A global view on the Higgs self coupling

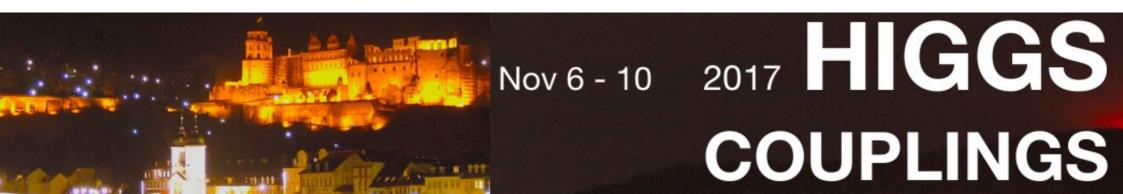
Thibaud Vantalon DESY - IFAE

Based on:

JHEP09(2017)069, S. Di Vita, C. Grojean, G. Panico, M.Riembau, T. Vantalon







Motivation

$$V_{\rm SM} = \frac{1}{2}m_h^2 + \lambda_3^{\rm SM}h^3 + \lambda_4^{\rm SM}h^4$$

$$\lambda_3^{\text{SM}} = \frac{m_h^2}{2v} \qquad \qquad \lambda_4^{\text{SM}} = \frac{m_h^2}{8v^2}$$

Standard model Higgs potential depends on only 2 parameters and is indirectly precisely measured

Direct measurements of h³ and h⁴ are challenging but an important consistency check.

- Stability of EW vacuum
- Baryogenesis through first order phase transition?

h³ challenging to measure at LHC

h⁴ out of reach of LHC

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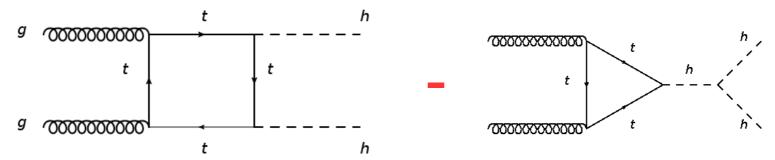
h4 out of reach of LHC

Double Higgs production

Small production cross section:

$$\frac{\sigma(pp \to hh)}{\sigma(pp \to h)} \sim 10^{-3}$$

Negative interference decrease cross section:



Most promising channel is a trade off between cleanness and statistic:

$$Br(h \to b\bar{b}) \times Br(h \to \gamma\gamma) \sim 60\% \times 0.1\%$$

HL-LHC @ 3 ab⁻¹, 95% CL $\kappa_{\lambda} \in [-0.8\,,\,7.7]$ ATL-PHYS_PUB_2017-001

Idea, since the bounds are so loose and trilinear enter at NLO in single Higgs process

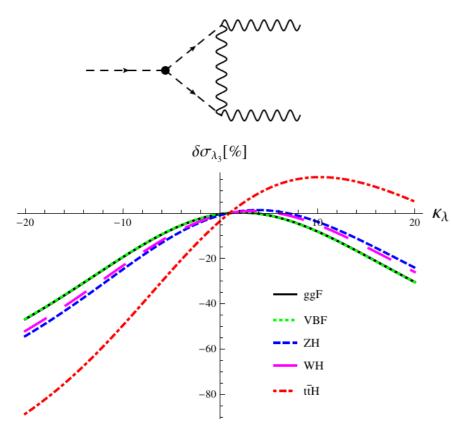
Can single Higgs process help?

McCullough, 1312.3322 Gorbahn, Haisch 1607.03773 Degrassi, et al. 1607.04251 Bizon, et al. 1610.05771

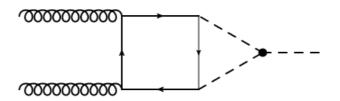
LHC from discovery to high precision

McCullough, 1312.3322 Gorbahn, Haisch 1607.03773 Degrassi, et al. 1607.04251 Bizon, et al. 1610.05771

The trilinear coupling enter at loop level in single Higgs observables



Degrassi, et al. 1607.04251



Only κ_{λ} deviate from SM:

(68% CL at 3ab⁻¹)

$$\kappa_{\lambda} \in [-0.7, 4.2]$$

Compared to an other double Higgs expected bound in $\mbox{HH} \rightarrow b \bar{b} \gamma \gamma$

Dim. 6 EFT

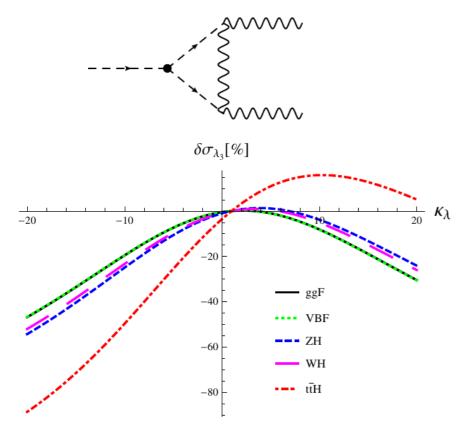
$$\kappa_{\lambda} \in [0, 2.8] \cup [4.5, 6.1]$$

