On the observable spectrum of theories with a Brout-Englert-Higgs effect

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& L. Egger, A. Maas
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Elementary fields: Quarks and Gluons

$$\mathcal{L} = -\frac{1}{4} G^{i\,\mu\nu} G^{i}_{\mu\nu} + \bar{q}^{a}\, \mathrm{i} \not{D}^{ab}\, q^{b} + m_{\mathrm{q}} \bar{q}^{a} q^{a}$$

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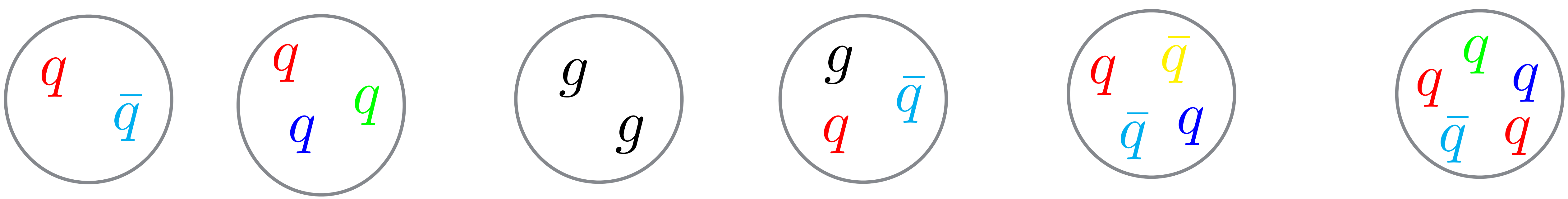
Observable states: hadrons



Mesons Baryons Glueballs? Hybrids? Tetraquarks? Pentaquarks?

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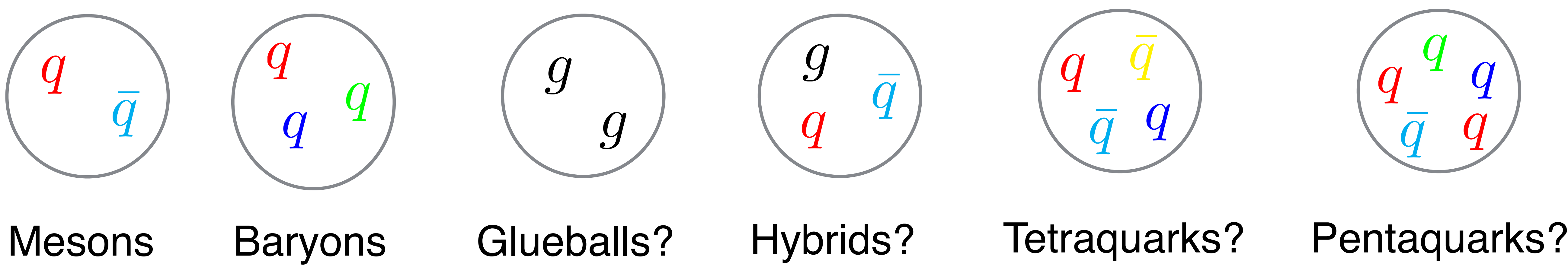

- Weak interaction?

Higgs-W Subsector

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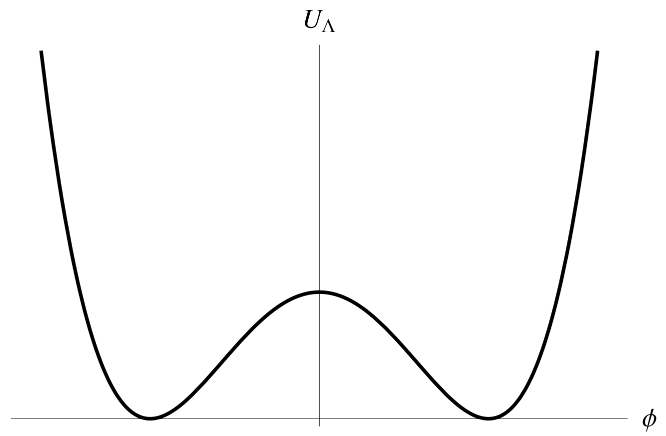

