BOOSTED GGF H(BB) FIRST SEARCH FOR BOOSTED HIGGS→BB WITH CMS

HIGGS COUPLINGS 2017 INSTITUTE FOR THEORETICAL PHYSICS HEIDELBERG UNIVERSITY HEIDELBERG, GERMANY

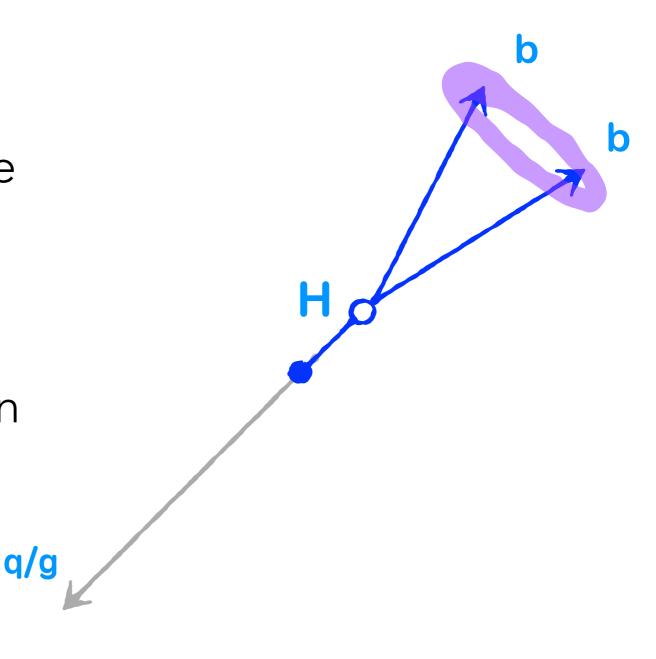
Javier Duarte Fermilab

NOVEMBER 9, 2017



OUTLINE

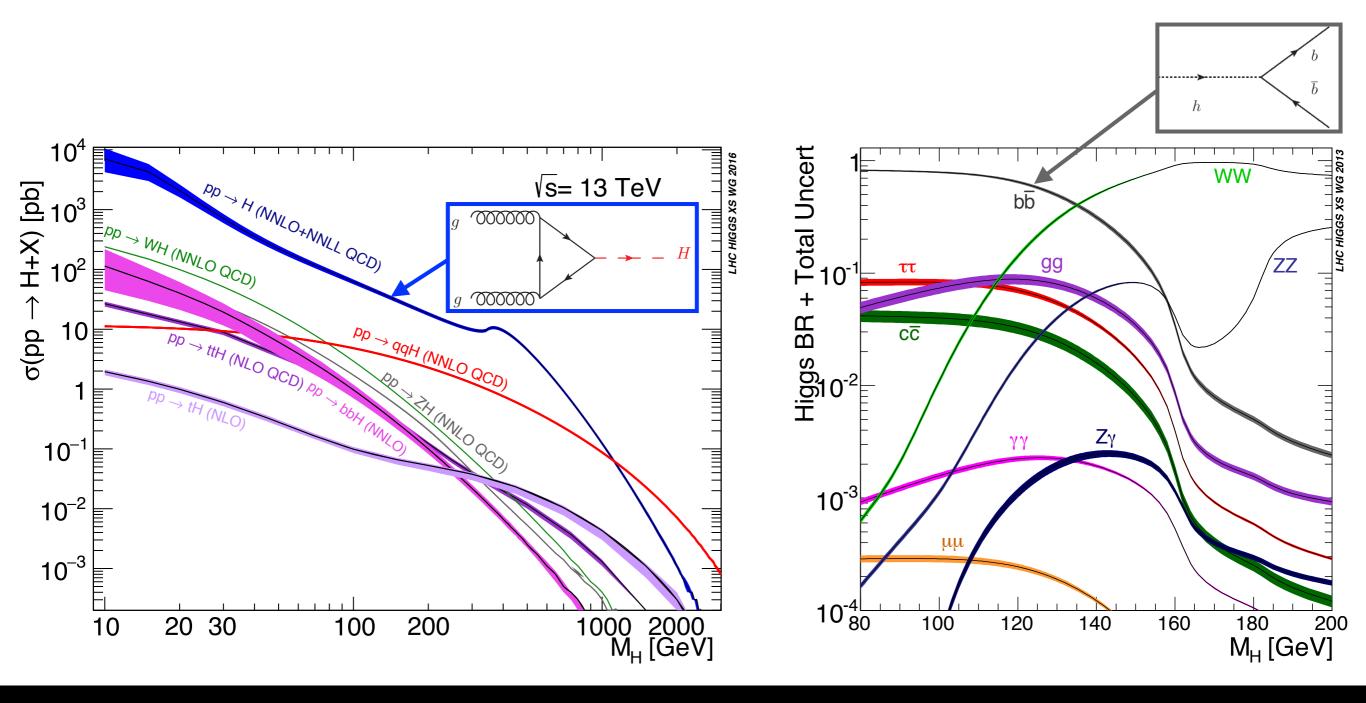
- Motivation
- Higgs tagging
 - Jet mass and substructure
 - Double-b tagging
- Event selection
- Data-driven QCD estimation
- Systematics
- Results
- Summary and outlook







HIGGS AS A DIJET SEARCH? Largest Higgs production/decay mode is $gg \rightarrow H \rightarrow bb$ (>50%)

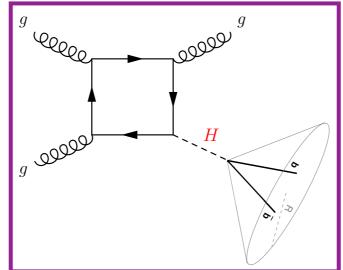


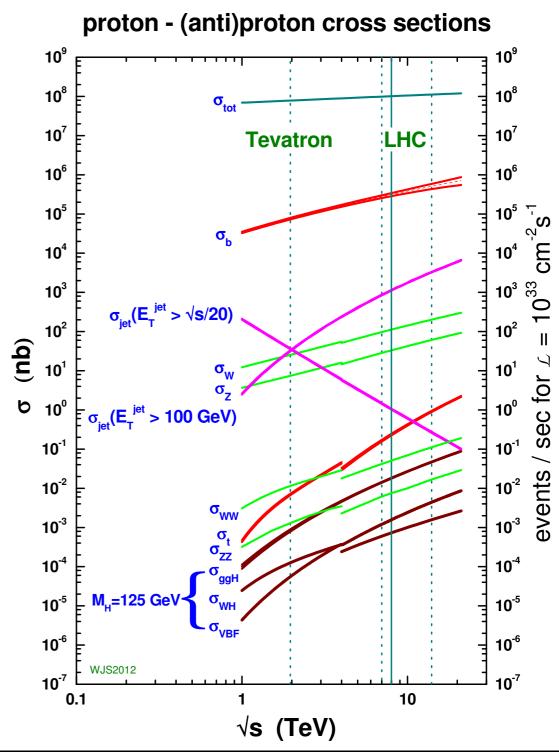




HIGGS AS A DIJET SEARCH?

- Largest Higgs production/decay mode is $gg \rightarrow H \rightarrow bb$ (>50%)
- But, background is also immense
 - QCD b production is 10⁷ times larger
- Using large Lorentz boost, machine learning, and jet substructure, we can achieve S/B ~ 1/10



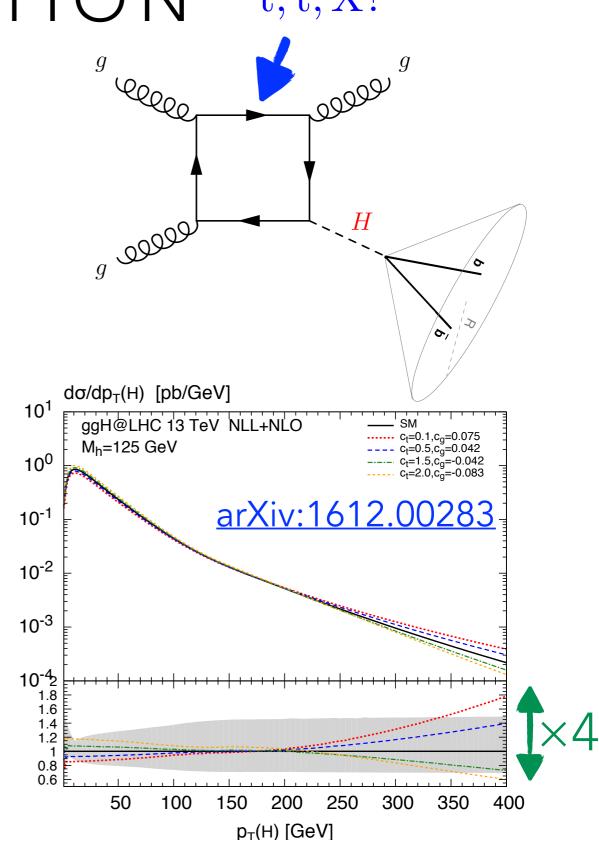






$\mathsf{MOTIVATION} \quad t, \widetilde{t}, X?$

- We can access this process in the boosted dijet topology
- Probing Higgs couplings at high momentum transfer (Q) accesses large new physics energy scale (Λ)



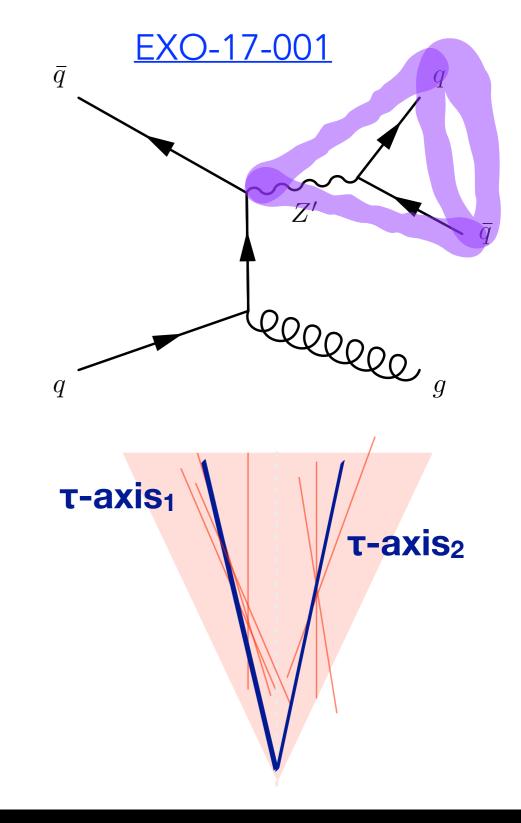




SO HOW CAN WE DO IT?

- Inspiration from boosted techniques?
 - Use an ISR jet to get above the trigger threshold
 - Require one boosted fat jet
 - Substructure and jet grooming to enhance S/B
 - VData-driven background estimate
- Inspiration from machine learning and b-tagging?
 - Double b-tagger selects fat jets containing two b-quarks







BOOSTED GGF H(BB) HIGGS TAGGING

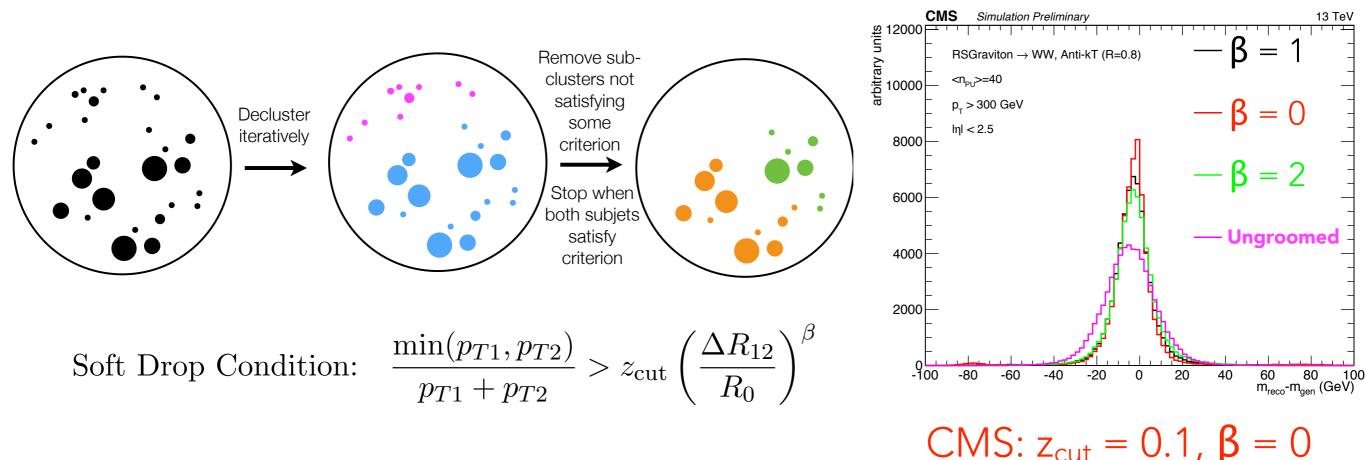




arXiv:1307.0007 arXiv:1402.2657

JET MASS

- Provides good separation between W/Z/H-jets and q/g jets
- Grooming removes soft and wide-angle radiation (soft drop / modified mass drop tagger)







arXiv:1609.07483 JET SUBSTRUCTURE

- How many "prongs" are in the jet?
- Generalized energy correlation functions are sensitive to N-point correlations within a jet
 - Two-pronged jets have $_2e_3 \ll (_1e_2)^2$
- Stable under grooming

$${}_{1}e_{2}^{\beta} = \frac{1}{p_{\mathrm{T}J}^{2}} \sum_{1 \le i < j \le n_{J}} p_{\mathrm{T}i}p_{\mathrm{T}j}\Delta R_{ij}^{\beta}$$

$${}_{2}e_{3}^{\beta} = \frac{1}{p_{\mathrm{T}J}^{3}} \sum_{1 \le i < j < k \le n_{J}} p_{\mathrm{T}i}p_{\mathrm{T}j}p_{\mathrm{T}k} \min\{\Delta R_{ij}^{\beta}\Delta R_{ik}^{\beta}, \Delta R_{ij}^{\beta}\Delta R_{jk}^{\beta}, \Delta R_{ik}^{\beta}\Delta R_{jk}^{\beta}\}$$

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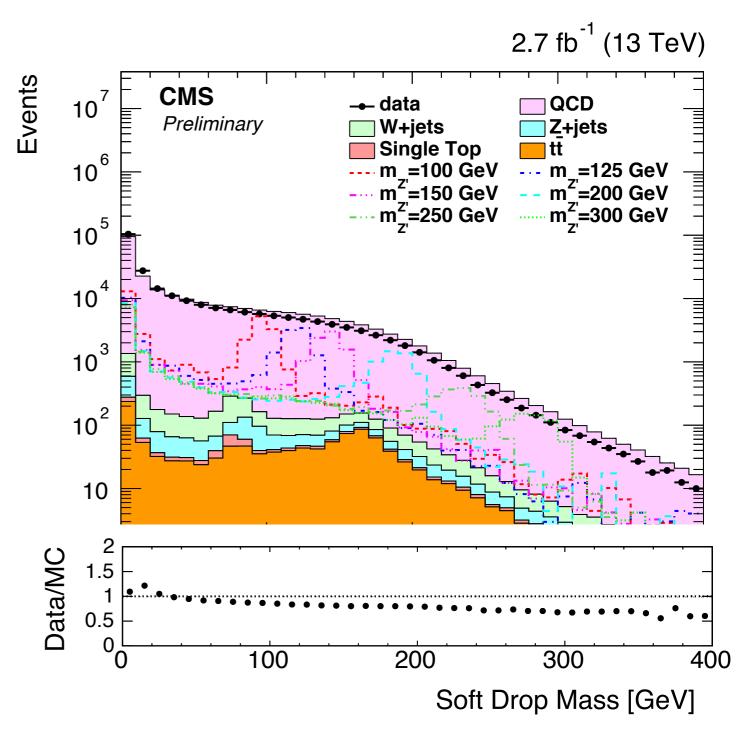
 $N_2^{\beta} = \frac{2e_3^{\rho}}{(1e_3^{\beta})^2} \beta = 1$

2-point 3-point

EXO-16-030

BACKGROUNDS

- Difficult to use the QCD Monte Carlo to predict the background in this phase space
- Fitting this mass distribution directly requires high order polynomial → large background uncertainties
- Can we try a data-driven sideband prediction?