Azatov et al. 1502.00539

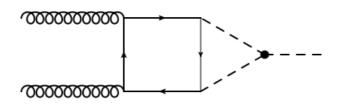
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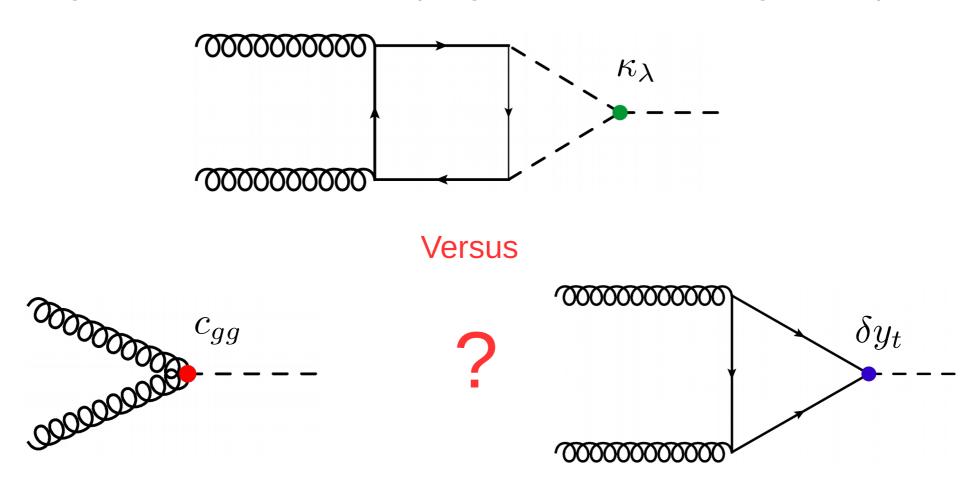
$$\kappa_{\lambda} \in [0, 2.8] \cup [4.5, 6.1]$$

Azatov et al. 1502.00539

But my comparison is not fair The bounds rely on different theoretical assumptions

Other deviations?

Setting on one anomalous coupling at a time is a strong assumption.



Is it possible to disentangle the different contributions?

The setup

Parametrization of dominating BSM effects in Higgs physics using dimension 6 Lagrangian in the "Higgs basis"

Assuming flavour universality and no CP violating operator



8 (+2) Independent operators that affect Higgs physics at leading order and have not been tested in existing precision measurements

6 parameters controlling deformations of the couplings to the SM gauge bosons

$$\delta c_z$$
, c_{zz} , $c_{z\square}$, $\hat{c}_{z\gamma}$, $\hat{c}_{\gamma\gamma}$, \hat{c}_{gg} ,

3 related to the deformations of the fermion Yukawa's

$$\delta y_t$$
, δy_d , δy_{τ} ,

1 distortion to the Higgs trilinear self-coupling



Inclusive observables

Global Chi squared fit of the signal strengths

We explore the sensitivity of HL-LHC at 3/ab, using the ATLAS projection.

ATL-PHYS-PUB-2014-016 ATL-PHYS-PUB-2016-008

ATL-PHYS-PUB-2016-018

+ Updated ggF uncertainties

We	assume	that i	in o	ur E	EFT	the	dim	6
leve	el is a go	od ap	prox	cima	ation.			

Higher order therm can be neglected so we linearized the signal strength in the wilson coefficient

Process		Combination	Theory	Experimental
	ggF	0.07	0.05	0.05
	VBF	0.22	0.16	0.15
$H \to \gamma \gamma$	$t\overline{t}H$	0.17	0.12	0.12
	WH	0.19	0.08	0.17
	ZH	0.28	0.07	0.27
	ggF	0.06	0.05	0.04
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$H \rightarrow VV VV$	VBF	0.15	0.12	0.09
$H o Z\gamma$	incl.	0.30	0.13	0.27
$H o b ar{b}$	WH	0.37	0.09	0.36
$H \rightarrow bb$	ZH	0.14	0.05	0.13
$H \rightarrow \tau^+ \tau^-$	VBF	0.19	0.12	0.15

$$\mu = \frac{\sigma_i}{(\sigma_i)_{\text{SM}}} \times \frac{\text{BR}[f]}{(\text{BR}[f])_{\text{SM}}}$$
$$\approx 1 + \delta\sigma + \delta \text{BR}$$

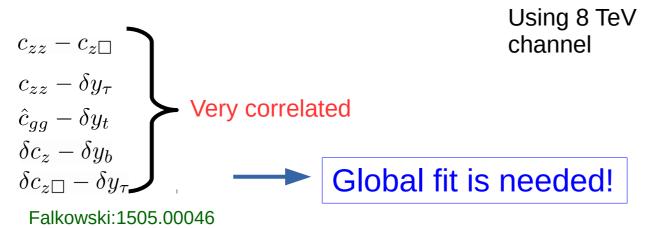
Single Higgs observable without the trilinear

Run 1 channel, Observable = SM exactly

$$\begin{pmatrix} \hat{c}_{gg} \\ \delta c_z \\ c_{zz} \\ c_{z} \\ \hat{c}_{z\gamma} \\ \delta y_t \\ \delta y_t \\ \delta y_\tau \end{pmatrix} = \pm \begin{pmatrix} 0.07 & (0.02) \\ 0.07 & (0.01) \\ 0.64 & (0.02) \\ 0.24 & (0.01) \\ 4.94 & (0.65) \\ 0.08 & (0.02) \\ 0.09 & (0.02) \\ 0.14 & (0.03) \\ 0.17 & (0.09) \end{pmatrix}$$

_									
1	-0.01	-0.02	0.03	0.08	0.01	-0.71	0.03	0.01	
	1	-0.45	0.36	-0.61	-0.33	0.18	0.89	0.53	
		1	-0.99	0.69	0.11	0.38	-0.47	-0.74	
			1	-0.58	-0.23	-0.42	0.42	0.71	
				1	-0.58	0.09	-0.46	-0.63	
					1	0.14	0.04	0.04	
						1	0.25	-0.08	
							1	0.57	
								4	

