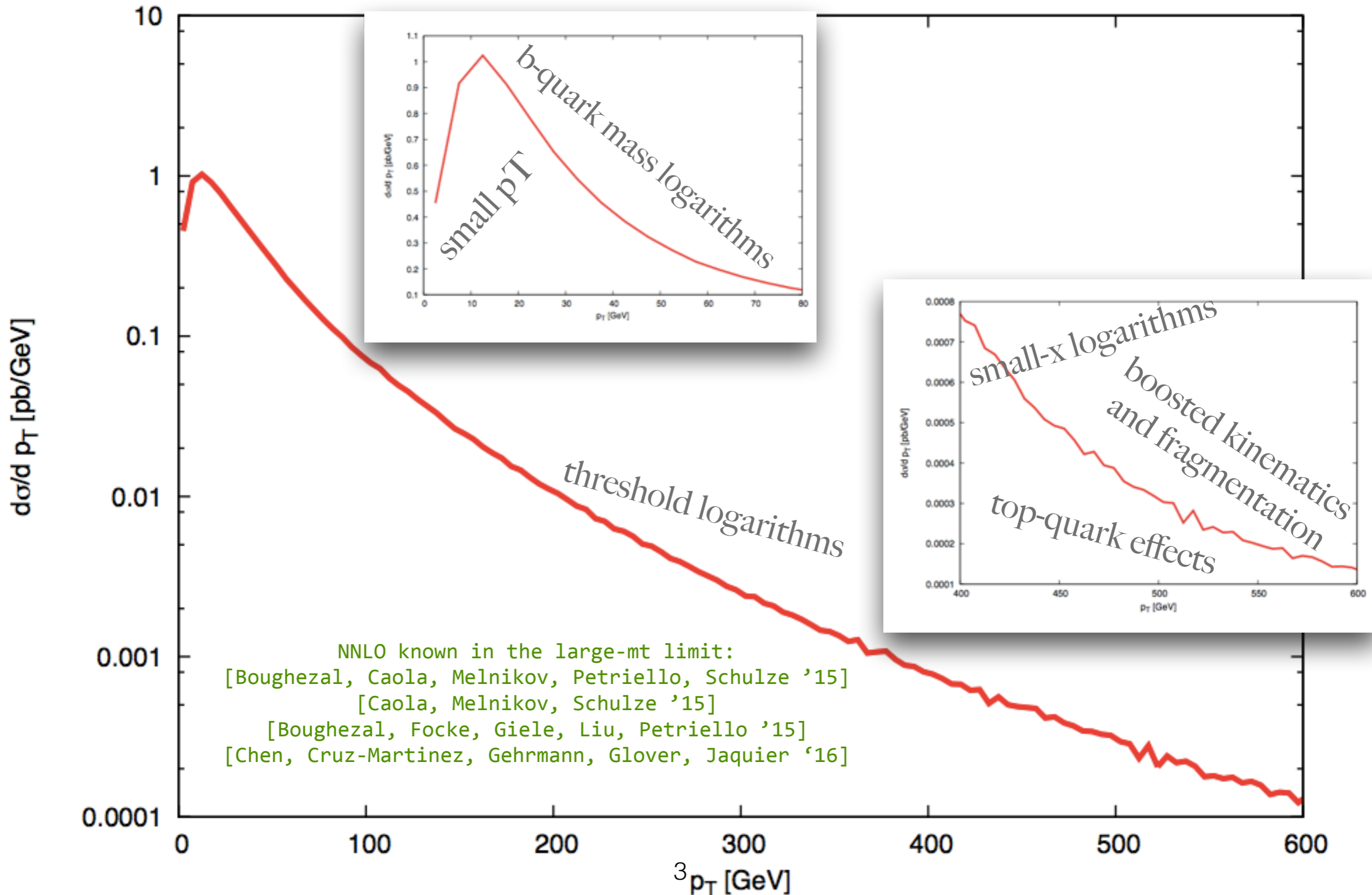




Resummation in Higgs physics

Pier Francesco Monni
CERN

e.g. inclusive Higgs p_T spectrum



Higgs at small transverse momentum

- Study of small- p_t region received a lot of attention in collider literature. Theoretically, it offers a clean environment to **test/calibrate exclusive generators** against more accurate predictions. Experimentally, shape is **sensitive to light-quark Yukawa** couplings
- Theoretically interesting observable. **Two mechanisms compete** in the $p_t \rightarrow 0$ limit
 - **Sudakov (exponential) suppression** when $k_{ti} \sim p_t$
 - **Azimuthal cancellations (power suppression, dominant)** when $k_{ti} \gg p_t$
- Standard solution obtained in impact-parameter space. Information on the radiation entirely lost

$$\delta^{(2)}(\vec{p}_t - (\vec{k}_{t1} + \dots + \vec{k}_{tn})) = \int \frac{d^2\vec{b}}{4\pi^2} e^{-i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^n e^{i\vec{b}\cdot\vec{k}_{ti}},$$

$$\frac{d^2\Sigma(p_t)}{d\Phi_B dp_t} = \sum_{c_1, c_2} \frac{d|M_B|_{c_1 c_2}^2}{d\Phi_B} \int b db p_t J_0(p_t b) \mathbf{f}^T(b_0/b) \mathbf{C}_{N_1}^{c_1; T}(\alpha_S(b_0/b)) H_{\text{CSS}}(M) \mathbf{C}_{N_2}^{c_2}(\alpha_S(b_0/b)) \mathbf{f}(b_0/b) \\ \times \exp \left\{ - \sum_{\ell=1}^2 \int_{b_0/b}^M \frac{dk_t}{k_t} \mathbf{R}'_{\text{CSS}, \ell}(k_t) \right\}.$$

[Parisi, Petronzio '79]
 [Collins et al. '85]
 [Bozzi et al. '05]
 [Becher et al. '10+'12]

- **Possible to obtain a more exclusive solution in momentum space ?**

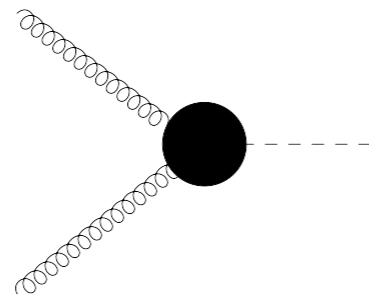
Formulation in momentum space

[PM et al. '16; Bizon et al. '17]
Also SCET formulation in [Ebert, Tackmann '16]

- Write all-order cross section as $(V(\{\tilde{p}\}, k_1, \dots, k_n) = |\vec{k}_{t1} + \dots + \vec{k}_{tn}|)$

$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^n [dk_i] |M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2 \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_n))$$

All-order form factor



Real emissions

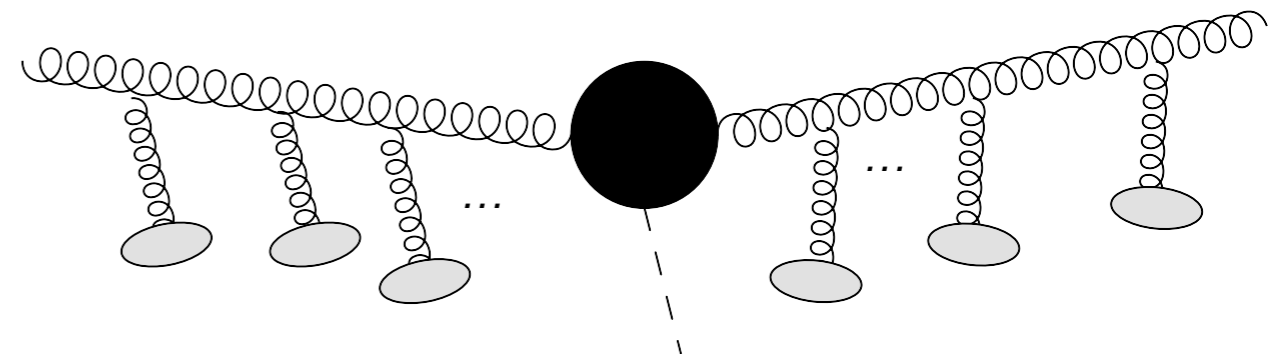
- Recast all-order squared ME for n real emissions as (each *correlated block* is dressed with loops)

$$|M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2 = |M_B(\tilde{p}_1, \tilde{p}_2)|^2 \times \frac{1}{n!} \left\{ \prod_{i=1}^n \left(|M(k_i)|^2 + \int [dk_a][dk_b] |\tilde{M}(k_a, k_b)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) + \int [dk_a][dk_b][dk_c] |\tilde{M}(k_a, k_b, k_c)|^2 \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \dots \right) \right\}$$

e.g. for $n=2$

$$|\tilde{M}(k_a)|^2 = \frac{|M(\tilde{p}_1, \tilde{p}_2, k_a)|^2}{|M_B(\tilde{p}_1, \tilde{p}_2)|^2} = |M(k_a)|^2,$$

