SMEFT beyond Leading Order and Pseudo Observables

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Outline

- Motivation: New Physics searches $\rightarrow$ Model independent searches?
- SMEFT beyond LO:
  - NLO SMEFT: renormalization
  - Sizeable QCD-EFT corrections
  - Theoretical Uncertainties
  - EFT in the experiment: 2 examples
- Pseudo Observables:
  - LEP example: wise and not-so-wise choices
  - IR behaviour of the SMEFT
  - Systematic approach to POs: Multi Pole Expansion (MPE)
  - POs for LHC

Motivation: New Physcis Searches

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The SM has been now *completed*

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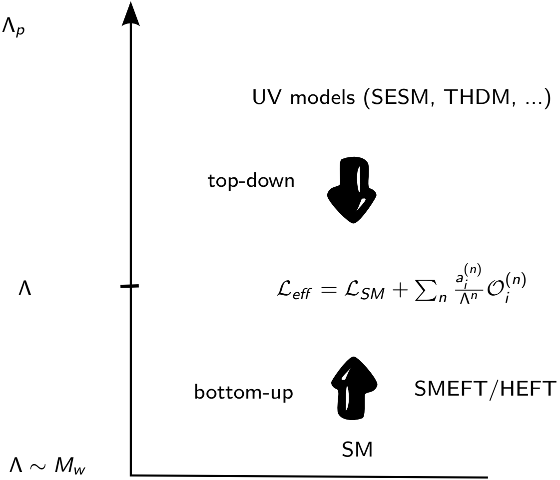
Solutions?

- We are far from a satisfactory, unique, solution.
- A proposal: Pseudo-Observables

Part I:

Effective Field Theories: the SMEFT at NLO


Λ_p
UV models (SESM, THDM, ...)
top-down
Λ
$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_n \frac{a_i^{(n)}}{\Lambda^n} \mathcal{O}_i^{(n)}$
bottom-up
SMEFT/HEFT
SM
$\Lambda \sim M_w$

A bottom approach: the SMEFT

Any Lagrangian for SMEFT can be written as:

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{a_5}{\Lambda}\mathcal{O}^{(5)} + \sum_i \frac{a_i}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j \frac{a_j}{\Lambda^4}\mathcal{O}_j^{(8)} + \dots$$

with some assumptions,

- Linear representation for the Higgs → Alternatively, use HEFT
- UV completion decouples at low energies → Alternatively use the top-down
- The a_i are arbitrary Wilson coefficients → basis dependent
- Assume SM symmetries → no CP violation, lepton/baryon number conservation etc.

Choice of the Dimension-6 basis

Warsaw Basis *(arXiv: 1008.4884)*

- First complete non-redundant basis in the literature
- Contains 59 operators: 76(2499) free parameters for $n_g = 1(3)$.
- Renormalization Group and 1-loop finite renormalization are known *(Alonso, Jenkins, Manohar, Trott, Ghezzi, RGA, Passarino, Uciratti, et al.)*

Alternatively:

- SILH basis
  (Giudice, Grojean, Pomarol, Rattazzi, hep-ph/0703164 ;
  Elias-Miro, Grojean, Gupta, Marzocca, 1312.2928)

Both bases are phenomenologically interesting, but they contain different parameters. Is there a way to make experimental measurements basis-independent?

The SMEFT renormalization is performed analogously to the SM one:

$$\underbrace{\{p_0\}}_{\text{bare}} = Z_{\{p\}} \underbrace{\{p\}}_{\text{ren.}}, \qquad \underbrace{\{\Phi_0\}}_{\text{bare}} = Z_{\{\Phi\}}^{1/2} \underbrace{\{\Phi\}}_{\text{ren.}} \tag{1}$$

with counterterms,

$$Z_i = 1 + \frac{g^2}{16\pi^2} \left(dZ_i^{(4)} + g_6 dZ_i^{(6)} \right) \Delta_{UV} \tag{2}$$

- For the SMEFT, we use on-shell renormalization for the SM parameters, and $\overline{MS}$ renormalization for the Wilson Coefficients.
- **Caveat:** The $\overline{MS}$ is a non-physical renormalization scheme and the Appelquist-Carazzone decoupling theorem does not hold any more, i.e. it has to be enforced using matching conditions.

