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Gauge and Yukawa beta functions in two-Higgs-doublet models

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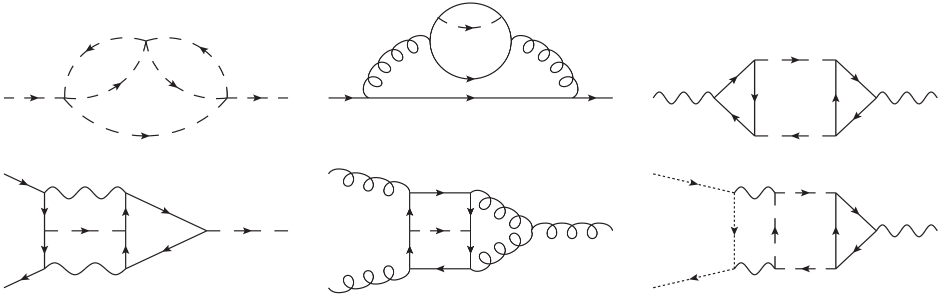
INSTITUT FÜR THEORETISCHE TEILCHENPHYSIK



QCD beta function

In $\overline{\mathrm{MS}}$-scheme strong coupling depends on renormalization scale μ

$$\beta_{\alpha_s} = \mu^2 \frac{d}{d\mu^2} \frac{\alpha_s}{\pi}$$

$$\beta_{\alpha_s} = - \sum_{i \geq 0} \beta_i \left(\frac{\alpha_s}{\pi} \right)^{i+2}$$

$$\beta_0 = \frac{1}{4} \left(11 - \frac{2}{3} n_f \right)$$

β_4 (5-loop) recently calculated:
[Baikov, Chetyrkin, Kühn 2016],
[Herzog, Ruijl, Ueda, Vermaseren 2017],
[Luthe, Maier, Marquard, Schroder 2017]

Stability of the electroweak vacuum

In the SM, λ becomes negative around 10^{11} GeV

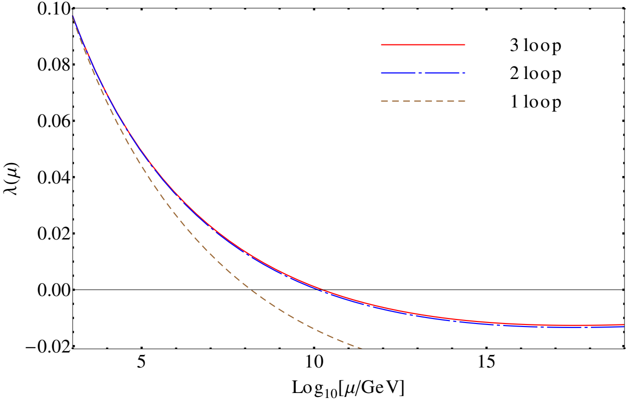


Figure: [Zoller 2014]

Stability of the electroweak vacuum

Three-loop contribution to beta function small, indicate validity of pert. expansion

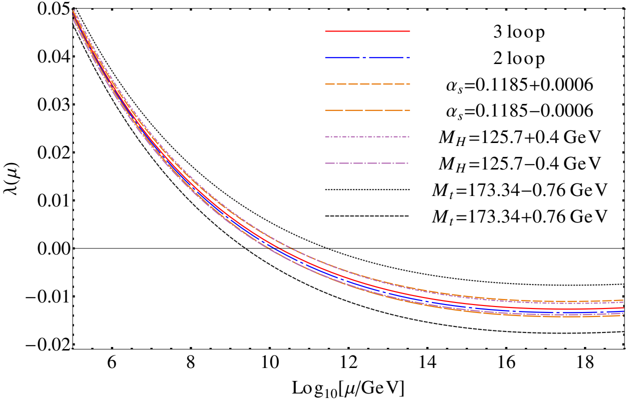


Figure: [Zoller 2014]

Beta functions in more complex theories

- In the SM there are 7 relevant couplings $\alpha_1, \alpha_2, \alpha_s, y_t, y_b, y_\tau, \lambda$
- Three-loop beta functions in the SM availabe for all couplings
  - Gauge couplings [Mihaila, Salomon, Steinhauser 2012], [Bednyakov, Pikelner, Velizhanin 2012]
  - Yukawa couplings [Bednyakov, Pikelner, Velizhanin 2012], Yukawa matrices [Bednyakov, Pikelner, Velizhanin 2014]
  - Higgs self-coupling [Chetyrkin, Zoller 2013], [Bednyakov, Pikelner, Velizhanin 2013]
- Goal of this work:
  1. extend to two-Higgs-doublet model
  2. cross-check SM results for Yukawa couplings