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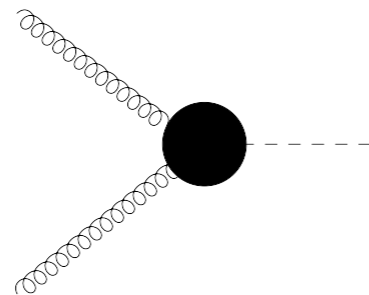
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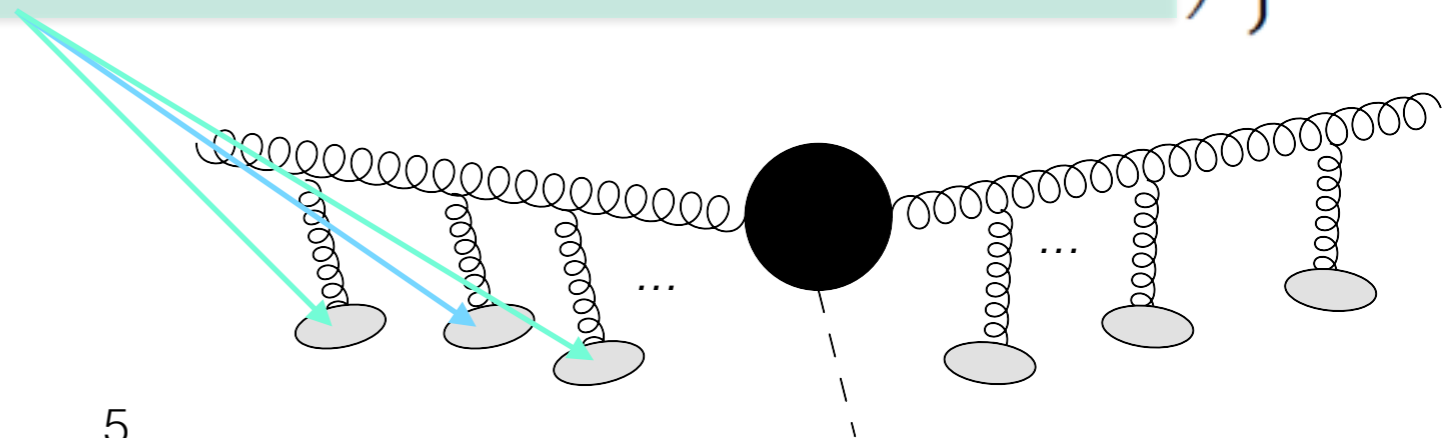
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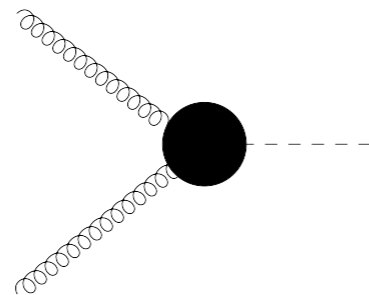
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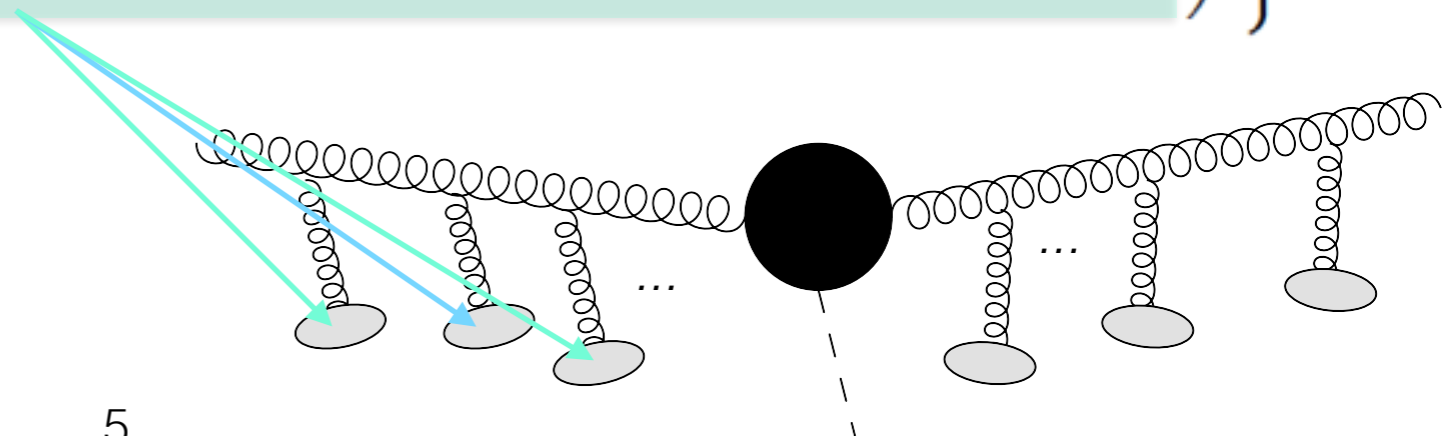
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Formulation in momentum space

- Subtraction of the IRC poles between $\sum_{n=0}^{\infty} \int \prod_{i=1}^n [dk_i] |M(\tilde{p}_1, \tilde{p}_2, k_1, \dots, k_n)|^2$ and $\mathcal{V}(\Phi_B)$:
 - introduce a phase-space resolution scale (slicing parameter) $Q_0 = \epsilon k_{t1}$
 - compute *unresolved* reals and *virtuals* analytically in D dimensions
 - compute *resolved* (reals only) in 4 dim. (**possible to generate MC events !**)
- Remarks:
 - *more* (although not completely) **exclusive generation of ISR**
 - possible to **formulate for more general rIRC-safe observables**
 - clear physical picture of the dynamics at small transverse momentum
 - reproduces b-space if integrated inclusively over the radiation
 - allows one to apply cuts on real radiation (a lot of care is required!); **access to multi-differential resummations**

Higgs p_T at $N^3LL+NNLO$

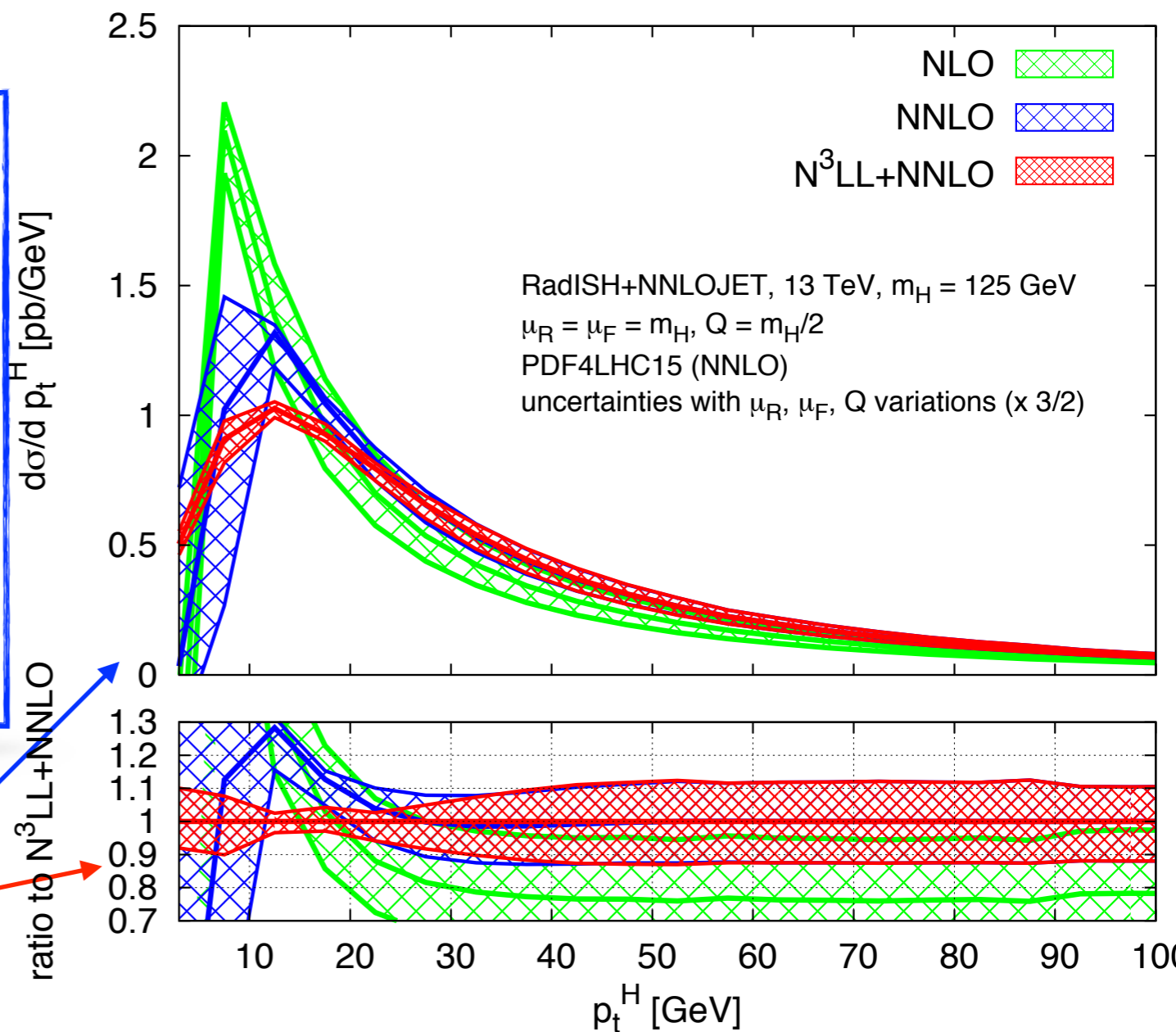
- Implementation in a MC code (**RadISH**) up to N^3LL
- Matching of the integrated distribution to N^3LO via a multiplicative matching, i.e.

[Anastasiou et al. '15-'16] [Boughezal et al. '15] [Caola et al. '15] [Chen et al. '16]

$$\sigma_{pp \rightarrow H}^{N^3LO} = \Sigma_{1\text{-jet}}^{NNLO}(p_t^H)$$

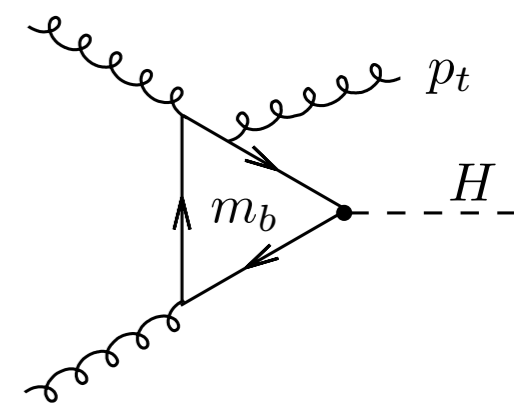
$$\Sigma_{MAT}(p_t, \Phi_B) = (\Sigma_{RES}(p_t, \Phi_B))^Z \frac{\Sigma_{FO}(p_t, \Phi_B)}{(\Sigma_{EXP}(p_t, \Phi_B))^Z}$$

$$Z = \left(1 - \left(\frac{p_t}{Q_{match}}\right)\right)^h \Theta(Q_{match} - p_t)$$



- Deviations from NNLO below 30 GeV
- Scale unc. $\sim 10\%$ down to very small transverse momentum

H+jet via virtual bottom quarks



- The small p_T region is affected by bottom-mediated production. Amplitudes are enhanced by (non-Sudakov) logarithmic terms due to the large mass gap

e.g. in the soft, abelian (i.e. no real emissions)

$$\frac{d\sigma_{pp \rightarrow H+j}}{dp_{\perp}^2} = \frac{d\sigma_{pp \rightarrow H+j}^{m_t \rightarrow \infty}}{dp_{\perp}^2} \left\{ 1 - \frac{3m_b^2}{m_H^2} L_{\text{eff}}^2 \left[1 - \frac{x_{\text{eff}}}{12} (1 - \tau^3 + \tau^4) \right. \right. \\ \left. \left. + \frac{x_{\text{eff}}^2}{48} \left(\frac{4}{15} - \tau^3 + 2\tau^4 - \frac{7\tau^5}{5} + \frac{2\tau^6}{5} \right) + \mathcal{O}(x^3) \right] + \mathcal{O}(m_b^4) \right\}$$

$$L_{\text{eff}} = \ln(m_H^2/m_b^2), \quad x_{\text{eff}} = \frac{\alpha_s C_F}{2\pi} L_{\text{eff}}^2$$

- Resummation known for double logarithms $\frac{m_b^2}{m_H^2} (\alpha_s L^2)^n$, $L = \{\ln(p_t^2/m_b^2), \ln(m_H^2/m_b^2)\}$

Effects beyond NLO are moderate in the SM, might be important for exclusion of some BSM scenarios with enhanced Yukawa couplings to light quarks [Melnikov, Penin '16]

