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HIGGSPLOSION

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IPPP Durham

- VVK & Michael Spannowsky 1704.03447, 1707.01531
- VVK 1705.04365
- VVK, J Reiness, M Spannowsky, P Waite 1709.08655
- & with Jakub Scholtz & Michael Spannowsky — to appear

100th Anniversary of Russian Revolution

7 November 2017

Now: an altogether different radical idea

- Before the Higgs discovery, massive Yang-Mills theory violated perturbative unitarity — problem with high-energy growth of 2 -> 2 processes

- Discovery of the (elementary) Higgs made the SM theory self-consistent

- The Higgs brings in the Hierarchy problem: radiative corrections push the Higgs mass to the new physics (high) scale: $m_h^2 \simeq m_0^2 + \delta m_{\rm new}^2$

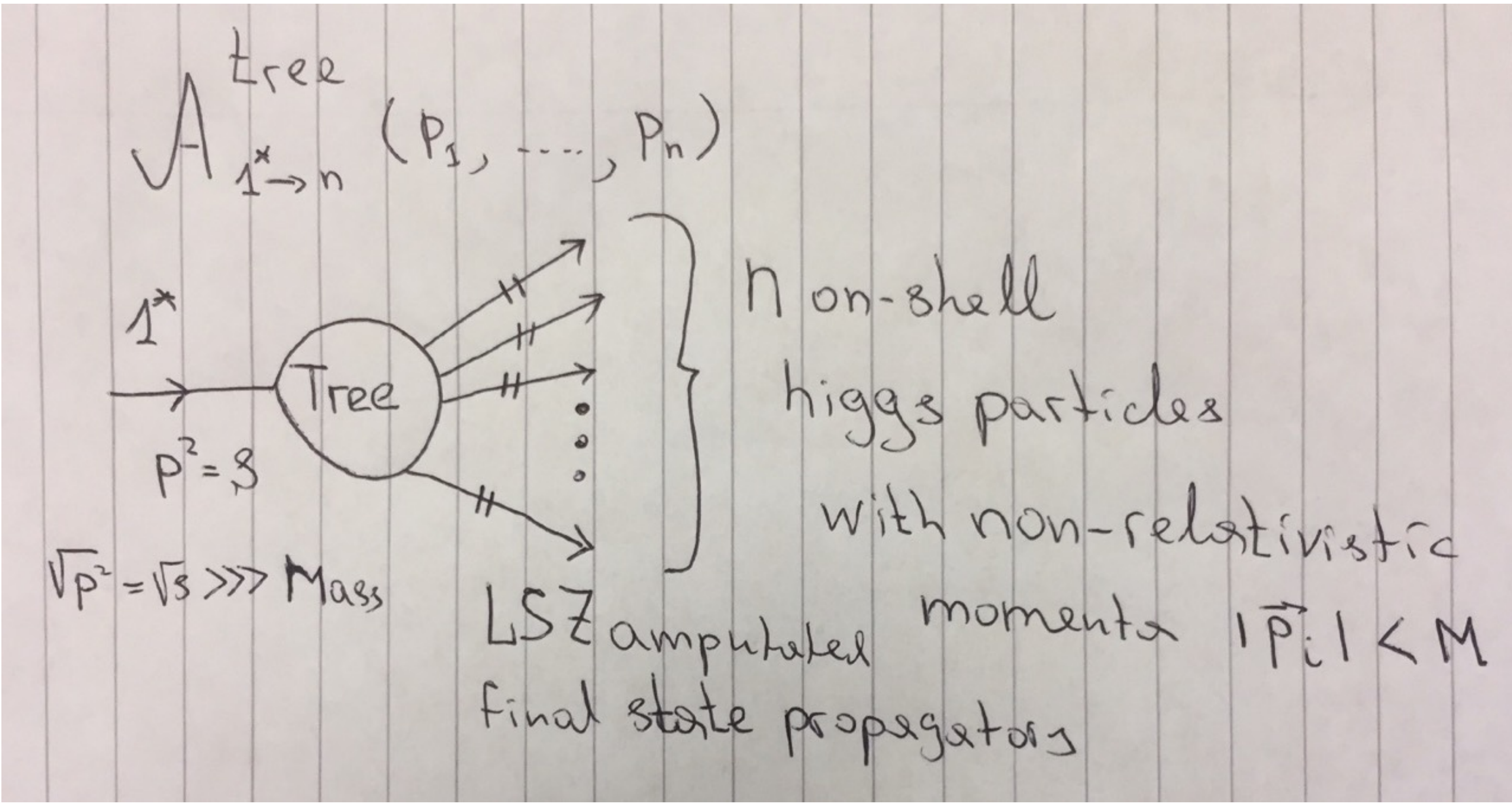
- In this talk: consider n~100s of Higgs bosons produced in the final state n lambda >> 1. Investigate scattering processes at ~ 100 TeV energies.

- HIGGSPLOSION: n-particle rates computed in a weakly-coupled theory can become unsuppressed above critical values of n and E. Perturbative and non-perturbative semi-classical calculations. n! ~ exponential growth with n or E. (Scale n~E/m).

- A new unitarity problem — caused by the elementary Higgs bosons — appears to occur (?) for processes with large final state multiplicities n >> 1

- HIGGSPLOSION offers a solution to both problems: it restores the unitarity of high-multiplicity processes and dynamically cuts off the values of the loop momenta contributing to the radiative corrections to the Higgs mass.

Compute 1 -> n amplitudes @LO with non-relativistic final-state momenta:



see classic 1992-1994 papers:
Brown; Voloshin;
Argyres, Kleiss, Papodopoulos
Libanov, Rubakov, Son, Troitski

more recently: VVK 1411.2925

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4}(h^2 - v^2)^2$$

prototype of the SM Higgs
in the unitary gauge

Tree-level $1^* \to n$ amplitudes in the limit $\varepsilon \to 0$ for any n are given by

$$\mathcal{A}_n(p_1, \dots p_n) = \boxed{n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}}} \left(1 - \frac{7}{6}n\varepsilon - \frac{1}{f}\frac{n}{n-1}\varepsilon + \mathcal{O}(\varepsilon^2)\right)$$

factorial growth

amplitude on the n-particle threshold

$$\varepsilon = \frac{1}{n\, M_h} E_n^{\rm kin} = \frac{1}{n}\frac{1}{2M_h^2}\sum_{i=1}^{n} \vec{p}_i^{\,2}$$

kinetic energy per particle per mass

In the large-n-non-relativistic limit the result is

$$\boxed{\mathcal{A}_n(p_1, \dots p_n) = n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}} \exp\left[-\frac{7}{6}n\varepsilon\right], \quad n \to \infty,\ \varepsilon \to 0,\ n\varepsilon = \text{fixed}}$$

Can now integrate over the n-particle phase-space

The cross-section and/or the n-particle partial decay Γ_n

$$\Gamma_n(s) \;=\; \int d\Phi_n \, \frac{1}{n!} \, |\mathcal{A}_{h^*\to n\times h}|^2$$

The n-particle Lorentz-invariant phase space volume element

$$\int d\Phi_n \;=\; (2\pi)^4 \delta^{(4)}(P_{\rm in} - \sum_{j=1}^{n} p_j) \, \prod_{j=1}^{n} \int \frac{d^3 p_j}{(2\pi)^3 \, 2p_j^0} \,,$$

in the large-n non-relativistic limit with $n\varepsilon_h$ fixed becomes,

$$\Phi_n \;\simeq\; \frac{1}{\sqrt{n}} \left(\frac{M_h^2}{2}\right)^n \, \exp\left[\frac{3n}{2}\left(\log\frac{\varepsilon_h}{3\pi}+1\right) \,+\, \frac{n\varepsilon_h}{4} \,+\, \mathcal{O}(n\varepsilon_h^2)\right]$$

We find:

$$\boxed{\Gamma_n^{\rm tree}(s) \;\sim\; \exp\left[n\left(\log\frac{\lambda n}{4} - 1\right) \,+\, \frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right) \,-\, \frac{25}{12}\, n\varepsilon \,+\, \mathcal{O}(n\varepsilon^2)\right]}$$

Son 1994;
Libanov, Rubakov, Troitskii 1997; more recently: VVK 1411.2925

- The n! growth of perturbative amplitudes is not entirely surprising: the number of contributing Feynman diagrams is known to grow factorially with n. [In scalar QFT there are no partial cancellations between individual diagrams (unlike QCD).]