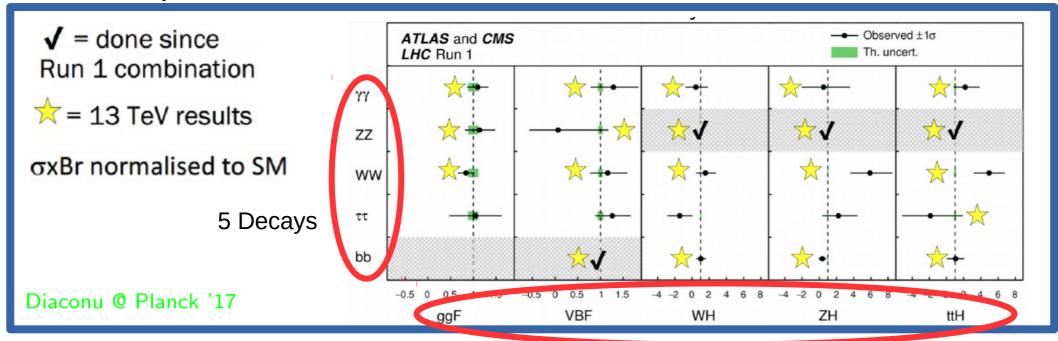
Global fit Fit with only 1 wilson



$\Delta \mu / \mu$	300 fb ⁻¹		3000 fb^{-1}	
	All unc. No theory unc.		All unc.	No theory unc.
$H \rightarrow \gamma \gamma \text{ (comb.)}$	0.13	0.09	0.09	0.04
(0j)	0.19	0.12	0.16	0.05
(1j)	0.27	0.14	0.23	0.05
(VBF-like)	0.47	0.43	0.22	0.15
(WH-like)	0.48	0.48	0.19	0.17
(ZH-like)	0.85	0.85	0.28	0.27
(ttH-like)	0.38	0.36	0.17	0.12
$H \rightarrow ZZ \text{ (comb.)}$	0.11	0.07	0.09	0.04
(VH-like)	0.35	0.34	0.13	0.12
(ttH-like)	0.49	0.48	0.20	0.16
(VBF-like)	0.36	0.33	0.21	0.16
(ggF-like)	0.12	0.07	0.11	0.04
$H \rightarrow WW \text{ (comb.)}$	0.13	0.08	0.11	0.05
(0j)	0.18	0.09	0.16	0.05
(VBF-like)	0.21	0.20	0.15	0.09
,				
$H \rightarrow b\bar{b}$ (comb.)	0.26	0.26	0.14	0.12
(WH-like)	0.57	0.56	0.37	0.36
(ZH-like)	0.29	0.29	0.14	0.13
$H \rightarrow \tau \tau \text{ (VBF-like)}$	0.21	0.18	0.19	0.15

Inclusive observables at 8 TeV

We have 10 quantities



Receiving modifications from 9+1 parameters

5 Productions

So, we should be able to constrain them by looking at the signal strengths

This is not possible

Only 9 Independent signal strength combinations (at the linear level)

$$\mu \approx 1 + \delta \sigma + \delta BR$$

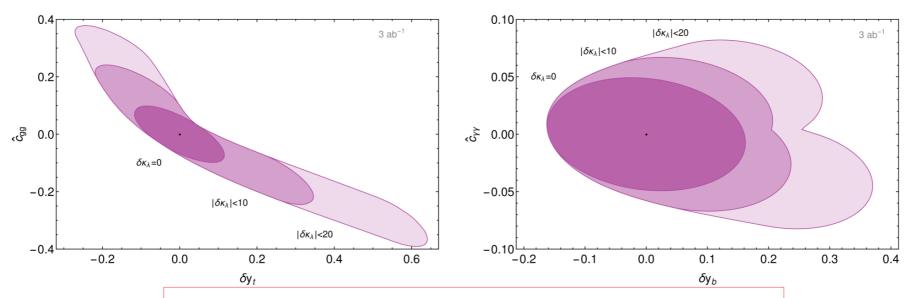
Shift in production can be compensated by opposite shift in decay

$$\delta\sigma = -\delta \mathrm{BR}$$
 ——— Unconstrained direction

Effect of the flat direction

Single Higgs without NLO effect validity

Incl. single Higgs data

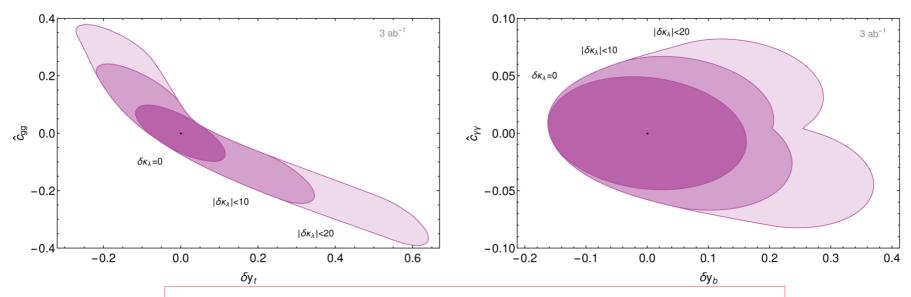


Only valid for reasonable value of the trilinear coupling

Effect of the flat direction

Single Higgs without NLO effect validity

Incl. single Higgs data



Only valid for reasonable value of the trilinear coupling

Valid in a SILH model

$$\delta c_z \sim v^2/f^2$$

$$\delta \kappa_\lambda \equiv \kappa_\lambda - 1 \sim v^2/f^2, \quad f \sim \frac{m*}{g*}$$

$$\delta c_z \sim \delta \kappa_\lambda$$

This is true for a broad class of model

A counter example

May not be valid for Higgs portal

$$\mathcal{L} \supset \theta g_* m_* H^{\dagger} H \varphi - \frac{m_*^4}{g_*^2} V(g_* \varphi / m_*)$$