- Extension to complete *virtual* corrections to H+0 j carried out more recently

$$M_{gg \rightarrow H}^{(m_b)} = -e^{-\frac{C_A}{\epsilon^2} \frac{\alpha_s}{2\pi}} {}_2F_2 \left(1, 1; 3/2, 2; (C_A - C_F) \frac{\alpha_s}{8\pi} \ln^2 \frac{m_b^2}{m_H^2} \right) \left(\frac{3}{2} \frac{m_b^2}{m_H^2} \ln^2 \frac{m_b^2}{m_H^2} \right) M_{gg \rightarrow H}^{(m_t \rightarrow \infty)}$$

[Liu, Penin '17]

- Two-loop virtual amplitudes and **full NLO distribution recently computed**

[Melnikov et al. '16 + Lindert et al. '17]

- Important to validate the performance of existing generators

A (practical) hassle #1: matching

- **Several ways to match resummed calculations to fixed order:** e.g. additive, multiplicative, logarithmic, ...; in spirit, problem analogous to N(N)LO+PS matching

$$\Sigma_{\text{RES}} + \Sigma_{\text{FO}} - \Sigma_{\text{RES}}^{(\text{expanded})} \qquad \Sigma_{\text{RES}} \left(\frac{\Sigma_{\text{FO}}}{\Sigma_{\text{RES}}^{(\text{expanded})}} \right)_{\text{expanded}} \qquad \ln \Sigma_{\text{RES}} + \left(\ln \Sigma_{\text{FO}} - \ln \Sigma_{\text{RES}}^{(\text{expanded})} \right)_{\text{expanded}}$$

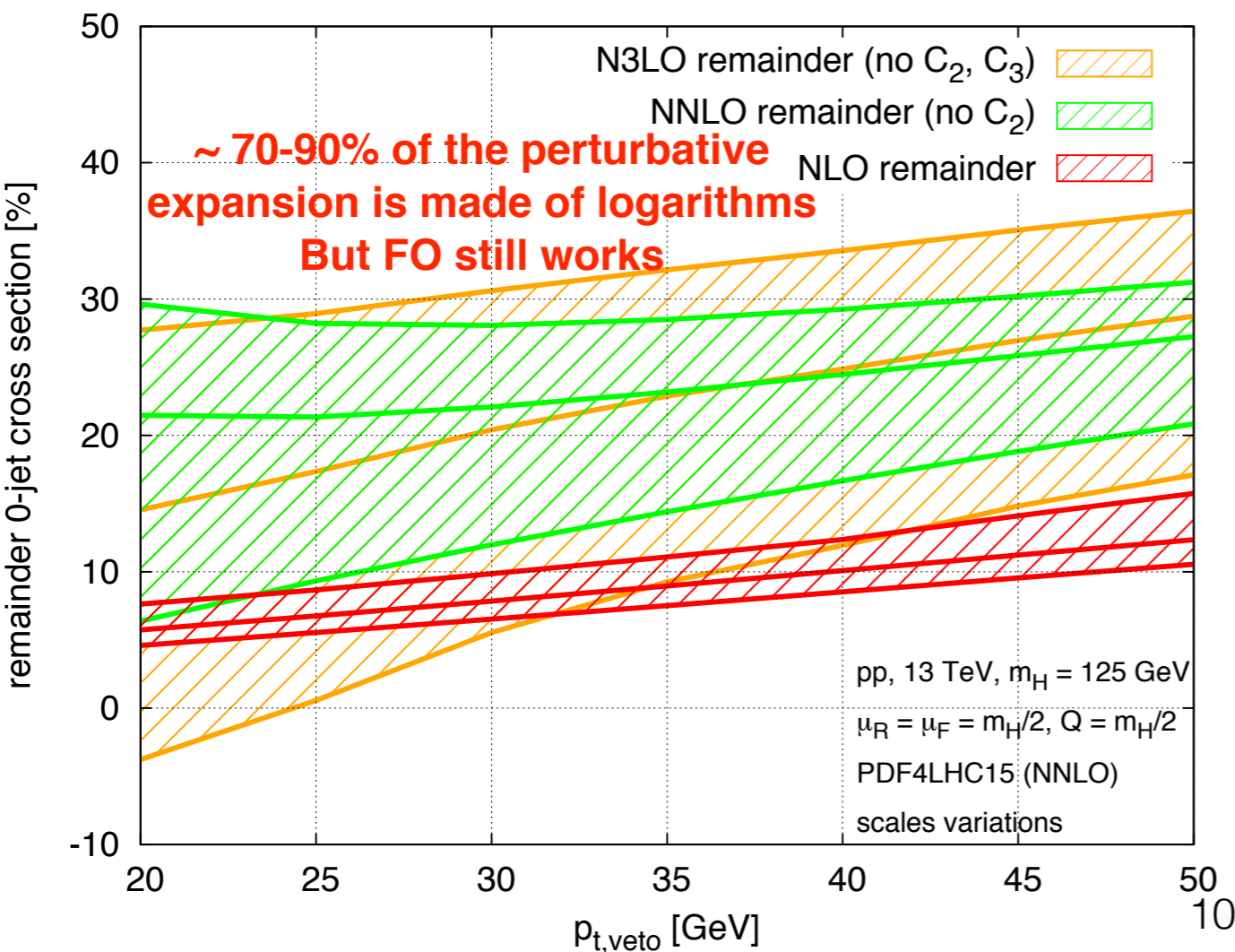
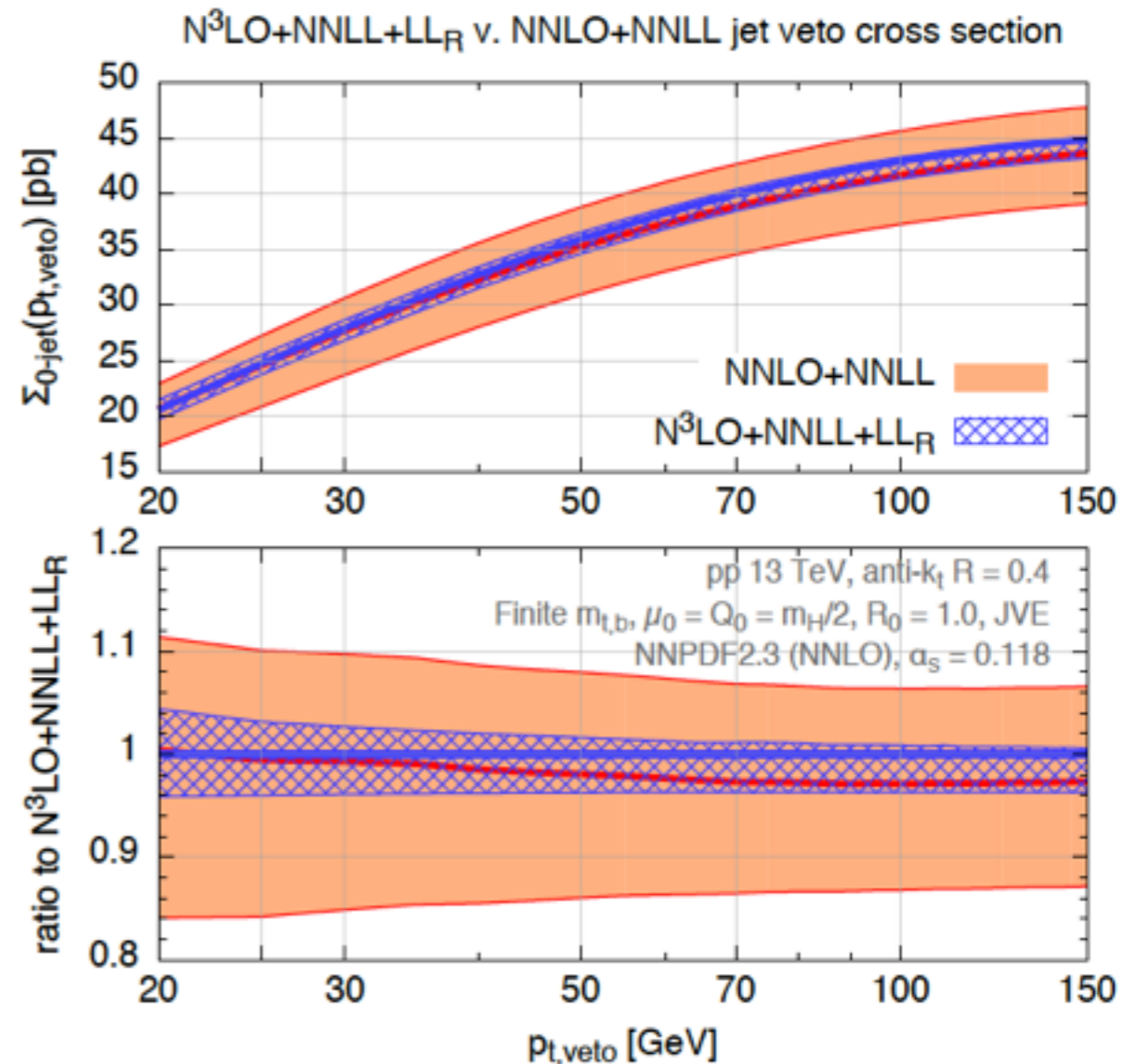
- **No constraint from the theory:** additive is simpler/cleaner, but **multiplicative solutions have a number of advantages**, e.g. numerical stability, constants determined from the fixed order
- **Not improvable with higher orders:** differences usually moderate between *judicious* choices of the scheme, but sometimes we're interested in the physics of the matching region where **fixed-order ~ resummation**
- The problem is real when high precision is demanded, as the choice of the matching scheme also slightly affects the perturbative scale uncertainty. Things could get worse in multiple-scale problems / joint resummations

An example: 0-jet cross section

[Banfi et al. '15]

Also [Banfi et al. '12] [Becher et al '13; Stewart et al.'13]

- ▶ Large uncertainty reduction with inclusion of N^3 LO
- ▶ Matching scheme variation included as extra source of uncertainty
- ▶ High precision allows one to identify pathological schemes



Possible future directions:

- ▶ Data driven: use precise measurements (e.g. Z pT) to select good schemes
- ▶ Inclusion of next-to-eikonal/power corrections can make transition more reliable

[Moult et al.'16] [Boughezal et al.'16]
[Bonocore et al.'16 + Del Duca et al.'17]

A (practical) hassle #2: unitarity

- Resummation must be turned off when the radiation approaches the hard scale(s) of the reaction, i.e. **the total cross section is preserved in the matching to fixed order**
- Commonly achieved by smoothly turning off the logarithms at the price of **adding *power-suppressed* corrections**. Many possible ways (modified logarithms, profile functions,...), **numerical differences in the matching region**, e.g.

$$\ln \frac{1}{v} \rightarrow \frac{1}{p} \ln \left(1 + \frac{1}{v^p} \right) \simeq \ln \frac{1}{v} + \frac{v^p}{p} + \dots$$