- Important to distinguish between the two types of large-n corrections:

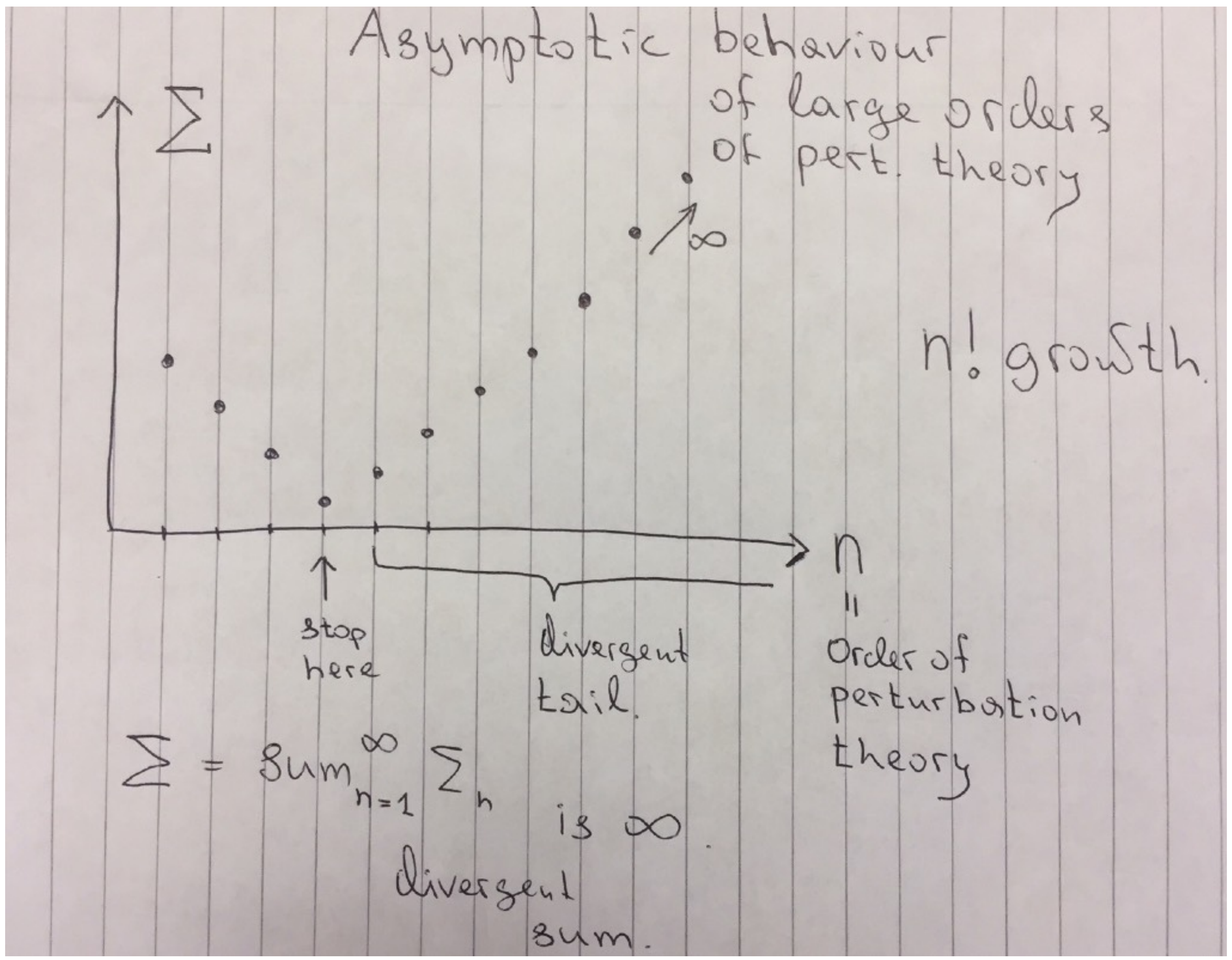
(a) *higher-order* perturbative corrections to some leading-order quantities

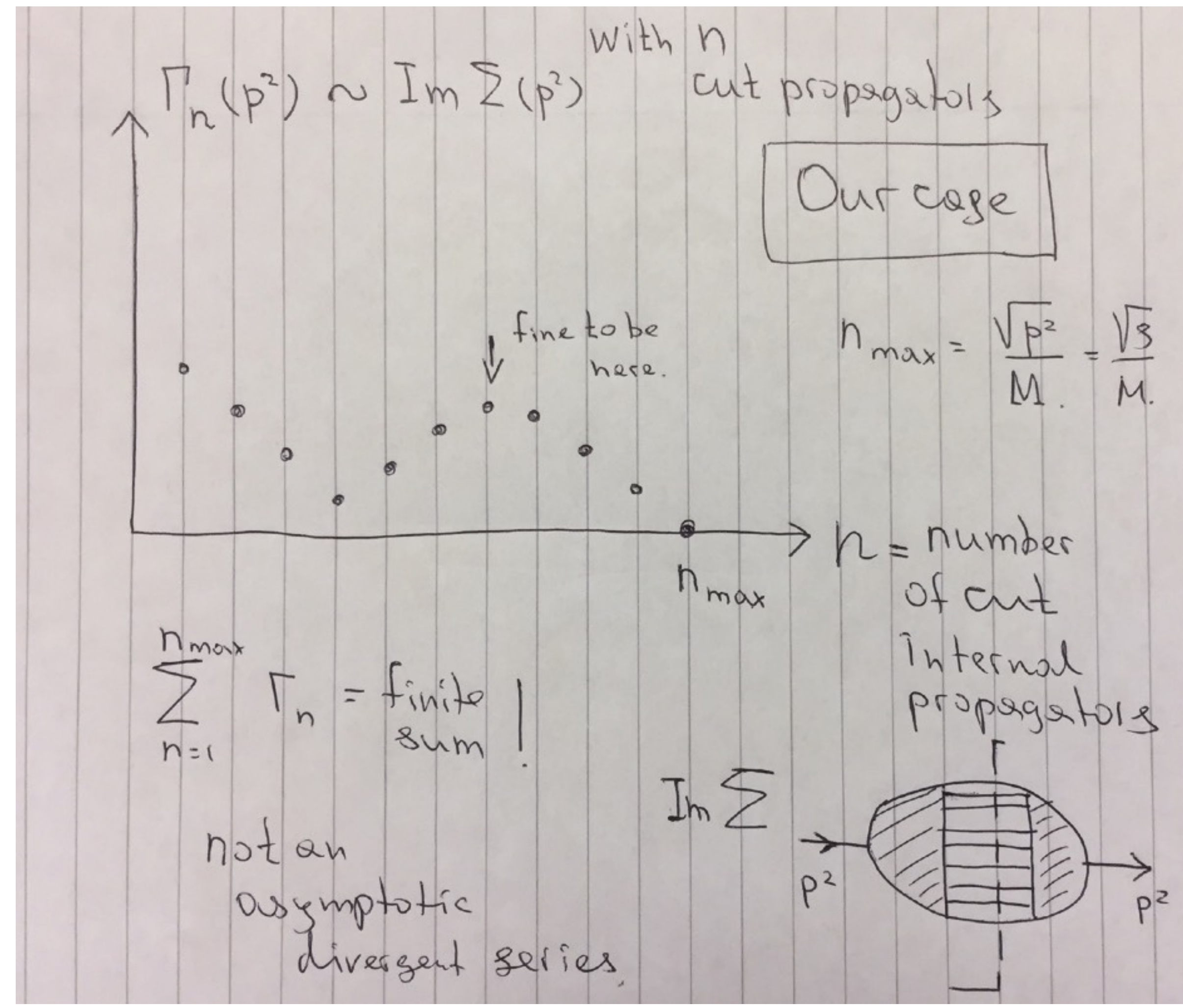
(b) our case where the *leading-order* tree-level contribution to the 1*->n Amplitude grows factorially with the particle multiplicity n of the final state.

- This was studied in the 90s in scalar QFTs

- But now realised that the characteristic energy scale for EW applications starts in the 50-100 TeV range. FCC would provide an exciting challenge to realise this in the context of the multi- Higgs and Massive Vector bosons production in the SM.