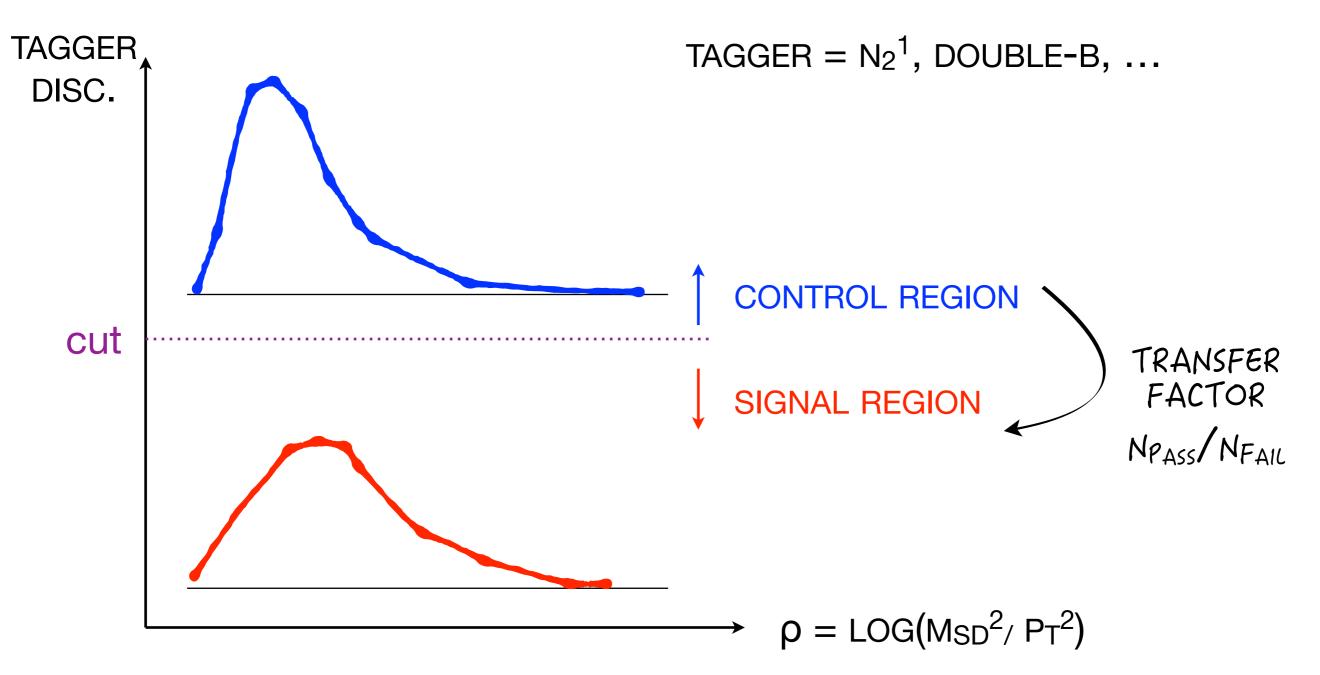






SIDEBAND QCD PREDICTION

- Can we predict the QCD jet mass distribution from region failing the tagger?
 - Potential problem: does tagger sculpt jet mass distribution?

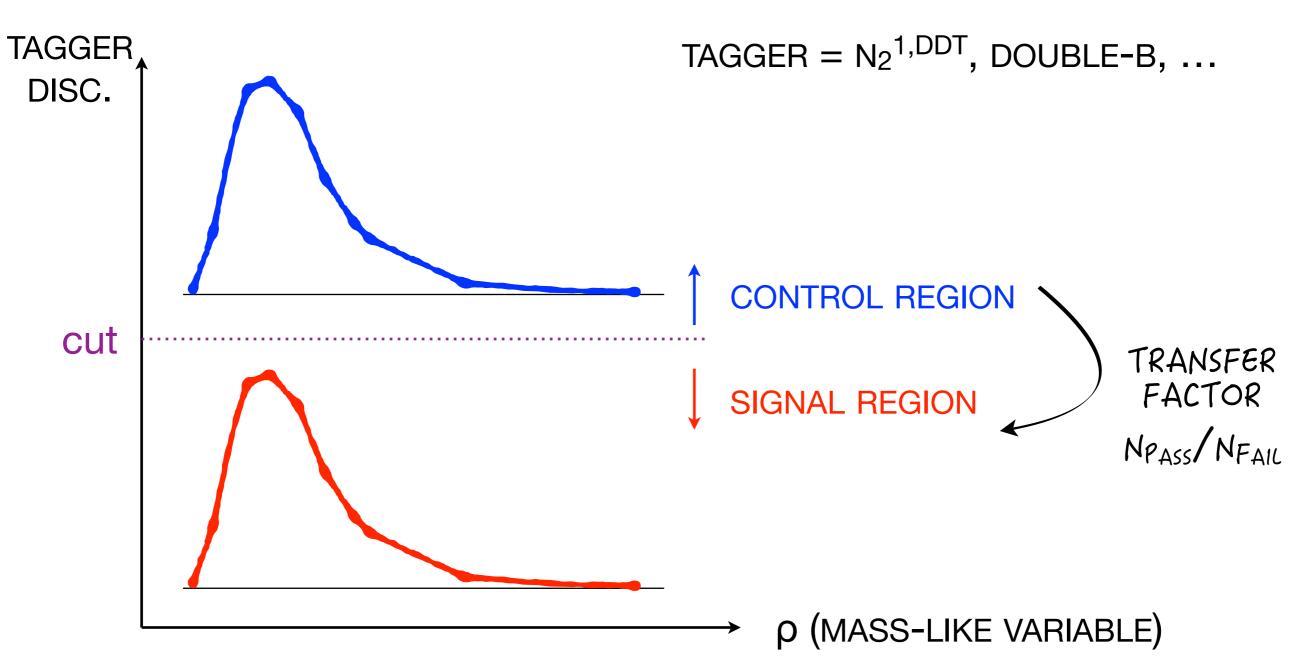






SIDEBAND QCD PREDICTION

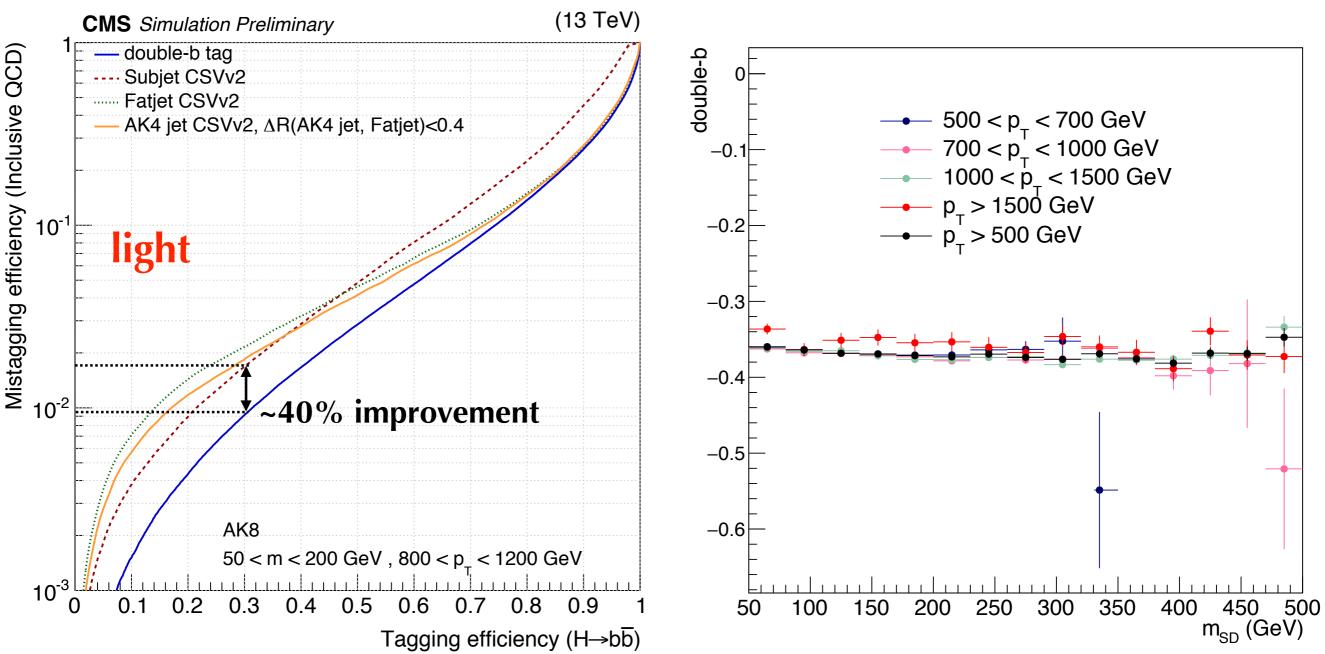
- Can we predict the QCD jet mass distribution from region failing the tagger?
 - Solution: use a tagger that is decorrelated from jet mass and $\ensuremath{p_{\mathsf{T}}}$







DOUBLE-B TAGGER



- Combines tracking and vertexing information in a multivariate classifier with 27 observables
- No strong correlations in double-b tagger versus m_{SD} or p_{T} in QCD background



BTV-15-002





EVENT SELECTION

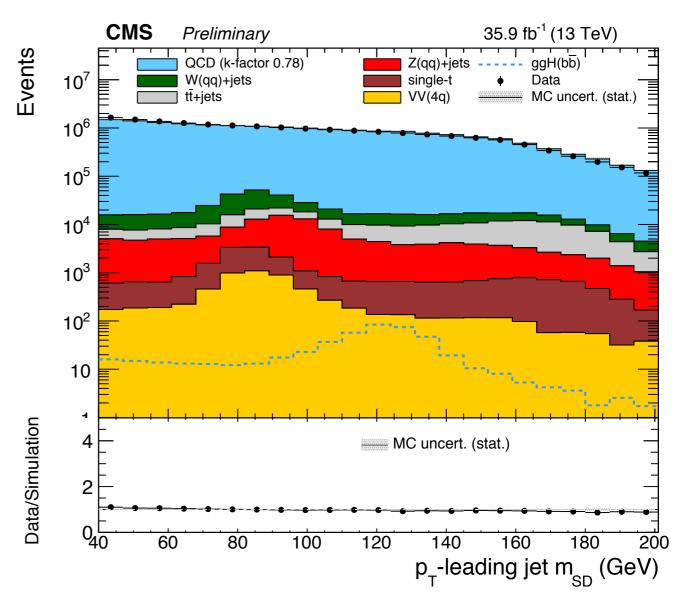
BOOSTED GGF H(BB)



<u>HIG-17-010</u>

EVENT SELECTION

- Online: high p_T single jet or large hadronic activity
 - $p_T > 360 \text{ GeV} (m > 30) \text{ or}$ $\Sigma p_T > 900 \text{ GeV}$
- Offline: Highest p_T jet
 - $p_T > 450 \text{ GeV}, |\eta| < 2.5$
 - jet mass $m_{SD} > 40 \text{ GeV}$
- lepton veto, p_T^{miss} veto
- $-6.0 < \rho = \log(m_{SD}^2/p_T^2) < -2.1$







<u>HIG-17-010</u>

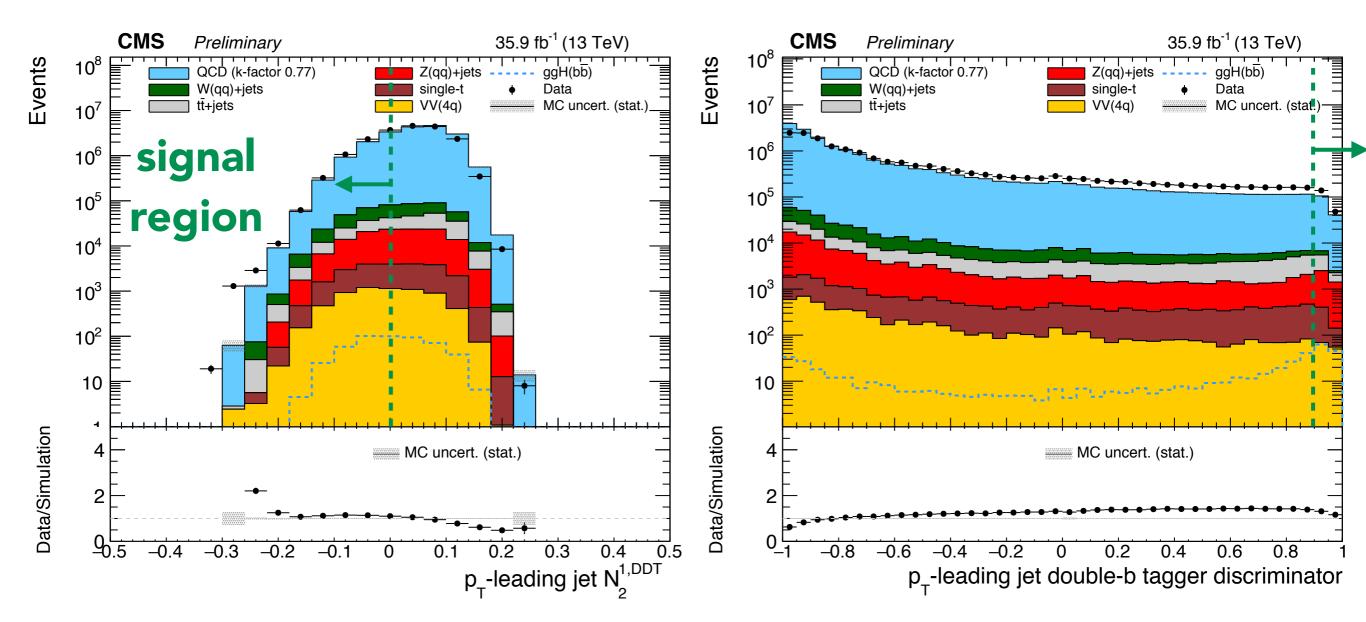
EVENT SELECTION

Substructure: two prong discrimination, ~50% sig. efficiency, 26% bkg. efficiency