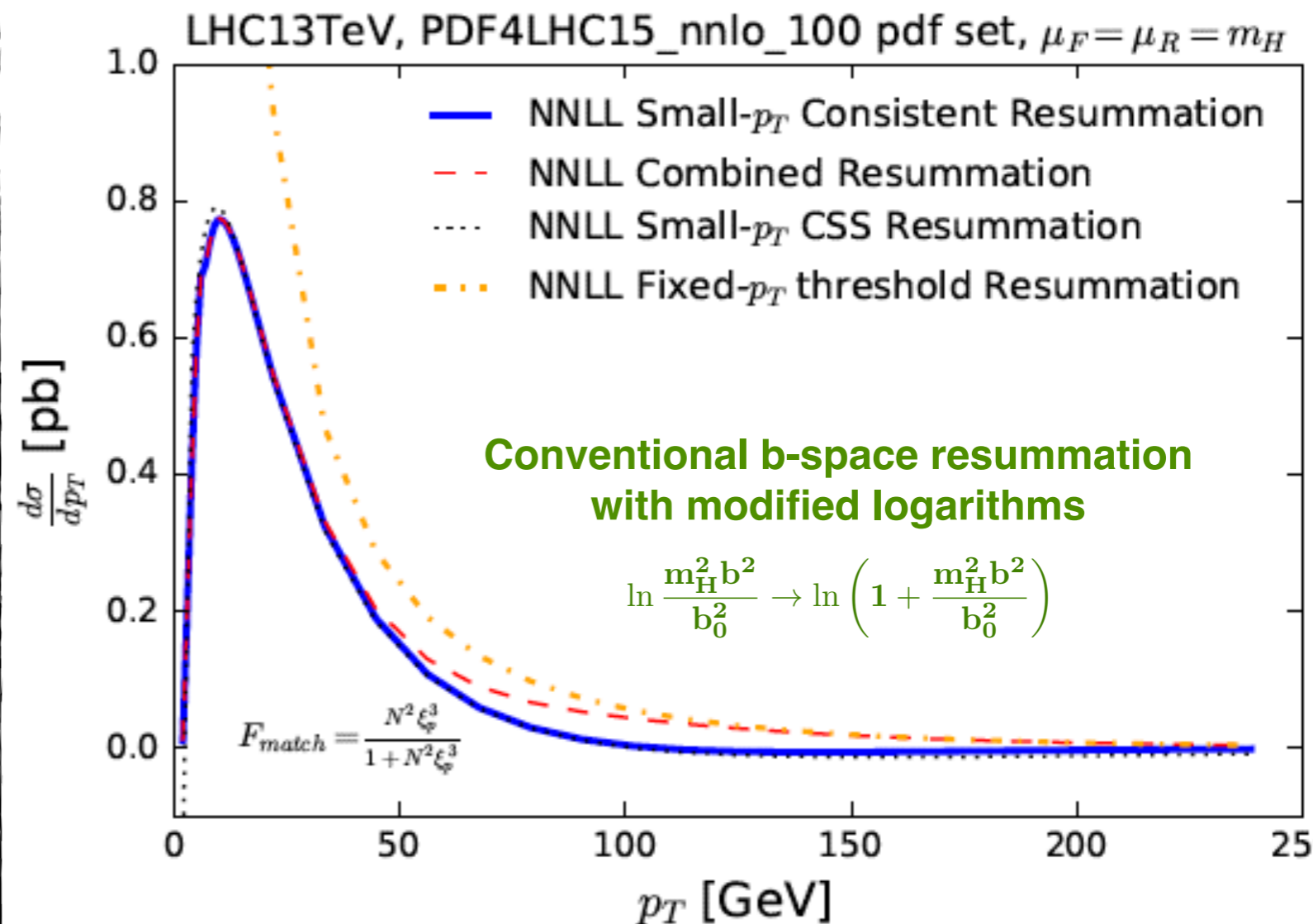
- Possible future improvements:
 - Recent progress in the computation of power corrections to exclusive observables could help adjust the form of the modified logarithms
 - Construct unitary resummation more exclusively with Monte Carlo methods
 - i.e. emission probabilities \sim total derivatives (like in common PS)
 - Can we allow for non-unitary effects in a controlled manner ? Accepted in some NLOPS matching/merging methods (e.g. POWHEG, MC@NLO+FxFx,...)

E.g. joint pt/threshold resummation

- ▶ Retain threshold effects in the radiation's phase space, **i.e. study the limit**

$$p_t^2 \simeq m_H^2 \left(1 - \frac{m_H^2}{\hat{s}} \right) \ll m_h^2$$

- ▶ Integrated distribution yields the known resummed XS + resummed threshold effects
- ▶ No need for *ad hoc* modification of resummed logarithms
- ▶ **Interesting to study the possibility to implement this idea in more exclusive cases**



[Muselli et al. '17]

See also:

[Lustermans et al.'16]

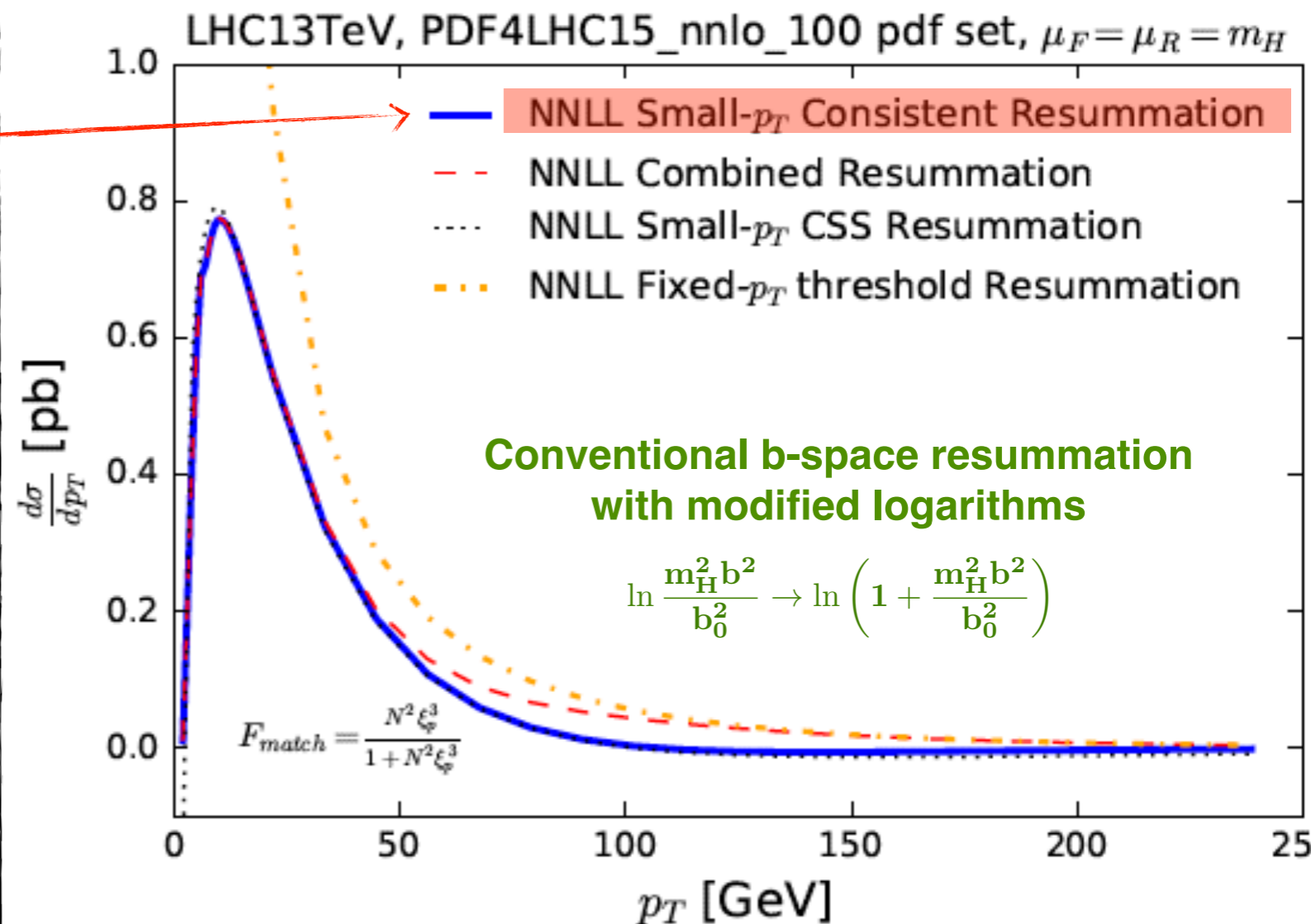
[Marzani, Theeuwes '17]