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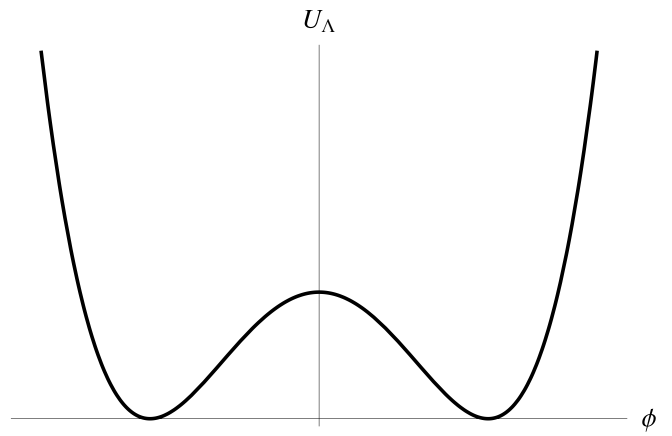

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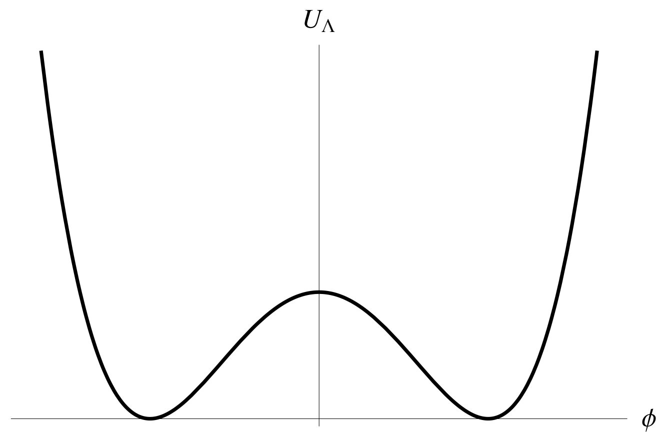

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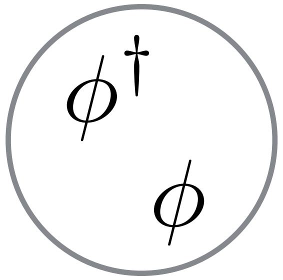
- Mass for the W's from the Higgs kinetic term

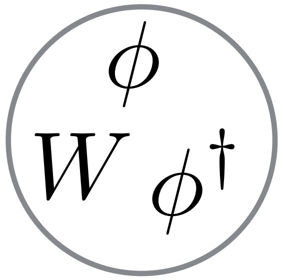
$$(D_\mu \phi)^\dagger D^\mu \phi = \frac{1}{2} \frac{g^2 v^2}{4} W^i_\mu W^{i\,\mu} + \ldots$$

Higgs-W Subsector

BUT!

- Elementary states are not gauge invariant
  Cannot be observable
- Gauge-invariant states are composite
  e.g., Higgs-Higgs, W-W, Higgs-W, ….





- Bound states are not asymptotic states in perturbation theory
- Why does perturbation theory works at all?
- Higgs condensate exist only in some gauges — Lee et al ’72, Osterwalder&Seiler’77
- “Local symmetry breaking” vs Elitzur’s theorem — Elitzur ‘81

Fröhlich-Morchio-Strocchi mechanism

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- Confirmed on the lattice for SU(2)-Higgs theory Maas ’12

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$$\mathcal{L} = \mathrm{Tr}[\partial_\mu X^\dagger \partial^\mu X] - U(\mathrm{Tr} X^\dagger X)$$

$$\text{where} \quad X = (\tilde{\phi} \;\; \phi) = \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix} = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} + O(\varphi)$$

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- Mapping of local to global multiplets

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Fröhlich et al ‘80
Egger,Maas,RS’17

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$$\omega\text{−meson} \;\; (\bar{u}u + \bar{d}d)$$

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$$\omega\text{−meson} \;\; (\bar{u}u + \bar{d}d)$$

- Not true for all mesons, e.g., pions $\pi^+ : \;\; \bar{\mathcal{O}}_2^{ud} \mathcal{O}_1^{ud} (\sim \bar{d}u)$

Beyond the standard model

Maas, Török ’16,
Maas, RS, Török ’17

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- SU(N) gauge theory + Higgs in fundamental representation

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J^P	Field	Mass	Degeneracy	Operator	Mass	Degeneracy
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1^-						

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- Nonperturbative check for N=3