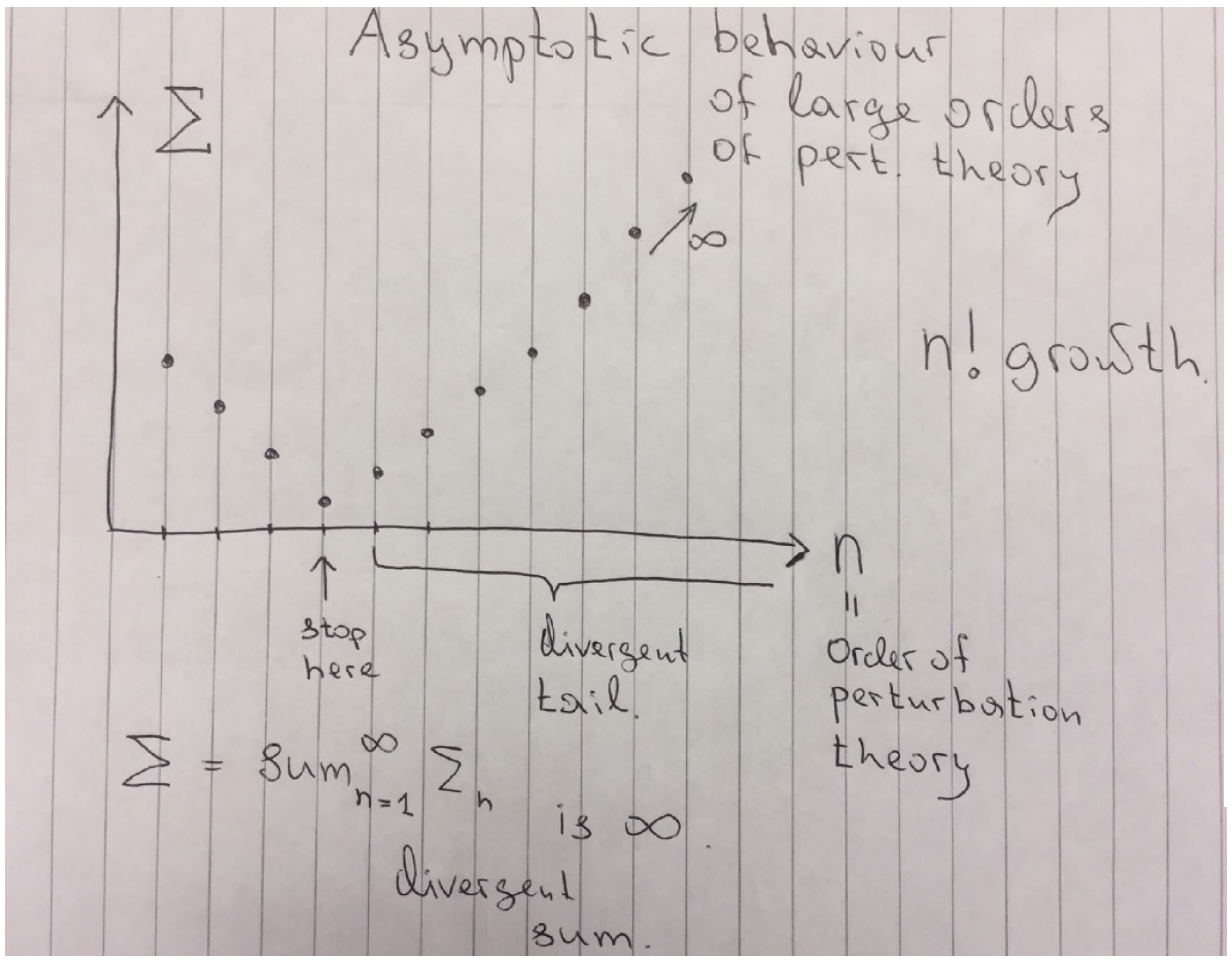
Contrast asymptotic growth of higher-order corrections in perturbation theory with the ~n! contributions to Gamma_n(s)

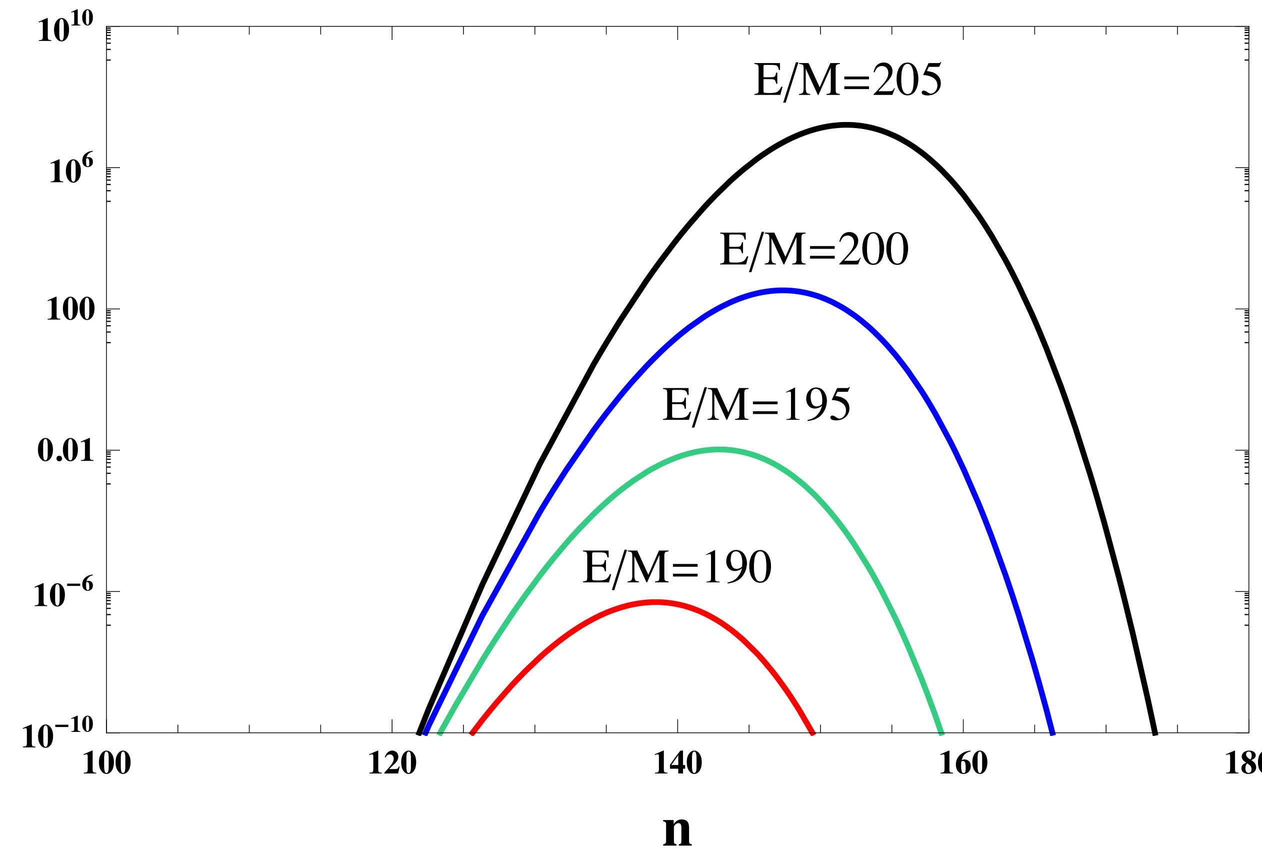




Not the same types of beasts

Contrast asymptotic growth of higher-order corrections in perturbation theory with the ~n! contributions to Gamma_n(s)





For Gamma_n(E) we'll find this

Not the same types of beasts

Perturbative as well as semi-classical calculations result in the exponential form for the n-particle width Gamma ~ exp[F_holy_grail]

- Libanov, Rubakov, Son, Troitsky; Son: 1994-1995

In the non-rel. limit for perturbative Higgs bosons only production we obtained:

bare cross-section [ignoring the width effect for now]

$$\boxed{\sigma_n \propto \exp\left[n\left(\log\frac{\lambda n}{4}-1\right)+\frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right)-\frac{25}{12}\,n\,\varepsilon\right]}$$

More generally, in the large-n limit with $\lambda n =$ fixed and $\varepsilon =$ fixed, one expects

$$\boxed{\sigma_n \propto \exp\left[\frac{1}{\lambda}\,F_{\rm h.g.}(\lambda n,\varepsilon)\right]}$$

[e.g. Libanov, Rubakov, Troitsky review 1997]

where the *holy grail* function $F_{\rm h.g.}$ is of the form,

$$\boxed{\frac{1}{\lambda}\,F_{\rm h.g.}(\lambda n,\varepsilon) = \frac{\lambda n}{\lambda}\left(f_0(\lambda n)+f(\varepsilon)\right)}$$

In our higgs model, i.e. the scalar theory with SSB,

$$\begin{aligned} f_0(\lambda n) &= \log\frac{\lambda n}{4}-1 && \text{at tree level} \\ f(\varepsilon) &\to \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right)-\frac{25}{12}\,\varepsilon && \text{for } \varepsilon \ll 1 \end{aligned}$$

Can also include *loop corrections* to amplitudes on thresholds:

The 1-loop corrected threshold amplitude for the pure n Higgs production:

$$\boxed{\phi^4 \text{ with SSB}: \quad \mathcal{A}_{1\to n}^{\text{tree+1loop}} = n!\,(2v)^{1-n}\left(1 + n(n-1)\frac{\sqrt{3}\lambda}{8\pi}\right)}$$

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

Libanov, Rubakov, Son, Troitsky 1994

$$\boxed{\mathcal{A}_{1\to n} = \mathcal{A}_{1\to n}^{\text{tree}} \times \exp\left[B\,\lambda n^2 + \mathcal{O}(\lambda n)\right]}$$

in the limit $\lambda \to 0$, $n \to \infty$ with λn fixed. Here B is determined from the 1-loop calculation (as above) – *Smith; Voloshin 1992*): $B = +\lambda n \frac{\sqrt{3}}{4\pi}$

$$\boxed{\begin{aligned} f_0(\lambda n) &= \log\frac{\lambda n}{4} - 1 + \lambda n \frac{\sqrt{3}}{4\pi} + \mathcal{O}(\lambda n)^2 \\ f(\varepsilon) &\to \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\varepsilon \qquad \text{for } \varepsilon \ll 1 \end{aligned}}$$

Semi-classical approach for computing the rate R(1->n,E)

- DT Son1995

Multi-particle decay rates Γ_n can also be computed using an alternative semi-classical method. This is an intrinsically non-perturbative approach, with no reference in its outset made to perturbation theory.