Will generate:

$$\delta c_z \sim \theta^2 g_*^2 \frac{v^2}{m_3^2}$$

$$\delta \kappa_{\lambda} \sim \theta^3 g_*^4 \frac{1}{\lambda_3^{\rm SM}} \frac{v^2}{m^2}$$

With a typical tuning of
$$\ \Delta \sim \frac{\theta^2 g_*^2}{\lambda_3^{SM}}$$

Perturbative expansion
$$\; \varepsilon \equiv \frac{\theta g_*^2 v^2}{m_*^2} \ll 1 \;$$

$$\theta \simeq 1, g_* \simeq 3 \text{ and } m_* \simeq 2.5 \text{ TeV}$$

$$\varepsilon \simeq 0.1 \,, \quad 1/\Delta \simeq 1.5\%$$

$$\delta c_z \simeq 0.1 \,, \quad \delta \kappa_\lambda \simeq 6$$

Hard to have model with large deviation only in $\delta \kappa$

Single Higgs fit valid for most model

Inclusive observables

Way out:

Extra constraints:

- 1 Higgs total width
- \$ Compare different energies
- 1 decay $\mu\mu$
- 2 Anomalous triple gauge couplings(aTGCs)
- **1** decay $Z\gamma$
- **L** Differential distributions
- 1 Add double Higgs

The less promising

- Compare different energies

Difference in signal strength small. Do no help much.

$$\frac{\sigma_{\text{WH}}}{\sigma_{\text{WH}}^{\text{SM}}} = 1 + \begin{pmatrix} 2.0\\2.0\\2.0\\2.0 \end{pmatrix} \delta c_z + \begin{pmatrix} 9.4\\10.1\\11.1\\12.1 \end{pmatrix} c_{z\square} + \begin{pmatrix} 4.4\\4.6\\5.0\\5.3 \end{pmatrix} c_{zz} + \begin{pmatrix} -0.83\\-0.94\\-1.09\\-1.25 \end{pmatrix} c_{z\gamma} + \begin{pmatrix} -0.44\\-0.48\\-0.53\\-0.59 \end{pmatrix} c_{\gamma\gamma} \begin{vmatrix} 8\\14\\33\\100 \end{vmatrix}$$

- Higgs total width

Not enough precision at 3ab⁻¹ to have a big effect.

Model independent determination challenging at LHC

- Decay to Muon

Constrain already provided by the decay to tau!

But we are not using all the data available at 14 TeV 3ab⁻¹

Anomalous TGCs

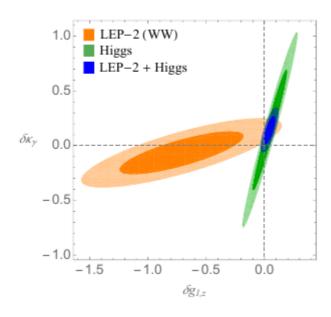
At dimension 6, the aTGCs can be written in terms of the Higgs basis parameters

$$\delta g_{1,z} = \frac{1}{2(g - g')} \left[c_{\gamma\gamma} e^2 g' + c_{z\gamma} \left(g^2 - g'^2 \right) g'^2 - c_{zz} \left(g^2 + g'^2 \right) g'^2 - c_{zz} \left(g^2 + g'^2 \right) g'^2 - c_{zz} \left(g^2 + g'^2 \right) g^2 \right],$$

$$\delta \kappa_{\gamma} = -\frac{g^2}{2} \left(c_{\gamma\gamma} \frac{e^2}{g^2 + g'^2} + c_{z\gamma} \frac{g^2 - g'^2}{g^2 + g'^2} - c_{zz} \right)$$

Falkowski et al '15

Need to be combined with Higgs data to improve precision!



 $H\to Z\gamma$

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	ggF	0.07	0.05	0.05
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FL-PHYS-PUB-2014-016

ATL-PHYS-PUB-2016-008

ATL-PHYS-PUB-2016-018

+ Updated ggF uncertainties

Correlation with new observables

$$\begin{pmatrix} \hat{c}_{gg} \\ \delta c_z \\ c_{zz} \\ c_{z\square} \\ \hat{c}_{z\gamma} \\ \delta y_t \\ \delta y_t \\ \delta y_\tau \end{pmatrix} = \pm \begin{pmatrix} 0.07 & (0.02) \\ 0.07 & (0.01) \\ 0.64 & (0.02) \\ 0.24 & (0.01) \\ 4.94 & (0.65) \\ 0.08 & (0.02) \\ 0.09 & (0.02) \\ 0.14 & (0.03) \\ 0.17 & (0.09) \end{pmatrix}$$