Beyond the standard model - SU(5) GUT

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$$\text{SU(5)} \xrightarrow{\langle\Sigma\rangle \sim w} \text{SU(3)xSU(2)xU(1)} \xrightarrow{\langle\phi\rangle \sim v} \text{U(1)} \qquad w \gg v$$

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0^+	h	m_{h}	1			
	φ^a	$\sim w$	6			
	σ_i	$\sim w$	8			
	$\tilde{\sigma}_i$	$\sim w$	3			
	$\bar{\sigma}_i$	$\sim w$	1			
1^-						

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	$\bar{\sigma}_i$	$\sim w$	1			
1^-	A^μ	0	1			
	$W^{\pm\mu}$	m_{W}	$1/\bar{1}$			
	Z^μ	m_{Z}	1			
	X^μ	$\sim w$	6			
	Y^μ	$\sim w$	6			

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- Global symmetry: U(1)xZ_2

J^P	Field	Mass	Degeneracy	Operator	Mass	Degeneracy
0^+	h	$m_{\rm h}$	1			
	φ^a	$\sim w$	6			
	σ_i	$\sim w$	8			
	$\tilde{\sigma}_i$	$\sim w$	3			
	$\bar{\sigma}_i$	$\sim w$	1			
1^-	A^μ	0	1			
	$W^{\pm\mu}$	$m_{\rm W}$	$1/\bar{1}$			
	Z^μ	$m_{\rm Z}$	1			
	X^μ	$\sim w$	6			
	Y^μ	$\sim w$	6			

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0^+	h	m_h	1	O_{0+}	m_h	1
	φ^a	$\sim w$	6	O_{0-}	m_h	1
	σ_i	$\sim w$	8	$O_{\pm 1,+}$	$\sim w$	$1/\bar{1}$
	$\tilde{\sigma}_i$	$\sim w$	3	$O_{\pm 1,-}$	$\sim w$	$1/\bar{1}$
	$\bar{\sigma}_i$	$\sim w$	1			
1^-	A^μ	0	1			
	$W^{\pm\mu}$	m_W	$1/\bar{1}$			
	Z^μ	m_Z	1			
	X^μ	$\sim w$	6			
	Y^μ	$\sim w$	6			

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0^+	h	$m_{\rm h}$	1	O_{0+}	$m_{\rm h}$	1
	φ^a	$\sim w$	6	O_{0-}	$m_{\rm h}$	1
	σ_i	$\sim w$	8	$O_{\pm1,+}$	$\sim w$	$1/\bar{1}$
	$\tilde{\sigma}_i$	$\sim w$	3	$O_{\pm1,-}$	$\sim w$	$1/\bar{1}$
	$\bar{\sigma}_i$	$\sim w$	1			
1^-	A^μ	0	1	O_{0+}	0	1
	$W^{\pm\mu}$	$m_{\rm W}$	$1/\bar{1}$	O_{0-}	0	1
	Z^μ	$m_{\rm Z}$	1	$O_{\pm1,+}$	$\sim w$	$1/\bar{1}$
	X^μ	$\sim w$	6	$O_{\pm1,+}$	$\sim w$	$1/\bar{1}$
	Y^μ	$\sim w$	6			

Summary

- Observable spectrum must be gauge invariant
- Non-Abelian gauge theory: composite operator
- FMS mechanism provides a mapping of the local to the global multiplets
- Gauge-invariant perturbation theory as a tool
- Same results in leading order for the standard model
- BSM model building may be affected
- Verification requieres non-perturbative methods

Generations as excitation spectra (speculation!)

Egger,Maas,RS'17

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$$X^\dagger Q_{\mathrm{E}} = \begin{pmatrix} \phi^T \epsilon^T Q_{\mathrm{E}} \\ \phi^\dagger Q_{\mathrm{E}} \end{pmatrix} = \begin{pmatrix} \phi_2 u_{\mathrm{E}} - \phi_1 d_{\mathrm{E}} \\ \phi_2^* d_{\mathrm{E}} + \phi_1^* u_{\mathrm{E}} \end{pmatrix}$$

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- higher excitations:

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- Ground state: $d = (\phi^\dagger Q_\mathrm{E})_g$
- higher excitations: $s = (\phi^\dagger Q_\mathrm{E})_*, \quad b = (\phi^\dagger Q_\mathrm{E})_{**}$