The path integral is computed in the steepest descent method, controlled by two large parameters, $1/\lambda \to \infty$ and $n \to \infty$.

$$\lambda \to 0\,, \quad n \to \infty\,, \quad \text{with} \quad \lambda n = \text{fixed}\,, \quad \varepsilon = \text{fixed}\,.$$

The semi-classical computation in the regime where,

$$\lambda n = \text{fixed} \ll 1\,, \quad \varepsilon = \text{fixed} \ll 1\,,$$

reproduces the tree-level perturbative results for non-relativistic final states.

Remarkably, this semi-classical calculation also reproduces the leading-order quantum corrections arising from resumming one-loop effects.

Semi-classical approach for computing the rate R(1->n,E)

The semiclassical approach is equally applicable and more relevant to the realisation of the non-perturbative Higgsplosion case where,

$$\lambda n = \text{fixed} \gg 1\,, \quad \varepsilon = \text{fixed} \ll 1\,.$$

This calculation was carried out for the spontaneously broken theory with the result given by,

- VVK 1705.04365

$$\mathcal{R}_n(\lambda; n, \varepsilon) \,=\, \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + 0.85\sqrt{\lambda n} + \frac{1}{2} + \frac{3}{2}\log\frac{\varepsilon}{3\pi} - \frac{25}{12}\,\varepsilon\right)\right],$$

Higher order corrections are suppressed by $\mathcal{O}(1/\sqrt{\lambda n})$ and powers of ε.

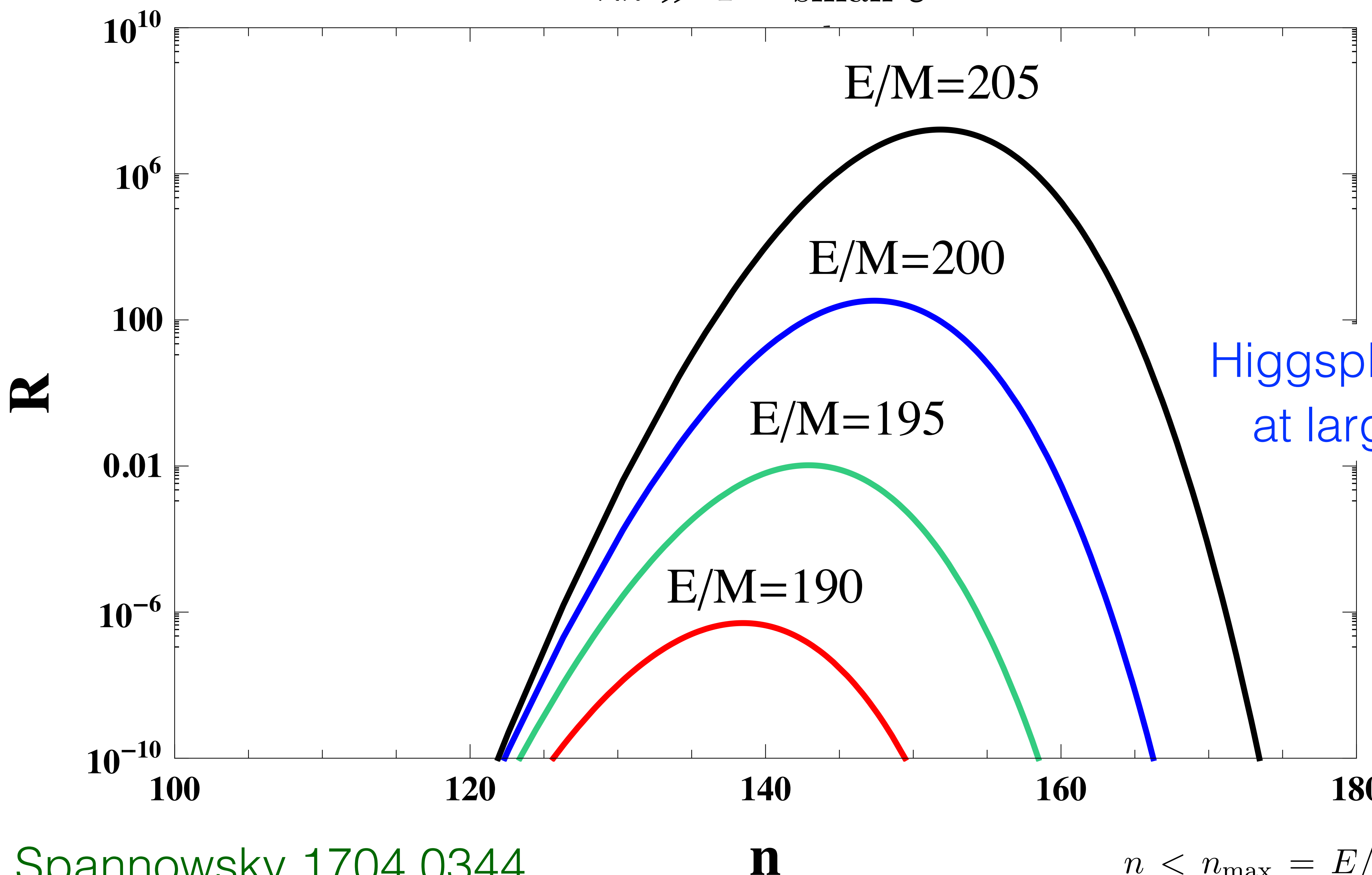
Thus we have computed the rate R in the large lambda n limit:

using the semi-classical approach and the thin-wall approximation

VVK 1705.04365

$$\mathcal{R} = \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + \boxed{3.02\sqrt{\frac{\lambda n}{4\pi}}} - 1 + \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right) - \frac{25}{12}\varepsilon\right)\right]$$

$\lambda n \gg 1 \quad$ small ε



Higgsplosion realised at large lambda n

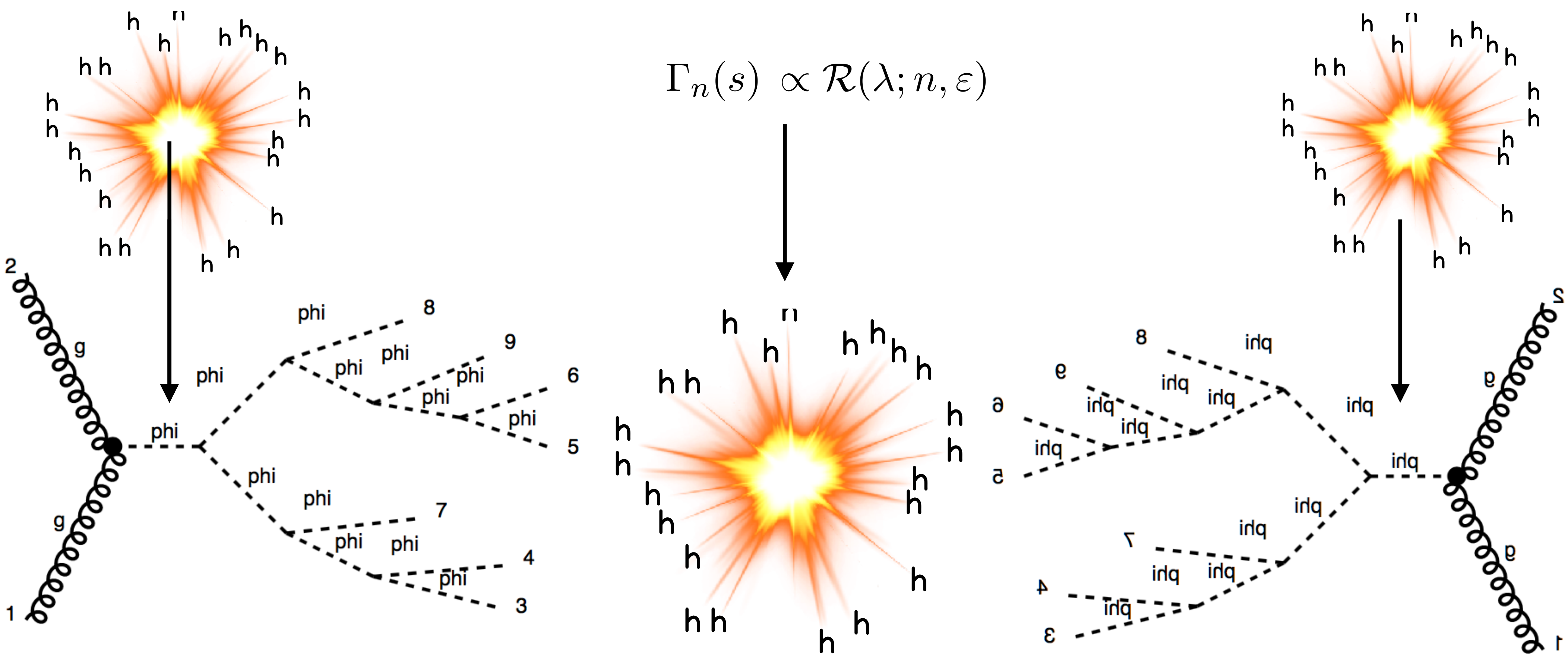
VVK & Spannowsky 1704.0344

$n < n_{\max} = E/M_h$

HIGGSPLOSION and HIGGSPERSION

$$\mathcal{M}_{gg\to h^*} \times \frac{i}{p^2 - m_h^2 - Re\tilde{\Sigma}(p^2) + i m_h \Gamma(p^2)} \times \mathcal{M}_{h^* \to n\times h}$$