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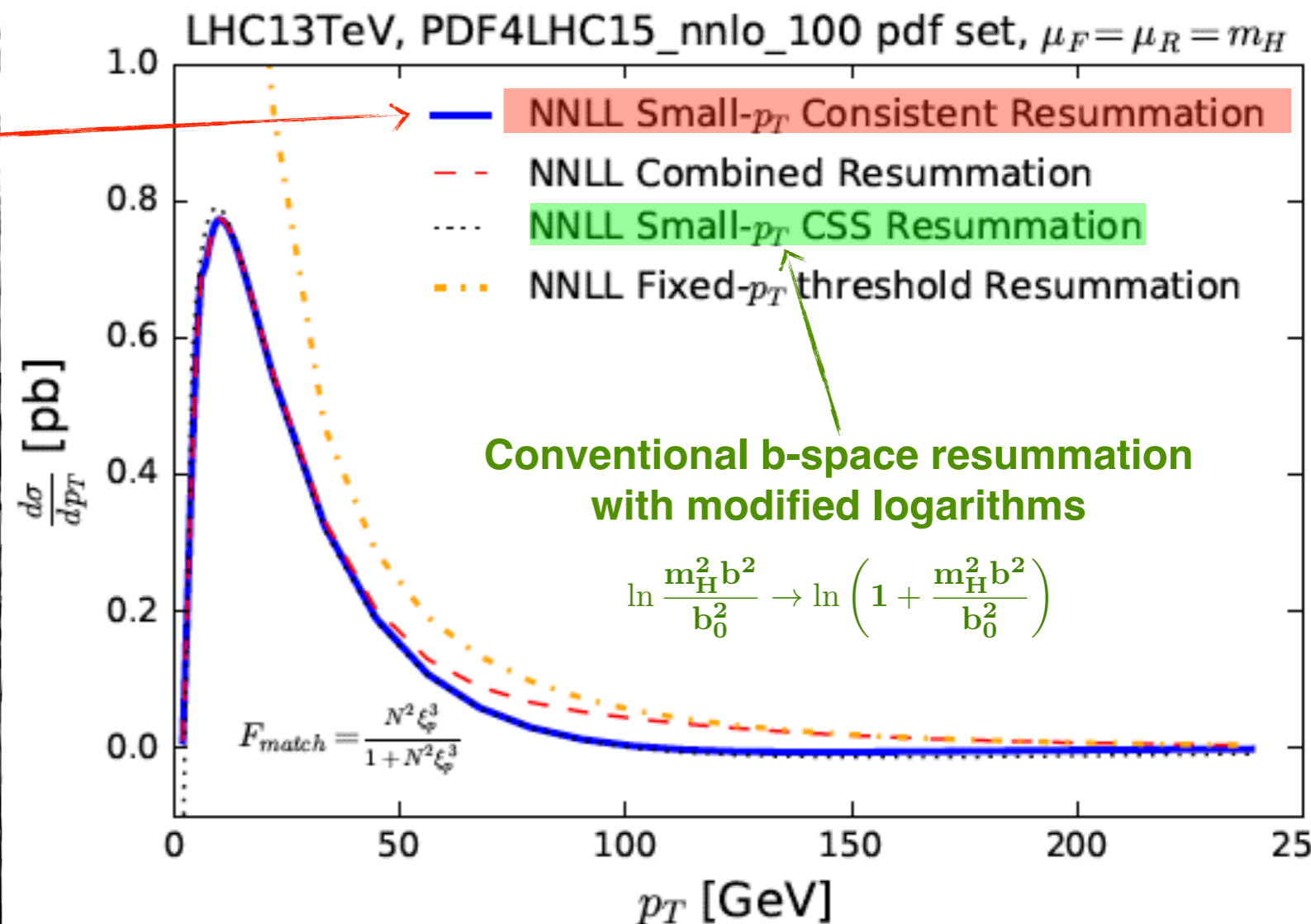
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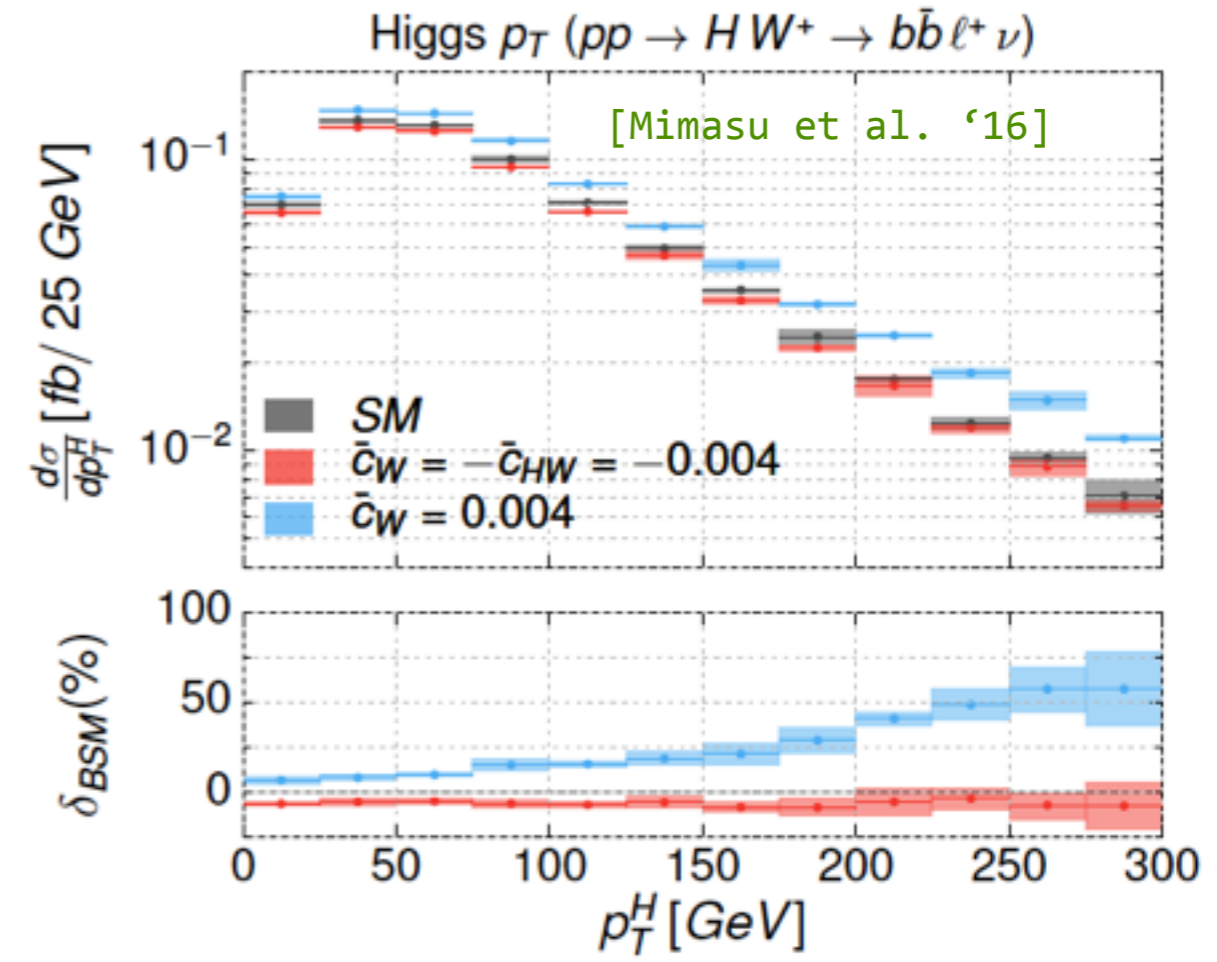
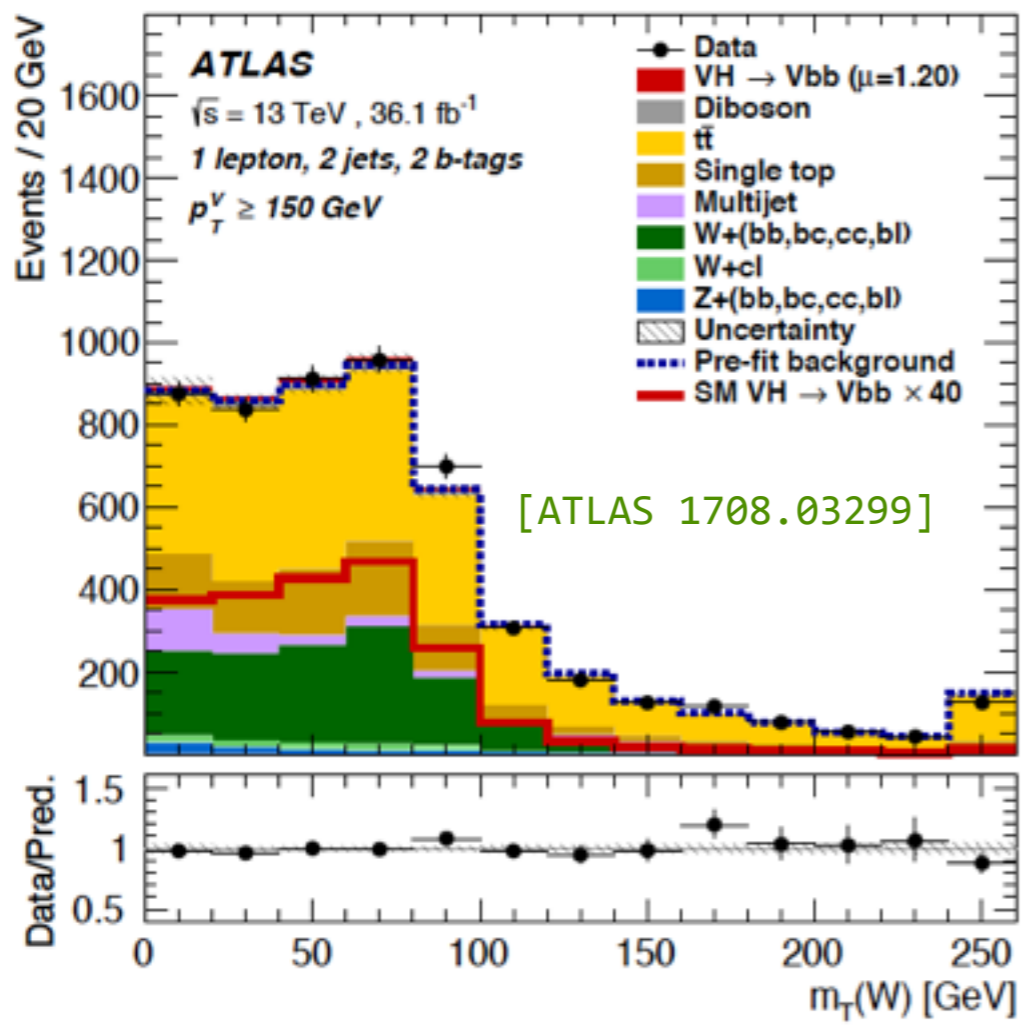
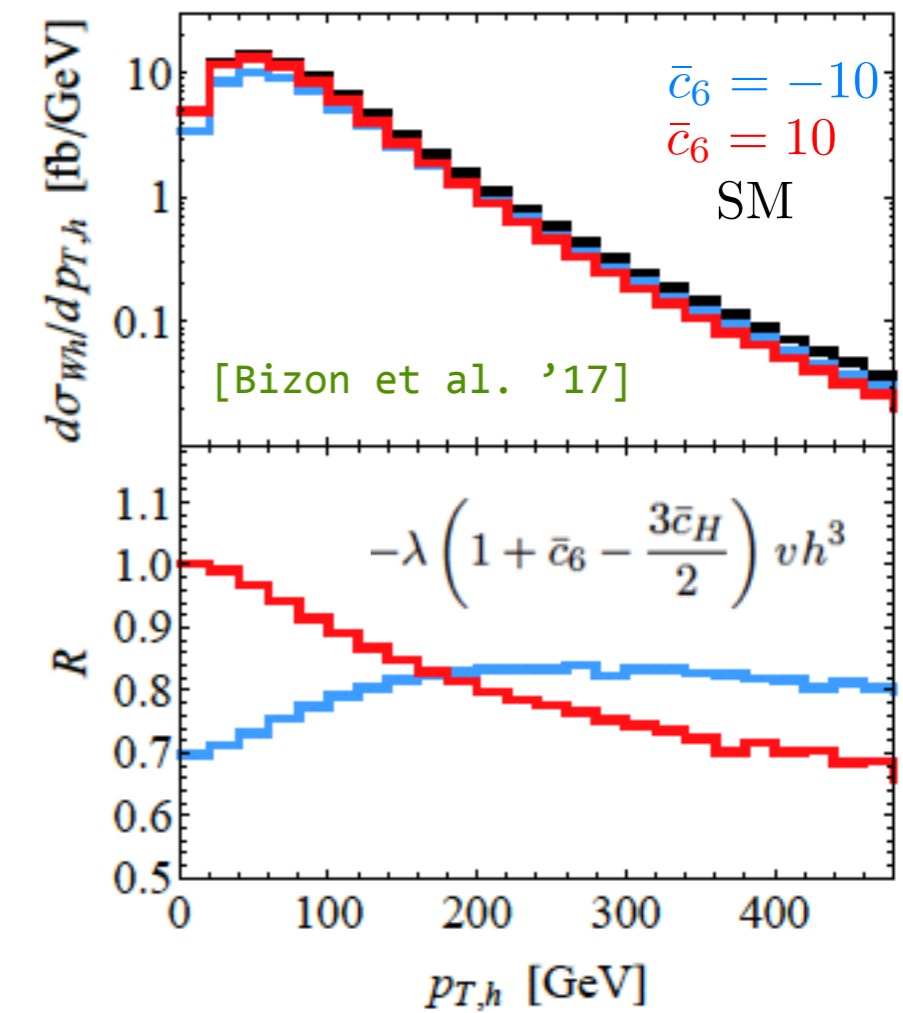
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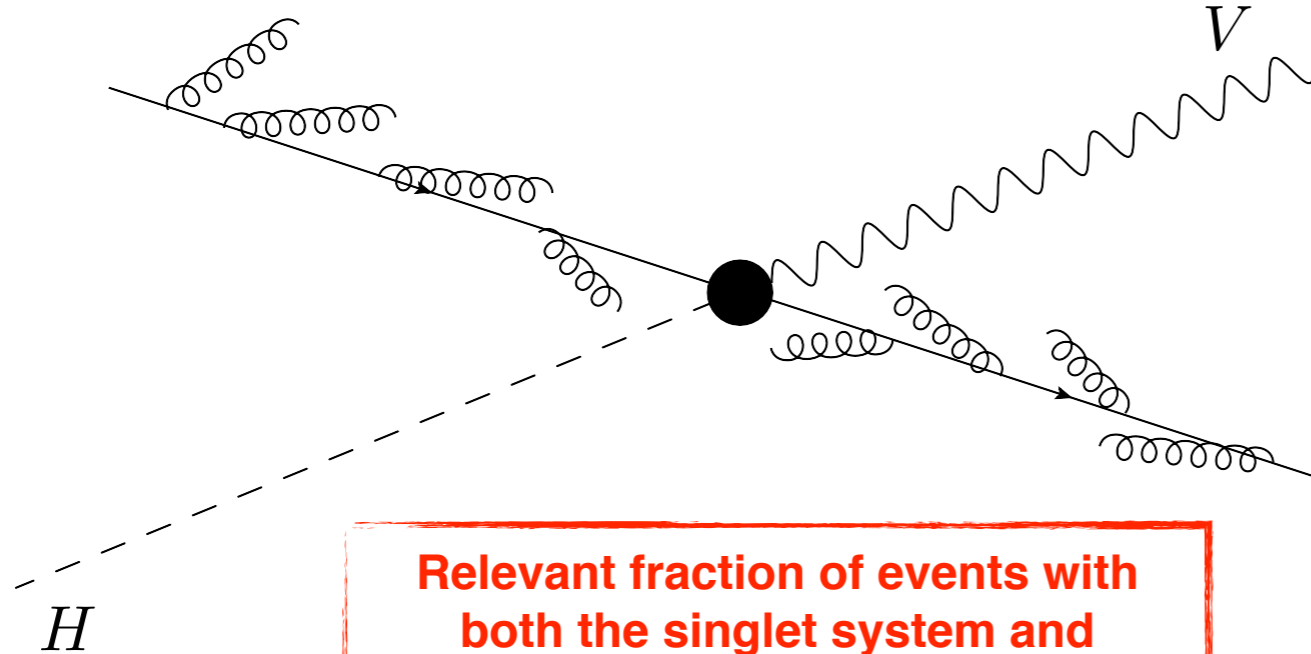
Boosted Higgs: VH(bb)