New channels help the correlations



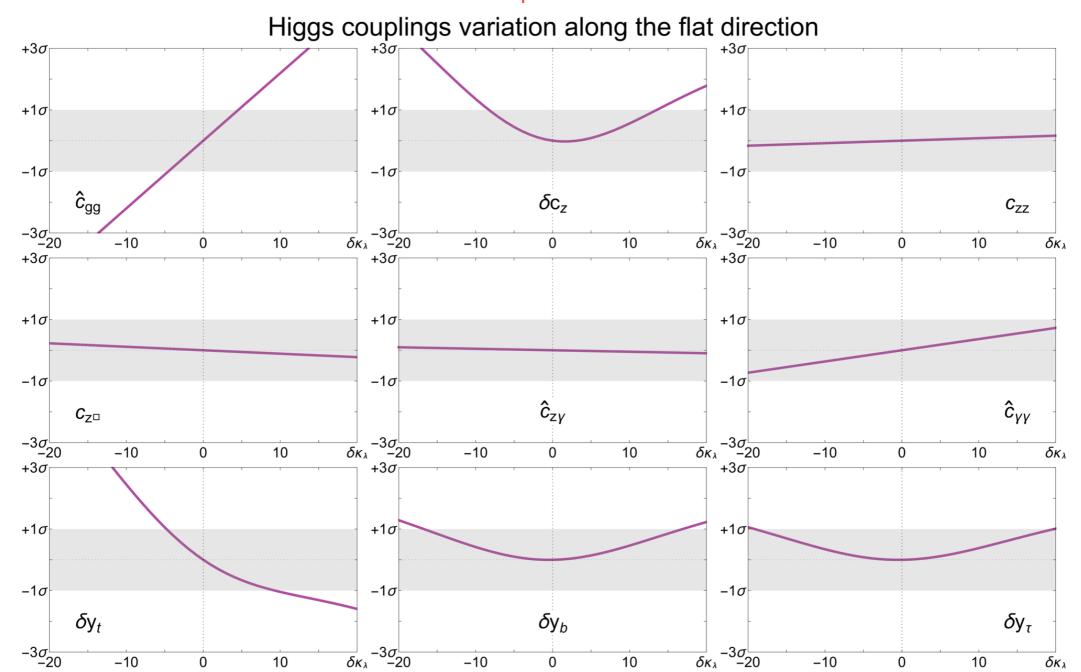
	\hat{c}_{gg}	١		0.07	(0.02)	
	δc_z			0.05	(0.01)	
İ	c_{zz}			0.05	(0.02)	
	c_z			0.02	(0.01)	
Ì	$\hat{c}_{z\gamma}$	$=\pm$		0.09	(0.09)	
	$\hat{c}_{\gamma\gamma}$			0.03	(0.02)	
	δy_t			0.08	(0.02)	
l	δy_b			0.12	(0.03)	
	$\delta y_{ au}$,	<i>]</i>		0.11	(0.09)	
'	,		'		,	/

$$\begin{bmatrix} 1 & 0.04 & -0.01 & -0.01 & 0.04 & 0.31 & -0.76 & 0.05 & 0.02 \\ 1 & -0.07 & -0.26 & 0.01 & 0.01 & 0.36 & \textbf{0.88} & 0.27 \\ 1 & -\textbf{0.87} & 0.13 & 0.20 & 0.03 & -0.07 & -0.06 \\ 1 & -0.09 & -0.09 & -0.09 & -0.17 & 0.08 \\ 1 & 0.05 & -0.02 & -0.02 & -0.03 \\ 1 & -0.32 & -0.19 & -0.12 \\ 1 & 0.50 & 0.28 \\ 1 & 0.36 \\ 1 \end{bmatrix}$$

Linear fit become a good approximation (If we can constrain the trilinear!)

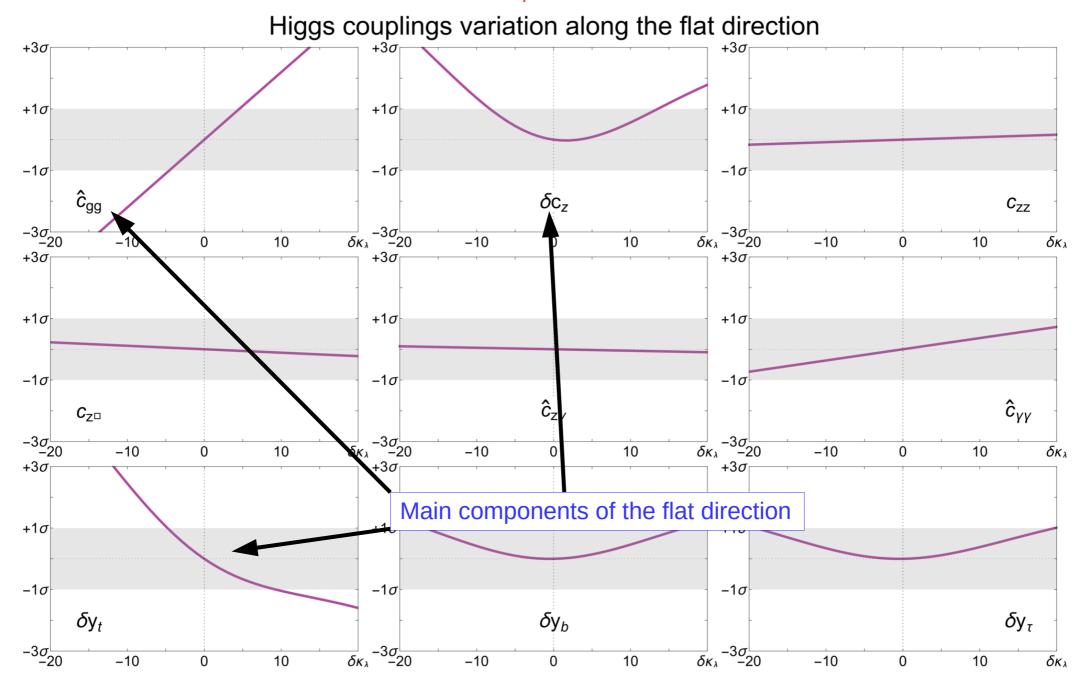
The flat direction

Value of all the couplings in function of $\delta \kappa_{\lambda}$ such that All the $\delta \mu$ =0