Include self-energy



$$\sigma^{\Delta}_{gg\to n\times h} \sim y_t^2 \frac{m_t^2}{m_h} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \times \frac{1}{(s - Re\Sigma(s))^2 + m_h^2\Gamma^2(s)} \times \Gamma_n(s)$$

VVK & Spannowsky 1704.0344

- Unitarity restored!

Summary of the main idea

The Dyson propagator (continued to Euclidean space) is,

$$\Delta_R(x_1,x_2) \,=\, \langle 0|\phi(x_1)\phi(x_2)|0\rangle \,=\, \int \frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2+m^2+\Sigma_R(p^2)}\,e^{ip_0\Delta\tau\,+\,i\vec{p}\Delta\vec{x}}\,.$$

When the theory enters the Higgsplosion regime, the self-energy undergoes a sharp exponential growth,

$$\Sigma_R(p^2) \,\sim\, \begin{cases} 0 & : \ \text{for } p^2 < E_*^2 \\ \infty & : \ \text{for } p^2 \geq E_*^2 \end{cases}$$

The loop momentum integral becomes cut off by Σ outside the ball of radius E_*

$$\Delta_R(x_1,x_2) \,=\, \int_{p^2\leq E_*^2} \frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2+m^2}\,e^{ip_0\Delta\tau\,+\,i\vec{p}\Delta\vec{x}}$$

$$\sim \begin{cases} 1/|\Delta x|^2 & : \ \text{for } 1/E_* \ll |\Delta x| \ll 1/m \\ E_*^2 & : \ \text{for } |\Delta x| \lesssim 1/E_* \end{cases}\,.$$

Summary of the main idea

A conventional wisdom: in the description of nature based on a local QFT, one should always be able to probe shorter and shorter distances with higher and higher energies.

Higgsplosion is a dynamical mechanism, or a new phase of the theory, which presents an obstacle to this principle at energies above E_*.

E_* is the new dynamical scale of the theory, where multi-particle decay rates become unsuppressed.

Schematically, $E_* = C\,\frac{m}{\lambda}$, where C is a model-dependent constant of $\mathcal{O}(100)$. This expression holds in the weak-coupling limit $\lambda \to 0$.

Higgsplosion

At energy scales above E_* the dynamics of the system is changed:

1. Distance scales below $|x| \lesssim 1/E_*$ cannot be resolved in interactions;
2. UV divergences are regulated;
3. The theory becomes asymptotically safe;
4. And the Hierarchy problem of the Standard Model is therefore absent.

Consider the scaling behaviour of the propagator of a massive scalar particle

$$\Delta(x) \, := \, \langle 0|T(\phi(x)\,\phi(0))|0\rangle \, \sim \begin{cases} m^2\, e^{-m|x|} & : \ \text{for } |x| \gg 1/m \\ 1/|x|^2 & : \ \text{for } 1/E_* \ll |x| \ll 1/m \\ E_*^2 & : \ \text{for } |x| \lesssim 1/E_* \end{cases} ,$$

where for $|x| \lesssim 1/E_*$ one enters the Higgsplosion regime.

This is a non-perturbative criterium. Can in principle be computed on a lattice.

Higgsplosion

Loop integrals are effectively cut off at E_* by the exploding width $\Gamma(p^2)$ of the propagating state into the high-multiplicity final states.

The incoming highly energetic state decays rapidly into the multi-particle state made out of soft quanta with momenta $k_i^2 \sim m^2 \lll E_*^2$.

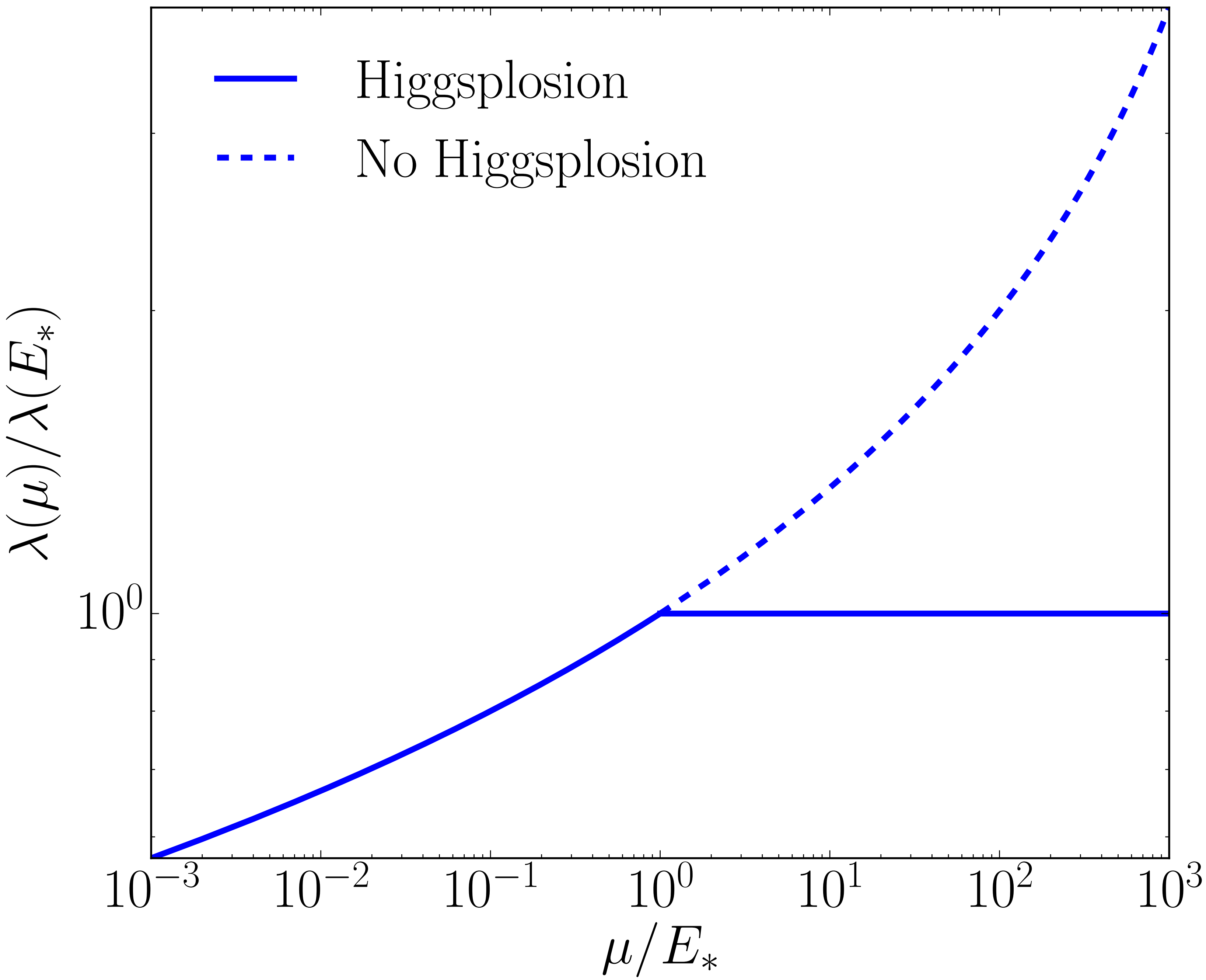
The width of the propagating degree of freedom becomes much greater than its mass: it is no longer a simple particle state.

In this sense, it has become a composite state made out of the n soft particle quanta of the same field ϕ.