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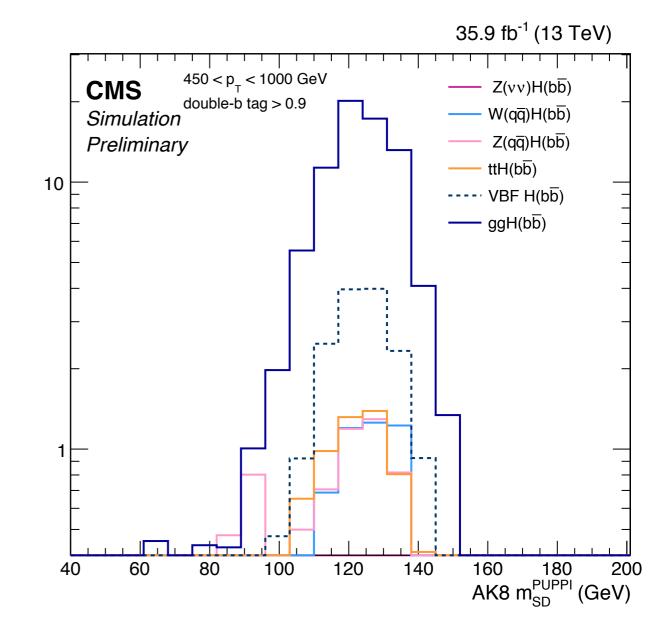
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Double-b tagger: ~30% sig. efficiency, 1% bkg. efficiency (tight working point)



HIG-17-010 SIGNAL COMPOSITION

- Analysis is inclusive in Higgs production mode
- Signal contributions after kinematic + substructure + double-b selection are
 - 74% ggF
 - 12% VBF
 - 8% VH
 - 6% ttH







BOOSTED GGF H(BB) BACKGROUND ESTIMATION





BACKGROUND STRATEGY

- Backgrounds estimated from data
 - **QCD** (90%): from failing double b-tag region × **TF**
 - tt+jets (3%): from 1µ
 control region
- Backgrounds estimated from MC including NLO QCD + EWK corrections and jet mass, resolution, and substructure tagging scale factors
 - W/Z+jets (5%)
 - single-t, VV (<1%)

QCD transfer factor

 $N_{\text{pass}}^{\text{QCD}}(m_{\text{SD}}, p_{\text{T}}) = R_{\text{p/f}}(\rho, p_{\text{T}}) \cdot N_{\text{fail}}^{\text{QCD}}(m_{\text{SD}}, p_{\text{T}})$

• If the double-b tagger were completely uncorrelated from jet mass and p_T , the transfer factor would be flat





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$$N_{\text{pass}}^{\text{QCD}}(m_{\text{SD}i}, p_{\text{T}j}) = \left(\sum_{k,\ell} a_{k\ell} \rho_{ij}^{k} p_{\text{T}j}^{\ell}\right) \cdot N_{\text{fail}}^{\text{QCD}}(m_{\text{SD}i}, p_{\text{T}j})$$

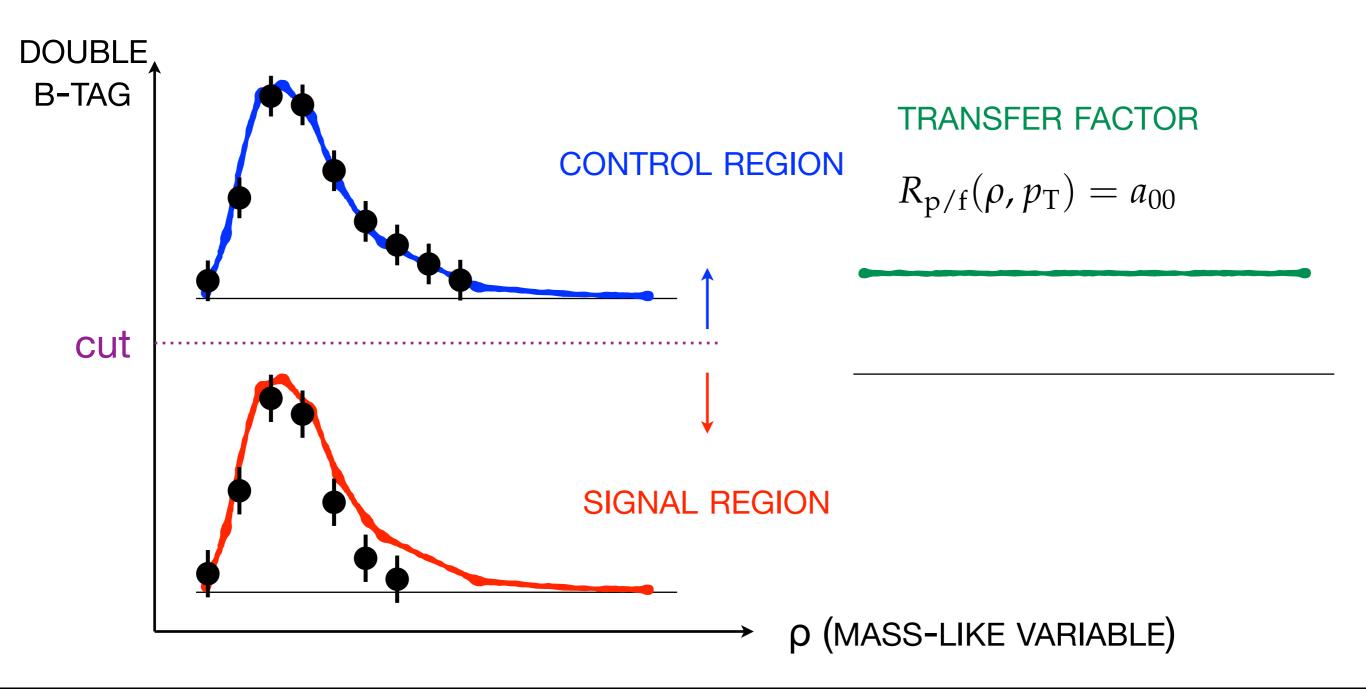
- If the double-b tagger were completely uncorrelated from jet mass and p_T , the transfer factor would be flat
- Taylor expand as a polynomial in ρ and p_T to parameterize any small correlations





FITTING TRANSFER FACTOR

• Pre-fit both regions have the same predicted shape

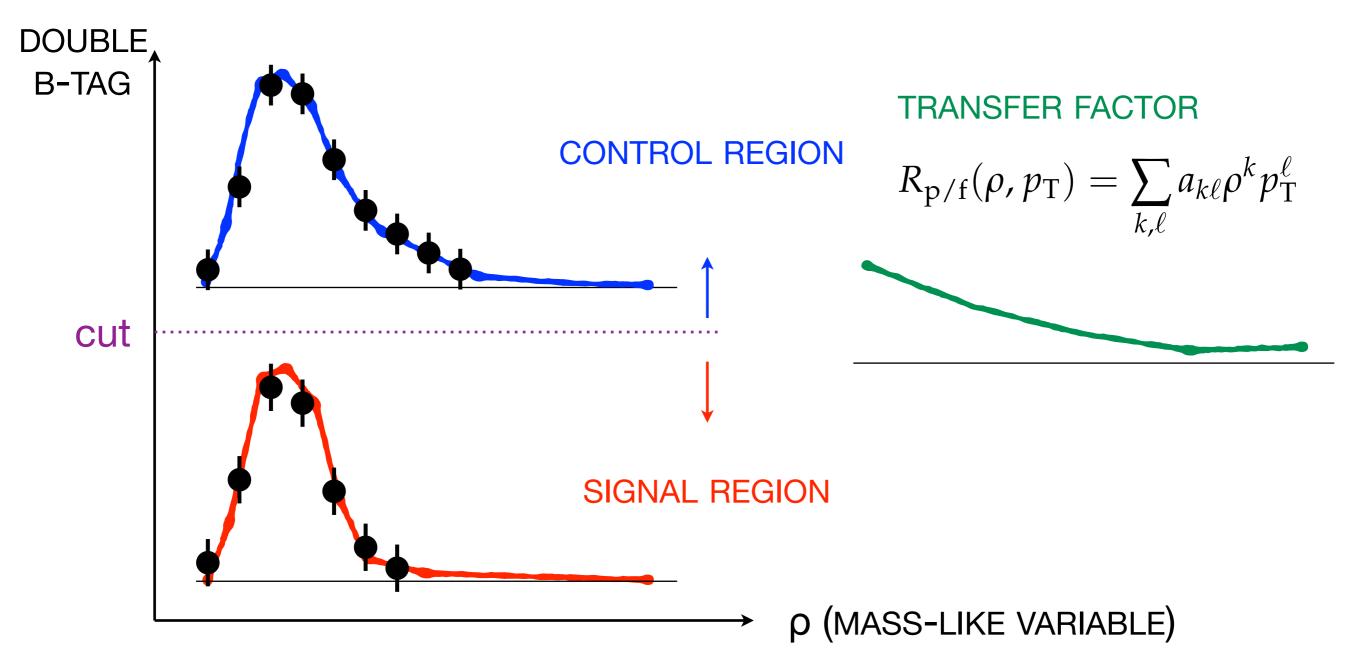






FITTING TRANSFER FACTOR

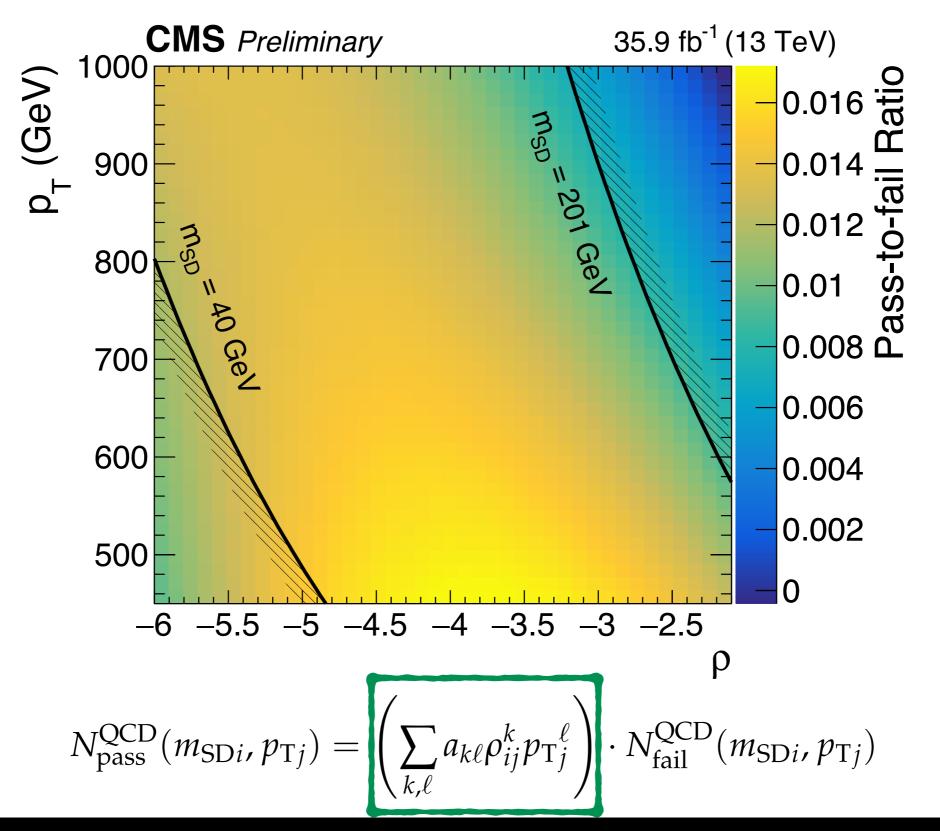
• Post-fit signal region has slightly different shape with the ratio given by the polynomial transfer factor







HIG-17-010 OCD TRANSFER FACTOR

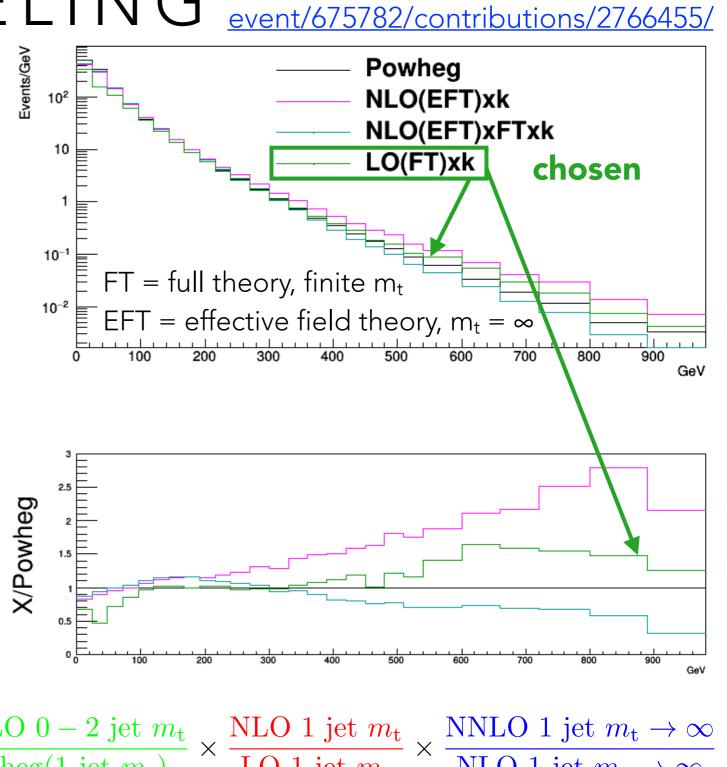






HIGGS PT MODELING

- LO H+0–2jet (MadGraph) Pythia CKKW-L merged, finite m_t
- NLO H+1jet finite m_t up to 1/m²_t
 expansion: <u>arXiv:1609.00367</u>
- NNLO H+1jet, m_t = ∞, p_T^H up to ~200 GeV <u>arXiv:1508.02684</u>
- Two factorized systematic uncertainties:
 - 30% overall normalization
 - 30% linear change in slope (no effect on overall norm.)