2HDM

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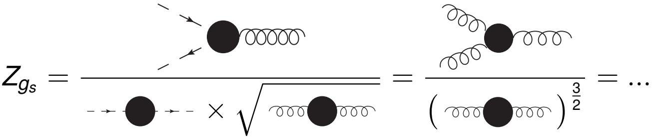
- Simple extension of the SM, adding one more scalar doublet
- Gauge structure remains as in SM
- Nontrivial changes in Yukawa sector and scalar potential

Renormalization

- Couplings renormalization constants computed via

$$Z_g = \frac{Z_V}{\prod_\Phi \sqrt{Z_\Phi}}$$

- Compute 2- and 3-point Green's functions up to three-loop
- Slavnov-Taylor identities relate renormalization constants



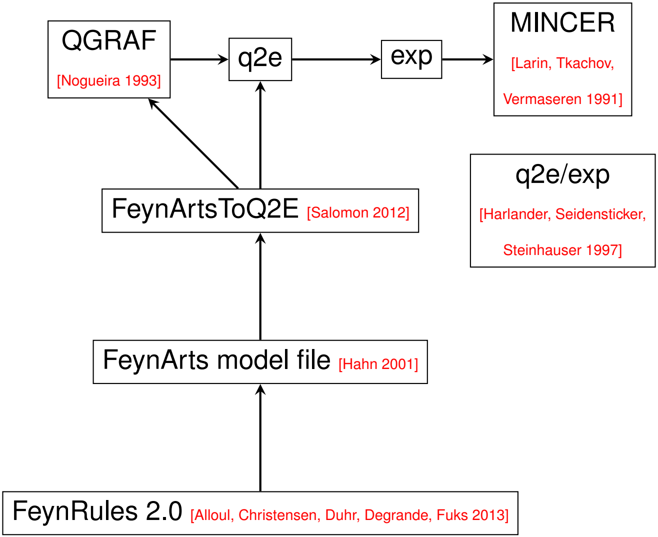
Calculation of Green's functions

- Pole parts of logarithmically divergent integrals are independent of masses and momenta
  $\rightarrow$ Calculation in unbroken phase (all fields massless)
  $\rightarrow$ Set one ext. momentum in 3-point functions to zero
  $\rightarrow$ Only massless 2-point integrals
- Keep all 3 gauge parameters, ξ_B, ξ_W, ξ_G
- γ_5 treated in a "semi-naive" way

$$\mathrm{tr}\left(\gamma_5\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\sigma\right) = -4i\epsilon^{\mu\nu\rho\sigma} + \mathcal{O}(\epsilon)$$

$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu'\nu'\rho'\sigma'} = g^{[\mu}_{[\mu'}\, g^{\ \nu}_{\ \nu'}\, g^{\ \rho}_{\ \rho'}\, g^{\ \sigma]}_{\ \sigma']}$$

Setup


QGRAF
[Nogueira 1993]
q2e
exp
MINCER
[Larin, Tkachov,
Vermaseren 1991]
q2e/exp
[Harlander, Seidensticker,
Steinhauser 1997]
FeynArtsToQ2E [Salomon 2012]
FeynArts model file [Hahn 2001]
FeynRules 2.0 [Alloul, Christensen, Duhr, Degrande, Fuks 2013]

2HDM

- We consider the most general 2HDM
- Yukawa matrices carry 2 different generation indices and one doublet index

$$\mathcal{L}_{\text{Yuk}} = -\sum_{i=1}^{2}\sum_{j,k=1}^{3}\Big(\overline{Q}_{L\,j}\tilde{\Phi}_i\left(Y^u\right)^{i}_{jk}u_{R\,k} + \overline{Q}_{L\,j}\Phi^i\left(Y^d\right)_{ijk}d_{R\,k}$$

$$+\overline{L}_{L\,j}\tilde{\Phi}_i\left(Y^\nu\right)^{i}_{jk}\nu_{R\,k} + \overline{L}_{L\,j}\Phi^i\left(Y^l\right)_{ijk}l_{R\,k} + \text{h.c.}\Big)$$

