff. & Weyl to realize
$$h_{ab} = \mathrm{diag}(-1,1) \qquad [\text{flat gauge}]$$

- Indeed:
$$\left.\begin{array}{ll} \text{Diff:} & \xi^a \to \xi^a(\xi^1,\xi^2) \\ \text{Weyl:} & h_{ab} \to \exp(2\omega(\xi^1,\xi^2)) \cdot h_{ab} \end{array}\right\} \text{ Can choose 3 arbitrary fcts.}$$

  Now, since h_{ab} only contains 3 indep. fcts. (h_{11}, h_{22}, $h_{12}=h_{21}$), we expect that we can use Diff. + Weyl to bring h to <u>any</u> fixed form.

<u>More explicitly:</u>

- Consider the "2d Einstein-Hilbert-term" $\sqrt{-h}\,R[h]$ (We do not - yet - add such a term to $\mathcal{L}_P$, but is useful to consider it here.)

<u>Aside:</u> $D_\mu v_\nu \equiv \partial_\mu v_\nu - \Gamma_{\mu\nu}{}^\rho v_\rho \;;\; \Gamma_{\mu\nu}{}^\rho \equiv \frac{1}{2} g^{\rho\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu})$

$$R_{\mu\nu\rho}{}^{\sigma} v_\sigma \equiv [D_\mu, D_\nu] v_\rho \quad ; \quad R_{\mu\rho} \equiv R_{\mu\nu\rho\sigma} g^{\nu\sigma} \quad ; \quad R \equiv R_{\mu\nu} g^{\mu\nu}$$

Riemann tensor — Ricci tensor — Ricci scalar

Now just let $g_{\mu\nu} \to h_{ab}$ etc. and set $d = 2$

- A straightforward calculation shows: If $h'_{ab} = e^{2\omega} h_{ab}$, then

$$\sqrt{-h'}\, R[h'] = \sqrt{-h}\left(R[h] - 2D^2\omega\right).$$

- Thus, we solve $2D^2\omega = R$ for ω, Weyl rescal h with ω, and obtain a new metric with vanishing Ricci scalar.
  [This is just like $D^2\varphi = \rho$ – the curved-space analogue of the (Poisson) equation for the electrostatic potential. On physical grounds we thus always expect to find a solution on a topologically trivial patch. Here, we actually need it for an infinite cylinder, where it is less obvious but still OK.]

- Fact ($\to$ problems): In $d = 2$ we have $R_{abcd} = \frac{1}{2}(h_{ab}h_{cd} - h_{ad}h_{bc}) \cdot R$

- Thus, we are in 2d flat space. $\Rightarrow$ locally, we can always find coordinates such that $h_{ab} = \text{diag}(-1, 1)$ (cf. various GR texts).
  [We actually again need this for the cylinder...]

From now on: Assume $h_{ab} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ by gauge choice.

$$\text{EOM:} \quad (\partial_\tau^2 - \partial_\sigma^2) X^\mu = 0$$

[Klein-Gordon, i.e. X^μ are free scalar fields]

- Use light-cone coordinates:

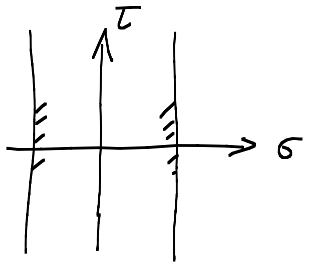
$$\sigma^\pm = \tau \pm \sigma \quad ; \quad ds^2 = -d\tau^2 + d\sigma^2 = -d\sigma^+ d\sigma^-$$

$$\Rightarrow h_{++} = h_{--} = 0 \;;\; h_{+-} = -\tfrac{1}{2} \;;\; h^{+-} = -2$$

$$\Box = h^{ab}\partial_a\partial_b = 2h^{+-}\partial_+\partial_- = -4\partial_+\partial_- \qquad \left[\partial_\pm \equiv \frac{\partial}{\partial\sigma^\pm}\right]$$

- EOM: $\partial_+ \partial_- X^\mu = 0$; general solution: $X^\mu = X_L^\mu(\sigma^+) + X_R^\mu(\sigma^-)$

- In addition: $X^\mu(\tau,\sigma) = X^\mu(\tau, \sigma+\pi)$



Since our theory is diff. invariant, we can choose this "width" of our cylinder.
[We follow GSW & BBS ; $\pi \to 2\pi$ is also common.]

- X^μ periodic in σ $\Rightarrow$ $\partial_\pm X^\mu$ periodic in σ $\Rightarrow$ $\partial_+ X_L^\mu$ and $\partial_- X_R^\mu$ periodic in σ

$\Rightarrow$ $\partial_+ X_L^\mu(\sigma^+) = \partial_+ X_L^\mu(\sigma^+ + \pi)$; $\partial_- X_R^\mu(\sigma^-) = \partial_- X_R^\mu(\sigma^- - \pi)$

$\Rightarrow$ $\partial_+ X_L^\mu$ & $\partial_- X_R^\mu$ can be decomposed in const. + $\Sigma\, e^{2in\sigma^\pm}$

$\Rightarrow$ X_L^μ & X_R^μ contain, in addition to exponentials, a linear term

$\Rightarrow$ <u>General solution:</u>

$$X_L^\mu = \frac{1}{2} x^\mu + \frac{\ell^2}{2} p^\mu \sigma^+ + \frac{i\ell}{2} \sum_{n\neq 0} \frac{1}{n} \tilde{\alpha}_n^\mu e^{-2in\sigma^+}$$

$$X_R^\mu = \frac{1}{2} x^\mu + \frac{\ell^2}{2} p^\mu \sigma^- + \frac{i\ell}{2} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-2in\sigma^-}$$

same coeff. by convention — same coeff. by periodicity of X^μ.

[$\ell \equiv \sqrt{2\alpha'} \equiv 1/\sqrt{\pi T}$ is the "string length" ; α' is the "Regge slope"]

Historical name, from "ST as a model for strong interactions".

- Reality $\Rightarrow$ x^μ, p^μ real ; $(\alpha_n^\mu)^* = \alpha_{-n}^\mu$ (same for $\tilde{\alpha}_n^\mu$)

- We have $X^\mu = \underbrace{x^\mu + \ell^2 p^\mu \tau}_{\text{linear motion of center-of-mass in target space}} + \underbrace{\cdots}_{\text{fluctuations}}$

linear motion of center-of-mass in target space

fluctuations, corresponding to left-moving (X_L) & right-moving (X_R) waves on a circle.