More details: <u>https://indico.cern.ch/</u>

$$ggF H(NNLO + m_t) = Powheg(1 \text{ jet } m_t) \times \frac{MG LO 0 - 2 \text{ jet } m_t}{Powheg(1 \text{ jet } m_t)} \times \frac{NLO 1 \text{ jet } m_t}{LO 1 \text{ jet } m_t} \times \frac{NNLO 1 \text{ jet } m_t \to \infty}{NLO 1 \text{ jet } m_t \to \infty}$$
$$CKKW \text{ merged} \quad factor of 2 \quad factor of 1.25$$

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HIGGS PT MODELING

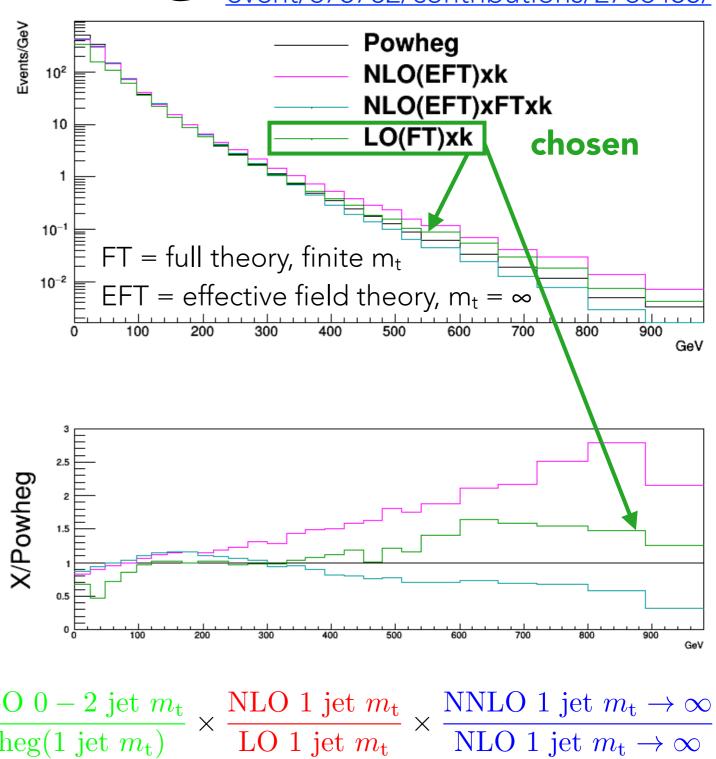
More details: <u>https://indico.cern.ch/</u> <u>event/675782/contributions/2766455/</u>

- We quote 31.7 fb ggF H(bb) cross section for leading reco. jet p_T > 450 GeV, |η| < 2.5
- Other (more recent) sources quote ~6-9 fb
- Differences come from:
 - Parton shower (35%)
 - Reconstruction (25%)
 - Leading p_T jet selection (25%)
- Corresponding ME-level cross section for Higgs $p_T > 450$ GeV is 15.1 fb

ggF H(NNLO +
$$m_t$$
) = Powheg(1 jet m_t) × $\frac{MG LO}{Powh}$

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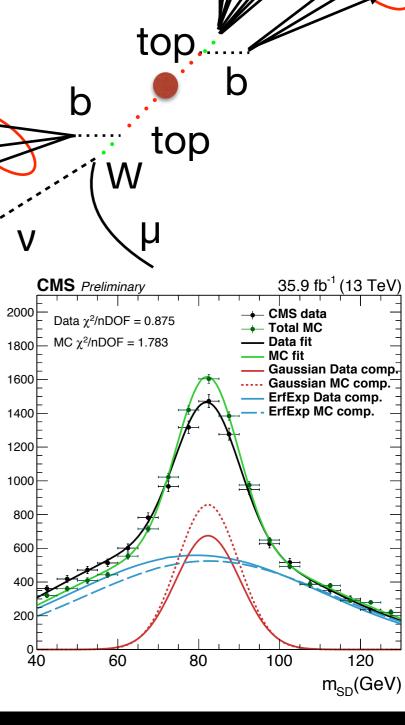


CKKW merged factor of 2 factor of 1.25



			1CS	
Systematic uncertainty source	Type (shape or normalization)	Relative size (or description)	(
QCD transfer factor	both	profile $a_{k\ell}$ and QCD normalization		
Luminosity	normalization	2.5%		
$(V-tag (N_2^{1,DDT}) efficiency)$	normalization	4.3%)	۱۸/	
Muon veto efficiency	normalization	0.5%	VV	
Electron veto efficiency	normalization	0.5%		
Trigger efficiency	normalization	4%	top	¥ //
Muon ID efficiency	shape	up to 0.2%	lop	·····
Muon isolation efficiency	shape	up to 0.1%		h
Muon trigger efficiency	shape	up to 8%	h 💭	D
tt normalization SF	normalization	from 1 <i>µ</i> CR: 8%		
t ī double-b mis-tag SF	normalization	from 1 <i>µ</i> CR: 15%		`
W/Z NLO QCD corrections	normalization	10%	top)
W/Z NLO EWK corrections	normalization	15% - 35%	🔨 İ \\\/	
W/Z NLO EWK ratio decorrelation	normalization	5% - 15%		
double-b tagging efficiency	normalization	4%		
Jet energy scale	normalization	up to 10%	$\mathbf{v} \ge \mathbf{U}$	
Jet energy resolution	normalization	up to 15%	V \P'	
Jet mass scale	shape	shift $m_{ m SD}$ peak by $\pm 0.4\%$		
Jet mass resolution	shape	smear m_{SD} distribution by $\pm 9\%$	CMS Preliminary	35
Jet mass scale $p_{\rm T}$	normalization	$0.4\%/100 \text{GeV} (p_{\text{T}})$	$\begin{array}{c} \mathbf{x} \\ \mathbf{y} \\ \mathbf{y} \\ \mathbf{z} \\ $	_+_ CMS
Monte Carlo statistics	normalization	-	$\Theta = \begin{bmatrix} Data \chi^{-1} DOF = 0.075 \\ How \chi^{2} DOF = 0.75 \end{bmatrix}$	_+ Tota
(H $p_{\rm T}$ correction (gluon fusion)	both	30%	III 1800 $\stackrel{\text{L}}{=}$ MC χ^2 /nDOF = 1.783	Data MC f

- Signal systematic uncertainties from merged W sample in semi-leptonic ttbar events (external constraint)
- SM candles: presence of W/Z(bb) in final jet mass distribution provides in-situ constraint
- Higgs p_T correction uncertainty of 30%







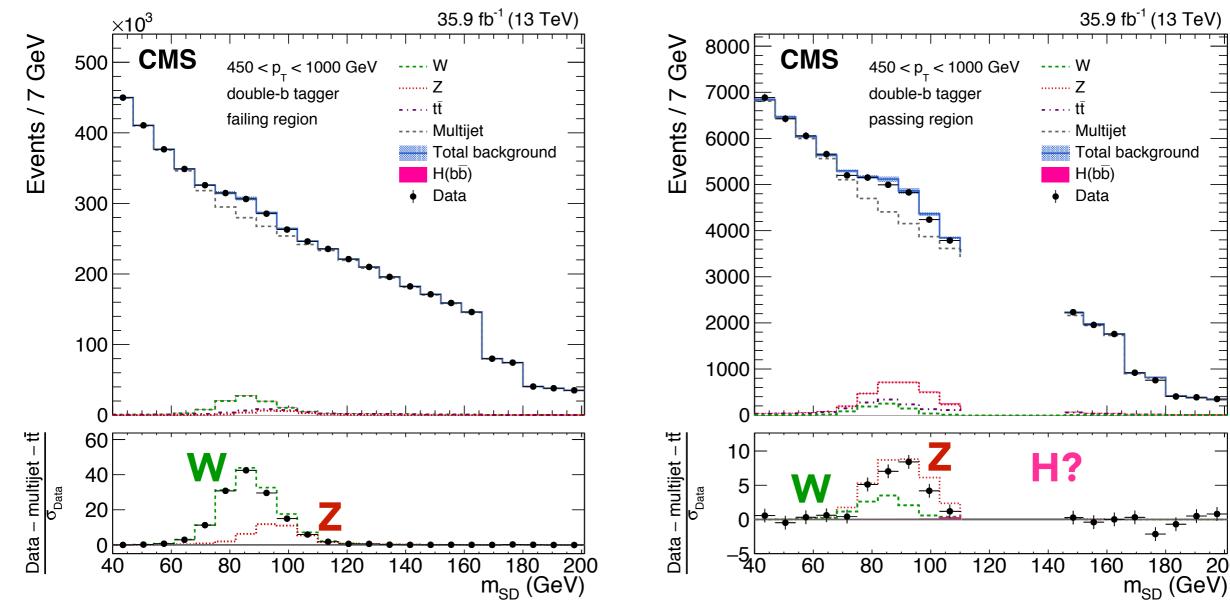


RESULTS

BOOSTED GGF H(BB)

<u>HIG-17-010</u> FIT RESULTS Simultaneous fit for Z(bb) and H(bb)

All p_T categories



SM candles: Z(bb) peak provides in-situ constraint of H(bb) signal systematics

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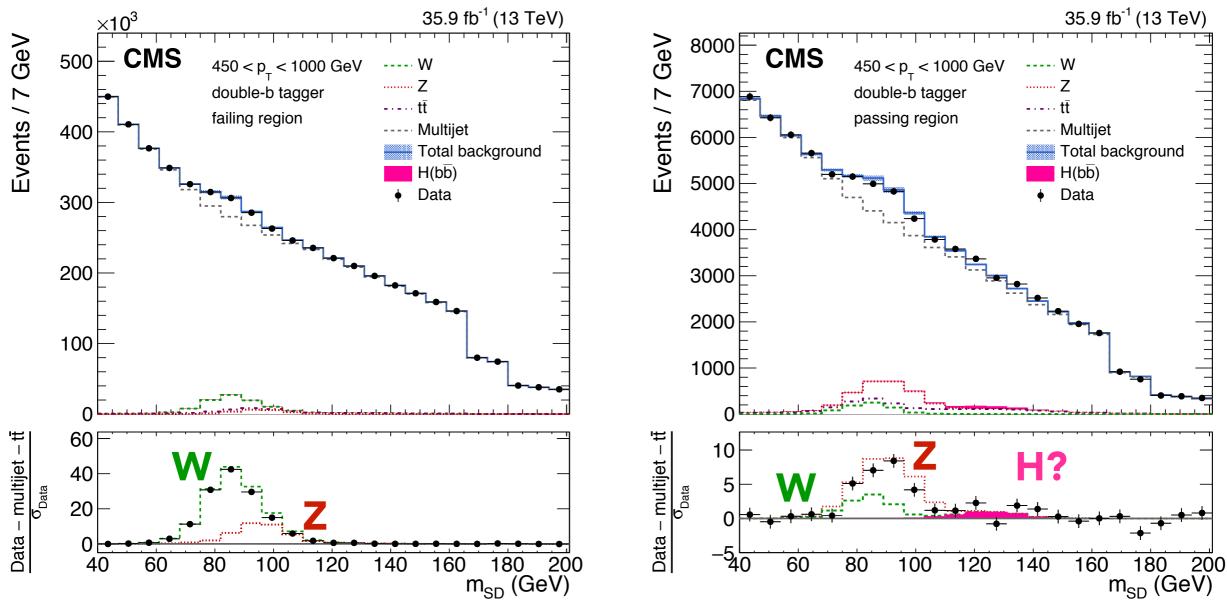
observed Z(bb) significance: 5.1 σ , $\mu_z = 0.78^{+0.23}$ -0.19



200

HIG-17-010 FIT RESULTS Simultaneous fit for Z(bb) and H(bb)

• All p_T categories



observed H(bb) significance:

1.5σ, μ_H = **2.3**^{+1.8}_{-1.6}

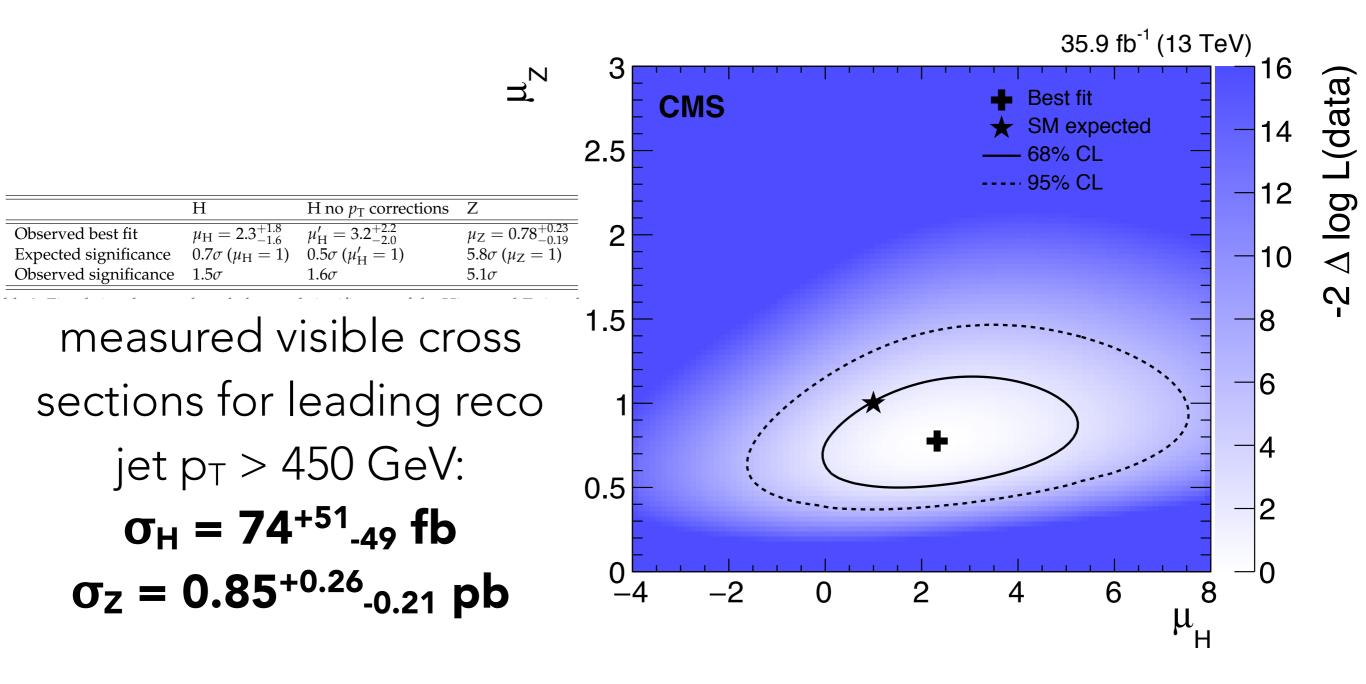




<u>HIG-17-010</u>

FIT RESULTS

• Two dimensional likelihood scan





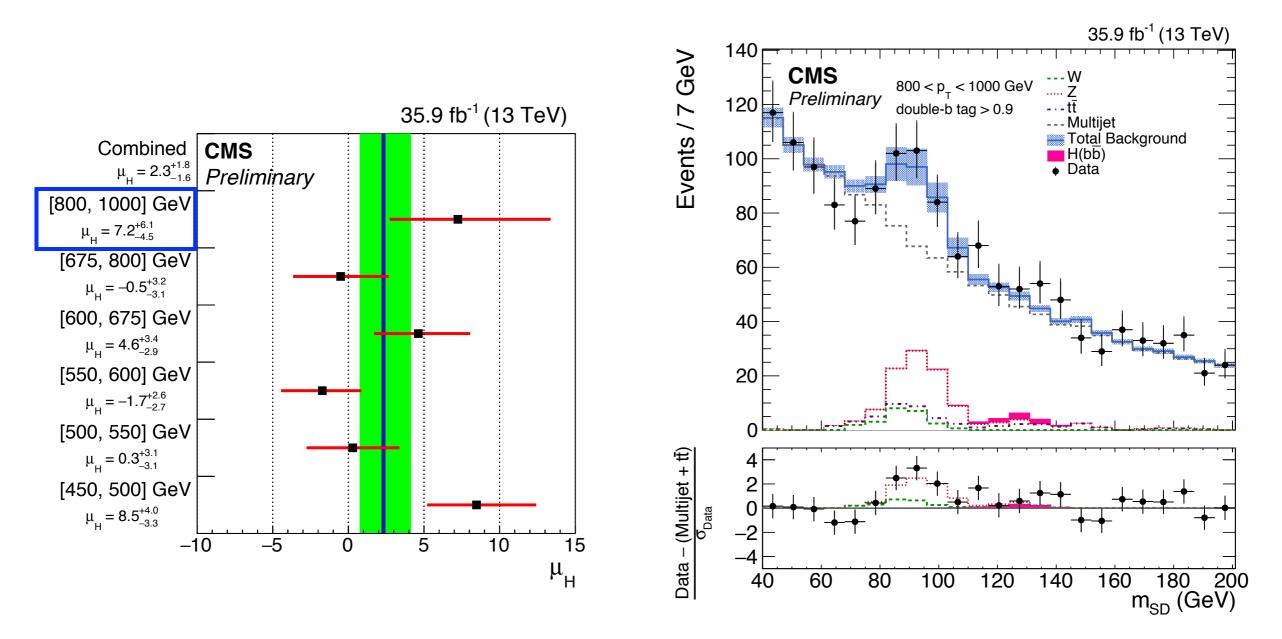


$\frac{\text{HIG-17-010}}{\text{P}_{T}} \quad \text{P}_{T} \quad \text{CATEGORIES}$

• Probing Higgs production up to $p_T = 1$ TeV!

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SUMMARY AND OUTLOOK

- First LHC search for $gg \rightarrow H \rightarrow bb$ in boosted topology
 - First observation of Z(bb) in single-jet topology,
 5.1σ observed (5.8σ expected)
 - Observed significance of H(bb) is 1.5σ (0.7 σ expected)
- Search probes previously unexplored regions of Higgs phase space
- New and generic strategy to search for boosted hadronic Higgs decays
 - Future prospects are bright: same search may have ${\sim}3\sigma$ expected H(bb) significance for 2016+2017+2018 data
 - Probing Higgs boson p_T up to **1 TeV** and beyond...



BOOSTED GGF H(BB)

BACKUP





QCD TRANSFER FACTOR

- If the double b-tagger were completely uncorrelated from jet mass and p_T, the transfer factor would be flat
- Taylor expand as a polynomial in *ρ* and p_T to parameterize any small correlations
- F-test determined 2nd order in *ρ* and 1st order in p_T is sufficient to fit the ratio

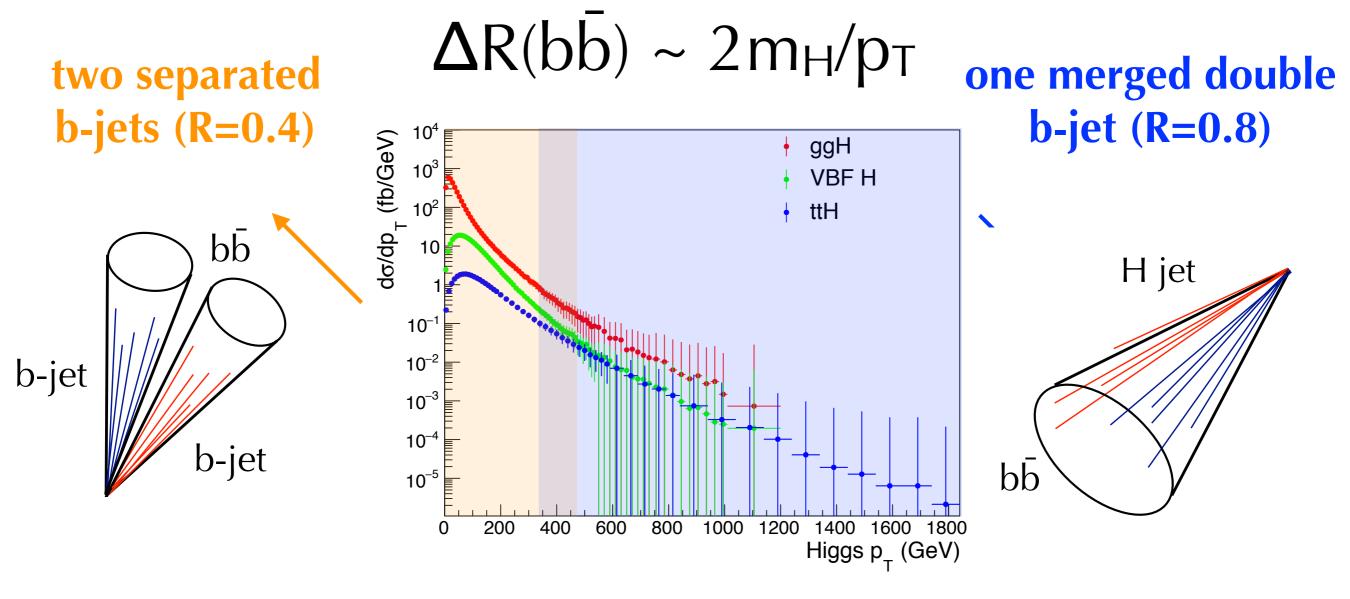
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BOOSTED H(BB)

- With large boost, both b quarks merge into a single large radius jets
- How can we best exploit the presence of the b-quarks in the jet in a tagger?

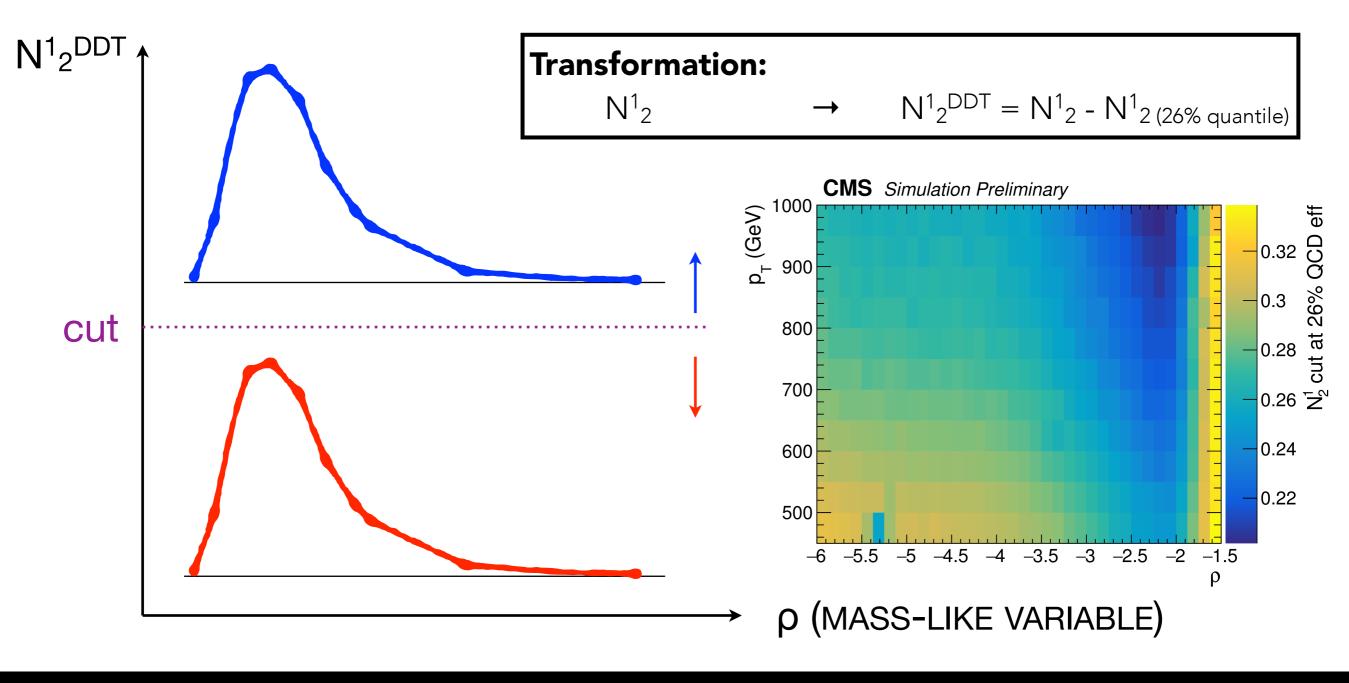






SIDEBAND QCD PREDICTION

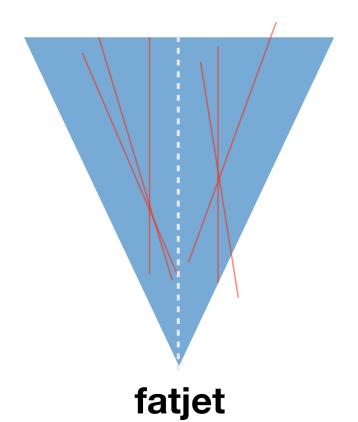
• Solution: define new substructure variable intended to be decorrelated from jet mass



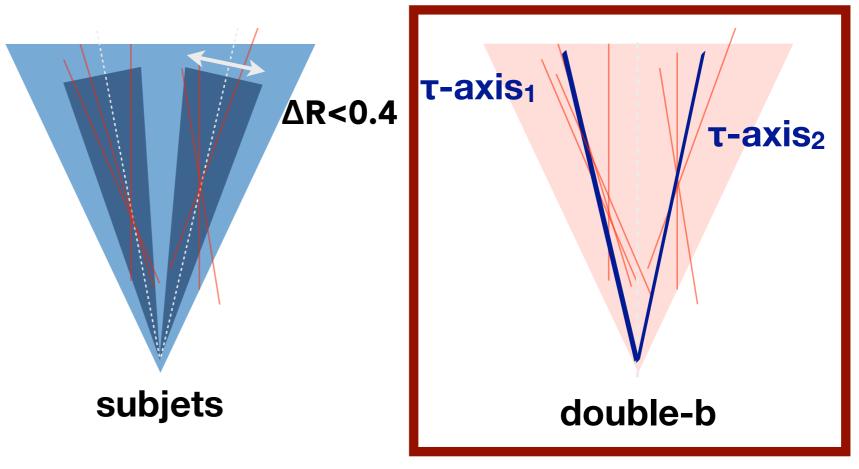




MULTIPLE APPROACHES



- Based on standard b-tagging algorithm
- Not designed for two b's in the same jet



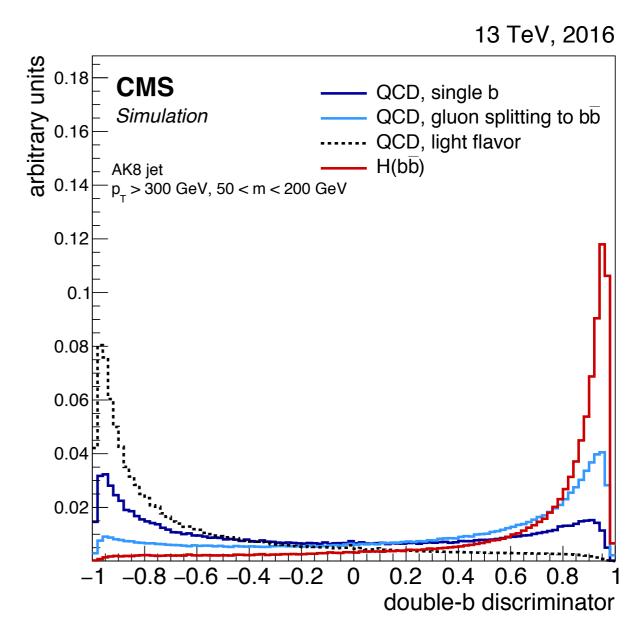
- Defines sub-jets
- Standard b-tagging applied to each subject
- Identifies two b hadron decay chains in the same fat jet
- Does not define subjects, but uses N-subjettiness axes





DOUBLE B-TAGGER

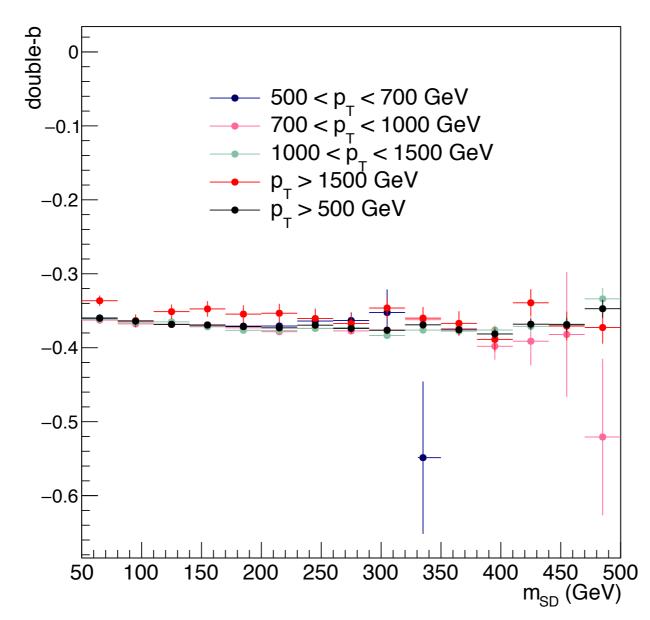
- Combines tracking and vertexing information in a multivariate classifier with 27 observables
- Targets the bb signal with additional aims:
 - jet mass and p_T independent
 - cover a very wide p_T range
 - inputs are chosen to avoid p_T correlation
 - e.g. no ΔR-like variables, no substructure info







CORRELATIONS?