- Regions with large momentum transfer are more *sensitive* to BSM effects, e.g. higher-dimensional operators
- Large background due to top-pair production in single-lepton category reduced by applying a veto on extra (>2) jets



See also [Buschmann et al. '14]

Boosted Higgs: VH(bb)



Relevant fraction of events with both the singlet system and accompanying jets at low pt

Variable	0-lepton	1-lepton	2-lepton
p_T^V	$\equiv E_T^{\text{miss}}$	×	×
E_T^{miss}	×	×	×
p_{T,b_1}	×	×	×
p_{T,b_2}	×	×	×
p_T	×	×	×
m_{bb}	×	×	×
$\Delta R(b_1, b_2)$	×	×	×
$ \Delta\eta(b_1, b_2) $	×	×	×
$\Delta\phi(V, bb)$	×	×	×
$ \Delta\eta(V, bb) $			×
m_{eff}	×		
$\min[\Delta\phi(\ell, b)]$		×	
m_T^W		×	
$m_{\ell\ell}$			×
m_{top}		×	
$ \Delta Y(V, bb) $		×	
Only in 3-jet events			
$p_T^{\text{jet}_3}$	×	×	×
m_{bbj}	×	×	×

Selection	0-lepton	1-lepton		2-lepton
		e sub-channel	μ sub-channel	
Trigger	E_T^{miss}	Single lepton	E_T^{miss}	Single lepton
Leptons	0 loose leptons with $p_T > 7$ GeV	1 tight electron $p_T > 27$ GeV	1 medium muon $p_T > 25$ GeV	2 loose leptons with $p_T > 7$ GeV ≥ 1 lepton with $p_T > 27$ GeV
E_T^{miss}	> 150 GeV	> 30 GeV	-	-
$m_{\ell\ell}$	-	-	-	$81 \text{ GeV} < m_{\ell\ell} < 101 \text{ GeV}$
Jets		Exactly 2 or 3 jets		Exactly 2 or ≥ 3 jets
Jet p_T			> 20 GeV	
b-jets		Exactly 2 b-tagged jets		
Leading b-tagged jet p_T		> 45 GeV		
H_T	> 120 (2 jets), > 150 GeV (3 jets)	-	-	-
$\min[\Delta\phi(E_T^{\text{miss}}, \text{jets})]$	$> 20^\circ$ (2 jets), $> 30^\circ$ (3 jets)	-	-	-
$\Delta\phi(E_T^{\text{miss}}, bb)$	$> 120^\circ$	-	-	-
$\Delta\phi(b_1, b_2)$	$< 140^\circ$	-	-	-
$\Delta\phi(E_T^{\text{miss}}, E_{T,\text{trk}}^{\text{miss}})$	$< 90^\circ$	-	-	-
p_T^V regions		> 150 GeV		$(75, 150]$ GeV, > 150 GeV
Signal regions	✓	$m_{bb} \geq 75$ GeV or $m_{\text{top}} \leq 225$ GeV		Same-flavour leptons Opposite-sign charge ($\mu\mu$ sub-channel)
Control regions	-	$m_{bb} < 75$ GeV and $m_{\text{top}} > 225$ GeV		Different-flavour leptons

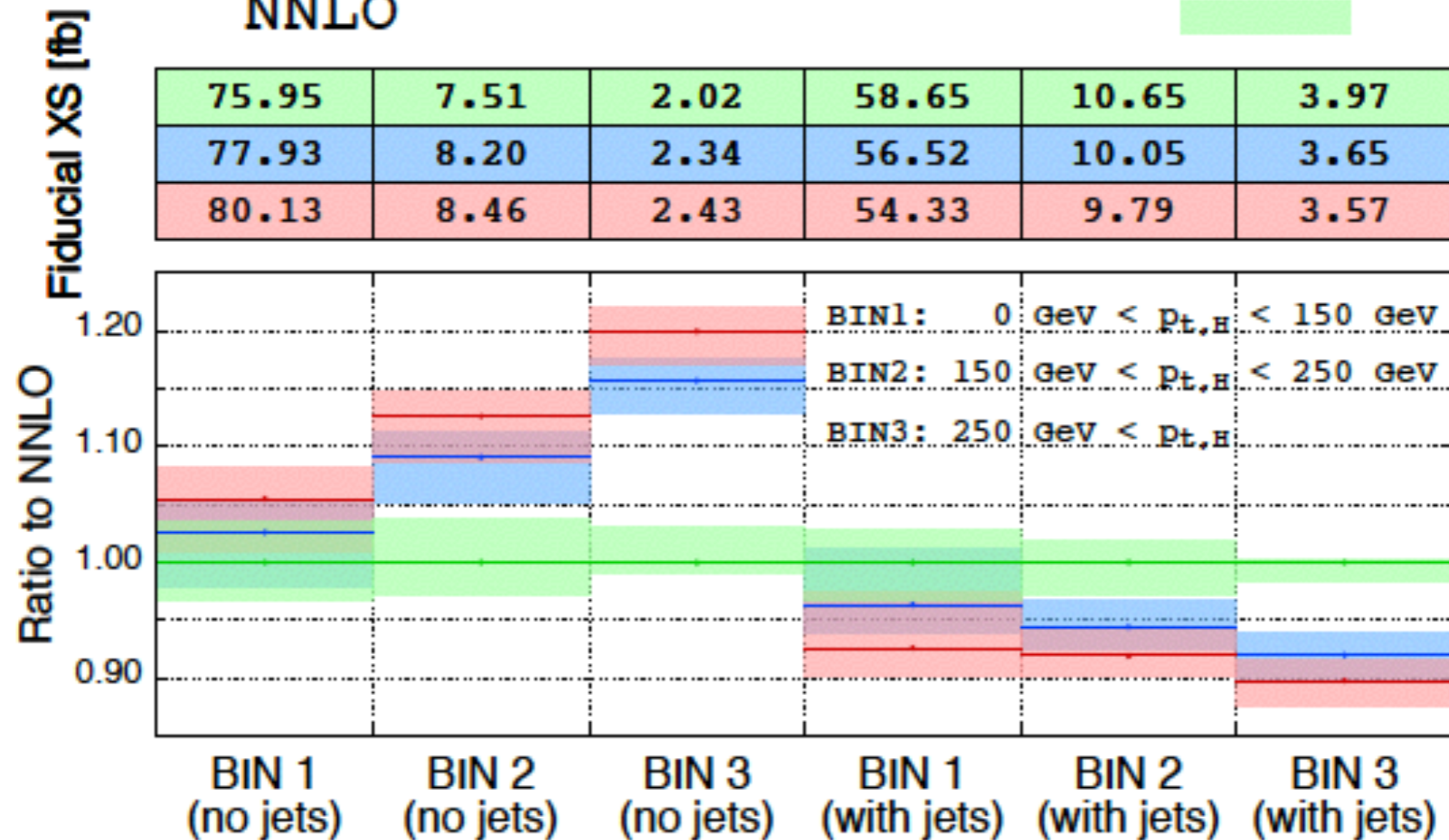
Boosted Higgs: VH(bb)

[Astill, Bizon, Re, Zanderighi '16]

HW-NNLOPS (Pythia8-part)

HW-NNLOPS (Pythia8-hadr)