When the physical quantities are known (i.e. there is a subtraction point), we can avoid using $\overline{MS}$ and use a finite renormalization scheme,

Finite renormalization:

- G_F renormalization scheme: The input parameter set is $\{G_F, \mathrm{M}_w, \mathrm{M}_z\}$

$$g_{\text{ren.}} = g_{\text{exp.}} + \frac{g^2_{\text{exp}}}{16\pi^2}\left(\mathrm{d}\mathcal{Z}^{(4)}_{\mathrm{g}} + \mathrm{g}_6 \mathrm{d}\mathcal{Z}^{(6)}_{\mathrm{g}}\right)$$

- α renormalization scheme: The input parameter set is $\{\alpha, G_F, \mathrm{M}_z\}$

$$g^2 s^2_\theta = 4\pi\alpha\left[1 - \frac{\alpha}{4\pi}\frac{\Pi_{AA}(0)}{s^2_\theta}\right]$$

Different choices of IPS lead to different predictions (already at tree level → (Brivio,Trott 1701.06424)) → important for the design of experimental fits

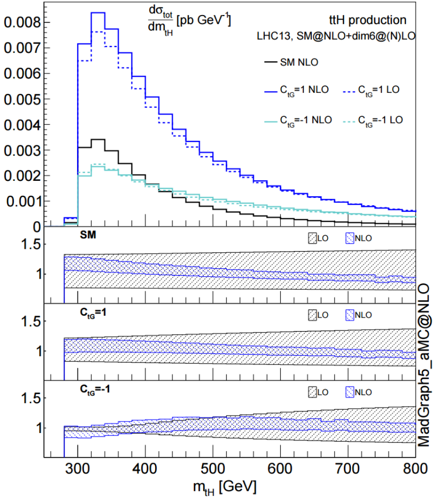
The one-loop structure of the theory is not predictable from the LO nor the RG

- NLO SMEFT might not lead to significant corrections, but has some important conceptual aspects that have to be understood. Mainly,
- When renormalizing 3-point functions, new relations between Wilson Coefficients appear,

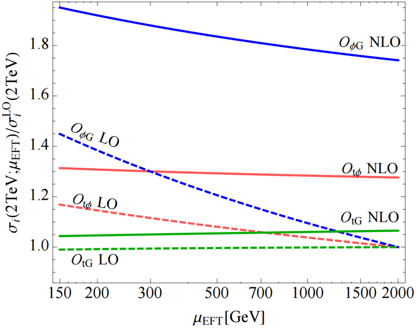
$$C_i = \sum_j Z^W_{ij} C^{ren.}_j, \qquad Z^W_{ij} = \delta_{ij} + \frac{g^2}{16\pi^2} dZ^W_{ij} \Delta_{UV}$$

- When designing the strategy for global fits for EFTs at LHC, non-trival dependencies between the Wilson coefficients should be taken into account.
- Additionally, new behaviours not encoded in the RG appear when doing full one-loop calculations, that may be sizeable.
- For ex. $h \to \gamma\gamma$ (Hartmann, Trott, 1505.02646; Ghezzi et al. 1505.03706)

In 1607.05330 (Maltoni,Vryonidou,Zhang), NLO-EFT corrections to $t\bar{t}h$ are presented,



- K factors for inclusive $\sigma \rightarrow K = 1. - 1.6$



Theoretical Uncertainties coming from EFT-MHO

The common argument:

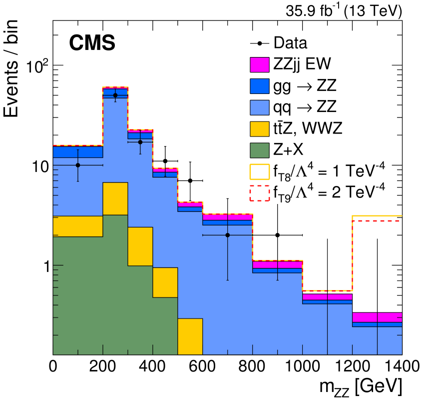
$$|\mathcal{A}_{\rm EFT}|^2 = |\mathcal{A}_{SM}|^2 + \underbrace{|\mathcal{A}_{\rm SM} \times \mathcal{A}_6^{(1)}|}_{\text{“linear EFT” } (1/\Lambda^2)} + \underbrace{|\mathcal{A}_6^{(1)}|^2}_{\text{“quadratic EFT”}} + \underbrace{|\mathcal{A}_{\rm SM} \times \mathcal{A}_6^{(2)}| + |\mathcal{A}_{\rm SM} \times \mathcal{A}_8^{(1)}|}_{\text{not available (th.uncertainty)}}$$