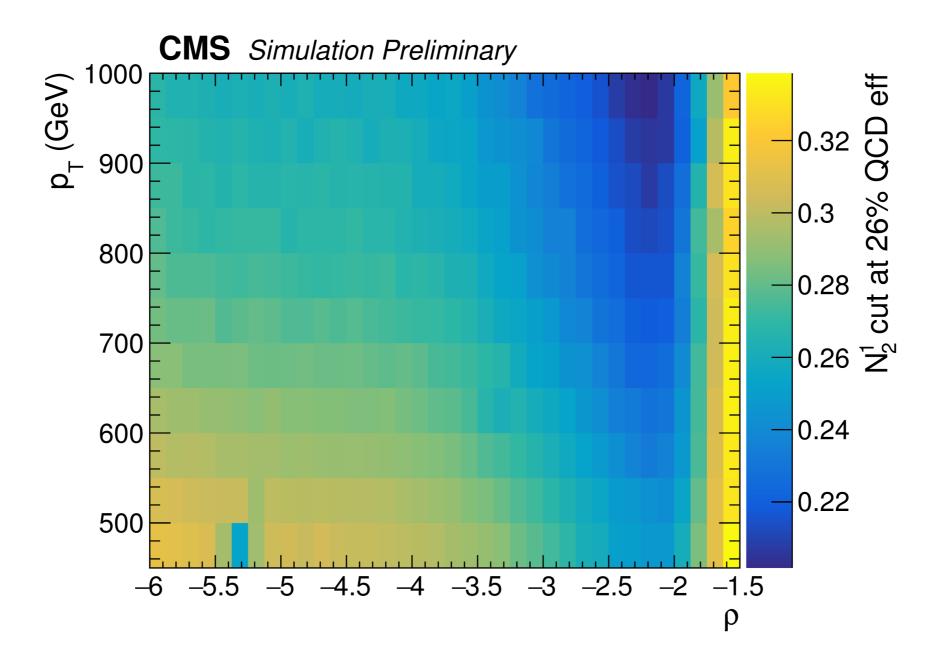


• No strong correlations in double-b tagger versus m_{SD} or p_{T} in QCD background

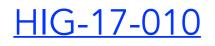


$N_{2}^{1}D^{D}$

• Cut value map used to transform N_2^1

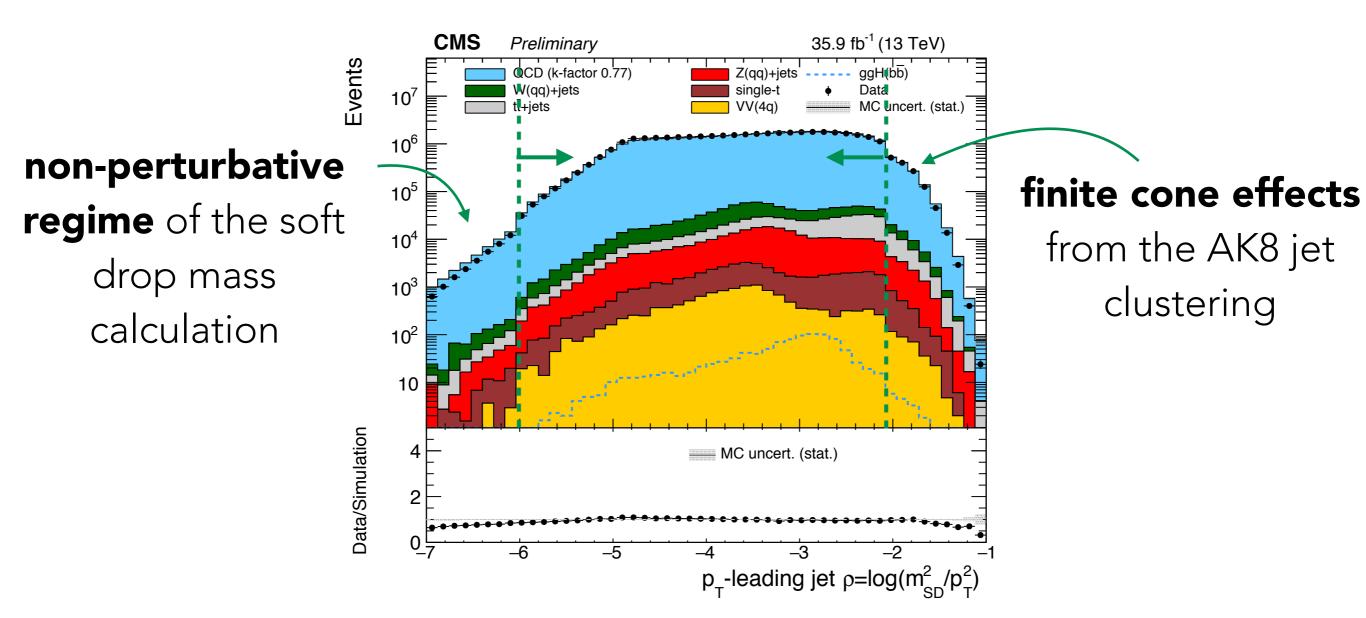






EVENT SELECTION

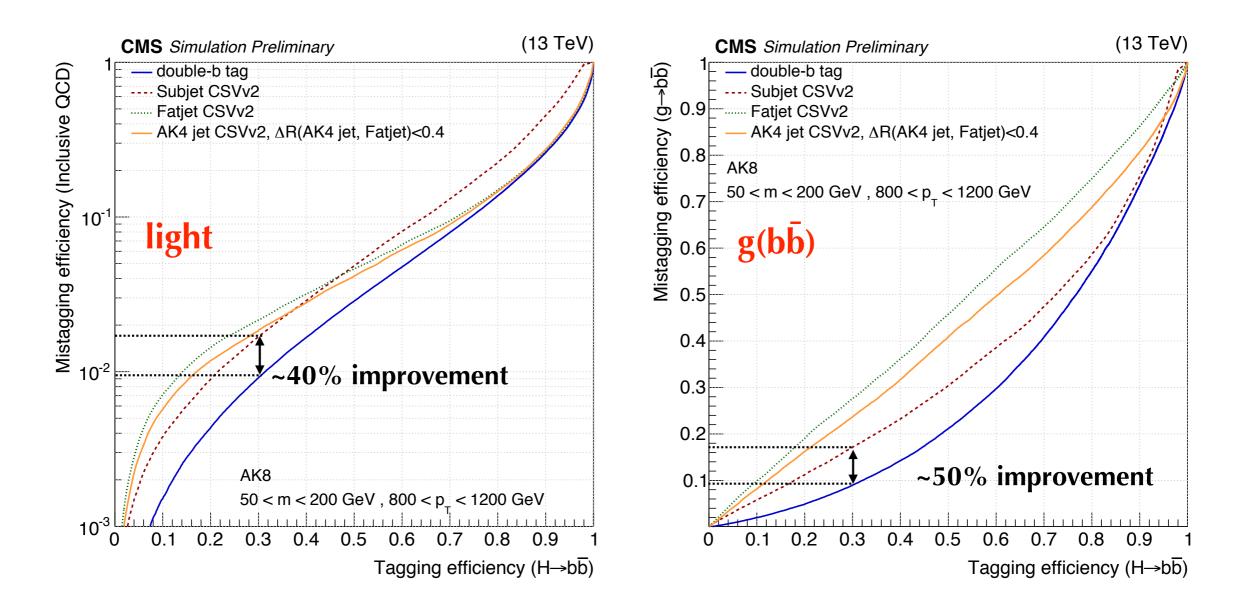
 $-6.0 < \rho < -2.1$







EFFICIENCY AND MIS-TAG

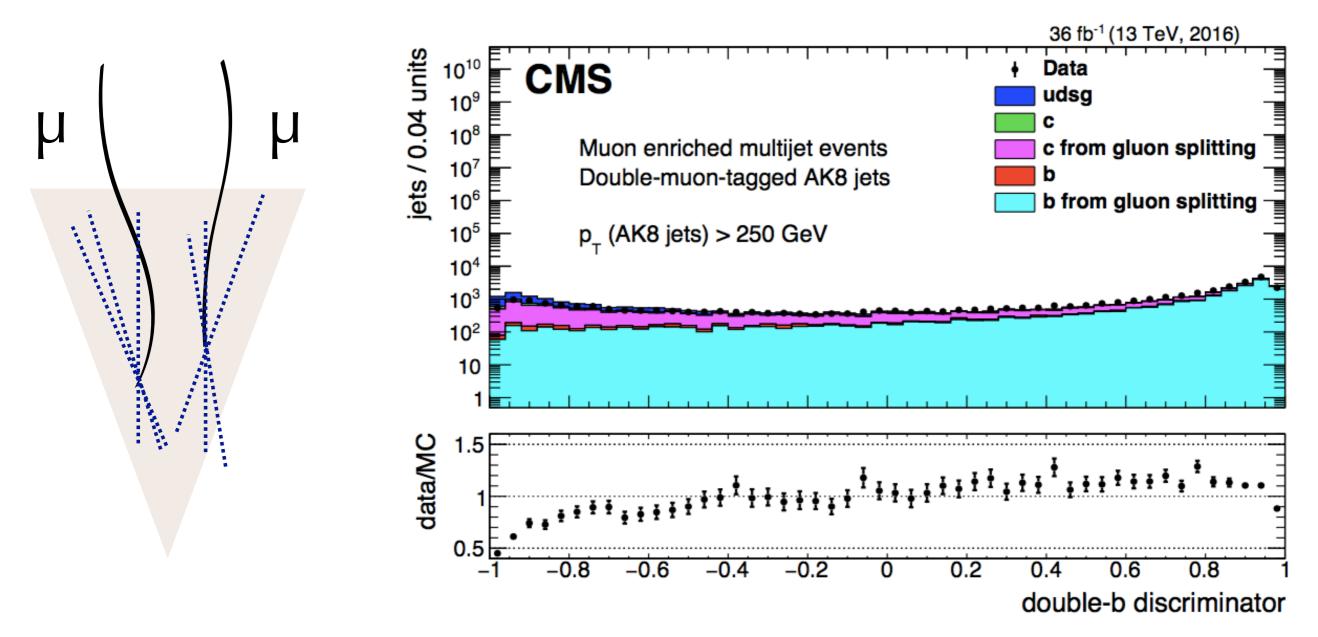


Mis-tag is reduced by more than 40% at 30% signal efficiency for a tight working point





EFFICIENCY IN DATA

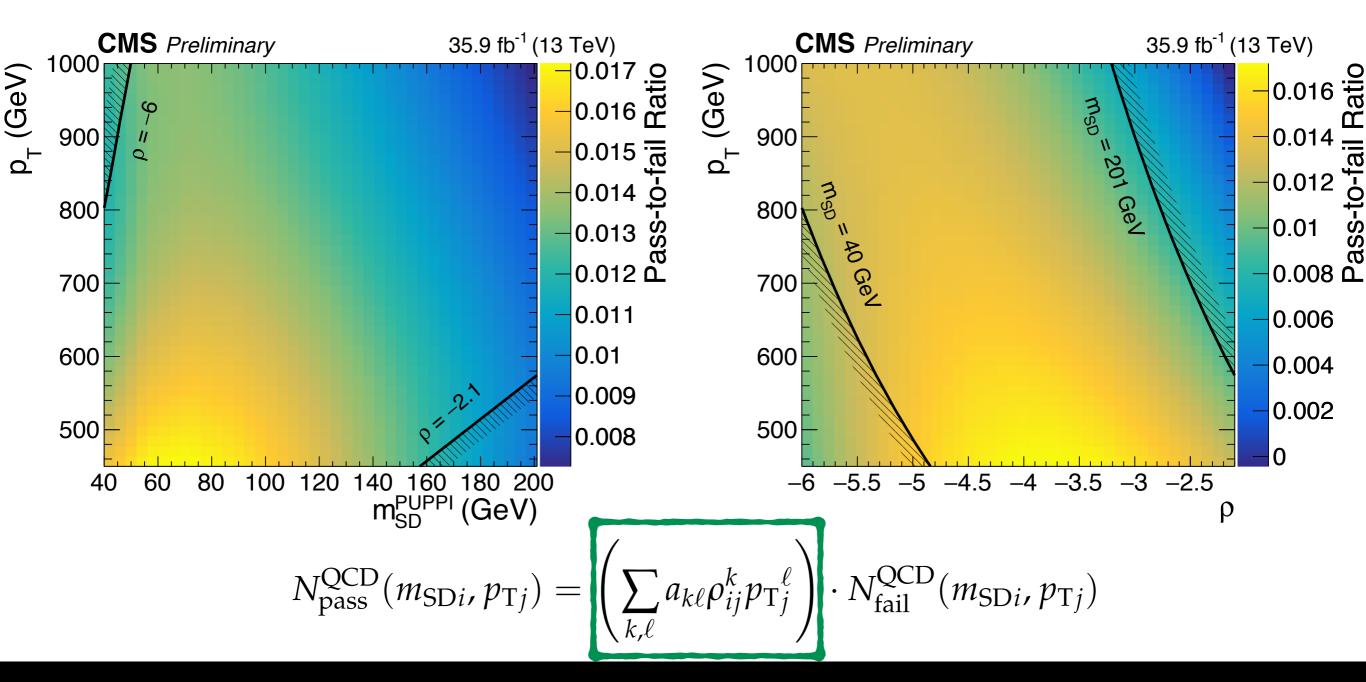


- Using g(bb) jet as proxy in double muon tagged jet sample
- Associated data/MC uncertainty 3-5%