VVK & Michael Spannowsky 1704.03447, 1707.01531

Asymptotic Safety

For all parameters of the theory (running coupling constants, masses, etc):

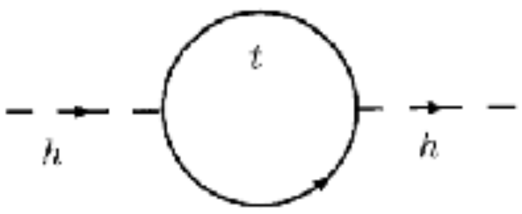


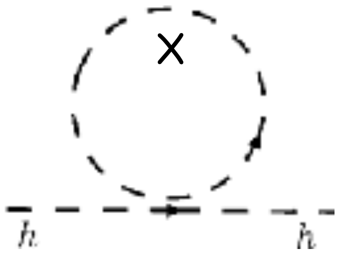
UV fixed point

Higgsploding the Hierarchy problem

X=heavy state

$$\mathcal{L}_X = \frac{1}{2}\partial^\mu X\,\partial_\mu X - \frac{1}{2}M_X^2\,X^2 - \frac{\lambda_P}{4}X^2h^2 - \mu X h^2$$





$$\Delta M_h^2 \sim \lambda_P \int \frac{d^4p}{16\pi^4}\,\frac{1}{p^2 + M_X^2 + \Sigma_X(p^2)} \propto \lambda_P\,\frac{E_\star^2}{M_X^2}\,E_\star^2 \quad \ll \lambda_P M_X^2\,.$$

Due to Higgsplosion the multi-particle contribution to the width of X explode at $p^2 = s_\star$ where $\sqrt{s_\star} \simeq \mathcal{O}(25)\mathrm{TeV}$

→ It provides a sharp UV cut-off in the integral, possibly at $s_\star \ll M_X^2$

Hence, the contribution to the Higgs mass amounts to

$$\text{For}\quad \Gamma(s_\star) \simeq M_X \quad \text{at} \quad s_\star \ll M_X^2 \quad \Longrightarrow \quad \Delta M_h^2 \propto \lambda_P\,\frac{s_\star}{M_X^2}\,s_\star \ll \lambda_P\,M_X^2$$

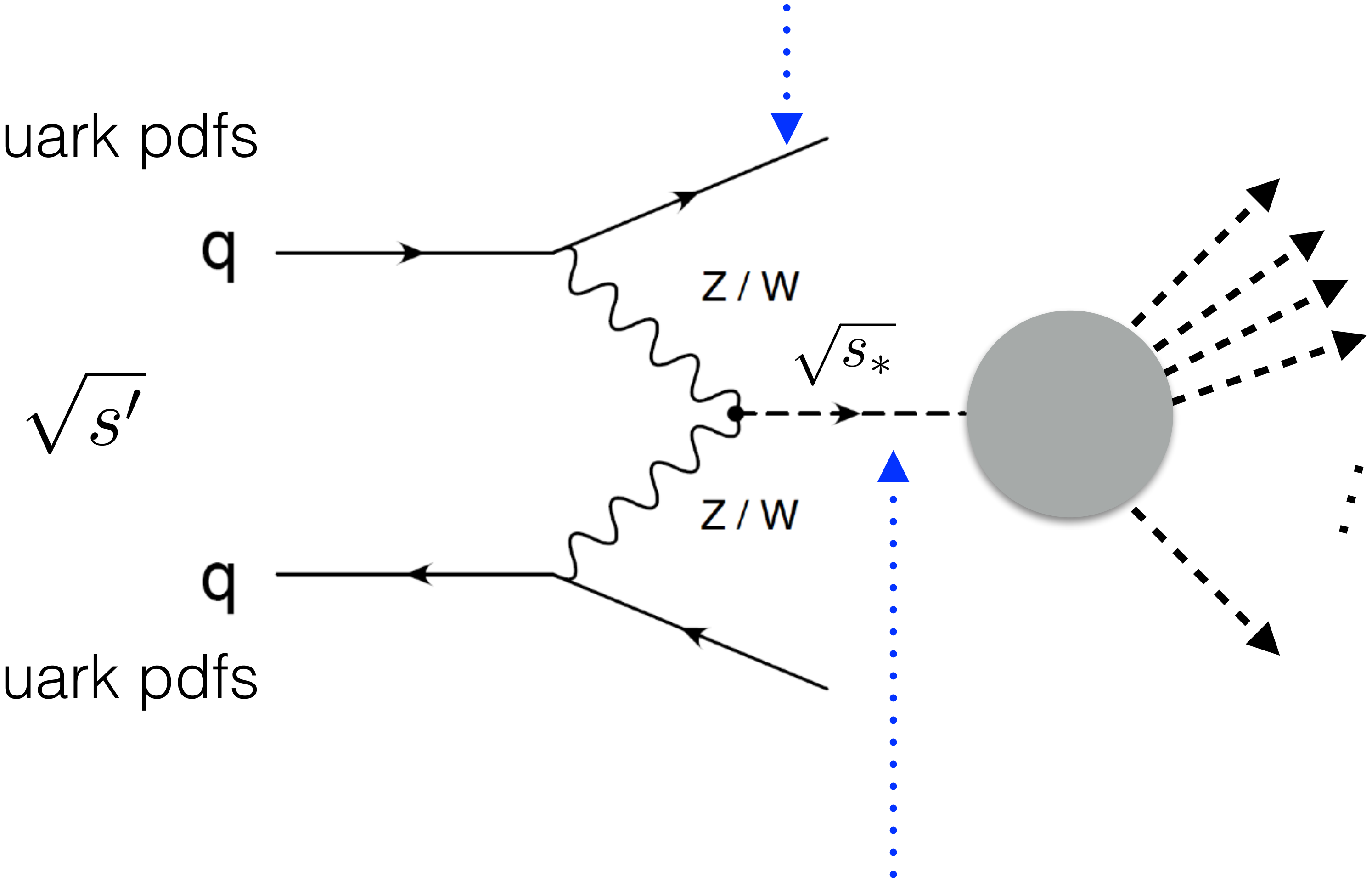
and thus mends the Hierarchy problem by $\left(\frac{\sqrt{s_\star}}{M_X}\right)^4 \simeq \left(\frac{25\,\mathrm{TeV}}{M_X}\right)^4$

Prospects of *direct* observation of Higgslposion

Vector boson fusion at high-energy pp colliders (FCC)

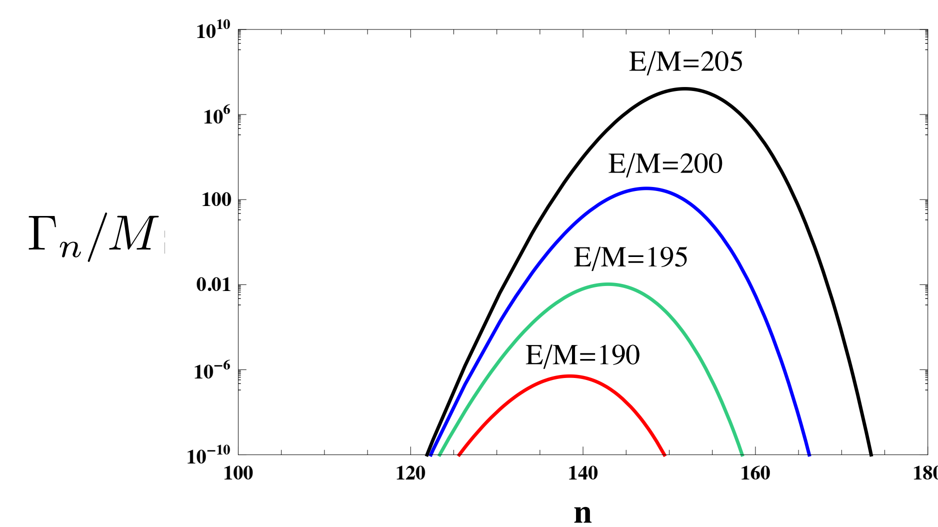
energy excess over $\sqrt{s_*}$ carried away by jets