- Type II:

$$\mathcal{L}_{\text{Yuk}} = -\sum_{j,k=1}^{3}\Big(\overline{Q}_{L\,j}\tilde{\Phi}_1\left(Y^u\right)_{jk}u_{R\,k} + \overline{Q}_{L\,j}\Phi^2\left(Y^d\right)_{jk}d_{R\,k}$$

$$+\overline{L}_{L\,j}\Phi^2\left(Y^l\right)_{jk}l_{R\,k} + \text{h.c.}\Big)$$

2HDM

- Quartic couplings carry 4 doublet indices

$$V(\Phi) = \sum_{i,j=1}^{2} (m^2)^i_j \left(\Phi^\dagger_i \Phi^j\right) + \frac{1}{2} \sum_{i,j,k,l=1}^{2} \lambda^{ik}_{jl} \left(\Phi^\dagger_i \Phi^j\right) \left(\Phi^\dagger_k \Phi^l\right)$$

- In $\mathbb{Z}_2$-symmetric models:

$$\begin{aligned} V(\Phi) = {} & m^2_{11} \left(\Phi^\dagger_1 \Phi^1\right) + m^2_{22} \left(\Phi^\dagger_2 \Phi^2\right) \\ & + \frac{\lambda_1}{2} \left(\Phi^\dagger_1 \Phi^1\right)^2 + \frac{\lambda_2}{2} \left(\Phi^\dagger_2 \Phi^2\right)^2 + \lambda_3 \left(\Phi^\dagger_1 \Phi^1\right) \left(\Phi^\dagger_2 \Phi^2\right) \\ & + \lambda_4 \left(\Phi^\dagger_1 \Phi^2\right) \left(\Phi^\dagger_2 \Phi^1\right) + \frac{1}{2} \left[\lambda_5 \left(\Phi^\dagger_1 \Phi^2\right)^2 + \mathrm{h.c.}\right] \end{aligned}$$

2HDM

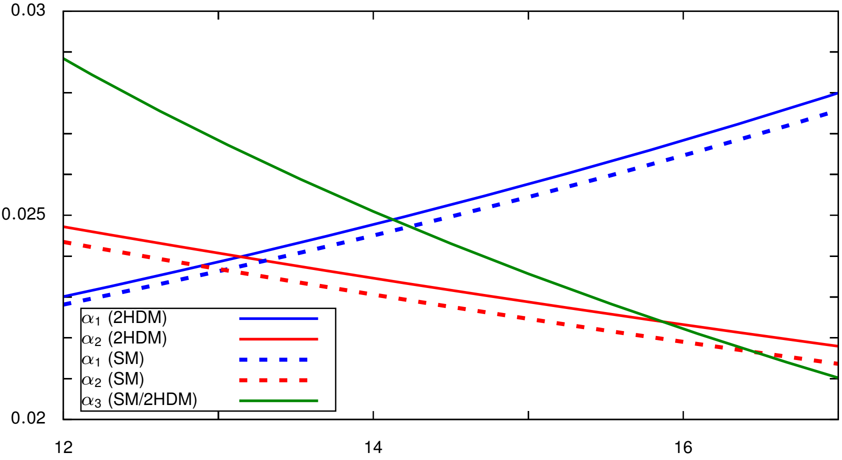
- Gauge coupling beta functions carry no open indices
  $\rightarrow Y^{u\,a}_{ij} Y^{u\dagger}_{aji}, \lambda^{ab}_{cd}\lambda^{cd}_{ab}$, ...
- Ren. constants of fermion fields are matrices in flavour space
  $\rightarrow Y^{u\,a}_{ij} Y^{u\dagger}_{ajk}, Y^{u\,a}_{ij} Y^{u\dagger}_{bjk} Y^{d}_{alm} Y^{d\dagger\,b}_{ml}$, ...
- Ren. constants of scalar fields are matrices in doublet space
  $\rightarrow Y^{u\,a}_{ij} Y^{u\dagger}_{bji}, \lambda^{ac}_{de}\lambda^{de}_{cb}$, ...