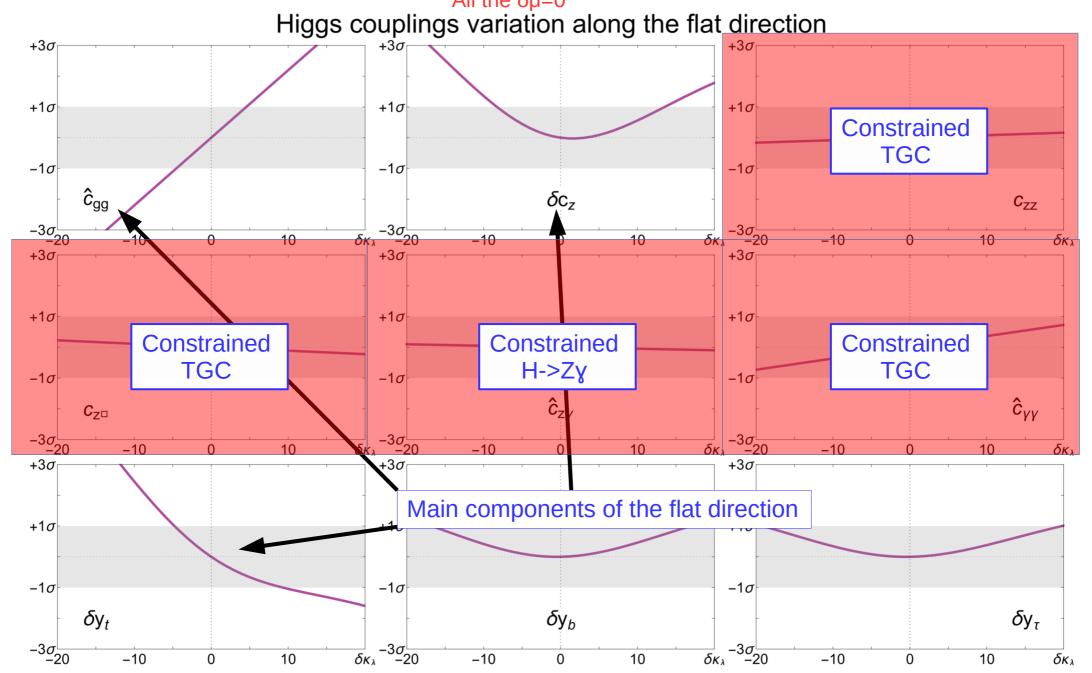
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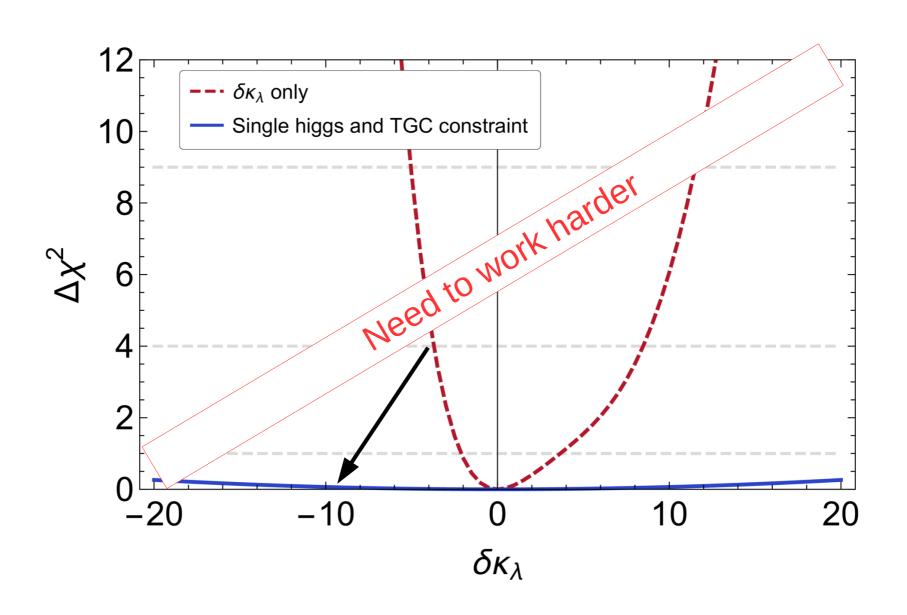


What we constrained

Value of all the couplings in function of $\delta \kappa_{\lambda}$ such that All the $\delta \mu$ =0



Not enough constraints



Differential Observables

Rough analysis looking at the prospects of differential observables

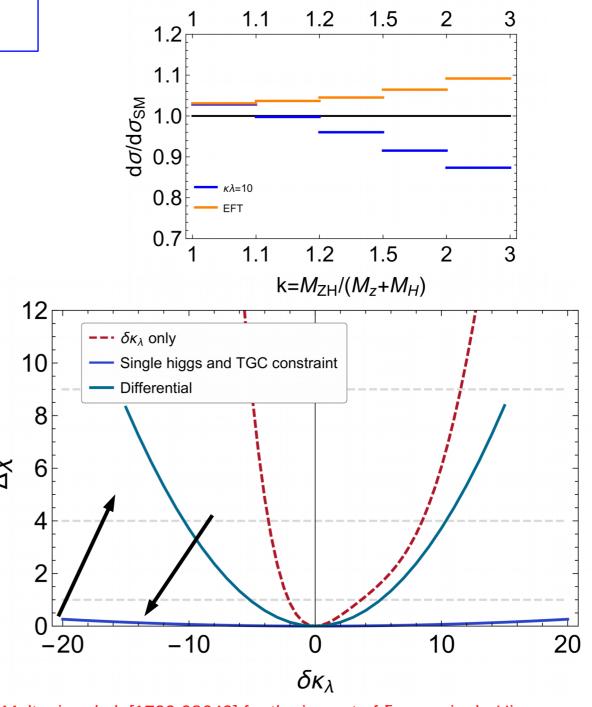
Cross section in each bin in terms of the EFT parameters computed using MadGraph.