NNLO

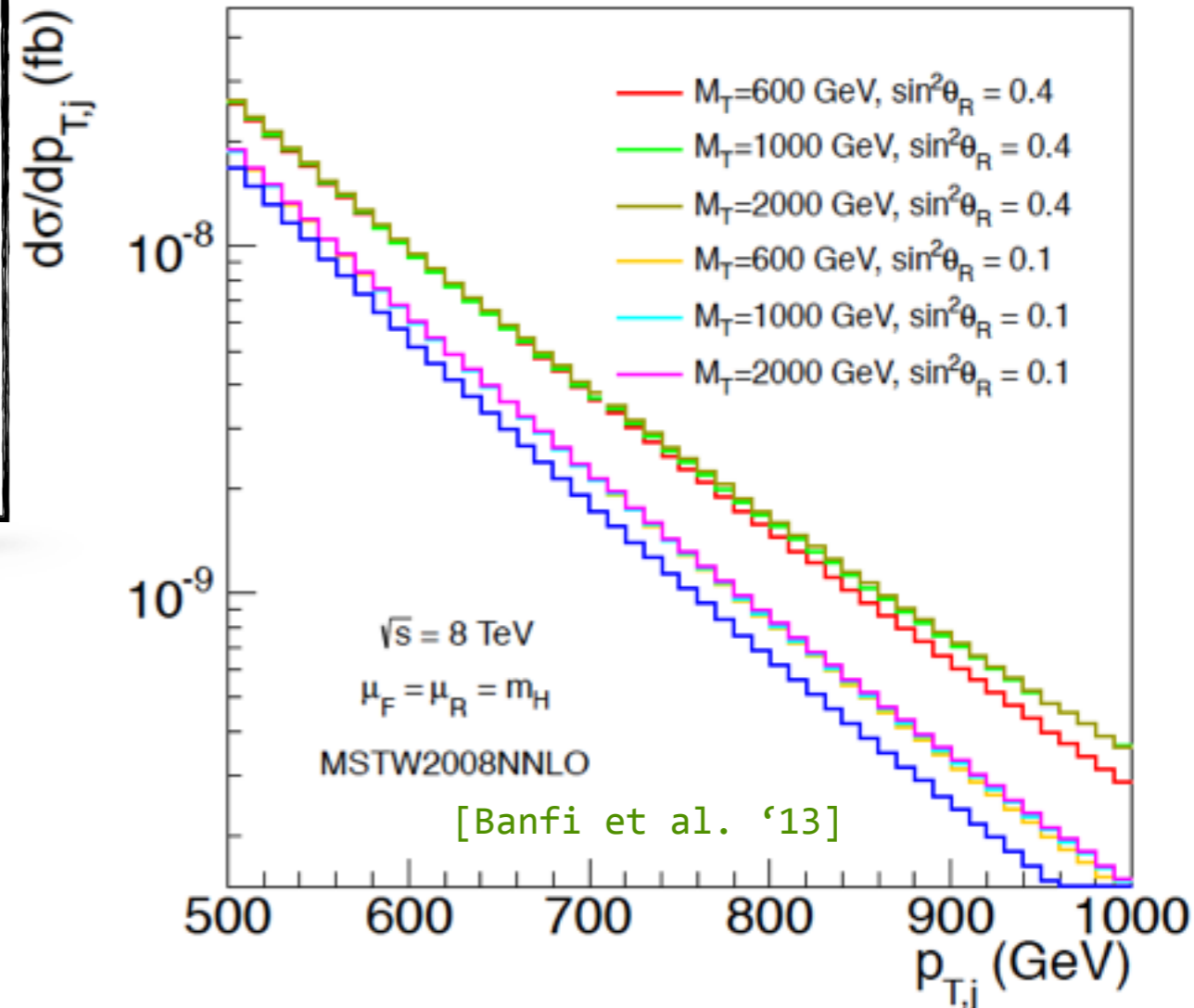


- Generators predict 10-20% corrections to NNLO from parton shower.
Are these all-order effects under control?

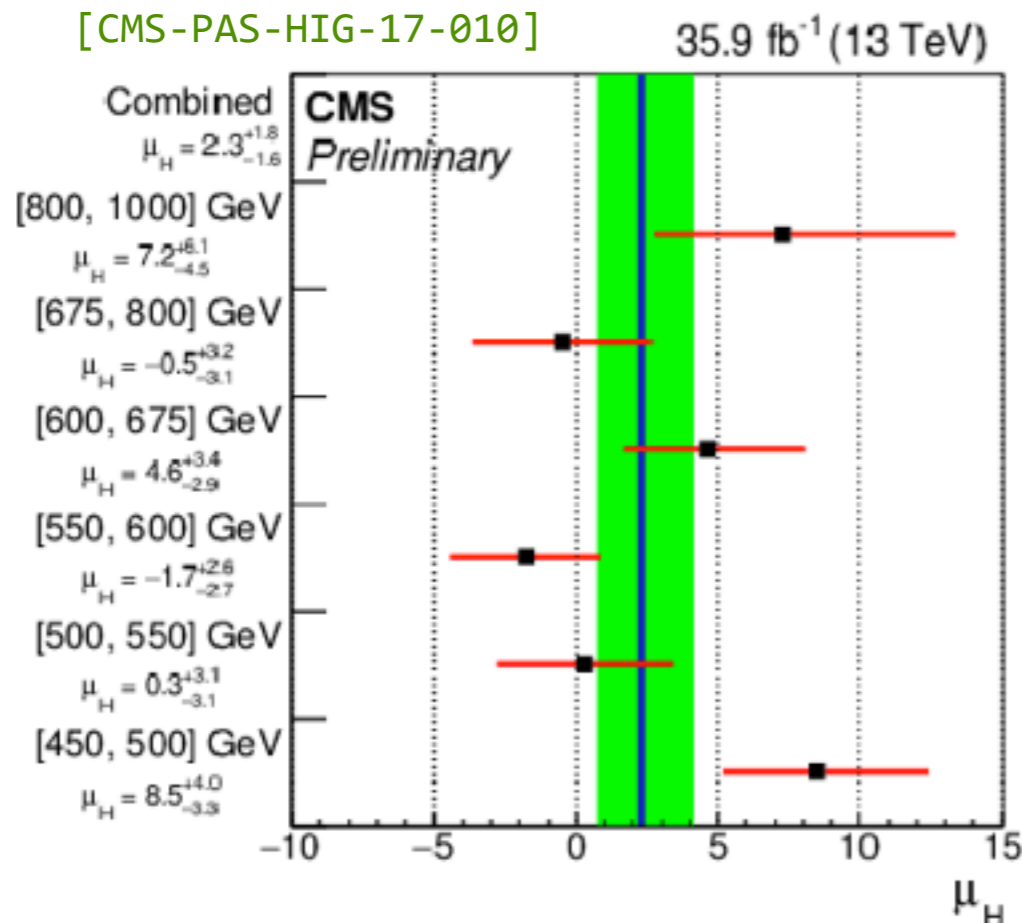
Boosted Higgs: gluon fusion

- Inclusive Higgs at high pt can probe indirectly virtual heavy states in the loop (**failure of HEFT picture**)
- Sensitivity increased through interference with the top loop + very low background
- Recently first measurement from CMS in this transverse momentum region

$$h\bar{t}t : \frac{m_t}{v} \cos^2(\theta_R), \quad h\bar{T}T : \frac{M_T}{v} \sin^2(\theta_R)$$



Up to very recently, no complete result beyond LO was available. Important to assess the size of radiative corrections



Boosted Higgs: small-x limit

- Boosted region can be approximated in the high-energy limit, i.e.

$$x = \frac{(\sqrt{m_H^2 + p_t^2} + p_t)^2}{\hat{s}} \ll 1$$

- Use expansion of the LL result to estimate NLO K factor

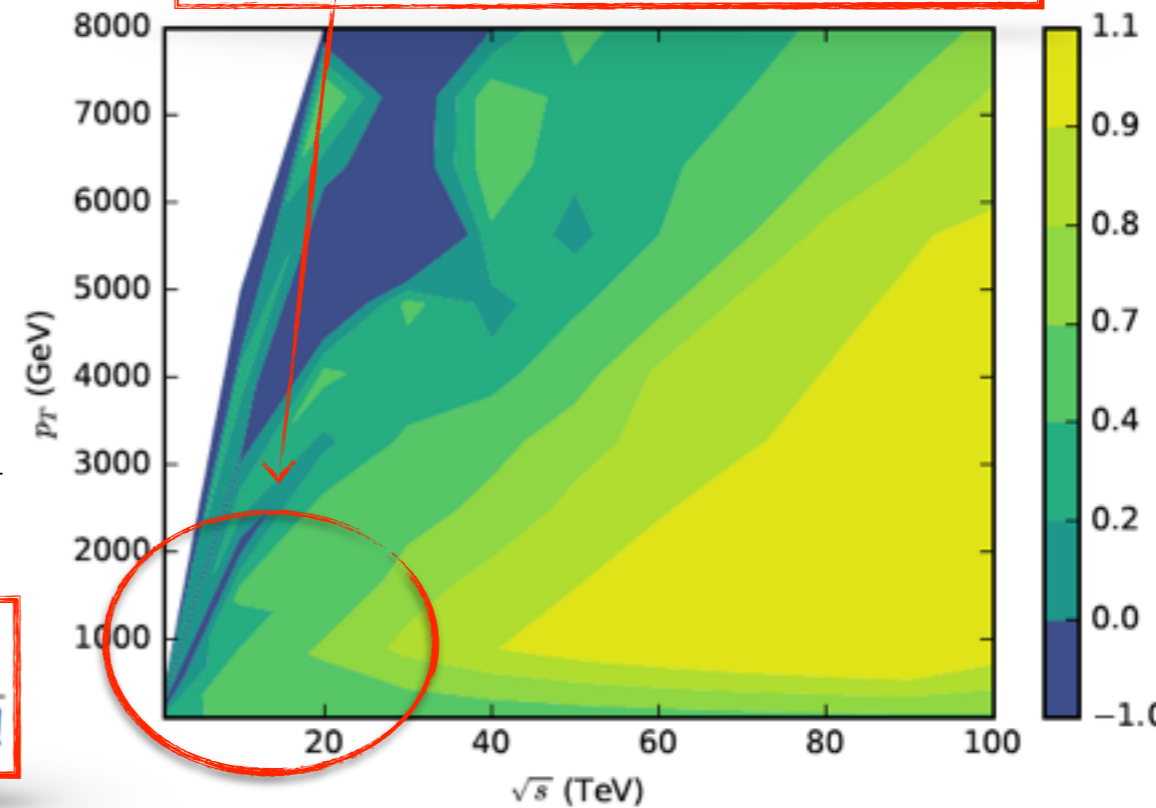
$$\frac{d\sigma}{d\xi_p}(x, \xi_p, y_t, y_b) = \sigma_0(y_t, y_b) \sum_{k=1}^{\infty} C_k(\xi_p, y_t, y_b) \alpha_s^k (-1)^{k+1} \frac{\ln^{k-1} x}{(k-1)!}$$

- Hard to estimate uncertainties reliably, but this provides a recipe to treat this region with more exclusive generators