β_{α_2}

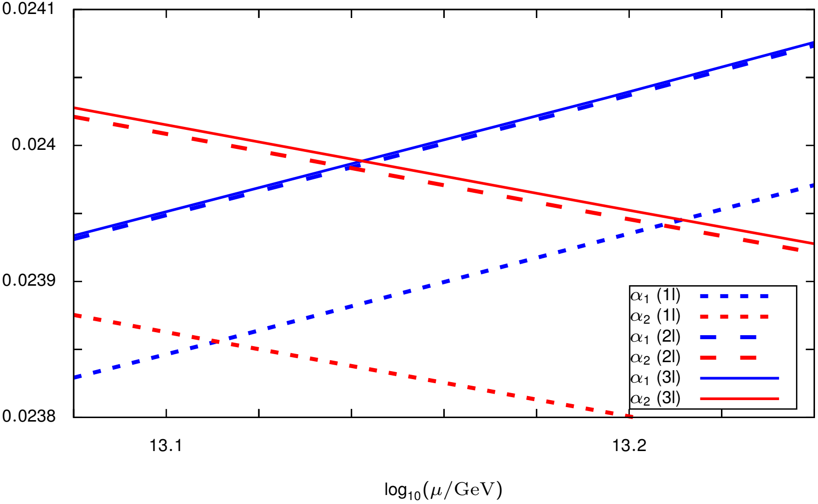
$$
\begin{aligned}
\beta_{\alpha_2} =& -\epsilon\frac{\alpha_2}{\pi} + \frac{\alpha_2^2}{(4\pi)^2}\left[-\frac{88}{3}+\frac{16}{3}n_g+\frac{2}{3}n_h\right] \\
&+ \frac{\alpha_2^2}{(4\pi)^3}\left[-\frac{544\alpha_2}{3}+n_g\left(\frac{4\alpha_1}{5}+\frac{196\alpha_2}{3}+16\alpha_3\right)+n_h\left(\frac{6\alpha_1}{5}+\frac{26\alpha_2}{3}\right)-2N_c(F)\sum_F \mathrm{T}F1\right] \\
&+ \frac{\alpha_2^2}{(4\pi)^4}\left[-\frac{45712\alpha_2^2}{27}+n_g^2\left(-\frac{44\alpha_1^2}{45}-\frac{1660\alpha_2^2}{27}-\frac{176\alpha_3^2}{9}\right)+n_h^2\left(-\frac{49\alpha_1^2}{200}-\frac{425\alpha_2^2}{216}\right)\right. \\
&\quad + n_g n_h\left(-\frac{91\alpha_1^2}{50}-\frac{1121\alpha_2^2}{54}\right)+n_h\left(\frac{261\alpha_1^2}{400}+\frac{561\alpha_1\alpha_2}{40}+\frac{65131\alpha_2^2}{432}\right) \\
&\quad + n_g\left(-\frac{7\alpha_1^2}{150}+\frac{13\alpha_1\alpha_2}{15}-\frac{4\alpha_1\alpha_3}{15}+\frac{52417\alpha_2^2}{54}+52\alpha_2\alpha_3+\frac{500\alpha_3^2}{3}\right) \\
&\quad + \frac{5}{2}\sum_{F,F'}N_c(F)N_c(F')\mathrm{T}F\mathrm{T}F'1+\frac{21}{2}\sum_Q \mathrm{T}QQ1+6\mathrm{T}UD1+\frac{7}{2}\sum_L \mathrm{T}LL1 \\
&\quad + 2\mathrm{T}NE1+\frac{15}{4}\sum_{Q,Q'}\mathrm{T}QQ'2+\frac{5}{4}\sum_{L,L'}\mathrm{T}LL'2-2\mathrm{LL6}-\mathrm{LL7} \\
&\quad - \frac{593\alpha_1}{40}\mathrm{T}U1-\frac{533\alpha_1}{40}\mathrm{T}D1-\frac{39\alpha_1}{8}\mathrm{T}N1-\frac{51\alpha_1}{8}\mathrm{T}E1+\frac{3\alpha_1}{5}\mathrm{L2} \\
&\quad \left. - \frac{243\alpha_2}{8}\sum_F N_c(F)\mathrm{T}F1+2\alpha_2\mathrm{L1}+\alpha_2\mathrm{L2}-28\alpha_3\sum_Q \mathrm{T}Q1\right],
\end{aligned}
$$