quark pdfs



quark pdfs

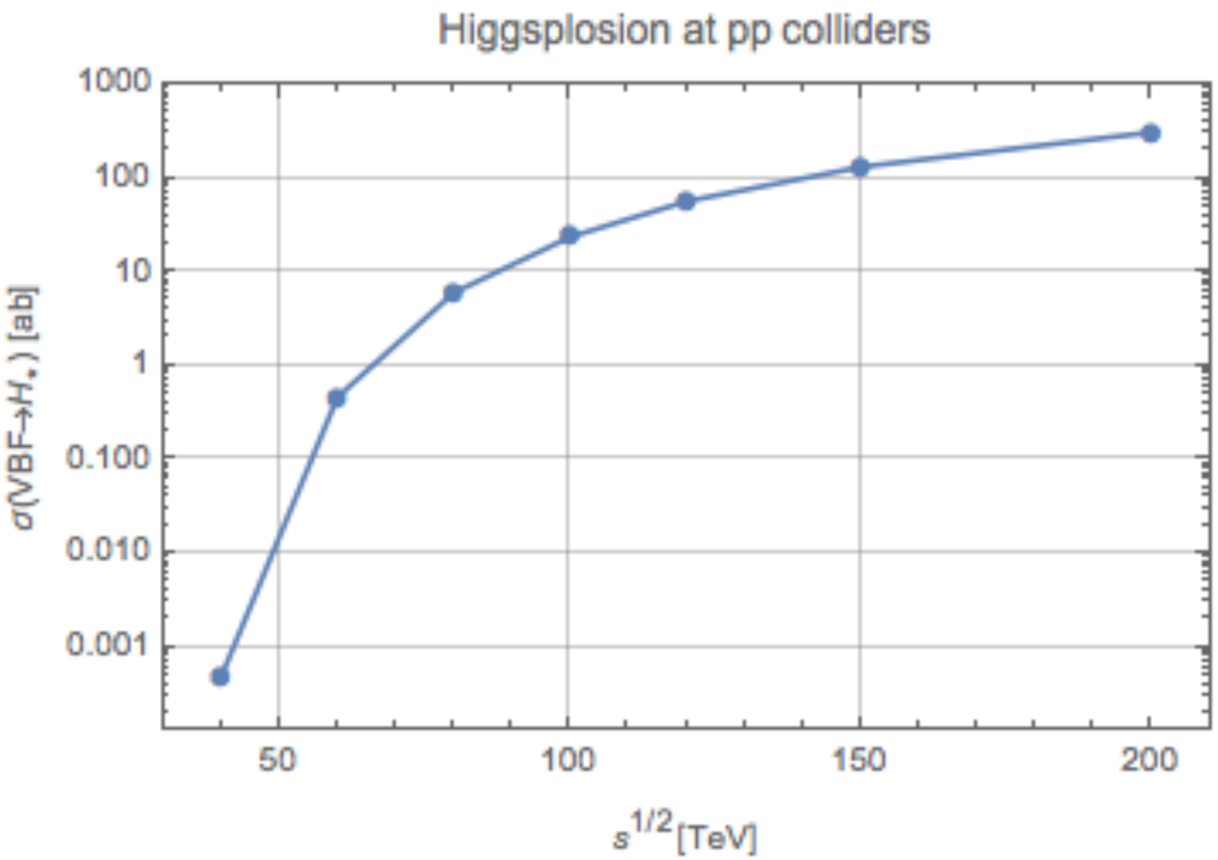
n non-relativistic Higgses
Higgsplosion at $\sqrt{s_*}$

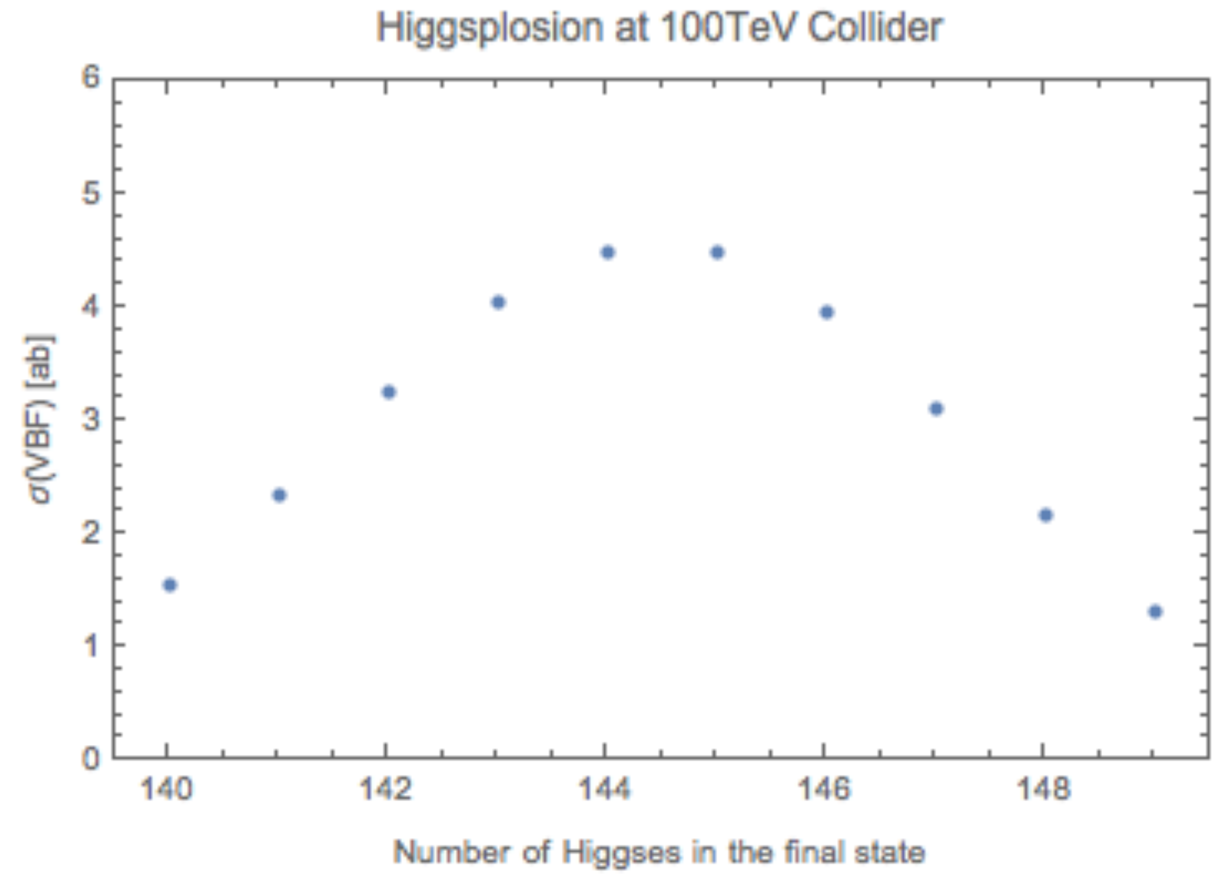


$$\frac{i}{s_* - m_h^2 - Re\tilde{\Sigma}(s_*) + i m_h \Gamma(s_*)}$$

Propagator with Higgspersion at $\sqrt{s_*}$

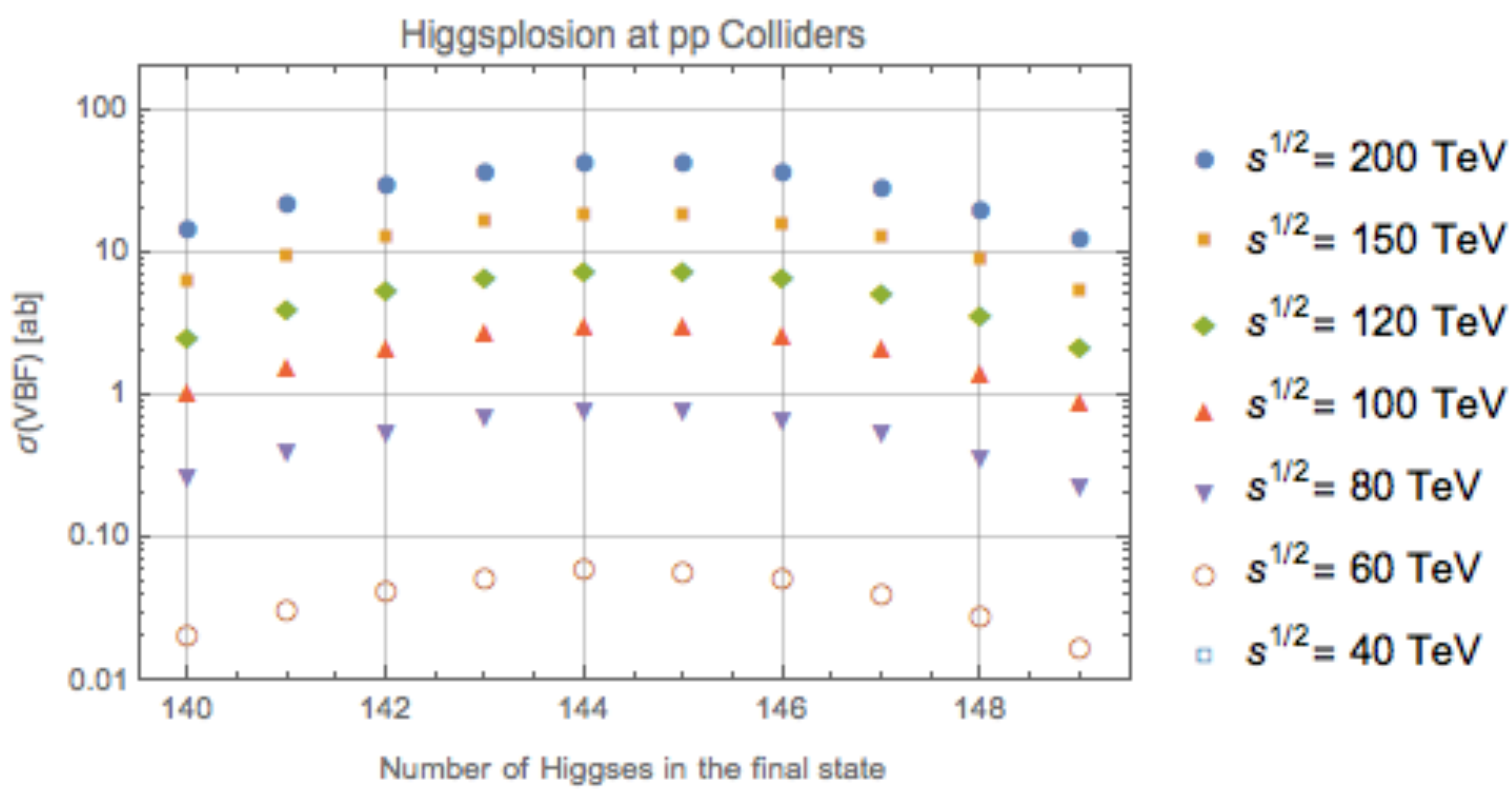
Vector boson fusion at high-energy pp colliders (FCC)