Contributions of order $1/\Lambda^4$

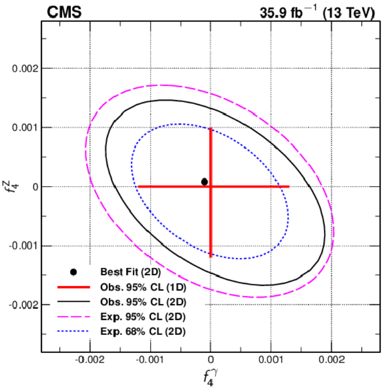
1. Quadratic terms in $|\mathcal{A}_{EFT}|^2$
2. For a $2 \to 2$ process: double $\dim = 6$ insertions at tree level
3. Loops with $\dim = 6$ vertices $\to$ NLO-SMEFT
4. Interference of $\dim = 8$ and SM: $|\mathcal{A}_{SM} \times \mathcal{A}_8|$
5. Negelcted terms in the application of EoM when building the basis

EFT in the experiment: 2 examples ($pp \to ZZjj$ and $pp \to ZZ$ in CMS)

- aQGCs (CMS-SMP-17-006)



- aTGCs (CMS-SMP-16-017)



- EFT interpretation in dim-8 operators:

Coupling	Exp. lower	Exp. upper	Obs. lower	Obs. upper	Unitarity bound
f_{T0}/Λ^4	−0.53	0.51	−0.46	0.44	2.5
f_{T1}/Λ^4	−0.72	0.71	−0.61	0.61	2.3
f_{T2}/Λ^4	−1.4	1.4	−1.2	1.2	2.4
f_{T8}/Λ^4	−0.99	0.99	−0.84	0.84	2.8
f_{T9}/Λ^4	−2.1	2.1	−1.8	1.8	2.9

- No agreement on the EFT interpretation!

Part II:

Pseudo Observables

- Pseudo-Observables (PO) where born in the frame of LEP experiment, as opposed to RO (Realistic-Observables). Designed to have two main features:
  - to allow comparison between experiments (independent of detector cuts)
  - to be as independent as possible of changes in the underlying theory.
- They represented a useful storage solution
- However there are fundamental differences between LEP and LHC . . .

POs at LEP

De-convolute the initial state QED radiation[1],

$$\sigma(s) = \int_0^{1-x_{cut}} \mathrm{dx}\ \underbrace{H(x,s)}_{\text{radiator}}\ \underbrace{\sigma_0((1-x)s)}_{\text{deconvoluted}}$$

- Around the Z peak, σ_0 only contains the Z resonant part of the amplitude.
- After we subtract the real emission and the non-resonant part from the process,

$$\sigma_{\bar{f}f}(s) = \sigma_0^{\bar{f}f} \frac{s^2\Gamma_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}, \qquad \sigma_0^{\bar{f}f} = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}$$

- The partial and total Z widths were defined as PO (i.e. $\Gamma_Z, \Gamma_f, \Gamma_e$)
- Other POs: forward-backward asymmetries, polarizations, EWPD ...
- LEP POs still today put strong constraints on BSM models

[1] *Assuming you can also do that in the experiment ...*

POs for LHC: motivation

- The POs can be obtained from the fiducial cross sections, by deconvoluting effects such as parton distribution functions and radiative corrections.
- Theory upgrades can be applied at the level of fiducial quantities, rather than starting from raw data.
- POs, being well defined objects from the theoretical point of view, can be then interpreted in terms of Wilson coefficients or Lagrangian parameters of some UV complete theory.
- It would be much more handy for model builders to interpret the nature of NP and parameters of the specific models from POs, rather than trying to extract them directly from fiducial or template cross sections.

Why are PO necessary? It's all about gauge invariance ...

Equivalence theorem (1972, Kallosh-Tyutin, and t 'Hooft-Veltman)

"In a renormalizable theory, a reparametrization of the fields leaves renormalized quantities invariant." This theorem is fundamental on proving the SM renormalizability, since gauge invariance is a particular case of field reparametrization.