Running to high scales

Type II 2HDM with $\tan\beta = 50$



Running to high scales



Issues with Yukawa beta functions

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- Square root for complex matrices ambiguous
- Ren. constants hermitian
  $\rightarrow \sqrt{Z} = U\sqrt{Z}_H$
  with U unitary and $\sqrt{Z}_H$ hermitian
- $\sqrt{Z}_H$ can be extracted from Green's functions
- Choosing U equal to unity leads to poles in ϵ in anomalous dimensions of scalar and fermion fields and beta functions
- $\gamma + \gamma^\dagger$ is finite
- One U per ren. constant

Solution 1

- Demand anomalous dimensions to be finite
- Fix U using this condition
  $\rightarrow$ Beta functions also are finite
- However, it is possible to also modify finite part
  $\rightarrow$ Beta function ambiguous

Solution 2

- Find invariants under unitary rotation in doublet and flavour space, e.g.

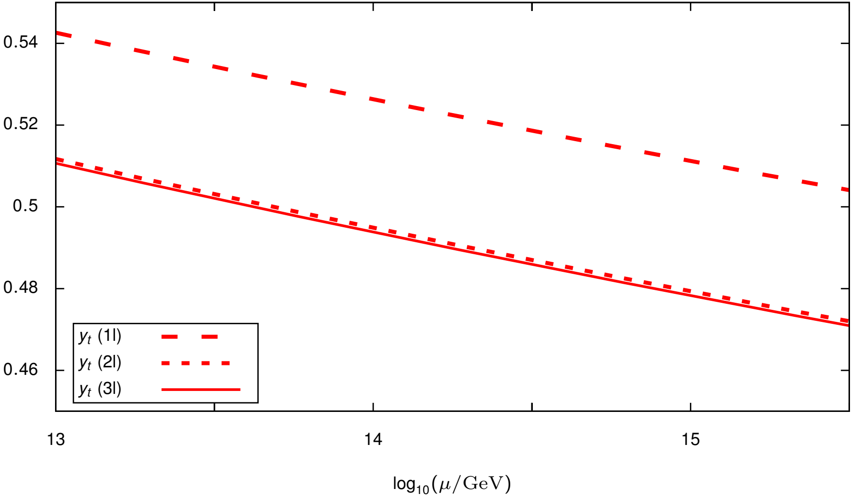
$$Y^{u\,a}_{ij} Y^{u\dagger}_{aji}, \lambda^{ab}_{cd} \lambda^{cd}_{ab}$$

- Express physical parameters by these invariants
- Choice of unitary part of the root does not influence anomalous dimensions
  $\rightarrow$ Evolution independent of U
- Anomalous dimensions finite

Results

- All renormalization constants local
- Dependence on gauge parameters drop out in ren. constants of couplings
- γ_5 does not introduce an ambiguity
- In the SM limit, full agreement with [Bednyakov, Pikelner, Velizhanin 2014] is found

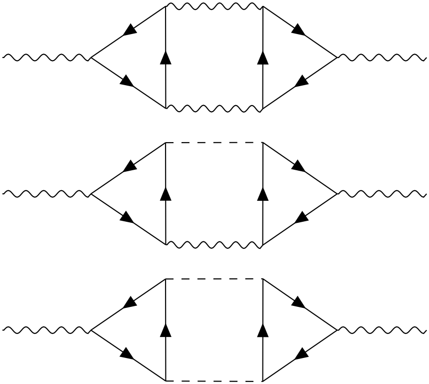
Running to high scales



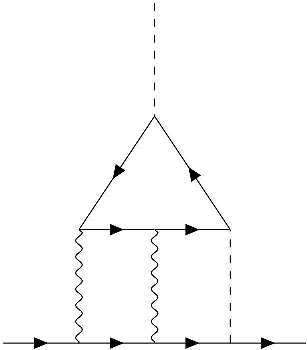
Conclusion

- General 2HDM
- Full flavour structure
- Three-loop gauge and Yukawa beta functions
- Issues with matrix-like couplings at higher orders understood
- SM results for Yukawa matrices confirmed

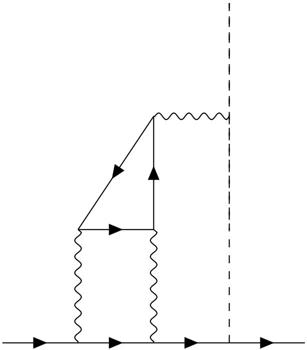
Difficulties with γ_5



Difficulties with γ_5



dangerous diagram



anomaly cancelation