using $p_{t\,\mathrm{jet}} > 40$ GeV

VVK, J Scholtz, M Spannowsky

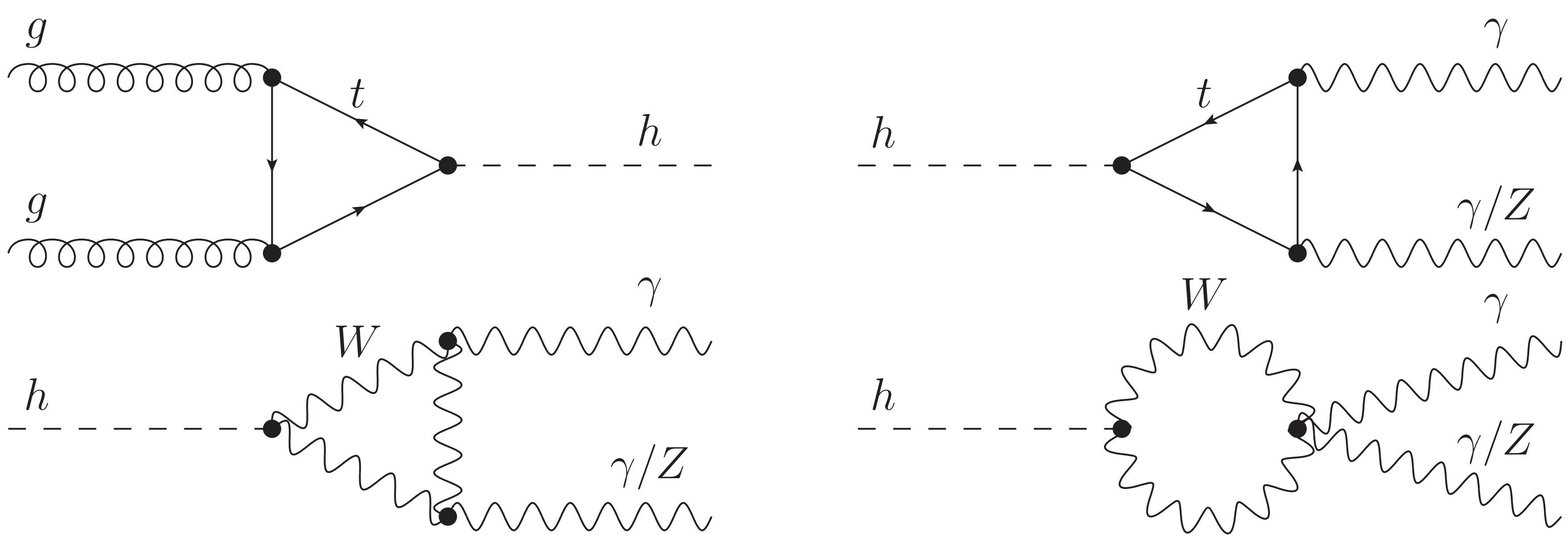


preliminary: no Higgs decays into SM dofs included;
& no vector bosons in final states yet

Effects of Higgsplosion on Precision Observables

- VVK, J Reiness, M Spannowsky, P Waite 1709.08655

Here focus on a class of observables which have no tree-level contributions

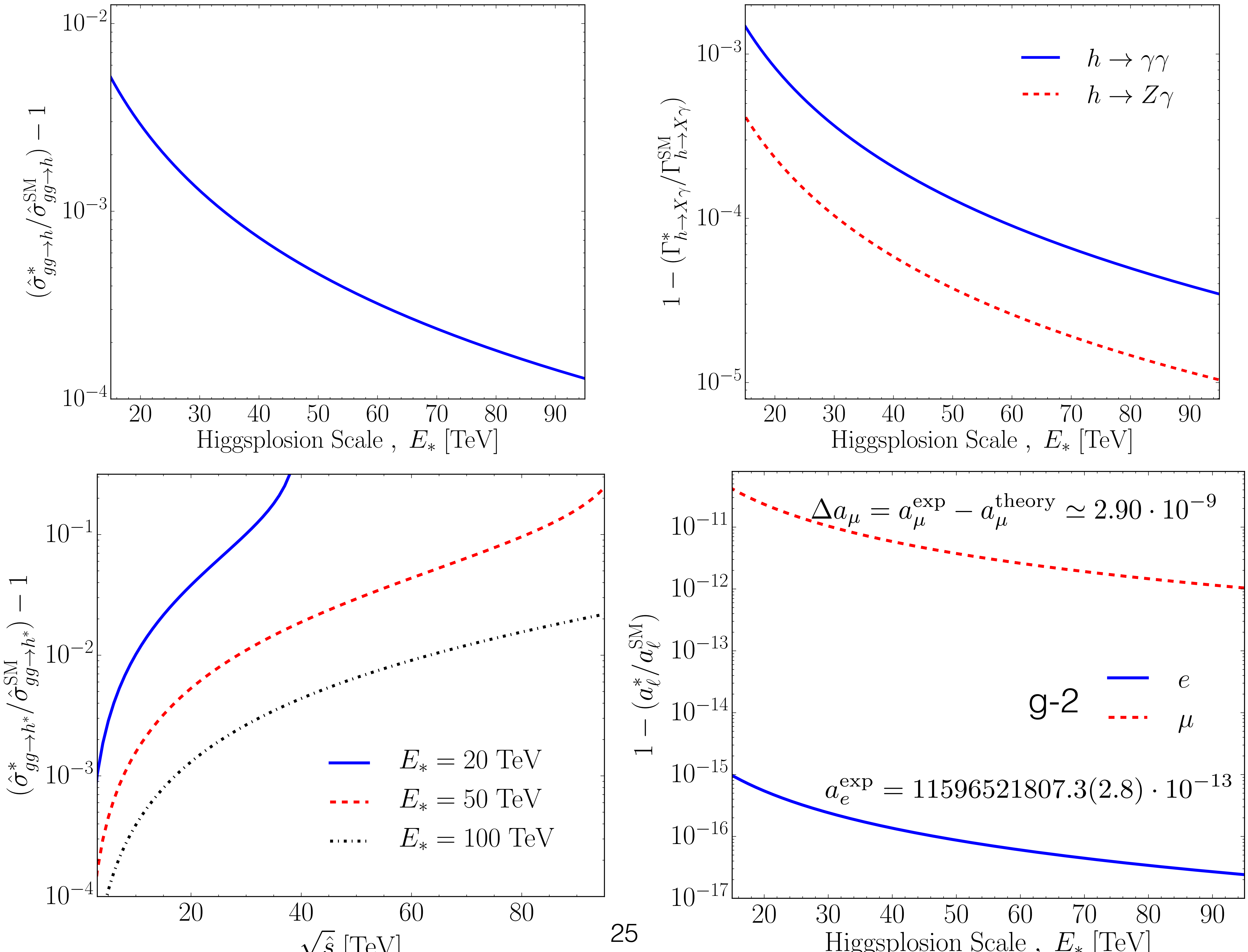


At LHC energies effects of Higgsplosion are small (next slide).

However O(1) effects can be achieved for these loop-induced processes if the interactions are probed close to ~ 2E*.

Effects of Higgsplosion on Precision Observables

VVK, J Reiness, M Spannowsky, P Waite 1709.08655



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Summary

- The Higgsplosion / Higgspersion mechanism makes theory UV finite (all loop momentum integrals are dynamically cut-off at scales above the Higgsplosion energy).
- UV-finiteness => all coupling constants slopes become flat above the Higgsplosion scale => automatic asymptotic safety
- [Below the Higgsplosion scale there is the usual logarithmic running]
- 1. Asymptotic Safety
- 2. No Landau poles for the U(1) and the Yukawa couplings
- 3. The Higgs self-coupling does not turn negative => stable EW vacuum
- No new physics degrees of freedom required — very minimal solution