- Effective Lagrangians are interesting and easy to use (straightforward to extract Feynman Rules, couplings etc.)
- But their parameters are not gauge invariant $\rightarrow$ by a field redefinition we can add or remove different interactions (i.e. by a different choice of basis)
- How can predictions from different effective Lagrangians be measured and compared by experiments then? $\rightarrow$ Looking at basis-independent quantities

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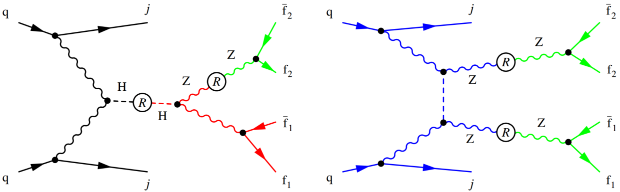
Gauge invariant quantities:

- (renormalized) S-Matrix elements
- Residues of the poles (Nielsen Identities)

The pole structure of the theory is agnostic to the basis chosen to parametrize the new physics effects $\to$ residues of the poles can be interpreted as POs (as long as they are inside the physical region)



- Two possible pole expansions: single-resonant and doubly-resonant

$$\sigma(qq \to \bar{f}_1 f_1 \bar{f}_2 f_2 jj) \overset{PO}{\longmapsto} \sigma(qq \to hjj)\,\mathrm{Br}(h \to Z\bar{f}_1 f_1)\,\mathrm{Br}(Z \to \bar{f}_2 f_2)$$

$$\sigma(qq \to \bar{f}_1 f_1 \bar{f}_2 f_2 jj) \overset{PO}{\longmapsto} \sigma(qq \to ZZjj)\,\mathrm{Br}(Z \to \bar{f}_1 f_1)\,\mathrm{Br}(Z \to \bar{f}_2 f_2)$$

A. David, G. Passarino 1510.00414, set of POs for Higgs production/decay: *Gonzalez-Alonso, Greljo, Isidori, Marzocca (1412.6038); Greljo, Isidori, Lindert, Marzocca (1512.06135)*

Interesting application: CP violation in $hf\bar{f}$

- Write the tree level amplitude decomposed in CP conserving and violating parts,

$$\mathcal{A}(h \to f\bar{f}) = -\frac{i}{\sqrt{2}}(y_S^f \bar{f} f + i y_P^f \bar{f}\gamma_5 f)$$

- The coefficients y_S^f and y_P^f are PO to be measured experimentally.
- In the well known κ framework as:

$$\kappa_f = \frac{y_S^f}{y_{S,SM}^f} \quad \delta_f^{CP} = \frac{y_P^f}{y_{S,SM}^f}$$

- Measurement of the total rate of Higgs BR to fermions, would not allow to differentiate between the two contributions,

$$\Gamma(h \to f\bar{f}) = \left[{\kappa_f}^2 + \left(\delta_f^{CP}\right)^2\right] \Gamma(h \to f\bar{f})^{SM}$$

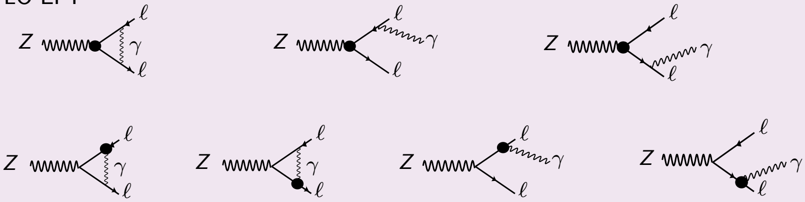
- To access separately the information about the CP violating part, the spins of the fermions need to be determined, e.g. by the measurements of angular distributions
- OBS. The CP violating part is not present in the EFT formalism !

POs: Caveats

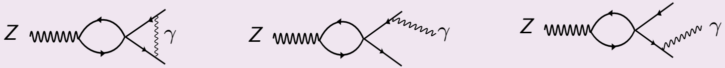
- PDFs $\rightarrow$ main difference wrt LEP
- Certain model dependency $\rightarrow$ in LEP this was no problem (not searching for new physics)
- Momentum expansion around physical poles is valid only in limited kinematic regions $\rightarrow$ specially problematic for production (the Higgs is not necessarily at threshold)

Example: Z decay to two charged leptons: Z $\rightarrow \ell\ell(\gamma)$

LO EFT



NLO EFT



Is it possible to use (deconvoluted) LEP data for SMEFT fits? What do we learn from this?

Conclusions and Future prospects

- NLO EFT $\rightarrow$ unveils interesting aspects of the underlying QFT structure
  $\rightarrow$ relevant for theoretical uncertainties
- A framework is needed where different EFT approaches: SMEFT/HEFT, LO/NLO, Warsaw/SILH, Top-down/bottom-up . . . can equally benefit from experimental measurements
- In that regard the PO approach is a satisfactory idea
- However it has several caveats that still have to be addressed before it's ready for LHC:
  - Model dependence introduce by the deconvolution
  - Validity of the MPE expansion in the interesting kinematic regions

Thank you!