Dependence on Higgs trilinear computed in Degrassi, et al. 1607.04251

Restore some power to the method, may be seen as complement to double Higgs

Maybe other differential observable can be more powerful

68% CL, 3ab⁻¹
$$\kappa_{\lambda} \in [-3.4, 6.4]$$



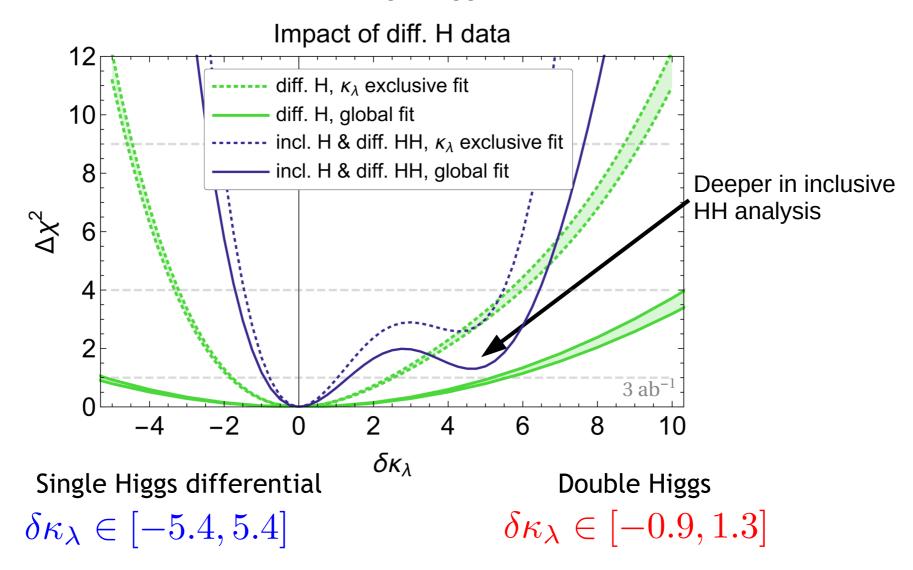
ZH

See Maltoni and al. [1709.08649] for the impact of $\delta \kappa_{\lambda}$ on single-Higgs differential distributions and for a κ -framework analysis

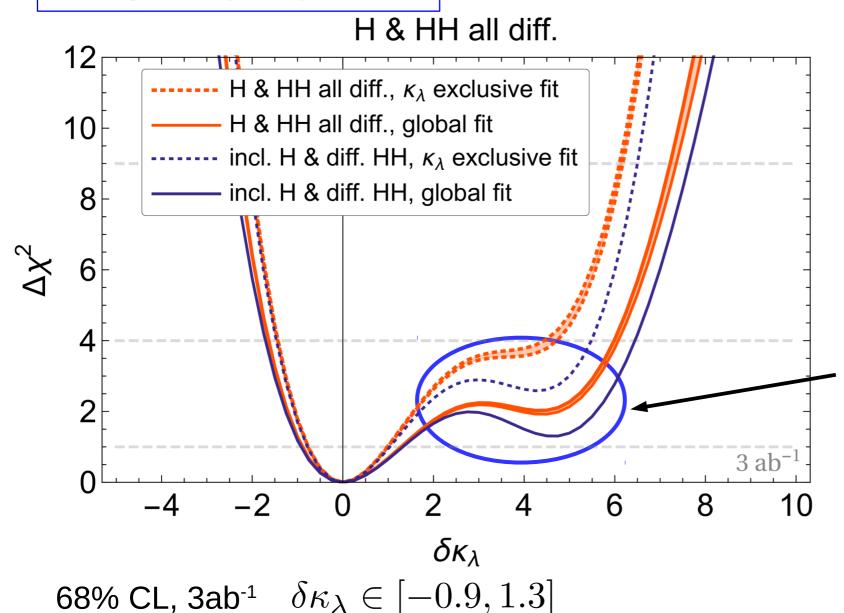
Differential Observables versus double Higgs

Double Higgs analysis more powerful

It also solves the flat direction issue in single Higgs



Everything together



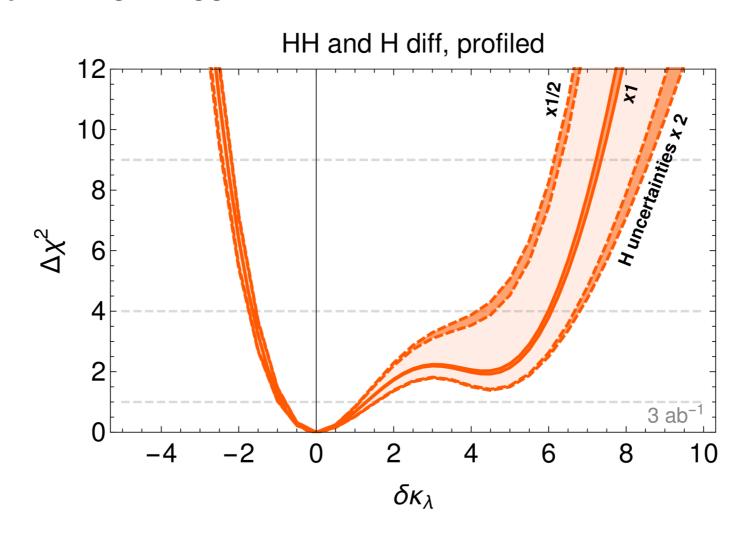
$$\begin{pmatrix} \hat{c}_{gg} \\ \delta c_z \\ c_{zz} \\ c_{z\Box} \\ \hat{c}_{z\gamma} \\ \hat{c}_{\gamma\gamma} \\ \delta y_t \\ \delta y_b \\ \delta y_\tau \\ \delta \kappa_\lambda \end{pmatrix} = \pm \begin{pmatrix} 0.06 \\ 0.04 \\ 0.04 \\ 0.02 \\ 0.09 \\ 0.03 \\ 0.06 \\ 0.07 \\ 0.11 \\ 1.0 \end{pmatrix}$$

Gaussian approx.