FINAL TRANSFER FACTOR

• Two views of the same transfer factor function





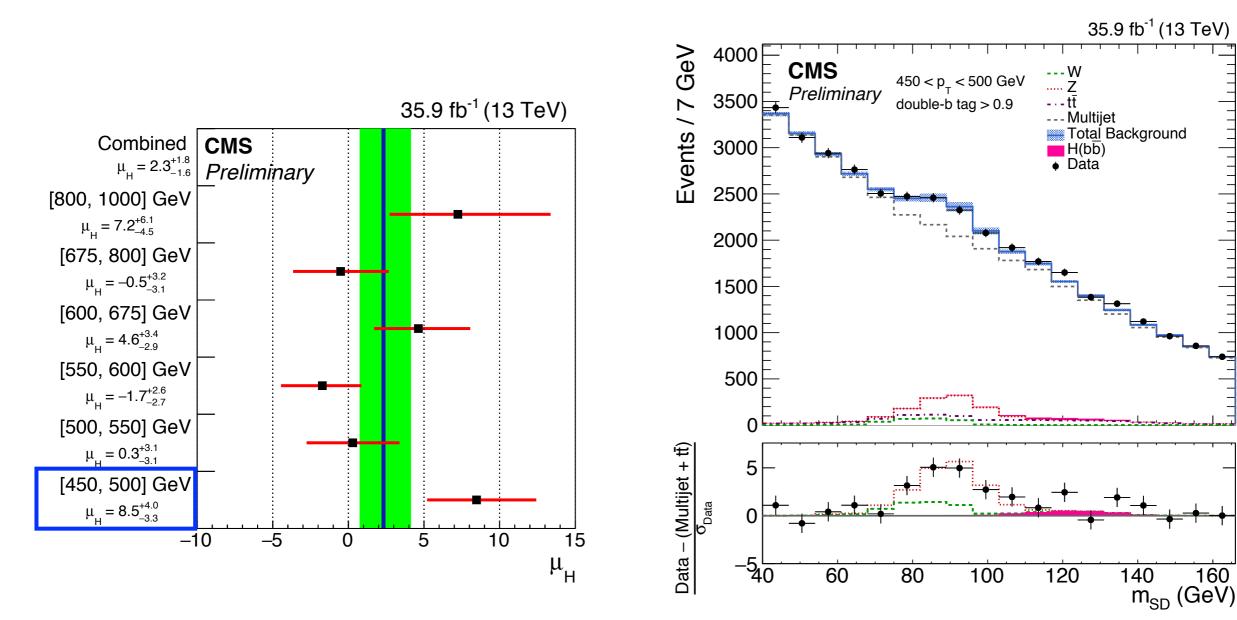


$P_T CATEGORIES$

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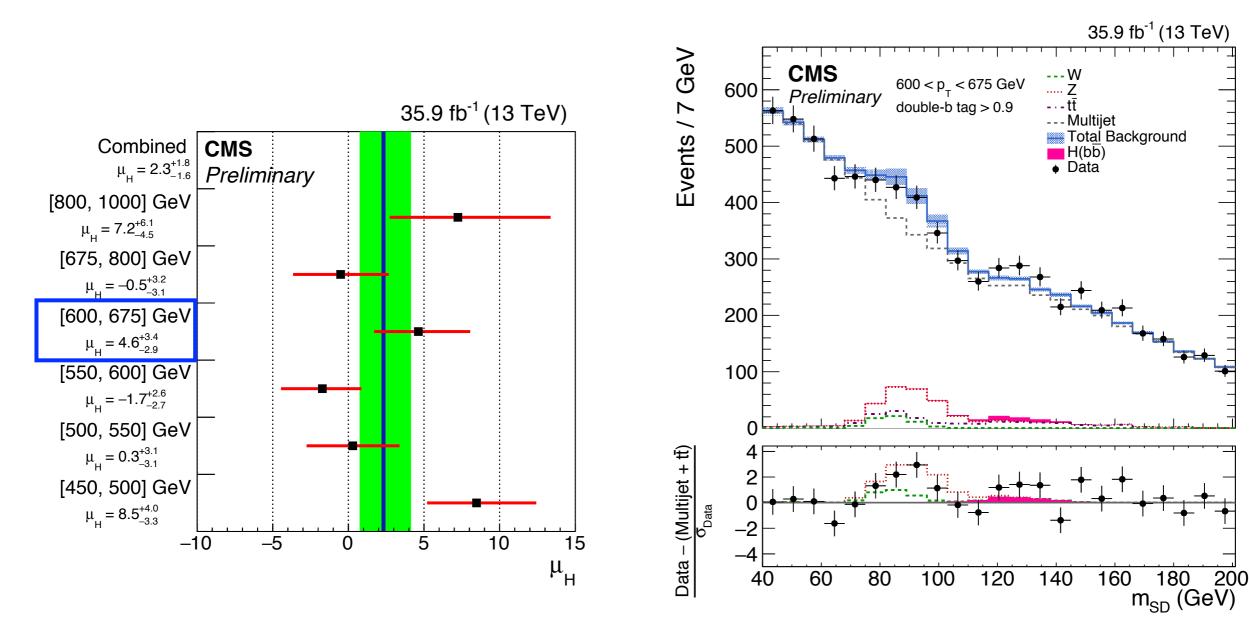
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$P_T CATEGORIES$

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LARGE HADRON COLLIDER

CMS



CERN

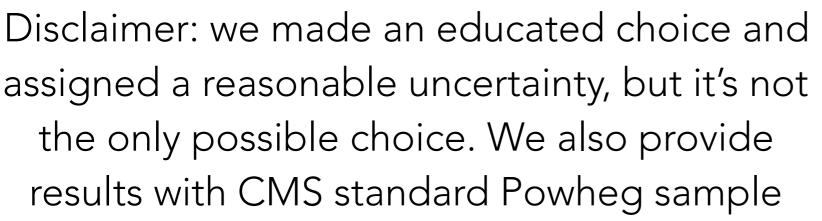
What is the best Higgs p_{τ} :Options

- The key is to identify two different effects
 - Finite top mass effect
 - NNLO differential corrections
- What are the orders known:
 - Differential EFT : NNLO H+1jet production
 - Finite top mass : almost NLO
 - At MC level EFT : NLO H+0/1/2jet
 - At MC level finite top mass : LO 0/1/2

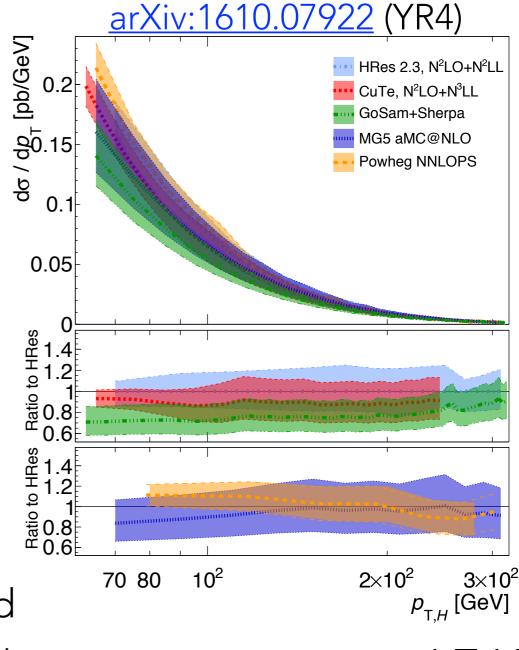




- Other CMS Higgs results use Powheg: 1 jet + m_t, <u>arXiv:</u> <u>1111.2854</u>
- We want to account for both effects of higher order corrections and finite top mass
 - No real NLO + finite top mass calculation available in the literature



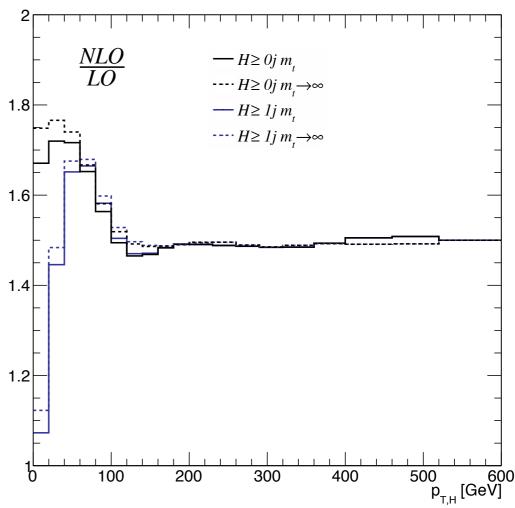




« 1 TeV



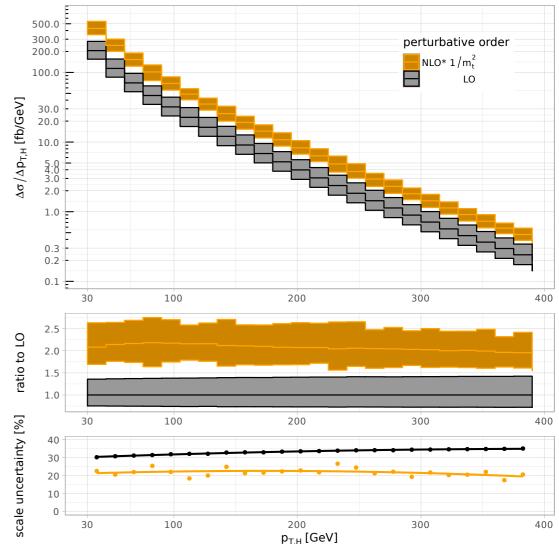
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 - No real NLO + finite top mass calculation available in the literature
- Adopt a factorized approach:
- LO H+0-2jet, finite m_t, p_t⁻ up to 600 GeV, including WW acceptance cuts <u>arXiv:1410.5806</u> → We build on this



ggF H(NNLO +
$$m_t$$
) = Powheg(1 jet m_t) × $\frac{MG \text{ LO } 0 - 2 \text{ jet } m_t}{Powheg(1 \text{ jet } m_t)}$
CKKW merged



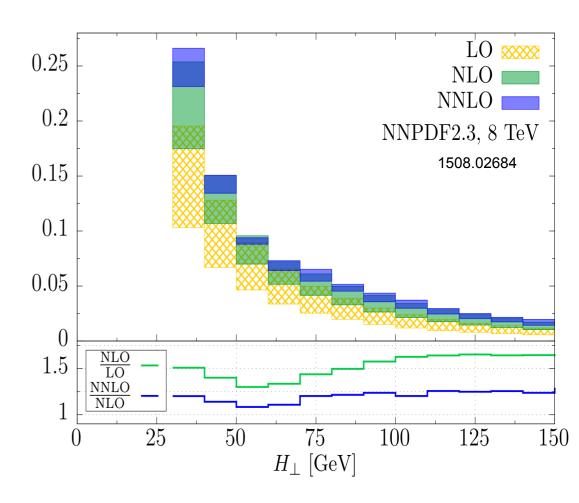
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- NLO H+1jet finite m_t up to 1/m_t expansion: <u>arXiv:</u> <u>1609.00367</u>



$$ggF H(NNLO + m_t) = Powheg(1 \text{ jet } m_t) \times \frac{MG \ LO \ 0 - 2 \ \text{jet } m_t}{Powheg(1 \ \text{jet } m_t)} \times \frac{NLO \ 1 \ \text{jet } m_t}{LO \ 1 \ \text{jet } m_t}$$
$$CKKW \ merged \qquad factor \ of \ 2$$



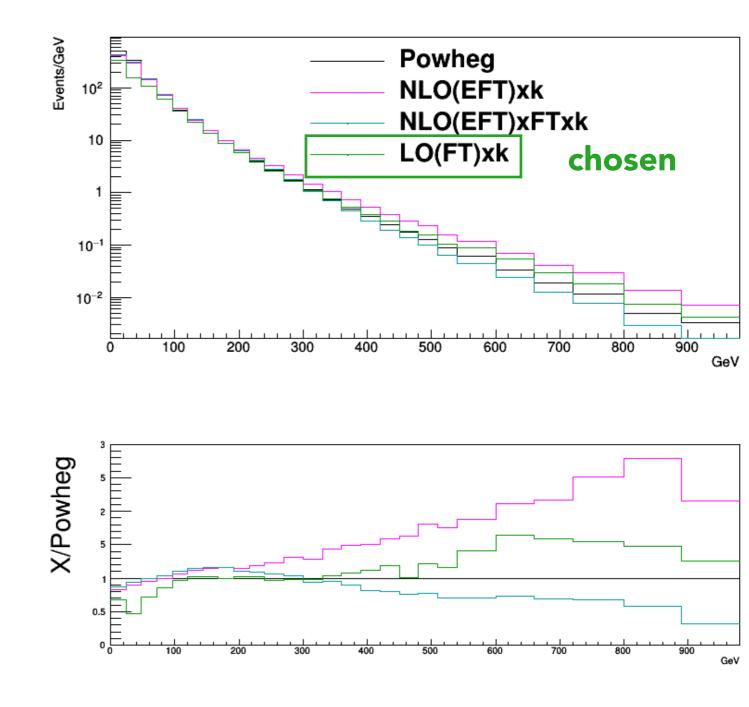
- Other CMS Higgs results use Powheg: 1 jet + m_t, <u>arXiv:</u> <u>1111.2854</u>
- We want to account for both effects of higher order corrections and finite top mass
 - No real NLO + finite top mass calculation available in the literature
- Adopt a factorized approach:
- LO H+0-2jet, finite m_t, p_t up to 600 GeV, including WW acceptance cuts <u>arXiv:1410.5806</u> → We build on this
- NLO H+1jet finite m_t up to 1/m_t expansion: <u>arXiv:</u> <u>1609.00367</u>
- NNLO H+1jet, m_t = ∞, p_T up to ~200 GeV, arXiv: 1408.5325, arXiv:1302.6216, arXiv:1504.07922, arXiv: 1505.03893, <u>arXiv:1508.02684</u>



$$ggF H(NNLO + m_t) = Powheg(1 \text{ jet } m_t) \times \frac{MG \ LO \ 0 - 2 \ \text{jet } m_t}{Powheg(1 \ \text{jet } m_t)} \times \frac{NLO \ 1 \ \text{jet } m_t}{LO \ 1 \ \text{jet } m_t} \times \frac{NNLO \ 1 \ \text{jet } m_t \to \infty}{NLO \ 1 \ \text{jet } m_t \to \infty}$$
$$CKKW \ merged \qquad factor \ of \ 2 \qquad factor \ of \ 1.25$$



- Pythia version of CKKW-L merged 0,1,2jet LO finite top mass
- ME generation in aMC@NLO (ptj > 20) with xqcut = 30 GeV
- CKKW shower is extended down to a merging scale of TMS = 20 GeV
- Two factorized systematic uncertainties:
 - 30% overall normalization
 - 30% linear change in slope (no effect on overall norm.)