THANK YOU FOR YOUR ATTENTION!
ANY QUESTIONS?
memegenerator.net

Additional Slides

Warsaw Basis. Bosonic Sector

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Warsaw Basis. Fermionic Sector

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

Amplitude for a $1 \to 2$ process

$$\mathcal{A}_{\mathrm{SMEFT}}(1 \to 2) = \sum_{n=N}^{\infty} g^n \mathcal{A}_n^{(4)} + \sum_{n=N_6}^{\infty} \sum_{l=0}^{n} \sum_{k=l}^{\infty} g^n g_{4+2k}^{l} \mathcal{A}_{nlk}^{(4+2k)}, \tag{3}$$

where g is the SU(2) coupling constant and we define $g_{4+2k} = 1/(\sqrt{2} G_F \Lambda^2)^k$.

EFT couplings

$$g_6 = \frac{1}{\sqrt{2} G_F \Lambda^2} \qquad g_6 = 0.0606 \left(\frac{\mathrm{TeV}}{\Lambda}\right)^2 \lesssim 1 \quad g_8 = \frac{1}{2 G_F^2 \Lambda^4} \equiv g_6^2 \qquad g_8 << 1 \tag{4}$$

What is a reasonable value for Λ? Usually we take $\Lambda \approx 1 - 2\mathrm{TeV}$

SMEFT amplitudes can be used as a tool to study the validity regime of the EFT perturbative expansion

(1 → 2 process)				
Higher dim. →				
Higher order ↓	$g\mathcal{A}_1^{(4)}$	$gg_6\mathcal{A}_{1,1,1}^{(6)}$	$gg_8\mathcal{A}_{1,1,2}^{(8)}$	...
	$g^3\mathcal{A}_3^{(4)}$	$g^3 g_6\mathcal{A}_{3,1,1}^{(6)}$	$g^3 g_8\mathcal{A}_{3,2,1}^{(8)}$	...
	...	...	...	...

The leading order for an EFT amplitude is unambiguous: $\mathcal{A} = \mathcal{A}_{\mathrm{SM}} + g_6\mathcal{A}_6^{(1)}$ where $\mathcal{A}_6^{(1)}$ has only one dimension-6 operator. When adding higher orders in PT to $\mathcal{A}$ we can use the following hierarchy:

$$\mathcal{A}_{\mathrm{EFT}} = \mathcal{A}_{SM} + \underbrace{g_6\mathcal{A}_6^{(1)}}_{\text{LO EFT}} + \underbrace{g_6^2\mathcal{A}_6^{(1)} + g_8\mathcal{A}_8^{(1)}}_{\text{NLO EFT}} + \ldots \tag{5}$$

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Example: Z decay to two charged leptons: $Z \to \ell\ell$

- After UV renormalization the LO amplitude for $Z \to \ell\ell$ is,

$$\mathcal{A}_\mu = g\,\mathcal{A}_\mu^{(4)} + g g_6\,\mathcal{A}_\mu^{(6)} \tag{6}$$

- The virtual and real contributions cancel exactly:

$$\Gamma(Z \to \bar{l}l)\big|_{\rm div} = -\frac{g^4}{384\pi^3} M_Z s_\omega^2 \mathcal{F}^{\rm virt}\left[\Gamma_0^{(4)}\left(1 + g_6\,\Delta\Gamma\right) + g_6\,\Gamma_0^{(6)}\right], \tag{7}$$

$$\Gamma(Z \to \bar{l}l\gamma)\big|_{\rm div} = \frac{g^4}{384\pi^3} M_Z s_\omega^2 \mathcal{F}^{\rm real}\left[\Gamma_0^{(4)}\left(1 + g_6\,\Delta\Gamma\right) + g_6\,\Gamma_0^{(6)}\right] \tag{8}$$

- Leading to an IR-safe final expression:

$$\Gamma^1_{\rm QED} = \frac{3\alpha}{4\pi}\frac{G_F M_Z^3}{24\sqrt{2}\pi}\left[(v_l^2 + 1)\left(1 + g_6\,\delta^{(6)}_{\rm QED}\right) + g_6\,\Delta^{(6)}_{\rm QED}\right] \tag{9}$$