Single Higgs help lifting this minimum (More clear for Inclusive double Higgs)

Robustness of the analysis

Sensibility to single Higgs uncertainties



What about leptonic the future...

Extension of arXiv:1704.02333 G. Durieux, C. Grojean, J. Gu, K. Wang

Possible future colliders will measure signal strength with high precision and open new channels McCullough, 1312.3322

$$e^-e^+ o
u ar{
u}h$$
 $e^-e^+ o
u ar{
u}hh$
 $e^-e^+ o
u hh$
 $$e^-e^+
ightarrow zh$$
 $e^-e^+
ightarrow zhh$
 $e^-e^+
ightarrow tar{t}h$
Maximum around threshold

Example: $\Delta\mu(e^-e^+ \to zh, h \to b\bar{b}) < 1\% @CLIC$

What can CLIC, ILC, CEPC, FCC-ee tell us about the trilinear?

What about leptonic the future...

Extension of arXiv:1704.02333 G. Durieux, C. Grojean, J. Gu, K. Wang

Possible future colliders will measure signal strength with high precision and open new channels

McCullough, 1312.3322

Just Wait 25 Minutes For Gauthier Durieux Talk

... and the hadronic

Some results for future proton colliders

Data at 33 TeV are naively extrapolated

	HL: h incl, hh incl	[-1, 1.5] U [3.9, 6.4]	[-1.8, 7.5]
	HL: h incl, hh diff	[-0.9, 1.3]	[-1.7, 6.4]
HE-LHC 33TeV	HE: h incl, hh incl	[-0.3, 0.3] U [5.0, 6.0]	[-0.5, 0.7] U [4.5, 6.7]
10ab ⁻¹	HL + HE	[-0.3, 0.3]	[-0.5, 0.6] U [4.8, 6.0]
1606.09408	FCC 100 TeV 30/ab h incl, hh diff	[-0.03, 0.03]	[-0.06, 0.06]

 $\delta \kappa_{\lambda}$ bound / scenario

Diff. analysis would help solve the second minima

- Uncertanties on single-H μ 's: naively extrapolated from HL-LHC
- Double-H EFT: interpolation between HL-LHC and FCC of Azatov et al '15

68%

95%

- NLO δκ, effect on single-H: courtesy of D.Pagani

Table presented by Stefano Di Vita @ Workshop on the physics of HL-LHC, and perspectives at HE-LHC

Conclusion

- At the inclusive level the trilinear corrections to single Higgs observables introduce a flat direction in the global fit.
- This flat direction degrades the precision achievable on the wilson coefficients. Some control on the trilinear is needed to solve this issue.
- Double Higgs is still the best way to extract Higgs trilinear and to restore the control over single Higgs fit.
- Most promising way to remove the flat direction without using double Higgs is to use differential distribution. More work in this direction is needed.

Work in progress

 Lepton colliders will give us more precision and observables to constraint the single Higgs. (G. Durieux talk)

More results in JHEP09(2017)069

Thank you

Our parametrisation:

Parametrization of dominating BSM effects in Higgs couplings:

$$\mathcal{L}^{\text{NP}} \supset \frac{h}{v} \left[\frac{\delta c_{w}}{2} \frac{g^{2} v^{2}}{2} W_{\mu}^{+} W^{-\mu} + \delta c_{z} \frac{(g^{2} + g'^{2}) v^{2}}{4} Z_{\mu} Z^{\mu} \right. \\ \left. + \frac{c_{ww}}{2} \frac{g^{2}}{2} W_{\mu\nu}^{+} W_{-\mu\nu} + c_{w\square} g^{2} \left(W_{\mu}^{+} \partial_{\nu} W_{+\mu\nu} + \text{h.c.} \right) \right. \\ \left. + \hat{c}_{\gamma\gamma} \frac{e^{2}}{4\pi^{2}} A_{\mu\nu} A^{\mu\nu} + c_{z\square} g^{2} Z_{\mu} \partial_{\nu} Z^{\mu\nu} + c_{\gamma\square} g g' Z_{\mu} \partial_{\nu} A^{\mu\nu} \right. \\ \left. + \frac{c_{zz}}{4\pi^{2}} \frac{g^{2} + g'^{2}}{4} Z_{\mu\nu} Z^{\mu\nu} + \hat{c}_{z\gamma} \frac{e \sqrt{g^{2} + g'^{2}}}{2\pi^{2}} Z_{\mu\nu} A^{\mu\nu} \right] \right. \\ \left. + \frac{g_{s}^{2}}{48\pi^{2}} \left(\hat{c}_{gg} \frac{h}{v} + \hat{c}_{gg}^{(2)} \frac{h^{2}}{2v^{2}} \right) G_{\mu\nu} G^{\mu\nu} \right. \\ \left. - \sum_{f} \left[m_{f} \left(\delta y_{f} \frac{h}{v} + \delta y_{f}^{(2)} \frac{h^{2}}{2v^{2}} \right) \bar{f}_{R} f_{L} + \text{h.c.} \right] \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{SM} v h^{3} \right. \\ \left. \left. + (\kappa_{\lambda} - 1) \lambda_{SM} v h^{3} \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{SM} v h^{3} \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{SM} v h^{3} \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{SM} v h^{3} \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{SM} v h^{3} \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{SM} v h^{3} \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{SM} v h^{3} \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{SM} v h^{3} \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{\Delta} v h^{3} v h^{3} v h^{3} v h^{3} v h^{3} \right. \\ \left. + (\kappa_{\lambda} - 1) \lambda_{$$