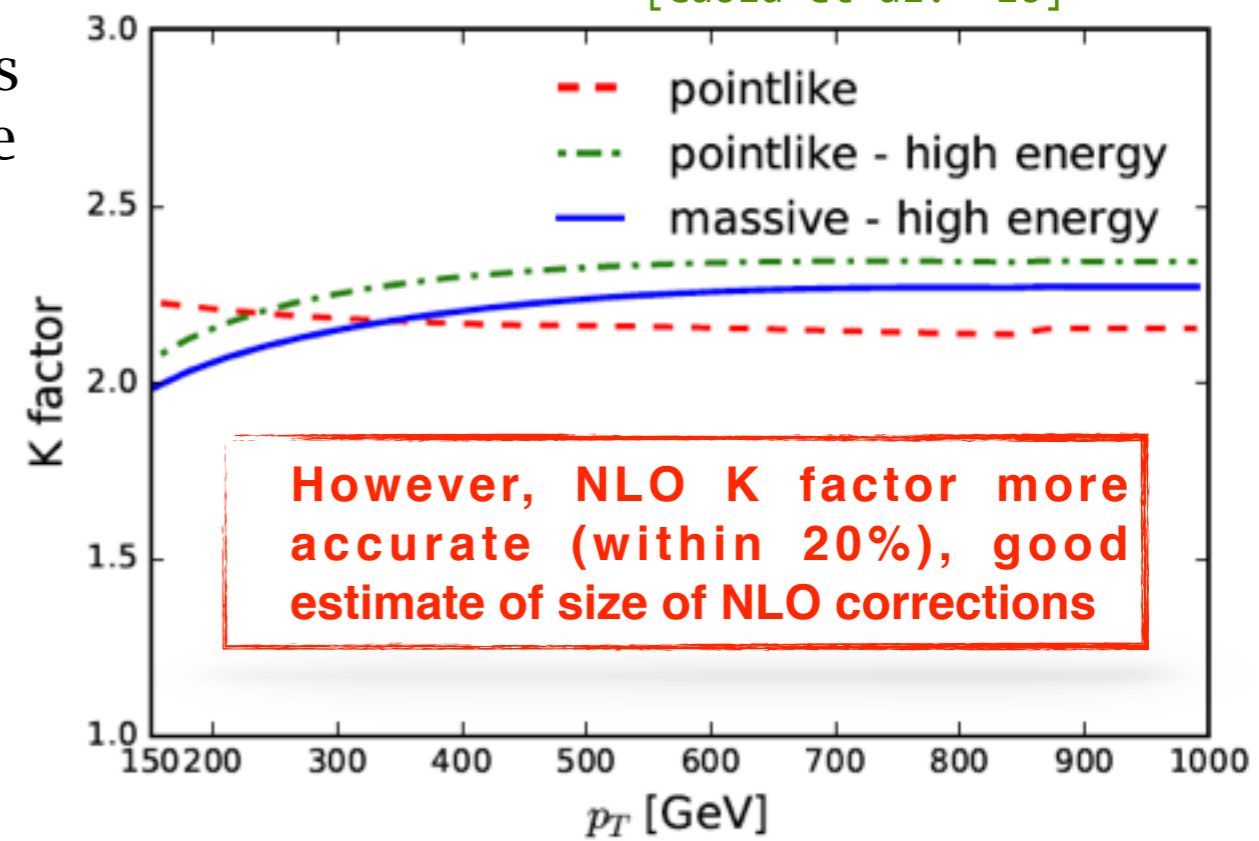
- Scaling also observed in multijet-merged MC predictions, very recently confirmed by the full NLO at large transverse momentum

See talks by K. Melnikov and C. Wever
See, e.g., also talk by F. Caola at
<https://indico.cern.ch/event/675782/>

- Ratio of NLO correction in HE limit to its full result, both in the large m_t approximation
- HE approx accurate only within ~30-40% at 13 TeV LHC



[Caola et al. '16]



Conclusions

- Several progresses in different aspects of resummation in the past few years
- Much more is to be done to keep up with the huge progress in fixed-order calculations and precise experimental data
- Inclusive observables (total XS, *two-scale* differential distributions) well known today, uncertainties under good control
- A lot of work and ideas still needed to handle multi-scale problems (masses, double differential) at all orders simultaneously
- Still *far* from understanding exclusive, multi-leg problems (e.g. VBF + 3j veto) with high perturbative (logarithmic) accuracy. Some progress recently in simpler observables
 - Exploiting synergy between available techniques could offer a way to explore new problems.

Momentum-space resummation

- The resummed differential distribution reads

$$\frac{d\Sigma(v)}{dp_t d\Phi_B} = \int_{c_1} \frac{dN_1}{2\pi i} \int_{c_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1, c_2} \frac{d|M_B|_{c_1 c_2}^2}{d\Phi_B} \mathbf{f}_{N_1}^T(\mu_0) \frac{d\hat{\Sigma}_{N_1, N_2}^{c_1, c_2}(v)}{dp_t} \mathbf{f}_{N_2}(\mu_0)$$

e.g. for all inclusive observables with $a=1$ (pt, phi*,...):

$$\begin{aligned} \hat{\Sigma}_{N_1, N_2}^{c_1, c_2}(v) &= \left[\mathbf{C}_{N_1}^{c_1; T}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}_{N_2}^{c_2}(\alpha_s(\mu_0)) \right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} \\ &\times e^{-\mathbf{R}(\epsilon k_{t1})} \exp \left\{ - \sum_{\ell=1}^2 \left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \Gamma_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \Gamma_{N_\ell}^{(C)}(\alpha_s(k_t)) \right) \right\} \\ &\sum_{\ell_1=1}^2 \left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \Gamma_{N_{\ell_1}}(\alpha_s(k_{t1})) + \Gamma_{N_{\ell_1}}^{(C)}(\alpha_s(k_{t1})) \right) \\ &\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \sum_{\ell_i=1}^2 \left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \Gamma_{N_{\ell_i}}(\alpha_s(k_{ti})) + \Gamma_{N_{\ell_i}}^{(C)}(\alpha_s(k_{ti})) \right) \\ &\times \Theta(v - V(\{\bar{p}\}, k_1, \dots, k_{n+1})), \end{aligned}$$

DGLAP anomalous dimensions

RGE evolution of coeff. functions

- Formulation equivalent to b-space result, up to a scheme change. Using the delta representation one finds

$$\frac{d\Sigma(v)}{dp_t d\Phi_B} = \int_{c_1} \frac{dN_1}{2\pi i} \int_{c_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1, c_2} \frac{d|M_B|_{c_1 c_2}^2}{d\Phi_B} \mathbf{f}_{N_1}^T(\mu_0) \frac{d\hat{\Sigma}_{N_1, N_2}^{c_1, c_2}(v)}{dp_t} \mathbf{f}_{N_2}(\mu_0) =$$

$$(1 - J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12} \frac{\partial^3}{\partial \ln(Mb/b_0)^3} \Theta(k_t - \frac{b_0}{b}) + \dots$$

$$= \sum_{c_1, c_2} \frac{d|M_B|_{c_1 c_2}^2}{d\Phi_B} \int b db p_t J_0(p_t b) \mathbf{f}^T(b_0/b) \mathbf{C}_{N_1}^{c_1; T}(\alpha_s(b_0/b)) H(M) \mathbf{C}_{N_2}^{c_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b) \\ \times \exp \left\{ - \sum_{\ell=1}^2 \int_0^M \frac{dk_t}{k_t} \mathbf{R}'_{\ell}(k_t) (1 - J_0(bk_t)) \right\}.$$

Momentum-space resummation

- Since $k_{ti}/k_{t1} = \zeta_i = \mathcal{O}(1)$ we can expand (although unnecessary) the integrands about $k_{ti} \sim k_{t1}$ to the desired accuracy for a more efficient evaluation
- At N3LL, only two resolved, hard-collinear emissions are relevant: Mellin inversion is analytic

$$\int dZ[\{R', k_i\}] G(\{\bar{p}\}, \{k_i\}) = \epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1}) G(\{\bar{p}\}, k_1, \dots, k_{n+1})$$

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\text{N}^3\text{LL}}(k_{t1}) \right) \int dZ[\{R', k_i\}] \Theta(v - V(\{\bar{p}\}, k_1, \dots, k_{n+1}))$$

$$+ \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int dZ[\{R', k_i\}] \int_0^1 \frac{d\zeta_s}{\zeta_s} \frac{d\phi_s}{2\pi} \left\{ \left(R'(k_{t1}) \mathcal{L}_{\text{NNLL}}(k_{t1}) - \partial_L \mathcal{L}_{\text{NNLL}}(k_{t1}) \right) \right. \\ \times \left(R''(k_{t1}) \ln \frac{1}{\zeta_s} + \frac{1}{2} R'''(k_{t1}) \ln^2 \frac{1}{\zeta_s} \right) - R'(k_{t1}) \left(\partial_L \mathcal{L}_{\text{NNLL}}(k_{t1}) - 2 \frac{\beta_0}{\pi} \alpha_s^2(k_{t1}) \hat{P}^{(0)} \otimes \mathcal{L}_{\text{NLL}}(k_{t1}) \ln \frac{1}{\zeta_s} \right) \\ \left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\text{NLL}}(k_{t1}) \right\} \left\{ \Theta(v - V(\{\bar{p}\}, k_1, \dots, k_{n+1}, k_s)) - \Theta(v - V(\{\bar{p}\}, k_1, \dots, k_{n+1})) \right\}$$

$$+ \frac{1}{2} \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} e^{-R(k_{t1})} \int dZ[\{R', k_i\}] \int_0^1 \frac{d\zeta_{s1}}{\zeta_{s1}} \frac{d\phi_{s1}}{2\pi} \int_0^1 \frac{d\zeta_{s2}}{\zeta_{s2}} \frac{d\phi_{s2}}{2\pi} R'(k_{t1}) \\ \times \left\{ \mathcal{L}_{\text{NLL}}(k_{t1}) (R''(k_{t1}))^2 \ln \frac{1}{\zeta_{s1}} \ln \frac{1}{\zeta_{s2}} - \partial_L \mathcal{L}_{\text{NLL}}(k_{t1}) R''(k_{t1}) \left(\ln \frac{1}{\zeta_{s1}} + \ln \frac{1}{\zeta_{s2}} \right) \right. \\ \left. + \frac{\alpha_s^2(k_{t1})}{\pi^2} \hat{P}^{(0)} \otimes \hat{P}^{(0)} \otimes \mathcal{L}_{\text{NLL}}(k_{t1}) \right\}$$

$$\times \left\{ \Theta(v - V(\{\bar{p}\}, k_1, \dots, k_{n+1}, k_{s1}, k_{s2})) - \Theta(v - V(\{\bar{p}\}, k_1, \dots, k_{n+1}, k_{s1})) - \right.$$

$$\left. \Theta(v - V(\{\bar{p}\}, k_1, \dots, k_{n+1}, k_{s2})) + \Theta(v - V(\{\bar{p}\}, k_1, \dots, k_{n+1})) \right\} + \mathcal{O} \left(\alpha_s^n \ln^{2n-6} \frac{1}{v} \right)$$

- This formula can be evaluated by means of fast Monte-Carlo methods (**RadISH**)

- Coefficient functions and hard-virtual corrections absorbed into the parton luminosity

- Valid for all inclusive observables with $a=1$. A similar formula holds for any $a>0$

$$V(\{\bar{p}\}, k) \equiv V(k) = d_\ell g_\ell(\phi) \left(\frac{k_t}{M} \right)^a$$

$$V(\{\bar{p}\}, k_1, \dots, k_n) = V(\{\bar{p}\}, k_1 + \dots + k_n)$$