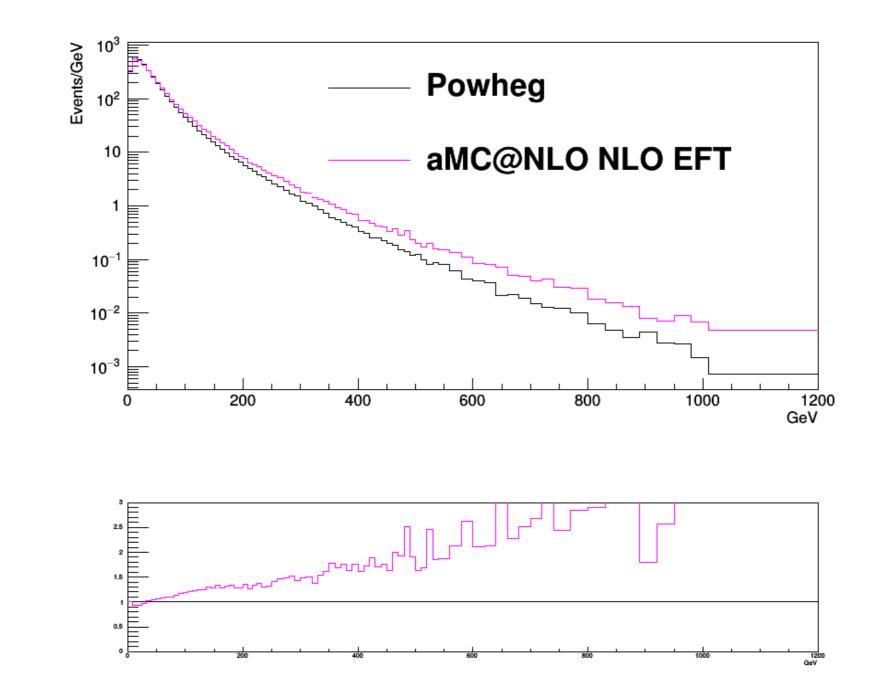






Going to EFT

When going to EFT large gain



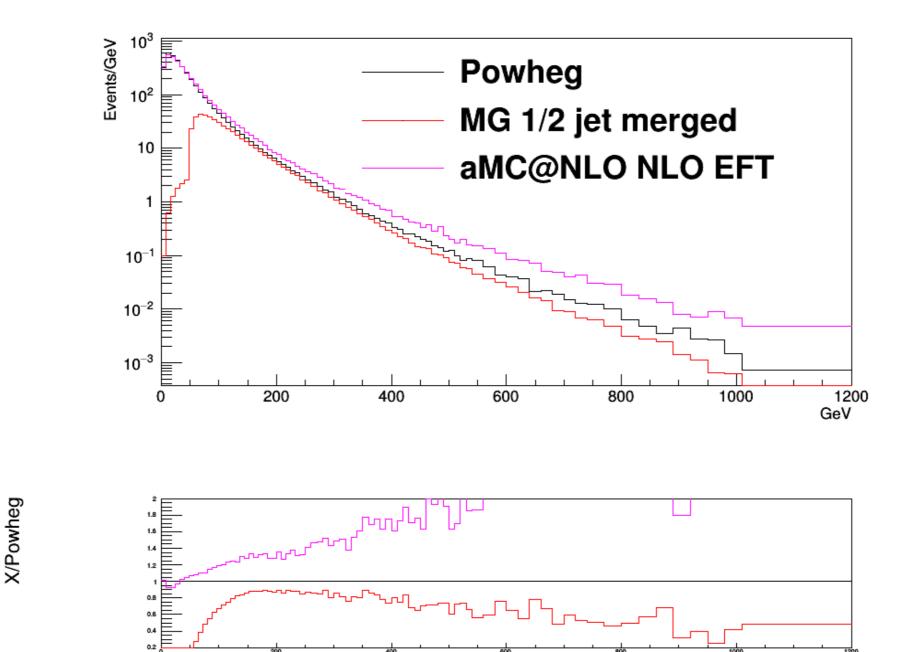


X/Powheg



Going to EFT

Adding the finite top mass merged LO its lower

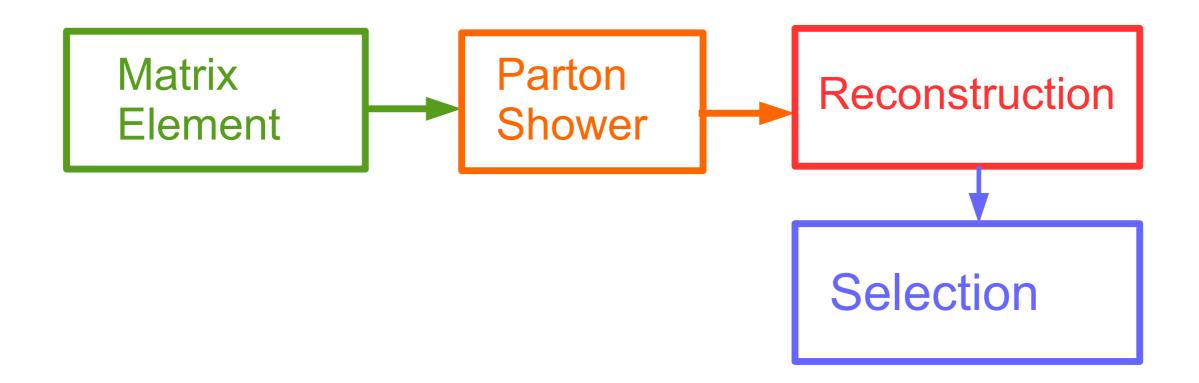






What are differences in our numbers?

Chain from ME to Reconstruction



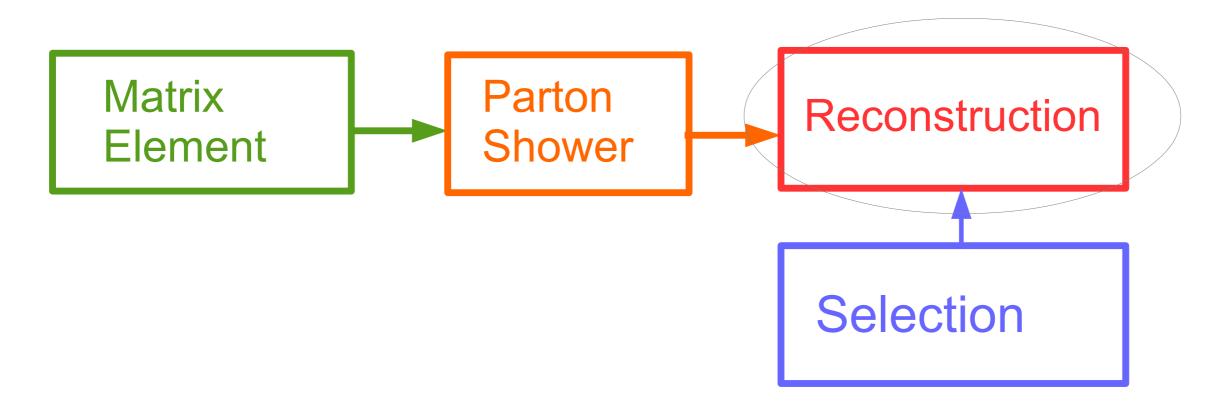
In the paper quote a number on : Selected Higgs Jets with $p_{\tau} > 450$ GeV





Back tracking

Chain from ME to Reconstruction



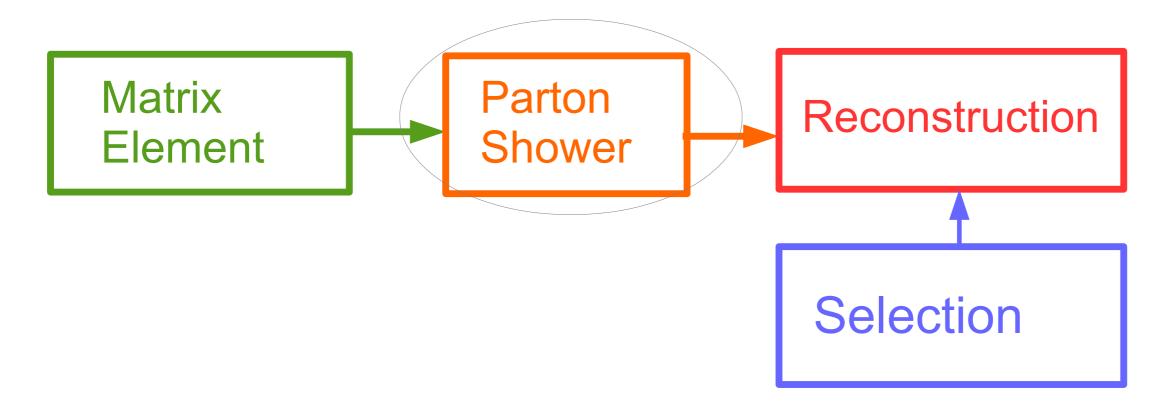
Our reconstructed cross section for our sample is : <u>26.2 fb</u> (-25% for other processes/selection)





Back tracking

Chain from ME to Reconstruction



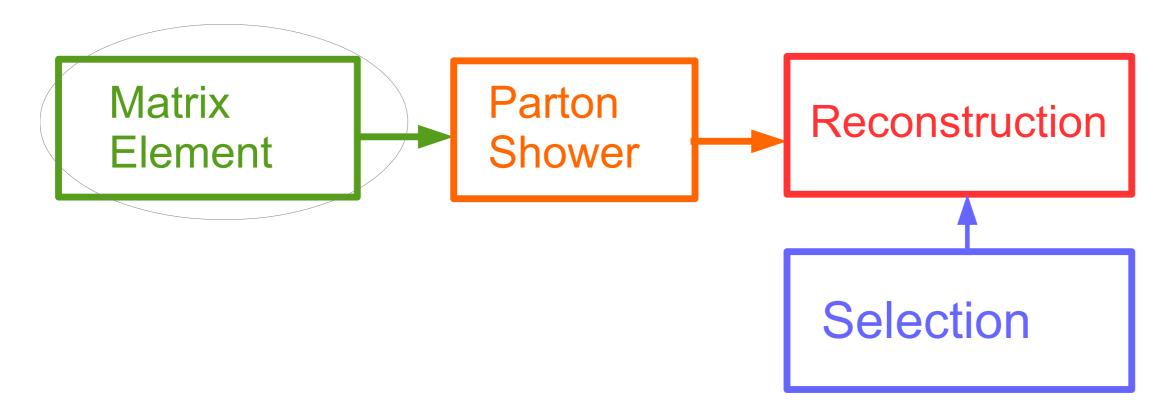
Our parton shower level cross section for p_T Higgs > 450 GeV is : 20.8 fb (-25% reco smearing pushes lower p_T Higgs to higher p_T)





Back tracking

Chain from ME to Reconstruction



Our ME level cross section for p_{T} Higgs > 450 GeV is : 15<u>.1 fb</u> (-35% parton shower pushes low p_{T} to higher p_{T}) Reminder table is : 8.9 fb

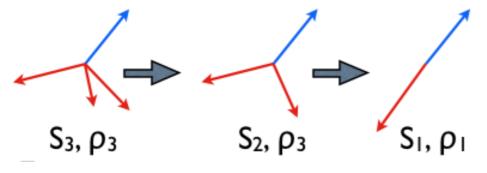




CKKW-L [L.Lönnblad, JHEP 05 (2002) 046, L.Lönnblad and S.Prestel, JHEP 03 (2012) 019

Idea: Reduce the dependence to the merging scale MS.

- Start by generate events with $N_1..N_2$ ME partons, hard and well separated
- Assume an event with n ME partons, reconstruct the possible shower histories, pick one according to the occurence probabilities



- Each clustering step *i* is characterized by the emission scale ρ_i , reweight by the product of $\alpha_s(\rho_i)/\alpha_s(ME)$
- For i=2..n (!=N₂):

Simon de Visscher (CERN)

ap

- Generate one emission ρ with ρ_i as starting scale.
- If $\rho > \rho_{i+1} \Rightarrow$ reject the event.
 - This is equivalent to the product of Sudakovs $\Pi(\rho_i, \rho_{i+1})$, i=2..n-1.
- if not HME: generate an emission at $\rho < \rho_n$, if $\rho > MS \Rightarrow$ reject the event.
- if HME, accept the event and start shower with ρ_n .

 $\checkmark Q (~$

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Essential parameters for matching/merging:

- ickkw
 - Applies the α_s reweighting at each QCD vertex in the ME calculation. K_T -MLM, Shower- K_T :1 CKKW-L, UMEPS:0
- xqcut
 - Defines the minimal K_T between the partons (+beam) at ME level.
- auto_ptj_mjj
 - Set to False: leaves the xqcut be the only cut applied to ME partons \Rightarrow ptj, mmjj=0
- maxjetflavor
 - QCD partons with pdgId < maxjetflavor are affected by xqcut ptj,etc... Otherwise, affected by ptb, mmbb, etc... That means that for a n-Flavour prediction, maxjetflavor = n



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Practical use: main89.cc

• main89ckkwl.cmnd: CKKWL. Essential parameters are

- Merging:TMS = XXX.
 - The merging scale
- Merging:Process = UUU
 - Type of process, e.g. pp>LEPTONS,NEUTRINOS
- Merging:nJetMax = WWW
 - Maximal number of additional jets in the matrix element
- Merging:doPTLundMerging = on
 - Set the merging scale definition to $P_{T,evol}$ (cfr definition in the manual)

• main89umeps.cmnd: UMEPS. Essential parameters are

- Merging:TMS = XXX.
- Merging:Process = (e.g.) pp>LEPTONS,NEUTRINOS
- Merging:nJetMax = WWW
- Merging:doUMEPSTree = on
 - Reweight events according to the UMEPS prescription for tree-level configurations)





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